Quantitative Investment Management
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Lecture 53
Forward vs Futures Prices

Welcome back. So, before we proceed, let us quickly recap the important points that we discussed in the last lecture regarding futures contracts.

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The first point is that the arbitrage free forward price of a forward contract or equivalently of futures contract, which we saw by virtue of the derivation that we did in the last lecture is given by the future value of this spot price at the risk-free rate. Number 2, forwards are private contracts, futures are exchange traded.

This is the fundamental difference between forward contracts and futures contracts. Forward contracts are customized they are tailor made, and they are negotiated between two parties, whereas futures contracts are released by the exchange in standardized fashion. And they are tradable on the exchange at which they are listed for trading. Futures carry no default risk performance is guaranteed by the clearing house.

As I mentioned, it is very necessary it is absolutely necessary for futures contracts to have two fundamental attributes. Number one, they need to be standardized. And number two, they need to be devoid of any default risk in order that uninhibited trading in these instruments does take place on the exchanges. So, the first thing is that they should be standardized. And the second thing is that the traders do not have to worry about the possibility of the counterparty defaulting on the futures contracts.

And that is done through the intervention of the clearing house of the exchange, which guarantees the performance of both legs of the contract. Clearing house protects itself. Now, because the clearing house guarantees performance of both legs of the contracts it needs to protect itself against the possibility of default by either party and it does through a very engineers a very beautiful mechanism, which comprises of marking to market together with margining. This DO of marking to market and margining operating in tandem ensure that the defaults in futures markets are negligible.

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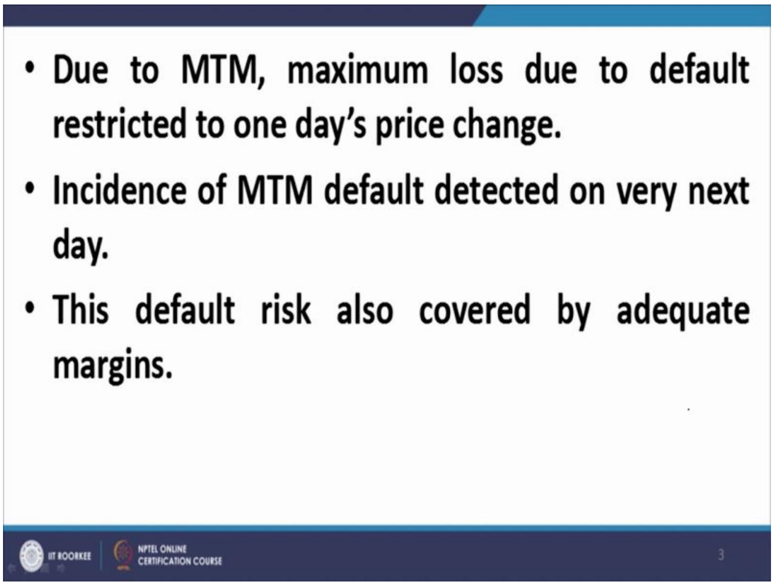

Due to marking to market, maximum loss due to default is restricted to one day's price change, because every day’s price change is transferred to the relevant margin accounts of the short party and the long party. And if there is a shortfall in margin, if the margin account balance falls below the maintenance margin then the margin call is initiated by the broker and that has to be met forthwith.

Therefore, if by the end of the next day, the margin call is not honored, the default is immediately detected. So, due to marking to market maximum loss due to default is restricted to one day's price change. In contrast, if you look at the forward contract, the forward contract entails the negotiation of the contract at t equal to 0 and then the settlement on the date of maturity of the contract at t equal to capital T.

In between the period between 0 to capital T there is no cash flow and there may not be any interaction also between the parties to the forward contract. As a result of which what happens is that when the forward contract becomes available for settlement, the cumulative profit or loss over the entire life of the forward contract is faced by the parties to the contract. And this may create a immense motivation to default if a party feels that it is being affected adversely due to the price changes of the underlying asset during the life of the forward contract.

The entire cumulative price change of the underlying asset becomes relevant when we talk about the forward contract. However, in the case of a futures contract, every day's price change in the futures market is transferred to the margin account. This is very important this is very fundamental. And this therefore reduces the motivation for the parties to the contract to indulge in defaults.

And whatever possibility is there, whatever possibility arises due to any eventuality is taken care of by a strong powerful margining system. So, incidence of MTM default detected on the very next day, as I explained just now. And then this default risk is also covered by adequate margins. So, the twin processes, the twin processes of marking to market together with margining ensure that the possibility of default in the futures market is minimal.

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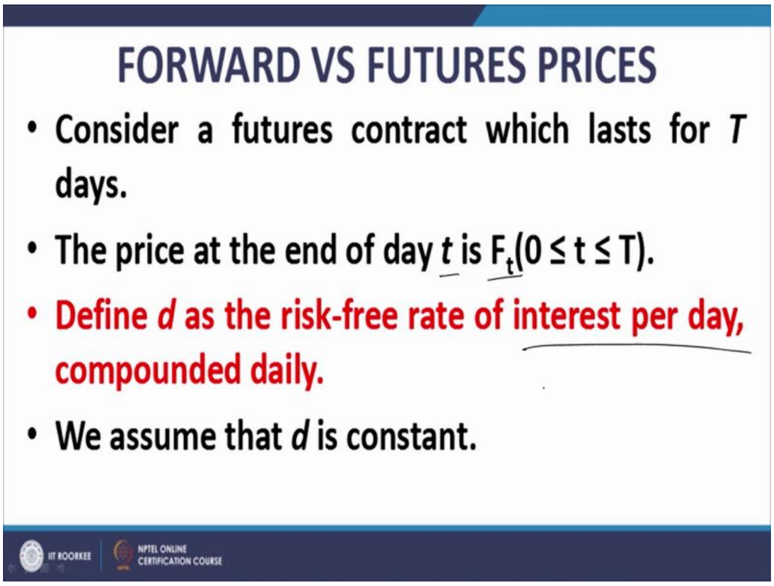

Now, let us look at a fresh topic and by virtue of this topic I want to bring to you the relationship between forward and futures prices. We have already seen that in the context of forward prices, the forward price of an asset is equal to the future value of the spot price worked out at the risk-free rate assuming that we ignore the possibility of default on the forward contract.

Now, we look at the relationship between forward contract prices and futures contract prices. We have already done, let me repeat the relationship between the forward price and the spot price, the forward price is equal to the future value of the spot price worked out at the risk-free rate. Now, let us look at the relationship between forward and futures price. For that purpose, we consider a futures contract which lasts for capital T days, capital T days represents or capital T represents the maturity of the futures contract.

The price at the end of day t, small t, small t is any arbitrary point in time between 0 and capital T. Let me repeat small t represents any arbitrary point in time between 0 and capital T and capital T is the date of maturity of the futures contract. So, let the price the futures price at the end of day small t equal to f of small t that is where t lies between 0 to capital T. We define d, small d as the risk-free rate of interest per day compounded daily.

Please note this fundamental feature of d, it is the interest rate per day. The normal practices to express the interest rate per annum, but for the sake of convenience for the sake of avoiding a proliferation of notation, we are using d as a per day rate of interest. That is, it is a rate of interest from one day to the next day for a borrowing from one day to the next day and it is compounded daily.

So, d has the special feature of being per day rate of interest compounded daily, we assume that d is constant. This is the fundamental premise of what is going to be shown here of the result that is going to be obtained that we assume that the interest rates over the life of the futures contracts are constant.

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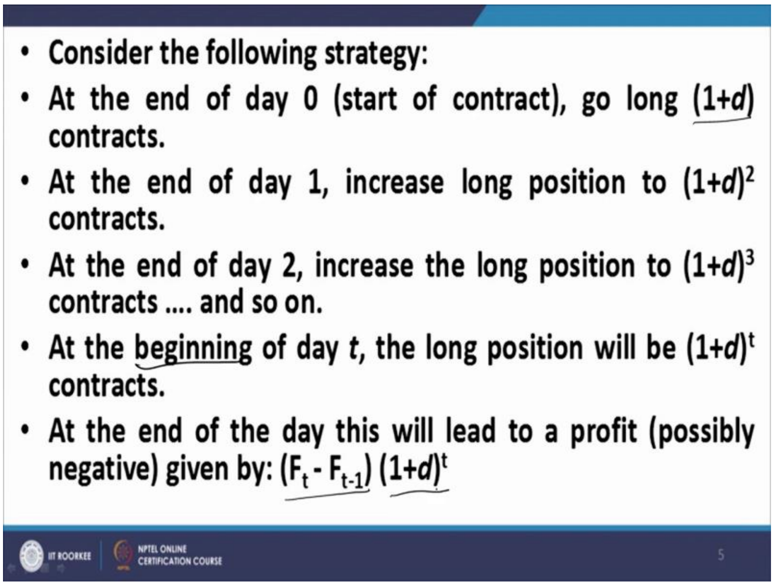

Now, we consider two strategies. Consider first is A strategy, let us call it the A strategy. In the context of A strategy what do we do, at the end of day 0, that is t equal to 0, that is the day of where you start taking a position in the futures contract, where you start initiating your position in the futures contract t equal to 0, you take a position in 1 plus d contracts that is contracts on 1 plus d units of the underlying asset.

Let me repeat, we are talking about 1 plus d units of the underlying asset futures contract on 1 plus d units of the underlying asset, this is at t equal to 0. This position is initiated at t equal to 0 end of day. Now, at the end of day 1, we increase this long position by taking further position in the futures contract and making the futures contract cover 1 plus d square number of units of the underlying asset.

At the end of day 2, we increase this long position to 1 plus d cube number of units of the underlying asset and so on. So, obviously at the beginning, please note this point, at the beginning of day small t the long position will be same as the end of day t minus 1 and which is equal to 1 plus d to the power small t number of units of the underlying asset will be covered by my futures position.

At the end of this day, this will lead to a profit possibly negative given by Ft minus Ft minus 1 into 1 plus d to the power small t. When the settlement will take place at the end of small t the amount that will be transferred to the margin account will be Ft minus Ft minus 1 per unit of the underlying asset.

So, because you have 1 plus d to the power small t units of the underlying asset covered by the futures contract on which the settlement is taking place at t equal to small t end of day, the total amount that will be transferred to the margin account on the t-th day, small t-th day will be given by Ft minus Ft minus 1 into the number of units which is 1 plus d to the power small t.

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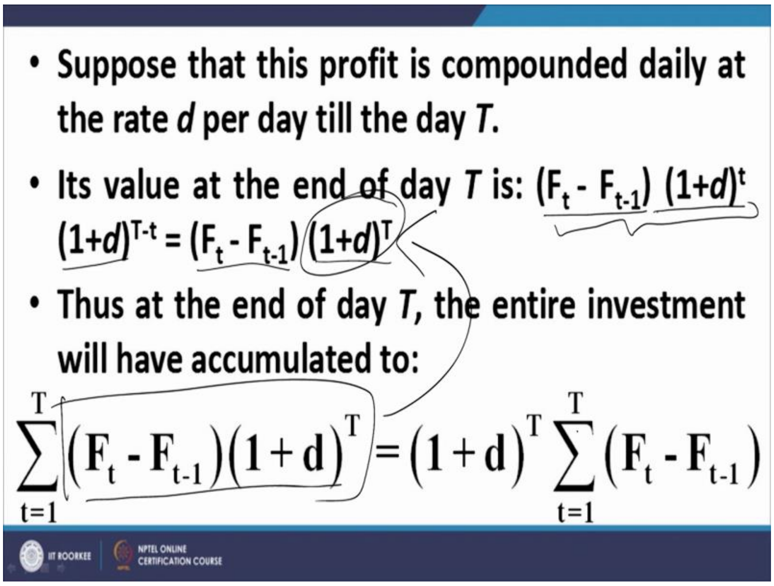

Suppose that this profit is compounded daily at the rate of d per day till the day T. Then what happens, then the value of this profit at the end of day capital T will be equal to Ft minus Ft minus 1 into 1 minus t to the power T this was the amount that was transferred on t-th day. And this will be compounded for how many days? It will be compounded for capital T minus small t days.

So, the future value at t equal to capital T-th day will be obtained by multiplying by 1 plus d to the power capital T minus small t. Please note, d is the daily interest rate firstly, and it is also compounded daily. So, when we work out the future value of Ft minus Ft minus 1 into 1 plus d to the power small t we will multiply it by the interest factor 1 plus d to the power T minus small t and that gives us this expression F small t minus F small t minus 1 into 1 plus d to the power capital T.

Please note that this factor is independent of small t that is it is independent of the any arbitrary point in time it you will have the same factor insofar as the compounding is concerned. Thus, at the end of day capital T the entire investment will have accumulated to now. Pease note, small t is any arbitrary day between t equal to 0 and t equal to capital T.

Therefore, when you do this exercise when you accumulate the future value of the profit per day every day and bring it in to t equal to capital T that is the date of maturity of the futures contract. What you will end up is given by this summation F small t minus F small t minus 1 into 1 plus d to the power capital T summation over small t.

And this, please note this, when you substitute the value of the expression that I now enclosed within the box from this expression that we have derived earlier what we get is 1 plus d to the power capital T being independent of small t can be taken outside that summation. And when I do the summation of F small t minus Ft minus 1 over t equal to 1 to capital T that is small the summation index varies between small t equal to 1 to capital T.

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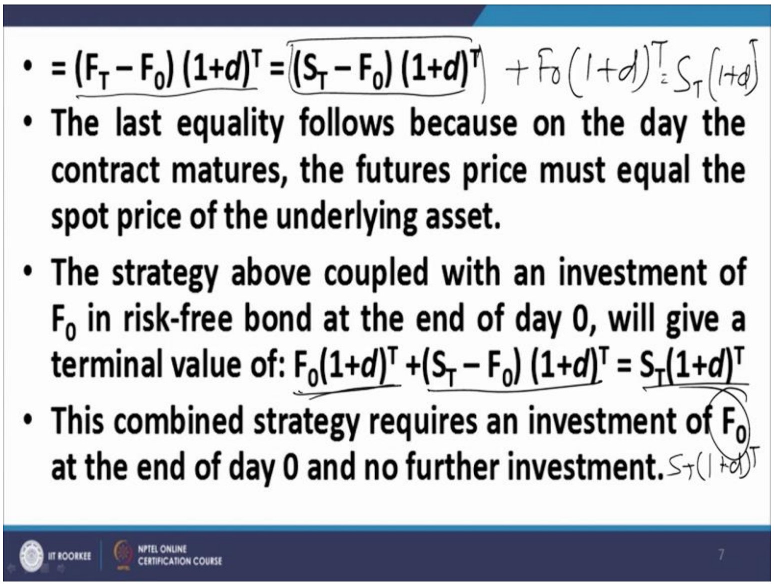

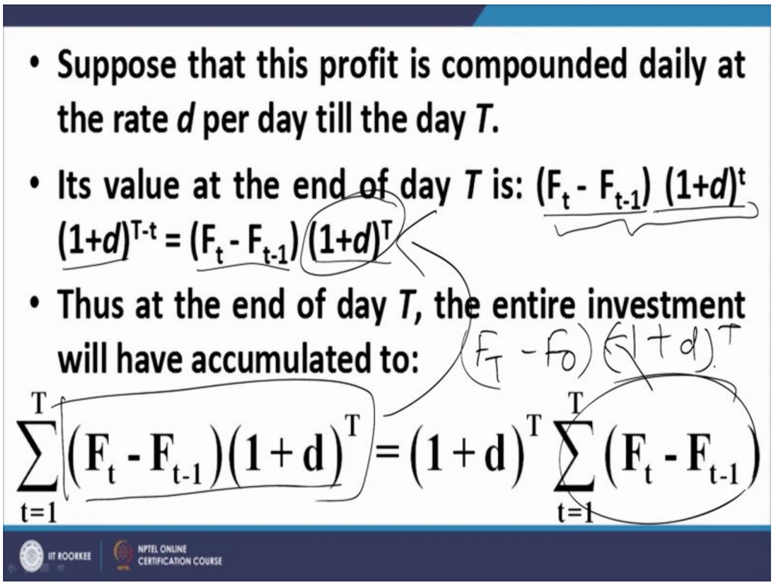

Then what we get is the expression that we have here FT minus, F capital T minus F0. Please note all the intermediate terms will cancel out. If I expand this term here, if I expand this term here, what will I get, I will get F1 minus F0 plus F2 minus F1 plus F3 minus F2 and so on. So, all the intermediate terms will cancel out and I will be left with only two terms the terminal term and the initial terms.

So, this whole summation becomes equal to F of capital T minus F of 0. And this is multiplied by what, the pre-factor that is 1 plus d to the power capital T. That is the expression that we have on the first equation on this slide. Now, the important thing is we invoke the convergence principle. What is the convergence principle?

The convergence principle tells us that on the date of maturity of the futures contract or for that matter of forward contract, the spot price of the contract or the futures price of the contract rather must be equal to the spot price of the asset to avoid any arbitrage happenings. In other words, let me repeat, on the data of maturity of the forward or futures contract the futures price or the forward price as the case may be must scale to the spot price of the asset in the spot market.

Why? Because otherwise arbitrage will take place and the phenomenon of arbitrage will then create a situation disequilibrium situation which will lead to the price dynamics and as a result of which the prices will converge that is the futures price and the spot price will converge. So, the last equality follows because on the day of the contract matures, the futures price must equal the spot price of the underlying asset.

Now, if you look at the expression that we have here, and to this expression, if I add an amount F0 into 1 plus d to the power T, then what I get is ST into 1 plus d to the power T. In other words, what I am trying to say is that, if I add to this particular process, the process of the strategy that I have been following so far in the futures market, if I add to this an investment of F0 in a risk-free bond at the end of day 0, then it will give me what, then this investment of F0 will give me a cash flow of F0 into 1 plus d to the power capital T on the date of maturity of the futures contract.

So, the combination of this futures positions that I have taken up, and that I have explained in the earlier paragraphs together with single position at t equal to 0 in a risk free bond of an amount equal to F0 at a rate d compounded daily will give us an amount equal to F0 into 1 plus d to the power T plus ST minus F0 into 1 plus d to the power T that is equal to ST into 1 plus d to the power T.

So, this let me repeat, the combination of the future strategy that I have followed in changing my futures position on a day to day basis, plus, an investment of F0 in a risk-free bond with a maturity of capital T will be equal to, will be worth S capital T into 1 plus d to the power capital T on the date of maturity of the futures contract.

So, the strategy above coupled with an investment of F0 and risk-free bond at the end of day 0 will give me a terminal value of the payoff from the bond investment and this is the payoff from the future strategy that I have elicited earlier and this gives us S capital T into 1 plus d to the power capital T as the total value of the combination of the futures positions and the bond position.

So, this combine strategy requires an initial investment of F0 and at the end of day 0 and no further investment and it is worth what, it is worth S capital T into 1 plus d to the power of capital T at t equal to capital T. Let me repeat the combined strategy involves an initial investment of F0 in the risk-free bond for a maturity of capital T and it gives us a value of a worth of S capital T into 1 plus d to the power capital T on the date of maturity of the futures contract. Now, let us look at this second strategy, let us call it a strategy B.

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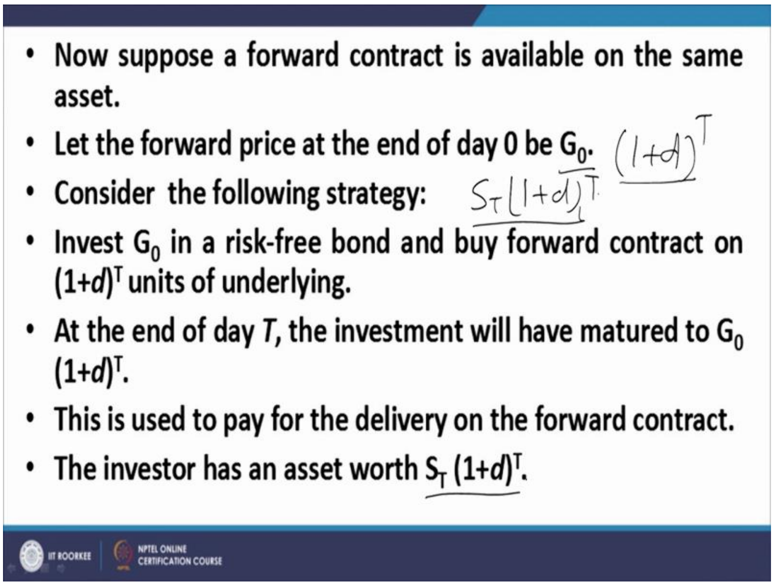

In the strategy B what do we do, we suppose, we take a long position in a forward contract at a forward price G0, at a forward price G0 per unit. And what is the number of units that we take the contract on that is equal to 1 plus d to the power T. In other words, we take forward position, we take a long position in a forward contract on 1 plus d to the power capital T units of the underlying and at the forward price G0. So, that is what is given here.

At the same time as the second part of the strategy, we invest in amount G0 in a risk-free bond for a maturity of capital T. So, again in strategy B also we have two legs of the strategy, number 1, we take a long position in a forward contract on 1 plus d to the power capital T number of units for maturity at capital T, and number 2, we invest in a bond an amount equal to G0, where G0 is the forward price at capital T for maturity an capital T for delivery for settlement at capital T per unit of the underlying.

So, we invest an amount equal to G0 in a risk-free bond with a maturity of capital T. Now, what will happen? On the date of maturity of the forward contract because we are long in the forward contract, we will receive 1 plus d to the power capital T number of units of the underlying asset. And these units in the market will be worth what, will be worth S capital T into 1 plus d to the power capital T.

Now, for getting this 1 plus d to the power capital T number of units, where will I get the money from, I will get the money from liquidating the investment of G0 that I made at t equal to 0, this amount G0 at t equal to 0 will now have become G0 into 1 plus d to the power capital T. And that is precisely the amount that is required in the context of my long position for to make payment for 1 plus d to the power capital T number of units of the underlying asset.

Let me repeat, the amount of investment G0 that I had made a t equal to 0 will grow to G0 into 1 plus d to the power capital T, on the date of maturity. And that is precisely my obligation under the long position on the forward contract for the receipt of 1 plus d to the power capital T number of units of the underlying asset, because G0 is the forward price.

So, I will pay G0 into 1 plus d to the power capital T to the party who is short and I will receive 1 plus d to the power capital T number of units of the underlying asset, which in the market would be worth ST into 1 plus d to the power capital T. So, in this strategy, what is happening, I am making an investment of G0 at t equal to 0 in a risk-free bond.

And at the end of this strategy, the value of the strategy is equal to ST into 1 plus d to the power capital T. now please note this ST into 1 plus d to the power capital T was also the terminal value of strategy A. So, we have, what we have establishes that the terminal value of strategy B is the same as that of strategy A, both of these strategies have the same terminal value.

And because there are no intermediate cash inflows or outflows from the system or to the system, that this implies that from arbitrage considerations, the cash outflow at t equal to 0 in both cases in strategy A and strategy B must be the same. So, F0 must be equal to G0. So, the futures price must equal the forward price. That is the bottom line of this analysis.

So, we have discussed this, let me read it out for you. The strategy B consists of, number 1, invest G0 in a risk-free bond and buy forward contract on 1 plus d to the power T units of the underlying this is the second part of the strategy. At the end of day capital T, the investment will mature to G0 into 1 plus d to the power capital T.

And this will be used for paying the obligation under the long forward contract and you will receive the 1 plus d to the power capital T number of units, which will be worth S T into 1 plus d to the power capital T.

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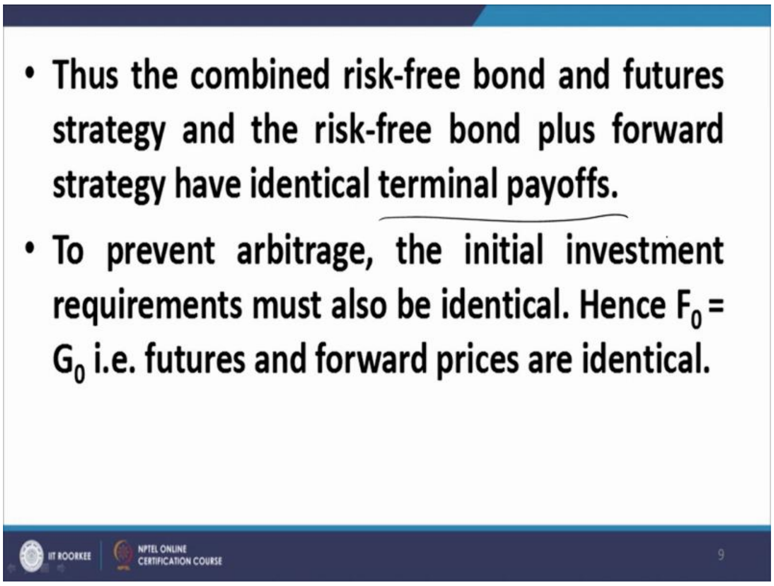

Thus, the combined risk-free bond and the future strategy and the risk-free bond plus the forward strategy have identical terminal payoffs. To prevent arbitrage, the initial investment requirements must also be identical. Hence, F0 is equal to G0 that is futures and forward prices are identical.

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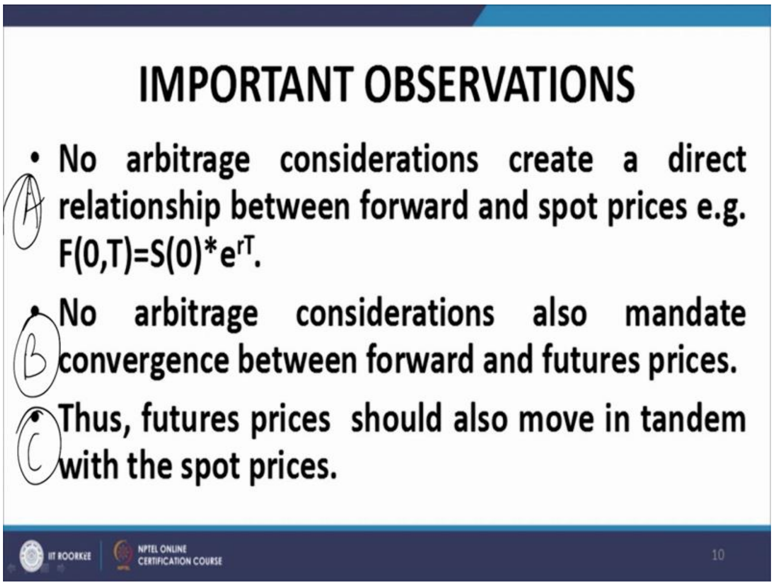

Now important observations. No arbitrage considerations or arbitrage free considerations create a direct relationship between forward and spot price of a underlying asset. We have already discussed that the forward price of the underlying asset is equal to the future value of the spot price worked out at the risk-free rate.

No arbitrage considerations also mandate convergence between forward and futures price, as you have seen from this arbitrary strategy, that I have explained in the last few minutes. Thus, futures price should also move in tandem with spot prices. This is the consequence of let us call it proposition A, let us call it proposition B. If you create a combination of proposition A and proposition B, the proposition C that emerges is that the futures price must move in tandem with the spot prices.

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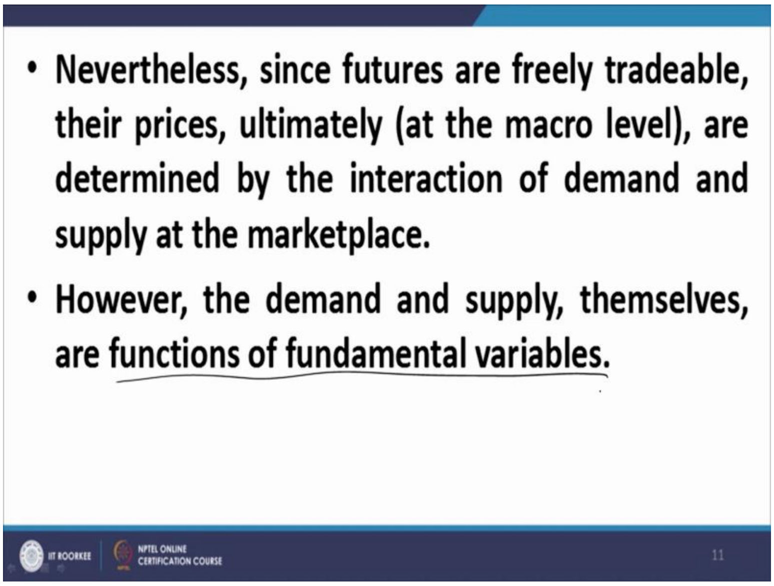

However, since futures are freely tradable, their prices, ultimately are determined by the interaction of demand and supply at the marketplace. The demand and supply itself is a function of fundamental variables. For example, if you are talking about a share, if a particular share is superior to another share, if a particular share A is superior to another share B in terms of its cash flow projections, as predicted by the market, then obviously the price that is commanded by A would be higher than the price commanded by B.

So, it is the fundamental variables like cash flow like riskiness and return basically or the constituents of that go in to the determination of cash flow and cash flow returns and the riskiness in the reliability of the returns. And that finally determine the demand and supply of shear which is A which is better than B will have a higher demand. And therefore, may come under higher price than a share B, which is of a lesser quality in terms of fundamentals.

So, the fundamentals manifest themselves, the bottom line of what I am trying to say is that the fundamentals manifest themselves in terms of the demand and supply process. And this demand and supply process is the superficial process, which results in the determination of the price of the asset or the price at which the asset has been traded in the market. So, let me repeat.

The fundamental variables are the variables that go into the determination of demand and supply because if a particular asset has strong fundamentals, it is going to be more in demand, if an asset has weak fundamentals is going to be less in demand. So, therefore, the prices that would be commanded by the asset which is more in demand would be higher than the price that is commanded by the asset which is lesser in demand.

So, strong fundamentals usually translate to higher prices and vice versa. So, the bottom line is that, let me repeat the bottom line is all the prima face (()(26:33), it is the supply and demand interaction, which leads to the determination of price. The process of supply and demand or the process or demand in particular is a manifestation of the fundamentals of the underlying assets about which we are talking.

So, nevertheless, since futures are freely tradable, the prices ultimately are determined by interaction of demand and supply in the marketplace. However, demand and supply themselves are functions of fundamentals variable, this is what I have tried to emphasize. The quality of the fundamental variables will lead to the demand of the increase or the decrease demand of the asset which will translate to higher or lower prices.

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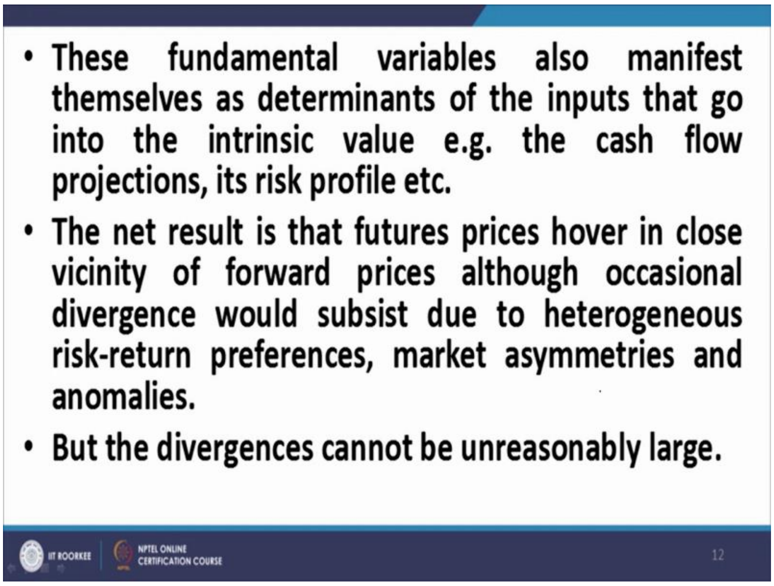

These fundamental variables also manifest themselves as determinants of inputs that go into the intrinsic value calculation. We have seen that when we work out the intrinsic value of an asset, we do the EPS analysis in the context of equity shares, and on that, we examine various fundamental variables. And on that basis, we move into the projections of cash flows and discount rates and then work on intrinsic value.

On that basis, the market starts trading the demand for that asset. If the asset has an high intrinsic value, there is more trading, there is more demand for that asset in the market and so on. So, the basic thing is, these fundamental variables also manifest themselves but in a latent manner insofar as the pricing of the asset is concerned. It is not that the pricing is an arbitrary process entirely. Although there is an element of randomness in prices.

But at the same time, the fundamental strength of the entity is also manifest in the price without doubt. These fundamental variables also manifest themselves as determinants of the inputs that go into the intrinsic value. For example, the cash flow projections, and its risk profile etcetera. The net result is that futures prices hover in close vicinity of forward prices although occasional divergence would subsist due to heterogeneous risk return preferences, market asymmetries and anomalies.

The net result is that futures prices hover in close vicinity of forward prices although occasional divergence would subsist due to heterogeneous risk return preferences, market asymmetries and anomalies. But the divergences cannot be unreasonably large.

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Now, we talk about a new topic, which is futures hedging, which I will take in the next lecturer. Thank you.