Quantitative Investment Management
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Lecture 51
Futures - 1

Welcome back. So, let us continue towards the end of the last lecture we have derived the Black-Scholes equation working on the premise of the Ito equation, we started with the Ito equation, wrote down the Ito equation for a derivative, keeping in view the infinitesimal model or the log normal model as you call it.

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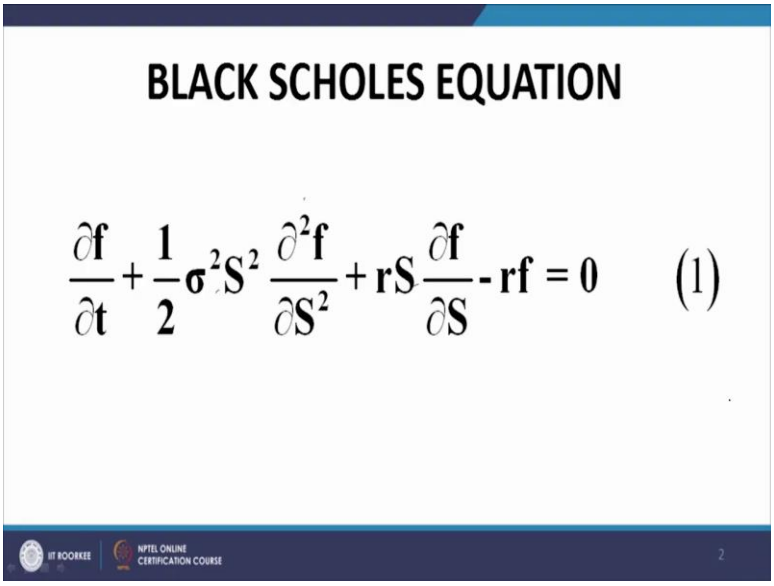

And on that basis, we arrived at the Black-Scholes equation, which is given on this slide. So, now, if we solve this equation, there are a number of ways of solving this equation, we can use the green function approach, we can use separation of variables, we can convert it by coordinate transformation to the heat equation and then solve it, we can use any of these approaches, we can use Fourier transforms as well.

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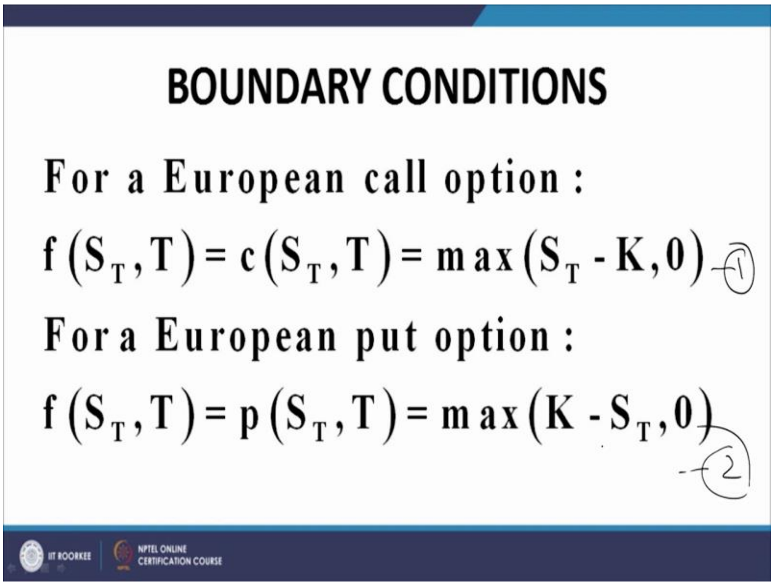

In any case, when we solve this equation using the boundary conditions, which we have on this slide, basically, these boundary conditions represent the fact that on the date of maturity of the option, we, please note we are talking about the European options, on the date of maturity of the options the value of the option must equal its payoff, that is what is represented by equation number 1 in the case of a call European call, then equation number 2 in the case of European put.

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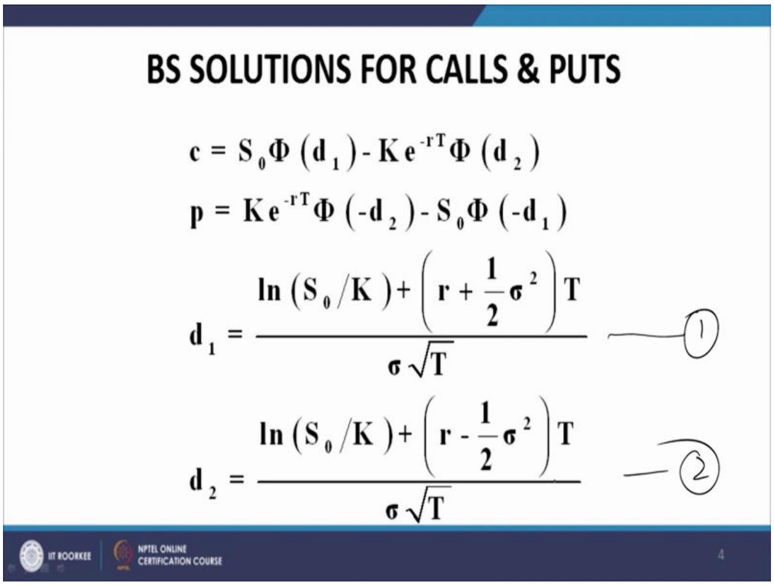

When this Black-Scholes equation is solved with these boundary conditions, what we arrive at as the solutions are given on the slide c is equal to S naught capital phi of d1 minus K e to the power minus rT phi of d2. And similarly, for the put option, put option is equal to, the value of the put option is equal to K e to the power minus rT phi of minus d2 minus S naught phi of minus d1, where d1 and d2 are defined by equations 1 and 2 respectively.

Please note r is the risk free rate of return in this equation, sigma is the volatility, time is the term to maturity, S naught is the current price of the underlying asset and K is the exercise price the strike price of the option.

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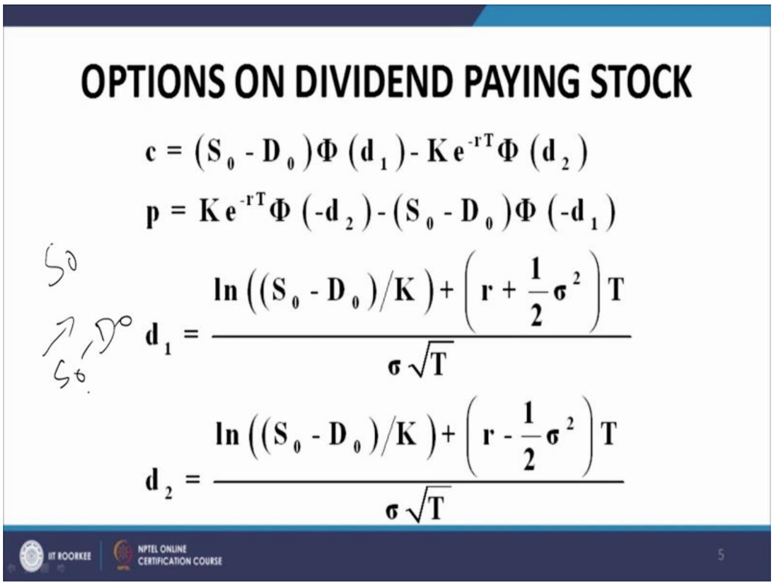

In the case of, see whatever derivations have been done so far have been done on the premise that the underlying asset pays no income during the lifetime of the option. If you recall the case of the forward contracts or the case of options when we talk about put call parity of options when there is a dividend payout envisage during the life of the options, the formulas get modified by the fact that S naught which is the current stock price gets replaced by S naught minus D naught everywhere, where D naught is the present value of the dividend payment. That is precisely what is done in this slide. In d1 and d2 also change and so do the values of c and p.

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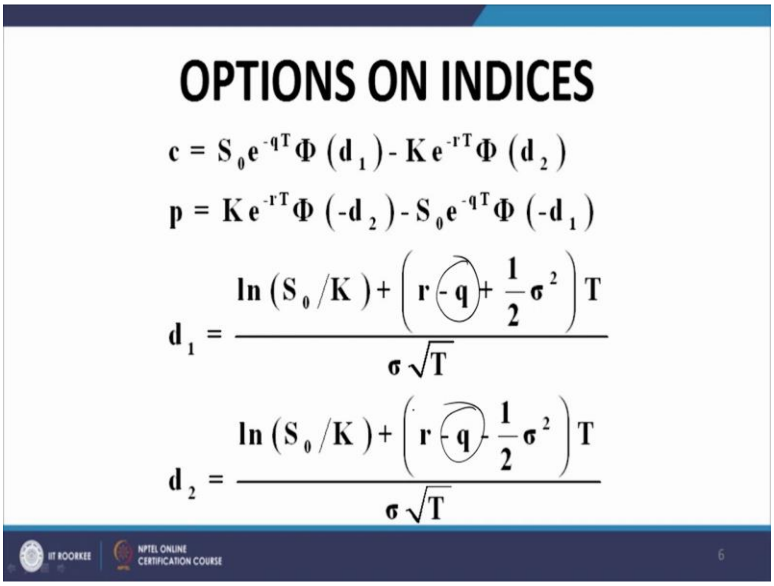

In the case of options on indices when we have a continuously compounded return on the underlying asset during the life of the option during the term of the option, we have this additional factor q coming into play as you can see, when we calculate d1 and d2. So, these are variations of the Black-Scholes formula in relation to various types of options.

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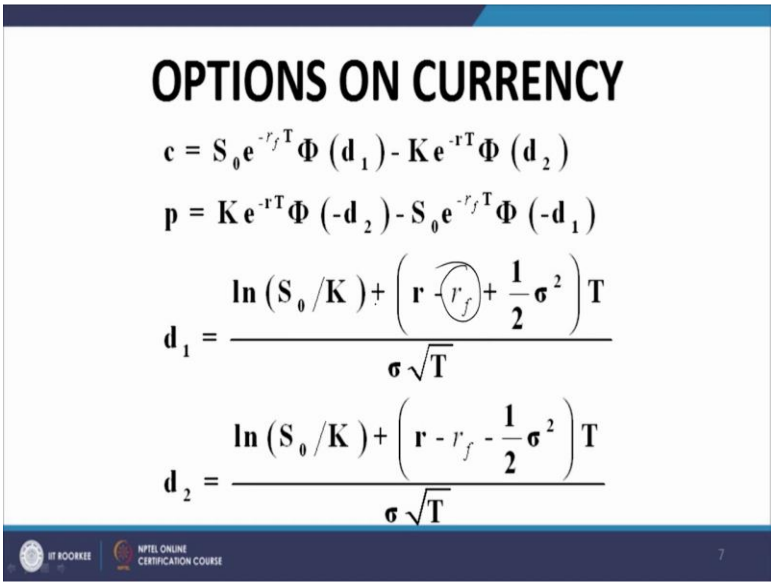

Similarly, for options on currencies, the factor q that I talked about just now is replaced by rf, where rf is the foreign rate of interest.

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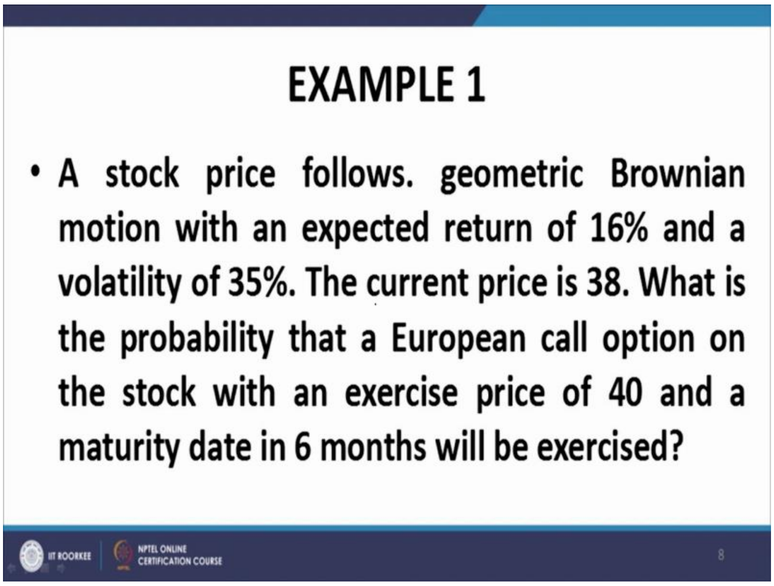

Now, let us do an example. A stock price follows geometric Brownian motion with an expected return of 16 percent and a volatility of 35 percent. The current price is 38. What is the probability that a European call option on the stock with an exercise price of 40 and maturity date in 6 months will be exercised? So, expected return mu is equal to 16 percent, volatility is equal to 35 percent and the current price is 38. The exercise price is 40. The option is a call option, and the maturity is 6 months. So, this is the data that is given to us.

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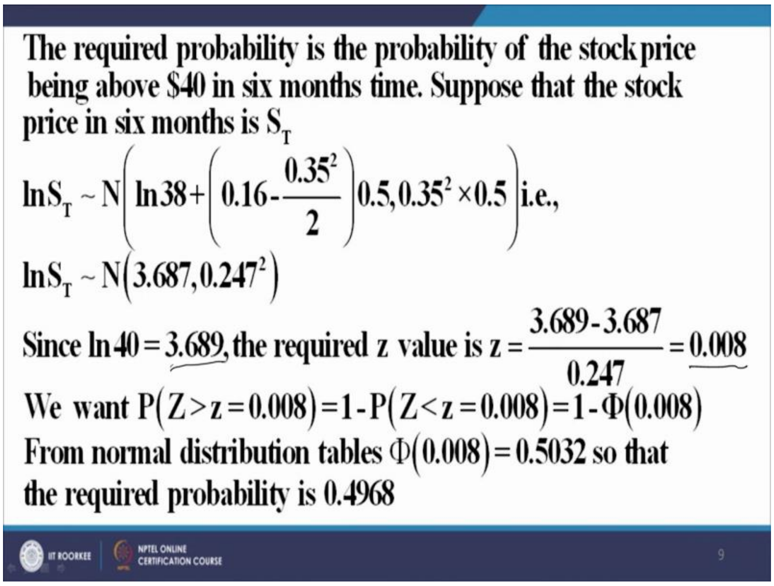

Now, obviously, an option will be exercised when the price on the date of maturity of the option in the case of a European option on the date of maturity of the option exceeds the strike price. So, in other words, given the data that I have just elucidated, we need to find out the probability that the stock price will be greater than 40 as on the data maturity of the options that is in 6 months' time.

But we know that log of ST is normally distributed with a mean equal to log of S naught plus mu minus sigma squared upon 2 into t and a variance of sigma squared into t. So, we are given all the information on all the parameters, we use that information, and what we find is that log of ST, where ST is the price of the stock as on the date of maturity of the option, ST is the price of the stock as on date of maturity of the option is normally distributed with the mean of 3.687 and a variance of 0.247 square that is the standard deviation is equal to 0.247.

Now, we need to find the probability of the stock finishing greater than 40. For that purpose, you work out the log of 40 and that is found out to be 3.689. Now, using the figure of 3.689 and the mean and standard deviation of the distribution, we can calculate the Z value, the Z value turns out to be equal to 0.008 as you can see on the slide.

And therefore, we need to find the probability that the standard normal variate takes a value greater than 0.008. This can be done using the normal distribution tables, and we find the probability to be 0.4968. So, this is an illustration of the use of the log normal property of stock prices.

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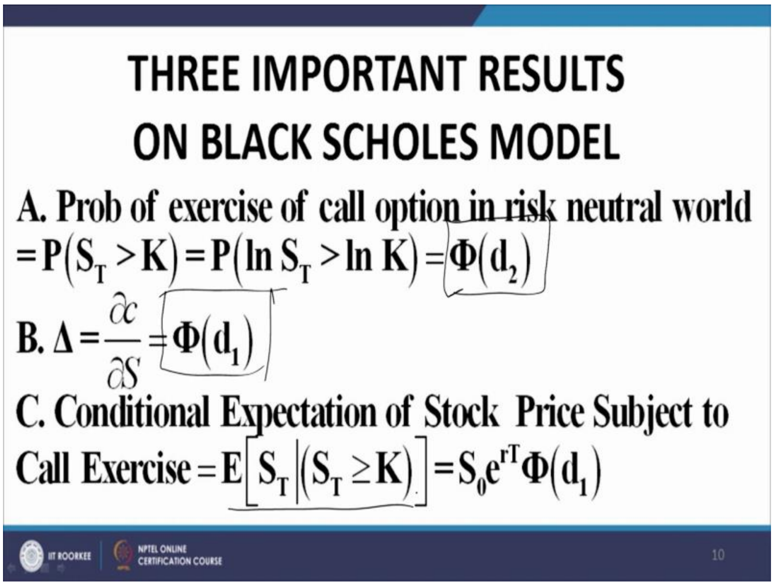

Now, I present on this slide three important results, I will not get into the proof of these results for the paucity, on account of the paucity of time shortage of time that we have in this course. But I will put them in the in the PowerPoint in that accompany this course, as well as in the notes which will be circulated along with this course for the benefit of learners.

I would also strongly recommend that if the learners could study from the course on financial derivatives that was run by me on the net NPTEL platform a couple of years ago, the proofs of all these results, together with the solution of the Black-Scholes equations, a derivation of the Fokker Planck equation and its solution, all these things are dealt with in considerable detail in that course on “Financial Derivative and Risk Management” that was run on the NPTEL platform a couple of years ago.

So, these are three results, first result is the probability of exercise of the call option in risk neutral world that is given by phi d2 when you work it out probability of ST greater than K because that is only when the option will be exercised. That is obviously equal to probability of log ST greater than log K because the logarithm is a monotonic function of the argument.

Therefore, what we have is probability of ST greater than K which is the probability of exercise of the call option is equal to probability log ST is greater than log K and which can be worked out to be equal to phi of d2, capital phi of d2. Please note capital phi represent the cumulative normal distribution. Delta value, you recall we constructed a delta neutral H, risk less portfolio consisting of one unit of the derivative and delta units of the underlying asset.

When we work out the delta value using the Black-Scholes solution, what we find is that delta value is equal to phi of d1, this is the second important result. And the third important result is the conditional expectation of stock price subject to call exercise is equal to S naught e to the power rT phi of d1. Let me repeat.

Conditional expectation of stock price subject to call exercise that is given by this equation, it considers or it is the expectation calculated with reference to all the values of the stock price which are greater than the exercise price the expected value of all the values of the stock price subject to the condition that those values of the stock price are greater than equal to K and that turns out to be equal to S naught e to the power rT phi of the d1.

So, these are three important results which can be derived from the Black-Scholes equation, we shall make use of these results in bringing to you an interpretation of the Black-Scholes equation, I will try to show by virtue of explaining this interpretation that the Black-Scholes equation is consistent with the principles of finance.

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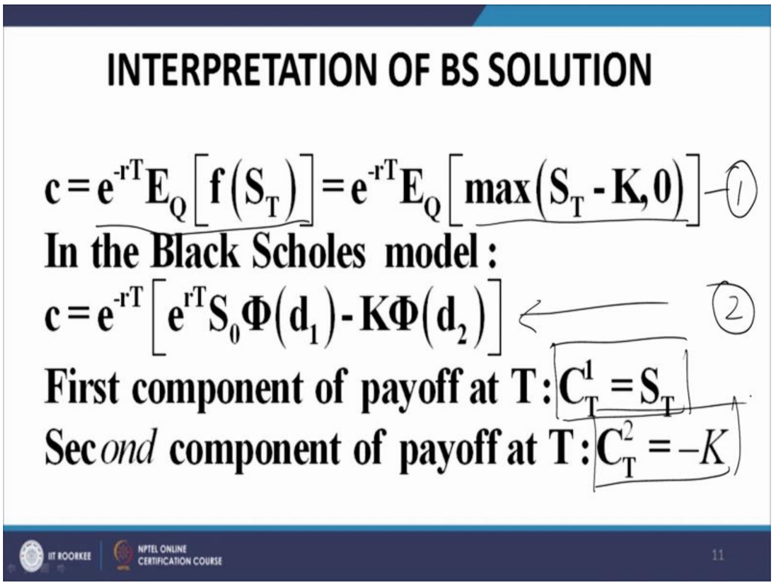

For that purpose, let us now move on to the interpretation. And now, we have seen, particularly when I talked about the binomial model of calculating the value of an option, that the value of a call is equal to the present value of the expected payoff from the option, that expected value being calculated the reference to risk neutral probabilities, that is what is given by this expression here which I have underlined e to the power minus rT EQ, Q is the probability measure representing risk neutral probabilities, f of ST, f of ST represents the payoff from the derivative which is obviously a function of ST, where ST is the stock price at maturity.

Now, in the case of a call option f of ST is equal to maximum of ST minus K comma 0. So, I can substitute f of ST by the expression that I have underlined maximum of ST minus K comma 0. In the Black-Scholes model, where this is the equation that we arrived at for c. So, we need to show that the two results that I have here number 1, let us call it number 1 the first equation, let us call that equation number 2.

We need to show that equation number 1 and 2 are mutually consistent firstly, and secondly, they are consistent with the fundamental principles of finance. Now, the first component of the payoff at capital T, please note capital T is the date of maturity of the call option, please also note that we are talking about European options. So, C1 of at time T is equal to ST this is the first component of the payoff and the second component of the payoff is minus K.

So, we write it as c2 of capital T that is equal to minus K. So, there are two components of the payoff from the derivative from the call option, the first component is ST and the second component is K.

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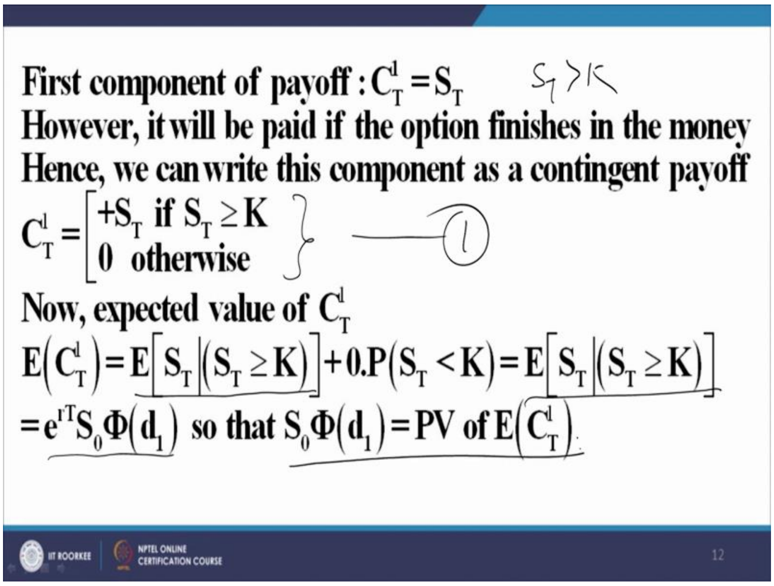

Now, let us restrict ourselves to the first component for the moment. The first component is given by C1 at time T that is equal to ST. This ST will be paid, if the option finishes in the money, that is if ST is greater than K, if ST is greater than K then this component of money will materialize. However, if ST is less than K, then this component of money will not materialize.

And this is a contingent payoff, which can be represented by the equation that we have here, let us call it equation number 1, C1 at capital T is equal to ST if ST is greater than equal to K and 0 otherwise. So, therefore, the expected value of C1 capital T is given by the expression that we have here E of C1 capital T that is equal to E of ST subject to ST is greater than equal to K plus 0, because, if ST is less than equal to K, then the payoff is 0 into probability that ST is less than K.

So, this part we have already worked out, this is a part of the results that I showed on the previous slide the third result and that is equal to E to the power rT S naught pi of d1. In other words, the present value of this component is equal to S naught phi of d1. Now, let us move to the second component.

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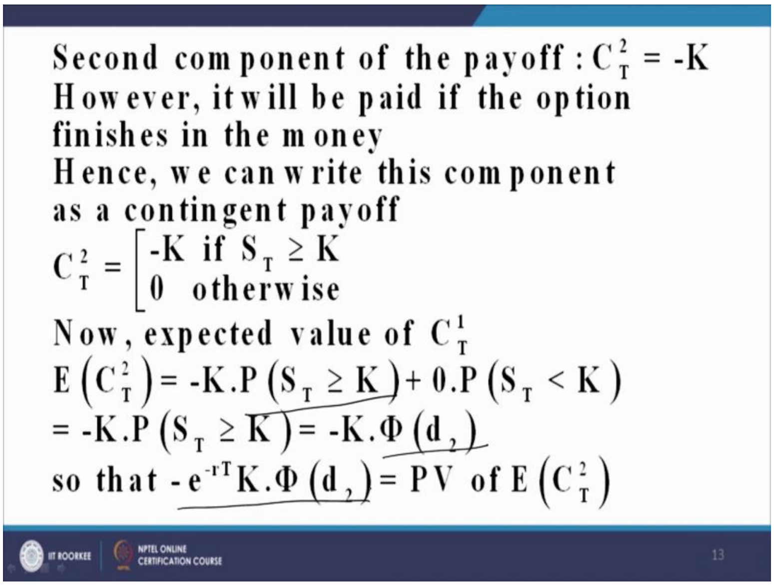

The second component of the payoff is minus K, this is C2 at time t, this will be paid if the option finishes in the money. Hence, just like the first component, we can write this component as a contingent payoff C2 T is equal to minus K if ST is greater than equal to K and is equal to 0 otherwise. Therefore, the expected value of the second component is worked out as minus K into probability that ST is greater than equal to K plus 0 into probability that ST is less than K.

Now, the probability ST is greater than equal to K, we have already worked out this was a result number 1, and we found that it was equal to capital phi of d2. So, from this expression we get that expected value of the second component at time T is equal to minus K into capital phi of d2. In other words, the present value of this component will be equal to minus K e to the power minus rT into phi of d2, this is the present value of the second component.

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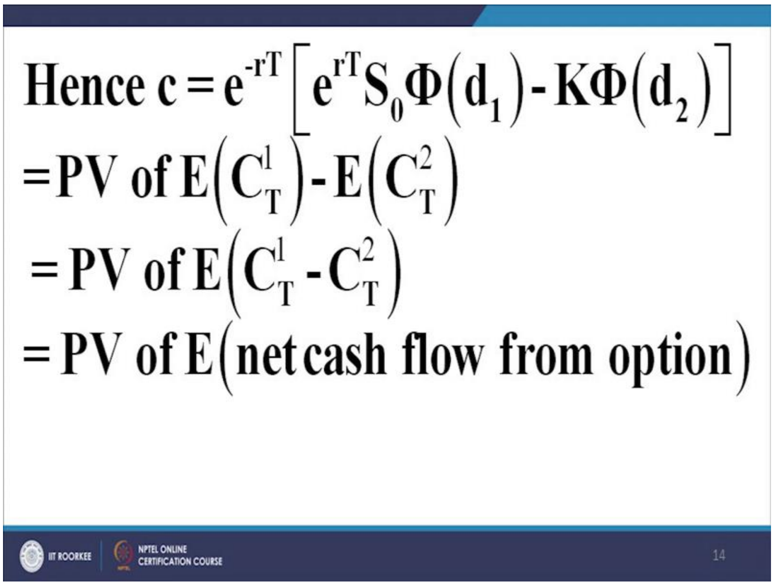

Therefore, the Black-Scholes formula which can be written as the expression that we have here, equation number 1 here on the slide, you can see here this can be written as the present value of E of C1 T minus E of C2, C2 at time T, this should be C2 here. So, present value of E of C1 T minus E of C2 T that is equal to present value of E of C1 T minus C2 T and that is equal to present value of net cash flow from the option.

So, which shows that this formula is consistent with the principles of finance, the value of the derivative is equal to the present value of the net expected cash flow from the option. Please note the word expected, this expectation operator is there. So, what we find is that, the value of the call option as per the Black-Scholes model is equal to the present value of the expected net cash flow from the option which is as it should be.

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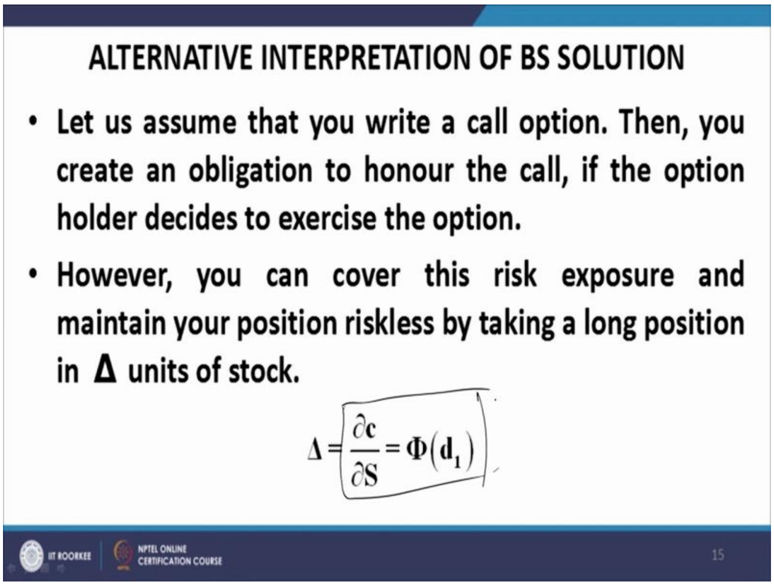

So, there is another interpretation to the Black-Scholes formula Black-Scholes solution. Let us assume that you write a call option, then you create an obligation to honor the call, if the option holder decides to exercise the option. You are the writer of the option, so you have the obligation to honor your leg of the option if the holder of the option decides to exercise the option.

However, you can cover this risk exposure and maintain your riskless position by taking a long position in delta units of the stock. This was the fundamental principle you may recall, on the basis of which we did the valuation both in the binomial model and the Black-Scholes model, we can create a riskless portfolio comprising of position in the derivative and then opposite position in the underlying asset that is precisely what is being said here.

So, because we are short in the option we will take a long position in the underlying asset, how much, that is equal to delta where delta is equal to the partial of c. Please note c is the premium or the price of the call option European call. And this we know, this part we know from the results that we discussed a few minutes back that the delta of the option is equal to phi of d1.

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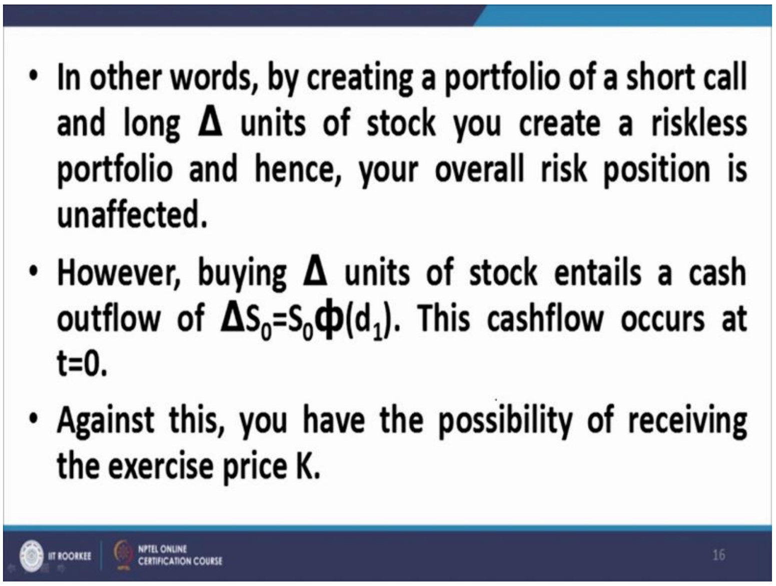

Now, in other words by creating a portfolio consisting of a short call and long delta units of the stock you create riskless portfolio and hence, your overall risk position is unaffected. However, buying delta units of the stock entails an expenditure that expenditure is equal to delta into S0 that is equal to S0 into capital phi of d1. This cash flow occurs at t equal to 0. Against this, you have the possibility of receiving the exercise price K on the date of maturity.

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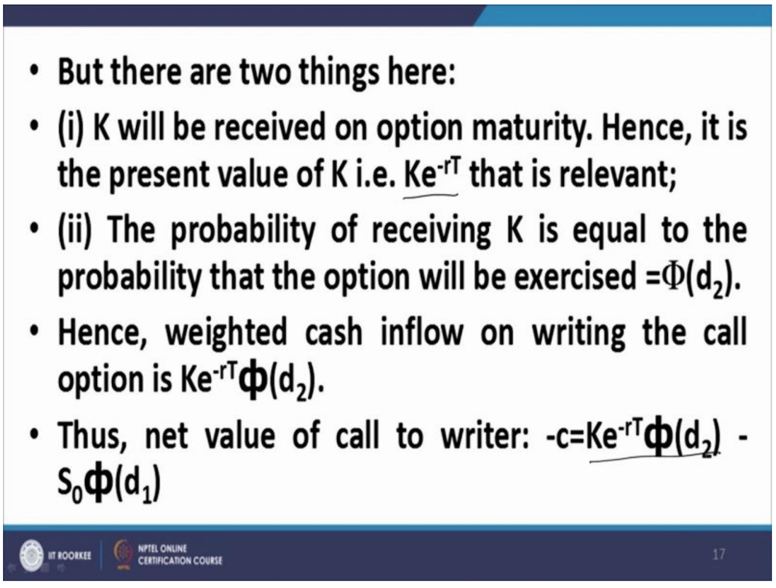

Now, there are two things here as I mentioned, K would be received by the option writer at maturity. Hence, the amount that is relevant when you are valuing the option at an earlier date is the present value of K that is equal to K e to the power minus rT. And the second thing is that you will receive this amount K only if the option is exercised, and the probability of the exercise of the option we have worked out is equal to phi of d2.

So, the relevant amount that needs to be considered when we are valuing the option at T equal to 0 is equal to K e to the power minus rT phi of d2. So, the net value of the option to the writer is what he is going to receive that is equal to K e to the power minus rT phi of d2 minus S naught phi of d1 which is its obligation against the option and this is equal to minus C because if this valuation is done from the perspective of the option writers. So, this is another perspective, this is another interpretation of the Black-Scholes model.

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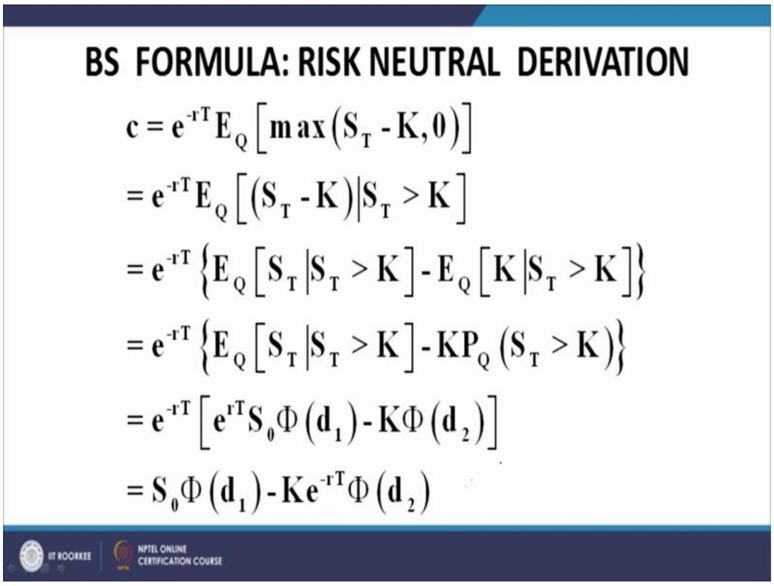

Then on this slide, as you know as when we talked about the binomial model, we derived the expression for the value of the option on two counts, number one by constructing a riskless portfolio and number two by risk neutral valuation. In this slide, I give you the risk neutral approach to the Black-Scholes model or the Black-Scholes formula for the European call.

And you can see here that this is also consistent with the with the Black-Scholes model that we arrived at using the delta approach, using the arbitrage free approach, using the construction of a riskless portfolio approach. The calculations are pretty elementary pretty straightforward, we have made use of the three results that I have mentioned at the beginning of today's class, and we arrive at the result which is exactly the same as the Black-Scholes model.

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Now, we move on to, this is all I have to offer in the context of options. I reiterate my request to the learners to please go through or to pre-supplement the material that is given in this course, with the material that is covered in the course on financial derivatives under the NPTEL program. All these things which I have summarized in this particular course insofar as they pertain to derivatives are dealt within considerable detail, it is a 12-week course.

And therefore, all these issues are dealt within considerable detail in that particular course, so that will provide you excellent supplementary material, I will also try to circulate transcripts of the course with the learners who are enrolled for this course. Anyway so, now, we move on to the next topic that is futures. So, what are financial futures?

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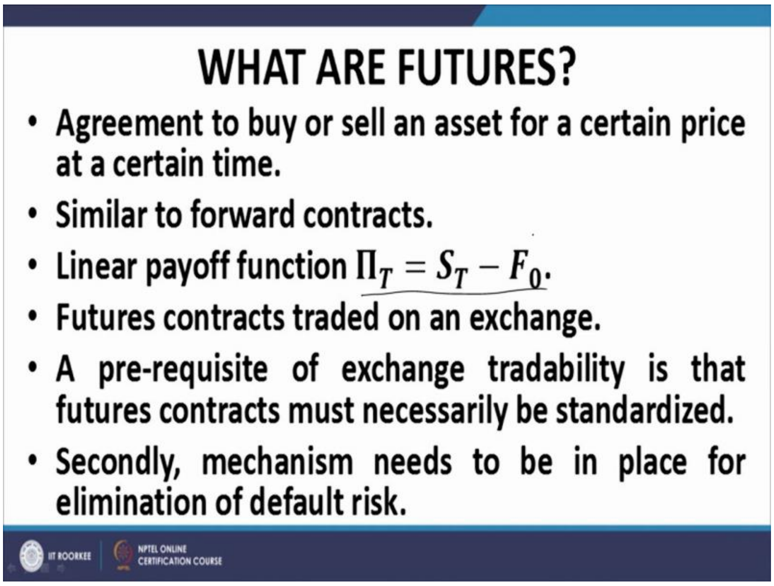

What are financial futures? Financial futures are agreement to buy or sell an asset for a certain price at a certain time just like the forward contract. So, as far as this feature is concerned, the emulator they are similar to the forward contract. So, that is what we say they are similar to forward contract.

They have a linear payoff function, which is given by this expression as you can see, F naught is the price that is contracted at t equal to 0 when you take a position in the futures contract, ST is the price of the stock of the underlying asset, as on the date of maturity of the futures, therefore, the payoff is equal to ST minus F naught from the perspective of the party who is long in the futures contract.

This is where we move away from the concept of forwards. This is what, this is the fundamental feature that distinguishes or discriminates between the forward contracts and the futures contract. Futures contracts are traded on an appropriate exchange or exchanges for that matter, whereas, forward contracts are private contract between parties, so that is the fundamental difference between the two types of instruments.

The prerequisite of exchange tradability is that the futures contract must necessarily be standardized. It is natural, if you want to have a free inhibited trading of an instrument, it is necessary that the instrument be standardized. So, that there is adequate liquidity in the market. You are able to, any party wanting to sell the instrument is able to locate a buyer with ease and that can only happen if the instruments are standardized to some extent.

So, it is very necessary in the context of futures, that futures be standardized, because of their future of incredibility. And secondly, there should be a mechanism for the elimination of default risk.

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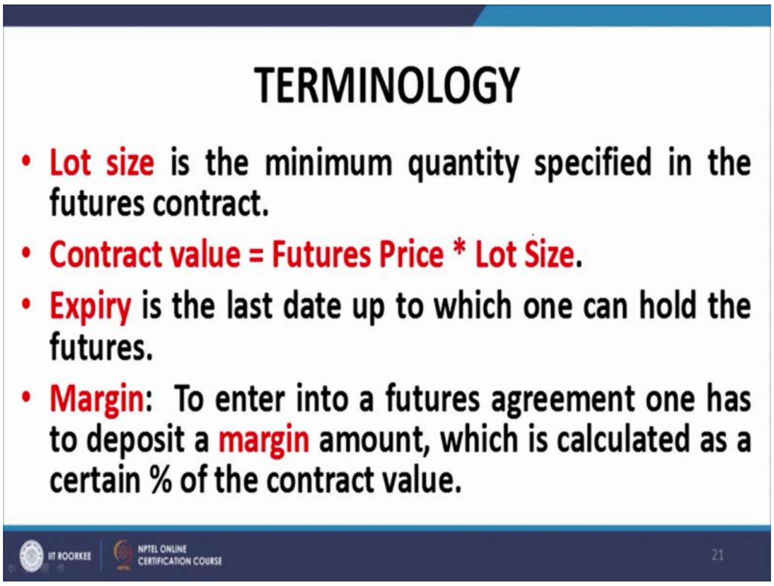

So, why is that necessary? That is necessary because when, in the context of futures, if you want that future should be freely tradable, then obviously, party A may transfer it to party, party A and B if they are if the initial part is to the contract, A may transfer it to C and C may transfer to D without the consent of B that would create chaos in the market if there was a significant default risk incorporated or present in the futures contract.

So, it is very necessary that in order to facilitate smooth unhabituated trading of futures, that the futures contract should be devoid of any kind of default risk. This is very fundamental. I will come back to this issue again in a minute, but for the moment, it is a necessary futures, it is an essential feature, essential ingredient that the futures contract should, number one, be standardized, and number two, devoid of default risk in order that they can be traded in an exchange with comfort and ease.

Now, we will talk about some terminology, a lot size is the minimum quantity specified in the futures contract. It is not necessarily the case that one futures contract is written on one unit of the underlying asset. There can be situations when a large number of units of the underlying asset are covered by one futures contract. For example, one futures contract on Great Britain Pounds is of 62,500 pounds.

So, this is a lot size requirement, a lot size represents the number of units of the underlying that are covered by one futures contract. That is, what is called a lot size. The value of the contract obviously is equal to the price per unit of the underlying into lot size. Expiry is the last date up to which one can hold the futures the maturity date the terminal date you may call it what you like.

Margin, to enter into a futures agreement one has to deposit a margin amount, which is calculated as a certain percentage of the contract value. Now, the necessity of this margin also I will come back to it, but for the moment, whenever you want to trade in a futures contract, whenever you are contacting your broker for taking a position in the futures contract, he will ask you to open a margin account and he will ask you to deposit a certain amount based on the contract value and the terms of the exchange at which the trading is going to take place of your order as margin to be deposited in that margin account.

Now, the settlement of the futures contract can be either physical or cash. Physical settlement is quite simple, physical settlement means that there will be actual delivery of the underlying asset by the party who is short in the futures contract to the party who is long in the futures contract. The party who is long in the futures contract will take the delivery, the party who is short in the futures contract will give the delivery of the underlying asset. This is physical settlement.

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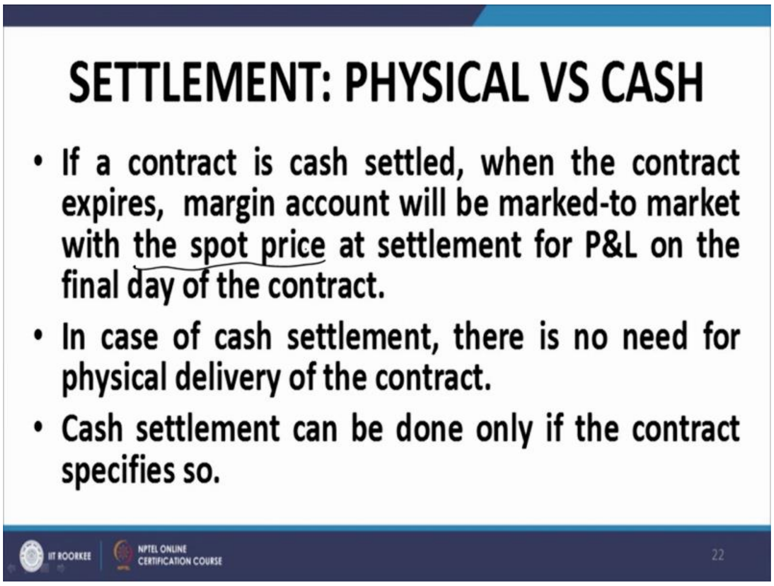

However, there are some underlying assets, particularly when we talk about commodities, which are not amenable to physical settlement, the physical settlement of such underlying assets may involve may entail significant costs and as a result of which an alternative method of settlement is sometimes adopted in exchanges in the context of certain underlying assets.

How, this is done is that on the date of maturity of the futures contract, the price of the futures contract is marked to the then prevailing spot price of the underlying asset and the differential between the previous day's futures price and the spot price on the data maturity of the futures contract is marked is transferred to the margin accounts in final settlement of the futures contract.

Let me repeat on the date of maturity of the futures contract, the contract is marked to the spot price of the underlying asset on that particular date that is on the date of maturity of the futures contract and the difference between the previous settlement price and the settlement price that has been arrived at by marking the price to this then prevailing spot price is transferred to the margin accounts.

So, that is how the cash settled, this is called cash settlement, that is how the cash settlement operates. Let me read it out for you. If a contract is cash settled, when the contract expires margin account will be marked to market with the spot price at settlement. This is the important part. This is the part that defines the settlement process, the spot price that is.

You mark the futures price to the current spot price on the date of maturity and the difference between the previous settlement price and this settlement price is transferred to the respective margin accounts of the parties to the contract to a settlement for profit and loss account on the final day of the contract. In the case of cash settlement, there is no need for physical delivery of the contract obviously, of the underlying asset, cash settlement can be done only if the contract is specified.

So, this is important, let me clarify to you that if a contract is to be cash settled if a futures is to be cash settled, it must be mentioned or it must be a part of the contract Abinitio that is when the contract is created, it must provide for cash settlement. Otherwise, you cannot have a situation where one fine morning one of the parties or both the parties even agree that the contract will be cash settled. If a contract is to be cash settled it needs to be mentioned in the issue document or the document by which the contract is created.

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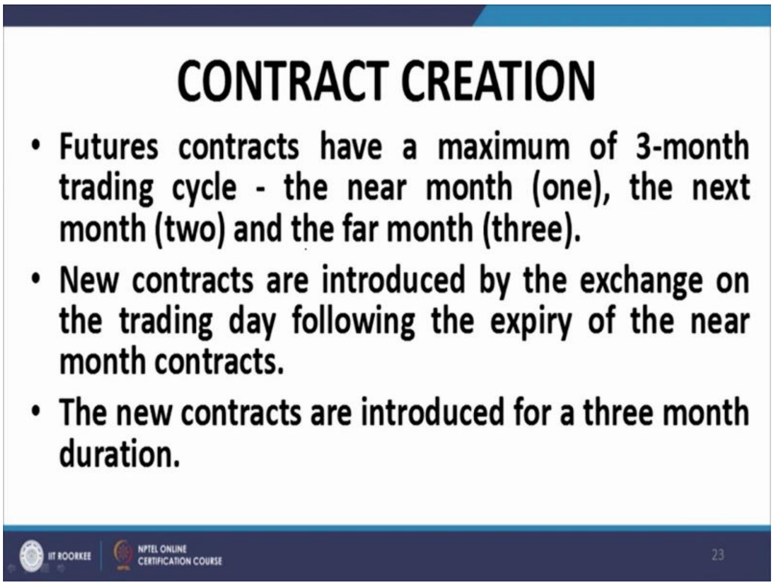

Contract creation, futures contracts have a maximum of three-month trading cycle, the near month, one month, the next month, that is the two month and the far month that is three-month settlement. So, futures contracts have a maximum of three-month trading cycle the near month, the next month and the far month. New contracts are introduced by the exchange on the trading day following the expiry of the near month contract.

So, on the very day the near month contract expires the next day fresh contracts are introduced for trading by the relevant clearing house and that is how the cycle continues. The new contracts are introduced for a three-month maturity. So, at any point in time, you will have three contracts being traded the near month contract the next month contract and the far month contract.

As soon as this near month contract reaches its maturity the next day a fresh contract will be created and released for trading. And this new contract will now become the far month contract. The next month contract will now become the near month contract and the far month contract will not become the next month contract. So, this way the trading cycle is continued. And at any point in time. There will be three contracts available for are trading in the market, one near month, one next month and one far month duration respectively.

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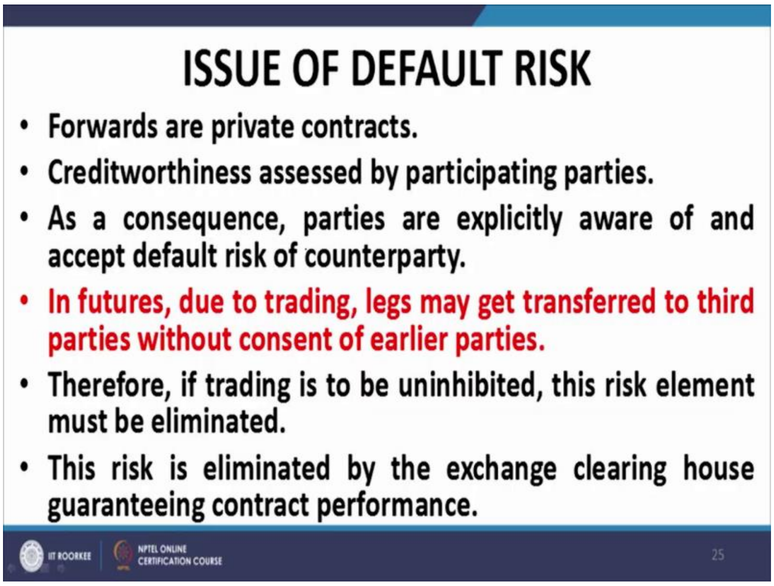

Now, there is a issue of default risk. I will come back in the next lecturer. Thank you.