Quantitative Investment Management
Professor J P Singh
Department of Management Studies
Indian Institute of Technology Roorkee
Lecture 35
Forward Pricing – Consumption Assets

Welcome back. So, in the last lecture, I introduced the concept of financial derivatives gave you a summarized version of the various types of derivatives that is futures, forwards, swaps and options. And then we moved over to a detailed exposition of forward contracts, defined forward contracts and then went on to the arbitrage free pricing that is embedded in forward contracts.

I followed that up with a description of the procedure that would involve or that would result in the siphoning out of arbitrary profits should there be deviation from the arbitrage free price of a given commodity in the forward market.

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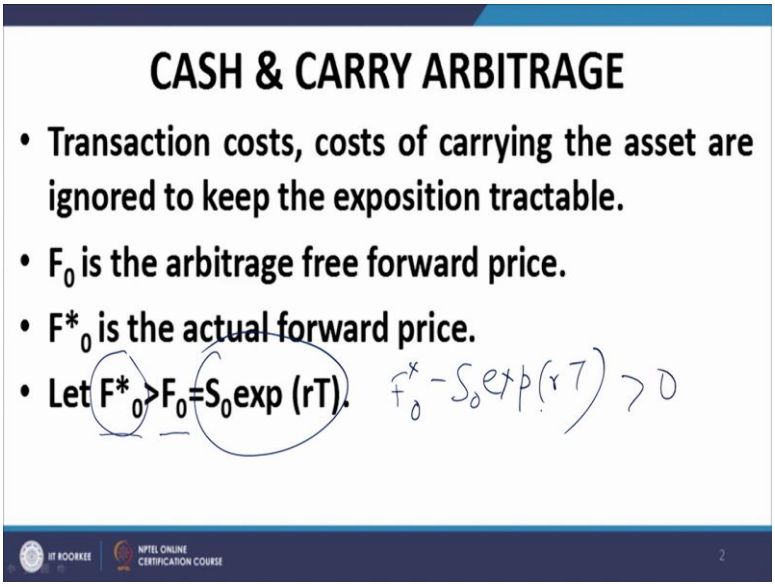

So, let us pick it up from there. Let us recap quickly the concept of cash and carry arbitrage and reverse cash and carry arbitrage, the cash and carry version of arbitrage will take place when the actual forward price exceeds the arbitrage free forward price. F star 0 is the actual forward price in the price in the foreign market whereas F0 is the arbitrage free price in the forward market.

So, we will simply take long position in F0, that implies that we will borrow an amount S0 which is the spot price of the underlying asset prevailing at t equal to 0 as of today rather, we

will use that S0 to buy one unit of the underlying asset in the spot market and hold the underlying asset with us.

Against this holding of the underlying asset will take a short forward position in other words, we will take a position in a forward contract which entails delivery of the asset and receipt of the forward price.

That is these are the set of transactions that would take place at t equal to 0 at t equal to capital T which is the maturity of the forward contract what will happen the forward contract would mature will deliver the underlying acid which you already have with us and received an amount equal to F star 0 please note this point, because this is the actual price at which the forward contract was entered into at t equal to 0 it was not entered into at F0 it was entered into at F star 0, which was the actual price prevailing in the forward market.

And against this against this receipt of F star 0 what will we do, we will repay the amount that we had borrowed together with interest thereon and that would equal to the amount S0 exponential rT assuming that r is continuously compounded rate of return.

So, that implies what that implies that F star 0 minus S0 e to the power rT is the residual cash flow and because F star 0 is greater than S0 e to the power rT, this quantity is greater than 0 and we extract a certain amount of profit out of this set of transactions.

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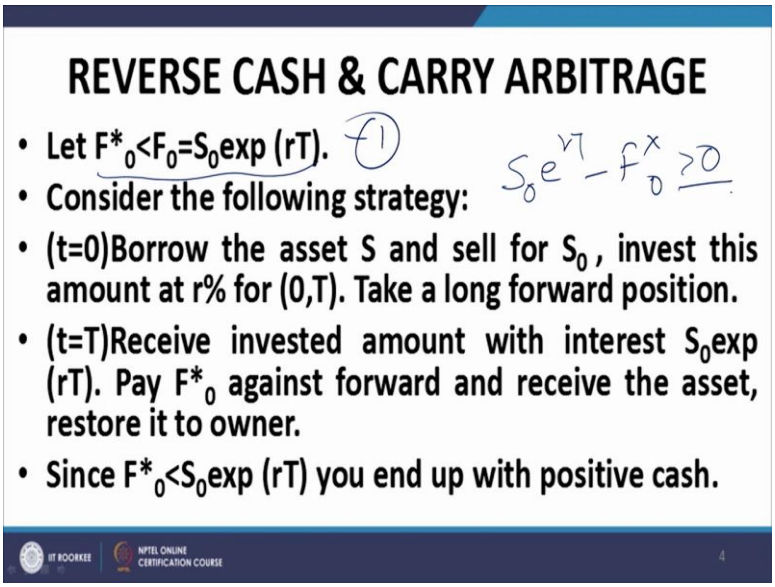

Then we talk about the reverse Cash and Carry arbitrage, the reverse Cash and Carry arbitrage will take place when the forward price in the market the actual forward price prevailing in the market is lower compared to the arbitrage free forward price that is F0.

In other words, F star 0 the actual forward price is less than F0 than arbitrage free forward price. So, in this case, what will be that set of sequence of transactions at t equal to 0 we will short the asset we will borrow the asset underlying asset from a party and we will sell the asset forthwith in the spot market in the cash market we will sell the asset and we will receive the price of spot price of the underlying asset that is S0, we will invest this amount S0 at the prevailing rate of interest for a period equal to the maturity of the forward contract in which we take a long position with the underlying as the asset that we have shorted.

So, t equal to 0 set of transactions, borrow the assets sell it in the market invest the proceeds, this is one set and second set is that you take a long position in a forward contract on the same asset that you have shorted and at maturity that is a t equal to capital T the maturity of the forward contract what will happen because you are long in the forward contract you will receive the asset against the payment of the price which is which is what which is F star 0 not F0.

You will receive the underlying asset you will deliver it to the party from whom you have borrowed the asset in compensation or in retirement of the obligation arising out of the borrowing of the asset at t equal to 0.

So, you will in addition to this, you will receive the amount that you have invested that is equal to S0 e to the power rT. So, your net cash flow will be equal to 0 e to the power rT minus F star 0 and this is greater than 0 by definition by virtue of this condition that we have condition number one this implies that this expression is greater than 0 and therefore, we siphon off a profit by by following this arbitrage prescription.

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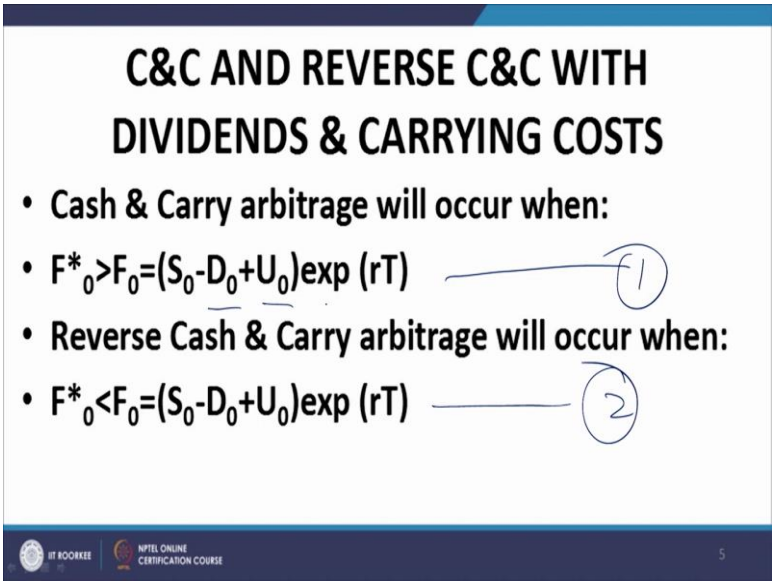

So, this is called reverse Cash and Carry arbitrage the former process when F star 0 was greater is called the cash and carry arbitrage. So, cash and carry arbitrage operates when the actual forward price is greater than the arbitrage free forward price and reverse Cash and Carry arbitrage operates when the actual forward price is lesser than the arbitrage free forward price.

So, in in the event that the carrying of the asset until the receipt of dividends or entails the payment of carrying costs, the conditions that we arrived at get modified in the cash and carry arbitrage will take place when equation one holds and reverse Cash and Carry arbitrage will operate when equation two holds.

So, the factors that is D0 and U0 are respectively what they are the present values of the dividend stream that you are going to receive while holding the asset with you while being long in the asset by possessing the asset with you during 0 to capital T where capital T is the maturity of the forward contract and U0 is the present value of carrying costs that may be involved that may be entailed by holding the asset by keeping the asset with you for the period 0 to capital T they are like go down rent like insurance premium and so on.

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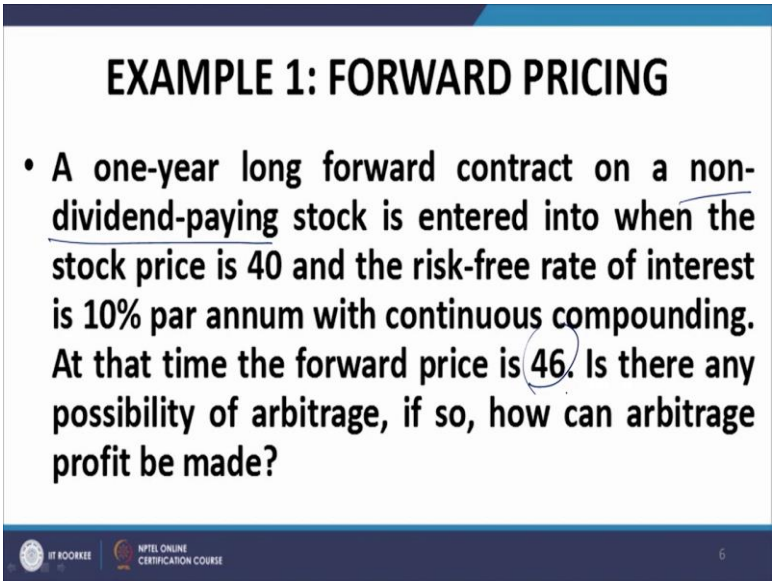

Let us do an example, a one year long forward contract on non dividend paying stocks important non dividend paying stock is entered into when the stock price is 40 and the risk free rate of interest is 10 percent per annum with continuous compounding at that time the forward price is 46.

Is there any possibility of arbitrage if so, how can arbitrage profit be made, to solve this problem what will we do we will work at the no arbitrage forward price and see whether the no arbitrage forward price agrees with the price that is actually prevailing in the forward market which is 46. And if there is a discrepancy between the arbitrage free forward price and the actual forward price, which is 46 we will infer that there is a possibility of arbitrage.

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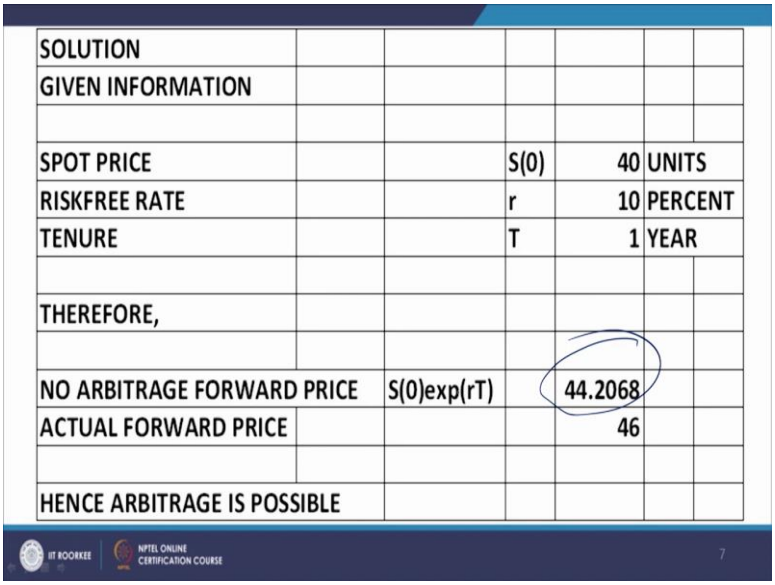

So, let us work out the arbitrage free forward price the calculations are given here quite straight forward problem this is and we find that the arbitrage free forward price turns out to be 44 point 2060. And the actual forward price is 46.

So, clearly there is a discrepancy between the arbitrage free forward price and the actual for price, the actual for price is higher, the arbitrage forward price is no or arbitrage free forward price rather is lower and what does that mean? That means that you will undertake or you can undertake Cash and Carry arbitrage and siphon off profits from the system.

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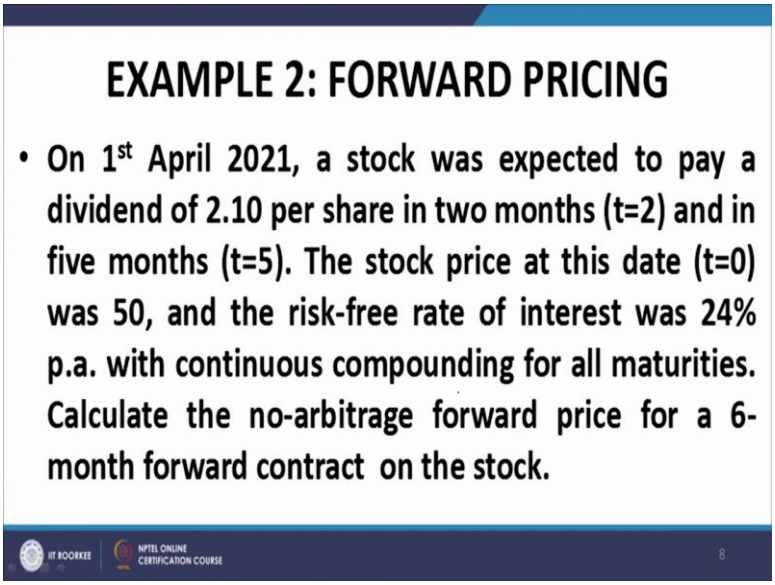

Another example, on April 1st, 2021, a stock was expected to pay a dividend of 2 point 10 per share in 2 months, t equal to 2 and in 5 months, that is t equal to 5. So, t equal to 0, a stock was expected to pay a dividend of 2 point 10 per share in 2 months as t equal to 2 months and then t equal to 5 months alike amount the same amount of 2 point 10.

So, there will be 2 installments of dividend during the life of the forward contract number one at t equal to 2 months and number 2 at t equal to 5 months and both of them will be of an amount equal to 2 point 10. The stock price at the state that is t equal to 0 that is 1st April, 2021 was 50 and the risk free rate of interest was 24 percent with continuous compounding. Calculate the no arbitrage or the arbitrator free forward price for a 6 month forward contract on the stock.

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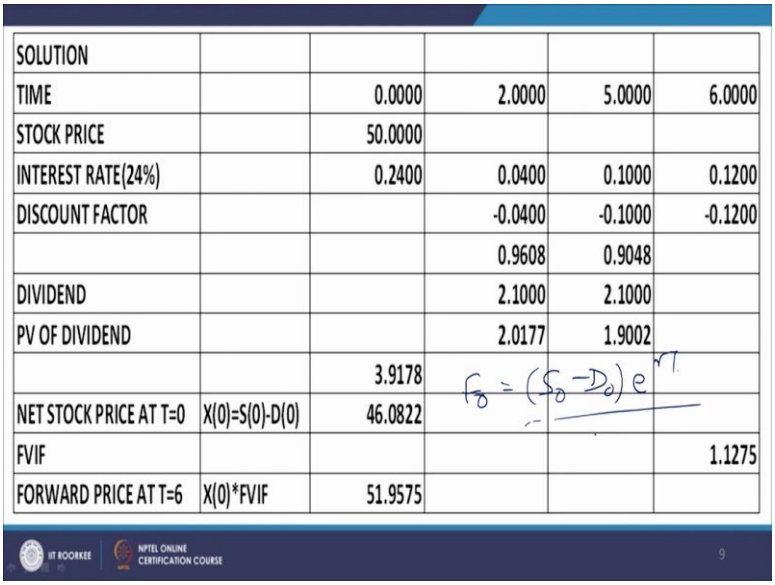

SOLUTION					
TIME		0.0000	2.0000	5.0000	6.0000
STOCK PRICE		50.0000			
INTEREST RATE(24%)		0.2400	0.0400	0.1000	0.1200
DISCOUNT FACTOR			-0.0400	-0.1000	-0.1200
			0.9608	0.9048	
DIVIDEND			2.1000	2.1000	
PV OF DIVIDEND			2.0177	1.9002	
		3.9178			
NET STOCK PRICE AT T=0	X(0)=S(0)-D(0)	46.0822			
FVIF					1.1275
FORWARD PRICE AT T=6	X(0)*FVIF	51.9575			

So, this is another simple problem the forward price works out to F0 is equal to S0 minus D0 e to the power rT. So, this is the calculations that are given in this Excel sheet are simply the calculation of this right hand side of this formula, we start with F0, and then we work out the present value of dividends, the present value of dividends turns out to be 3 point 9178. So, the net spot price or S0 minus D0 you may say is equal to 46 point 0822 and the future value of this net spot price turns out to be 51 point 9575, which is the forward price.

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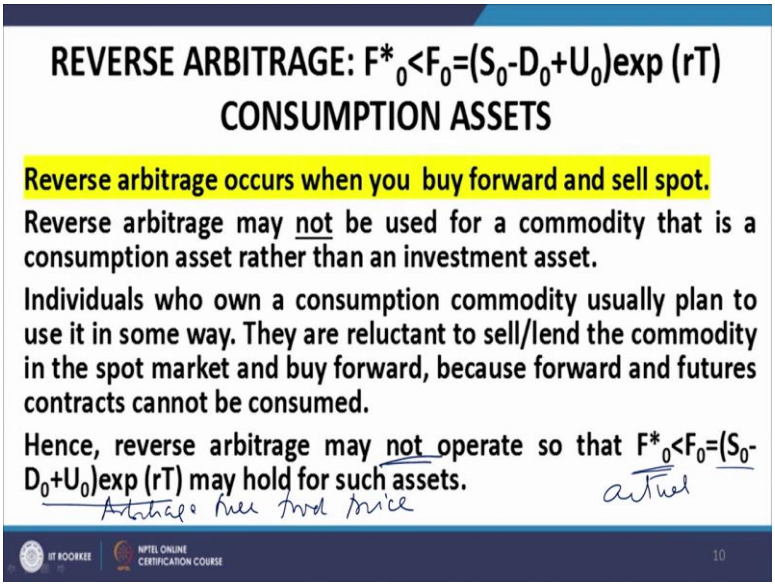

Now, we talk about another a very interesting point, you would recall that when I started talking about the pricing of a forward contracts or calculation of the forward price, based on

arbitrage free or arbitrage free condition, I partitioned I segregated assets into two types investment assets, which are held for the purpose of investment by a majority of the community and consumption assets, which are held for the purpose of personal consumption by the majority of the people in the trading market or the majority of the people involved in buying the asset.

So, why was this discrimination necessary? Why was this classification of significant segregation necessary in the context of calculation of forward price? Let us try to understand this, how did we arrive at the forward price we arrived at the forward price from arbitrage free considerations.

And we also found that in the event, that the forward price is greater and the arbitrage free price is lower, then what will happen we will sell in the forward market and we will buy in the spot market and undertake Cash and Carry arbitrage and thereby, that differential will tend to decline and become almost equal or close to equality.

Conversely, if the forward price is lower, or the price in the forward market is lower, and the spot price is higher, we will short the asset in the spot market, sell it in the spot market invest the proceeds and take a long position in the forward contract at maturity, what we will do is we will buy the asset in the under the forward contract and replenish it to the person who was lent the asset at t equal to 0 2 as for in relation to shorting of the asset, this is called reverse Cash and Carry arbitrage.

So, if the forward price is lower this if the actual forward price is lower, again, the 2 will tend to neutralize as more and more people enter into reverse Cash and Carry equity. What happens if the force of neutralizing the price differential the force of eliminating the price differential by extracting out the profits in the case of reverse arbitrage does not operate or operates with it is substantially mitigated.

What will happen in that situation, in that situation, what will happen is that the there can be situations there can be circumstances in which the forward price or the price in the forward market continues to remain lower than the price in the spot, spot market or the arbitrage free worked out price on the basis of the spot market and notwithstanding that equilibrium has been attained.

In other words, there may be situations in which equilibrium was there, but equilibrium is there, notwithstanding the fact that the price in the forward market is lower compared to the arbitrage free forward price, because the process of reverse arbitrage is substantially mitigated.

What can be the circumstances what would that would mitigate or substantially mitigate the possibility of reverse arbitrage because it is reverse arbitrage, which is going to bring the forward price upwards and this is the arbitrage free price low downwards, so that they become equal at equilibrium that that force operates in the case of investment assets.

But in the case of consumption assets, what is the reason that we can have a situation where the forward price continues to remain lower sustained at a lower level relative to the arbitrage free forward price, because the reverse arbitrage process is not substantially activated, let us say, the reason is very fundamental consumptions are, consumption assets are held by the parties, and by the people who are taking long positions in the asset for purpose of consumption.

And because these assets are held for the purpose of consumption, the temptation is not to part with them, and not to allow other people to use this as a not to allow the other people to borrow these assets and short these assets or sell them in the market. The temptation is to retain that so, as yourself, the retaining of the assets with yourself gives you a sense of convenient sense of comfort in because you are going to use or you plan to use those assets in the very near future.

So, if for example, if you are holding a deposit of holding a inventory of coal and ball you lend the asset to somebody, tomorrow, if the supply of coal coal gets curtailed, then you could have a difficult situation or your operations could be adversely affected.

So, to protect yourself against that you are more happy or more comfortable retaining the amount of coal notwithstanding the fact that there are there are parties who are willing to buy that coal and for shorting their position.

So, that is the important part in the case of consumption as it the temptation the desire the motivation to retain the assets, commands a premium, and that premium manifests itself as a differential such that the arbitrage free forward price could be higher than the actual price in the forward market.

Let me quickly read the read through this now. Reverse arbitrage occurs when you buy forward and sell spot reverse arbitrage may not be used for a commodity that is a consumption asset, rather than an investment asset, because of the temptation to retain the asset, you do not want to part with it, as it because you see, the shorting process entails that you part with the asset, you lend the asset to somebody and that somebody then sells the asset in the market and it takes a short position.

So, that process of lending the asset is beset with inconvenience beset with a temptation not to do so, because if you have the asset with you, you feel more comfortable, you feel that your operations are insulated or protected against the scarcity of the asset in the in the near future.

So, reverse arbitrage may not be used for a commodity that is a consumption asset rather than an investment asset. Individuals who own a consumption commodity usually plan to use it in somewhere this is fundamental, they are reluctant to sell or oblig lend the commodity in the spot market and buy forward because they want to keep it with them this gives them a feeling of security because forward and futures contracts cannot be consumed.

Hence, reverse arbitrage may not operate may not operate and as a result of is because reverse arbitrage is not operating efficiently, we find that there can be circumstances where the actual forward price is lesser than what than the arbitrage free forward price arbitrage free forward price.

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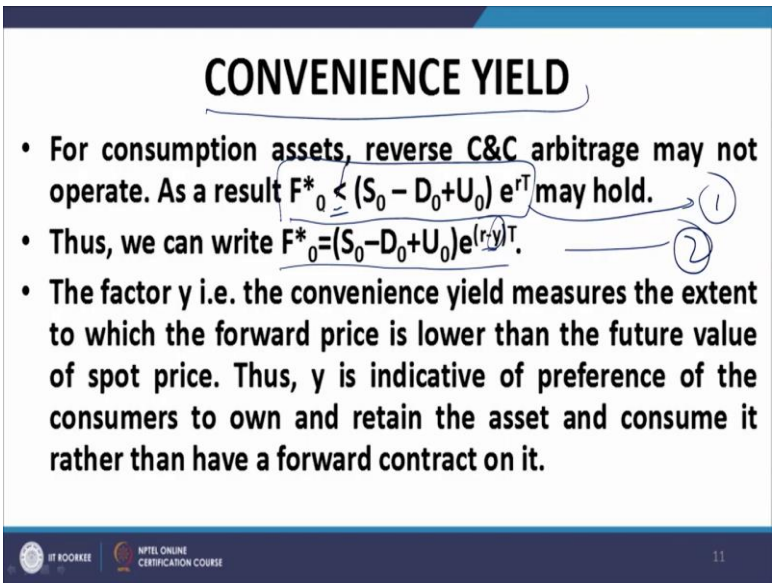

So, this condition can hold in the case of assets which are consumption assets allied to this concept allied to the situation environment where we can have a situation where the actual forward price is lesser than the arbitrage free forward price, we define a quantity called the convenience yield which is a measure of what is the level of differential that could prevail between the actual forward price and the arbitrage free forward price.

How do we define this level of convenience? How do we define a measure of this level of (conveni) convenience measure of this motivation temptation to retain the assets with yourself rather than allowing them to be borrowed by somebody and then buying them in the forward market that is given by this equation.

We introduce a parameter y how is this parameter y introduced, let us try to understand. In the case of the consumption assets as I mentioned, we can have a situation where this holds. Let us call it equation number 1 where equation number 1 with that is where the price in the forward market is lesser actual price in the forward market is lesser and the arbitrage free price is higher this allows us to define a quantity y by this equation that is equation number 2.

This is, because the right left hand side is lesser than the right hand side we can incorporate a quantity on the right hand side such that that by introducing introduction of that factor in the in the right hand side this quantity y on the right hand side we are able to introduce or replace this less than quantity by an equality.

In order to replace this less than quantity by equality what we do is we introduce a factor y and this factor y is called the convenience yield and what is the what is the significance what is the interpretation of this convenience yield, it is clear that higher this factor y higher is the differential between the actual forward price and the arbitrage free forward price that means, what that means higher is the temptation higher is the motivation to hold the asset with you rather than parting the asset and then buying it in the forward market.

So, the factor y that is the convenience yield measure the extent to which the forward price is lower than the future value of this spot price that is the arbitrage free price. Thus y is indicative of the preference of the consumers to own and retain the asset higher the value of y, higher is the motivation higher is the preference of the consumers to own and retain the asset and consume it rather than allowing it to be borrowed and then buying it in the forward market.

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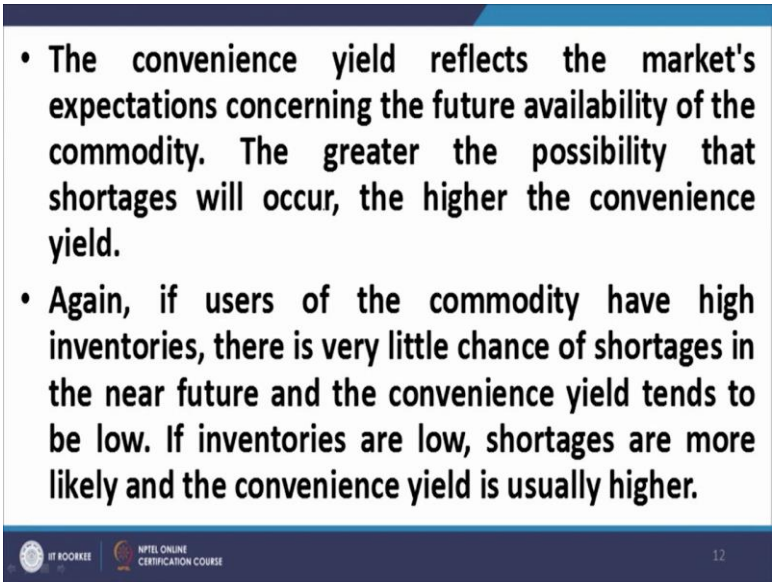

The convenience yield reflects the market's expectations concerning the future availability of the commodity the greater the possibility that shortages will occur, the higher is the convenience yields, that is quite obvious that a greater is a possibility of shortage more is the temptation to retain the asset with you and higher would be the convenience yield.

Again if users of the commodity have high inventories, there is very little chance of shortages in the market in the near future and in the convenience yield tends to be low. If inventories are low shortages are more likely and the convenience yield will be higher.

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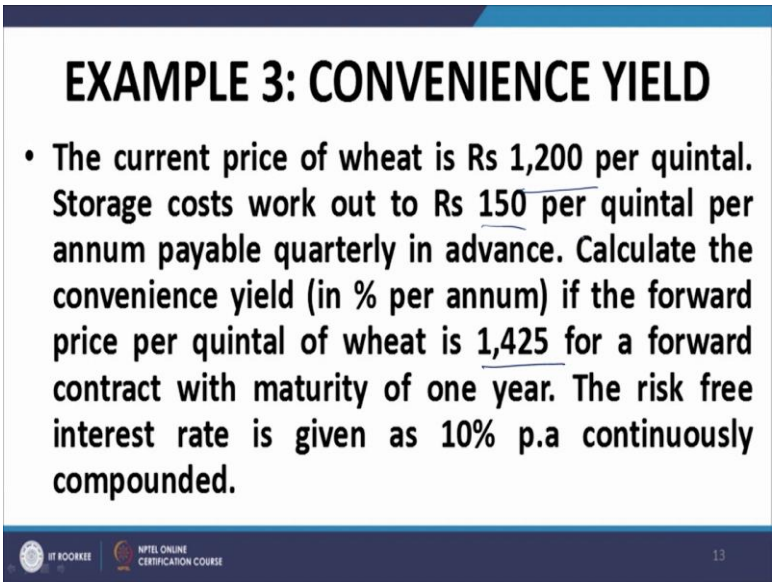

This is an example on convenience yield as to this the current price of wheat is at rupees 1200 per quintal storage costs work out rupees 150 per quintal per annum per quintal per annum, payable quarterly in advance calculate the convenience yield in percentages per annum, if the forward price per quintal of wheat is 1,425, the spot price is 1200 carrying costs are 150 for the year in 4 installments so equal installments and payable quarterly in advance and the forward actual forward price is 1425 for a contract of maturity 1 year and the continuously compounded interest rate is 10 percent per annum.

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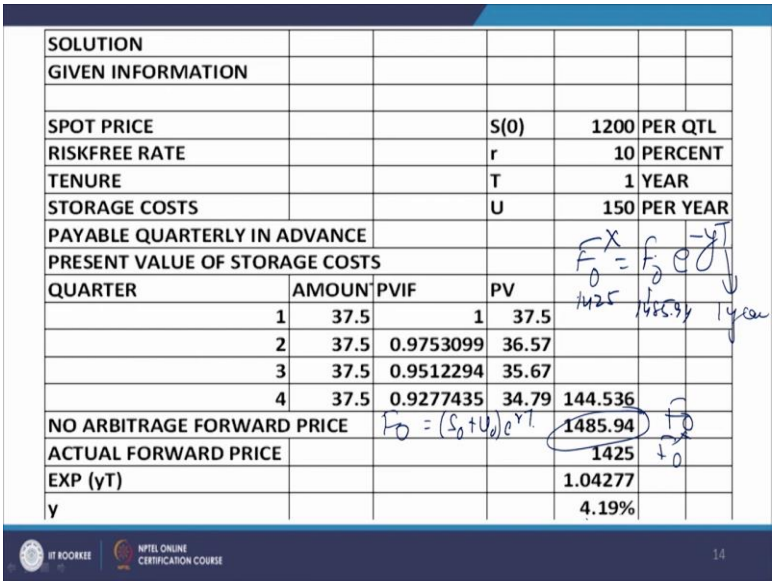

So, first of all, we work out the arbitrage free forward price the arbitrage free forward price is worked out in this Excel sheet and it is the arbitrage free forward price works out to 1485 point 94 this is what is F0 and this is F star 0.

So, using these two expressions, we can, what do what is the relationship how do we define convenience yield we define convenience yield as F star 0 is equal to F0 e to the power minus y T, F0 e to the power minus yT their f star 0 is 1,425 F0 is equal to 1485 point 94, 1485 point 94 and y is the unknown quantity and t is equal to 1 year, this is equal to 1 year.

So, knowing all these quantities, we can find out that F y is equal to y is equal to 4 point 19 percent. So, and as far as this 1485 is concerned, we find, we work it out as F0 this is our F0 in fact 1485 as I have shown here, so, F0 is equal to F0 plus U0 e to the power rT there is no D0 here no equation of dividends, but there are carrying costs.

Now, we work out the present value of carrying costs the spot value is given the spot price is given as 200. So, knowing all the quantities we can calculate F0, we use this F0 over here and we work out the value of y which is 4 point 19 percent.

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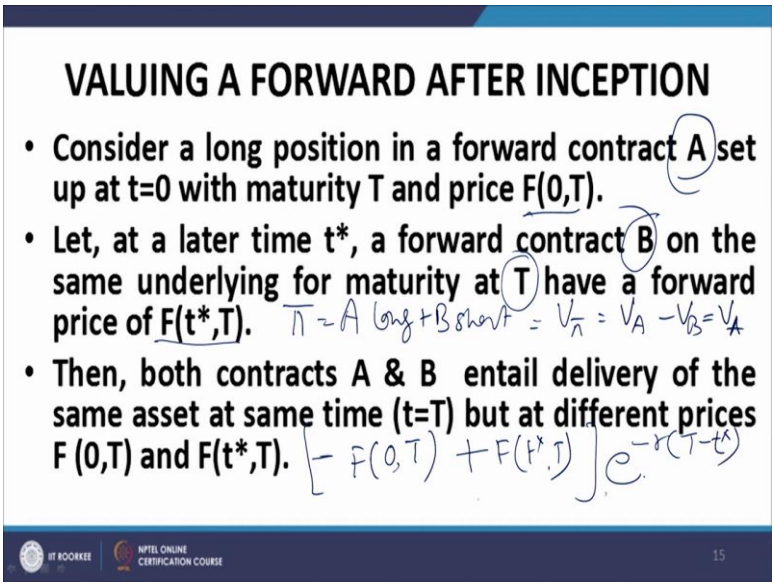

Now, we come to a very interesting point which I briefly alluded to at the when I started talking about forward contracts, why forward contracts or derivatives? Let us try to understand the quantitative aspect of whatever explained at that point in time, we consider a long position in a forward contract A set up at t equal to 0 let us let us assume the t equal to 0 is 1st January of a particular year and the maturity is on 31st December that is capital T and the price at which we enter into the forward contract at t equal to 0 F0 tF.

So, this is the this is the name of the contract this is the contract A, the contract is set up at t equal to 0 let us assume it is 1st January maturity is 31st December of that year, which is capital T and the price is F0 comma t later at time t star let us say it is 1st April forward contract B on the same underlying with maturity at t that means it is having the same maturity is not same term to maturity is having the same maturity date as A please note this point, it is not a maturity of 1 year not maturity from 1st January to 31st it is having its maturity at 31st December that is capital T.

So, please note this point please note that the length of the forward contract is not the same, it is the point point of maturity that is the same and then obviously, the underlying is the same and the price that prevails at t equal to t star is given by this quantity Ft star comma capital T.

So, let me repeat we have a contract a which is initiated at t equal to 0 and which has the maturity of capital T let us say t equal to 0 is 1st January t equal to capital T is 31st December of the same year and the price at which the contract is entered into is equal to F0 comma capital T then we have another contract B which is initiated at t equal to t star let us say it is 1st of April of that year and it has the same maturity date as the contract A that is it has the maturity of 31st December of the same year.

In other words, the length of the contract is 9 months it is not 1 year the length is different from the contract A, the maturity date is the same the underlying is same and the price at which the contract is entered into is F t star comma t.

Now, what will happen let us try to understand let us say we construct a portfolio which is long in A when which is short in B we construct a portfolio at t equal to t t star at the point at which we have to value contract A our objective what is our objective, our objective is to ascribe a value to the contract A so, what do we do we construct a portfolio comprising of the contract A long and contract B short.

So, we construct a portfolio let us say pi consisting of A long plus B short. So, what will be its value its value will be V of pi is equal to V of A minus V of B because why this minus sign because B is short A is long B is short, but please note the value of a contract on the date of its inception is 0.

So, VB is equal to 0. So, this is equal to VA. So, this part is quite simple. Let us look at the payoff from this portfolio at t equal to capital T that is 31st of December what will happen the short position will under the forward contracts will deliver the asset and the long position will receive the asset so the asset quantity is cancel out.

Suppose the underlying is 1 dollar than the short position that is the contract B or the party having the short position under contract B will deliver 1 dollar to the party who is long in a along in the dollar and that means A will receive the dollar and B will deliver the dollar. So, the dollar content cancels out.

Now, whatever the price the price at which A will receive the dollar is equal to F0 comma t and the price at which B will deliver the dollar is equal to Ft star comma t because there is long so, A will pay the price and B will receive the price. So, the net cash flow from the portfolio will be plus of this and minus of this.

And this is the net cash flow at the maturity of the contract and the value of this contract at t equal to t star will be equal to its present value which will be equal to when you put it here e to the power minus r into capital T minus t star. So, this is the value of the contract at t equal to t star.

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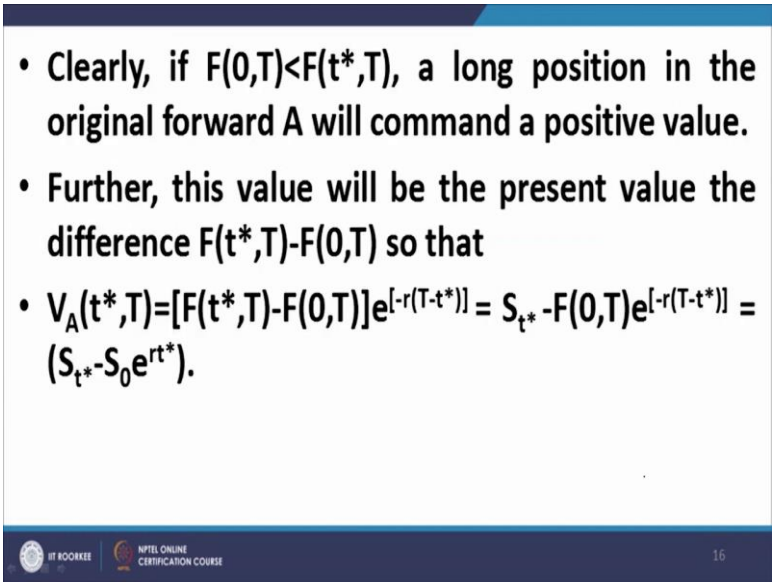

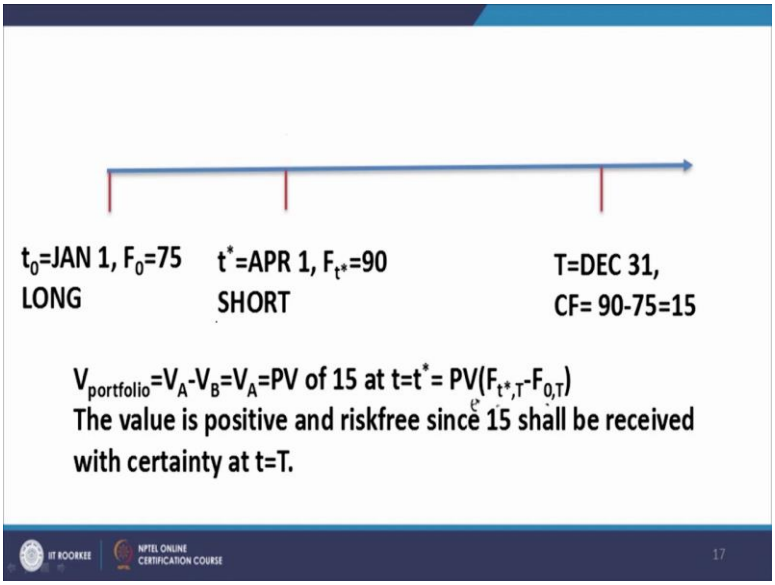

Let us look at an example quickly. So, here is the example. We will continue in the next lecturer.