Quantitative Investment Management
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Lecture 32
Floaters, Caps and Floors

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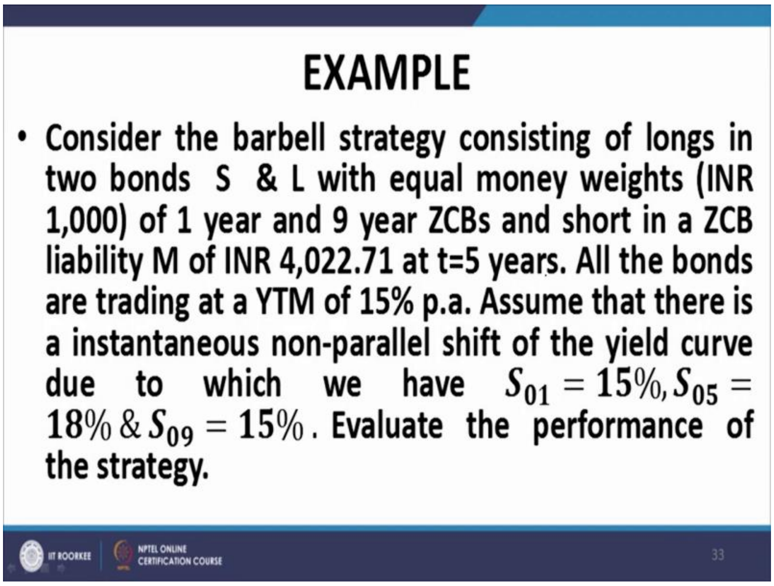

So let us continue with the example that will illustrate the use of the butterfly, we consider the barbell strategy consisting of long positions in two bonds S and L with equal money weights INR 1000 of one year and 9 year zero coupon bonds, so this is the constitution of the barbell and we have a short body, the short body is a zero coupon bond that is a liability M of INR 4022.71 at t equal to 5 years.

All the bonds are trading at a YTM of 15 per annum so if you work out the value of this liability at t equal to 0 that is today it turns out to be exactly 2000 that is equal to the investment that is required for the long barbell, in other words by shorting this particular bond of value 2000 at t equal to 0 the proceeds that we are getting are deployed by investing in the barbell strategy.

All the bonds are trading at a YTM of 15 percent per annum assume that there is an instantaneous non-parallel shift of the yield curve due to which this spot rates change as follows S01 is equal to 15 percent S05 is equal to 18 percent S09 is equal to 15 percent, so the t equal to 0 spot rates, the initials spot rates were 15 percent for all maturities that is indicated by this YTM of 15 percent for all the bonds and then and there is a shift in the yield curve as a consequence of which while the short and long end rates remain unchanged. The middle rates increases by from 15 percent to 18 percent, the intermediate rate increases from 15 percent to 18 percent.

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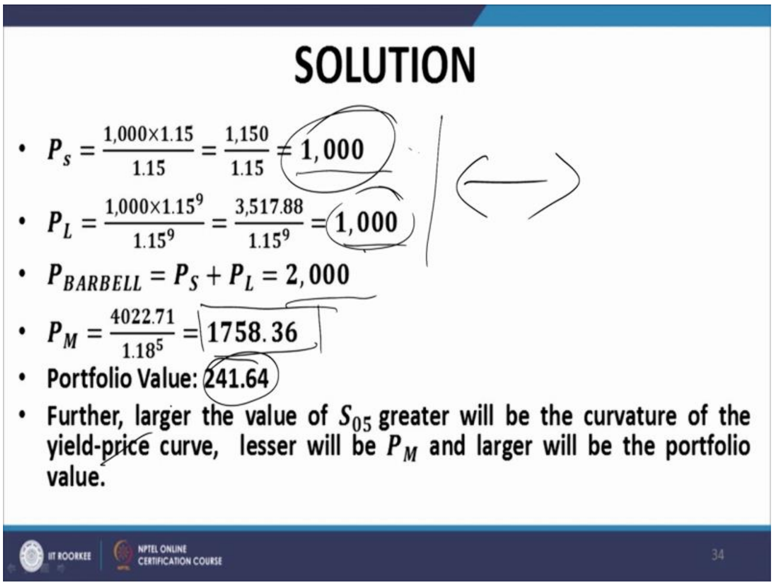

So let us evaluate the strategy, for the S bond that is the short maturity bond, the value of the bond terms is 1000 at t equal to 0 and for the long majority bond also it is 1000 we know that and therefore the value of the barbell before the shift is equal to 2000 and the value of the mid maturity bond is equal to 1758.36 after this shift, before the shift as I mentioned it was 2000 after the shift it has changed to 1758.36 please note this very important point that there is no change in the 1 year and the 9 year rates and consequently this values of 1000 before the shift and 1000 before the shift for the short maturity bond and the long maturity bond comprising the barbell remain the same at the after this shift.

So after the shift PS is continues to be 1000 after to the shift PL continues to be 1000 and P barbell is equal to 2000 after the shift as well however PM which was 2000 before the shift has declined to 1758.36 after the shift then thereby leading to a profit or a portfolio appreciation in value up 241.64, further larger the value of S05 greater will be the curvature of the yield curve, spot yield curve rather lesser will be PM and larger will be the portfolio value.

So if the 18 percent figure that we have assumed for S05 if we assume it to be 24 percent or so this figure will increase further so that is how the strategy of a butterfly operates which has a long position in the barbell and a short position in the body or the bullet.

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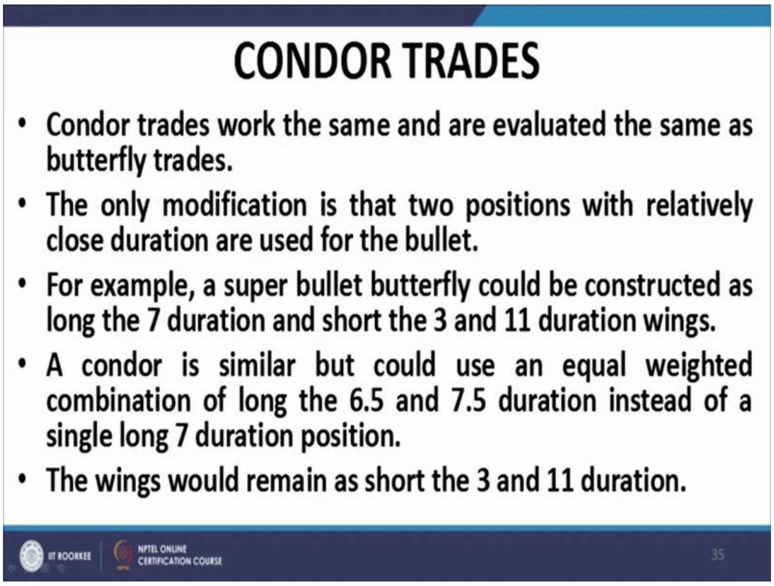

Now condor trades, a condor trades are the variant of the butterfly strategy condor trades work the same and are evaluated the same as butterfly trades the only modification is that two positions with relatively close duration are used for the bullet, for example a super bullet butterfly could be constructed as long the 7 duration and short the 3 and 11 duration wings, a condor is similar but could use an equal weighted combination of long 6.5 and 7.5 duration instead of a single long 7 duration portfolio, the wings would remain as short the 3 and 11 duration.

So it is a minor variation of the butterfly strategy, the butterfly strategy usually has a bullet in the middle, the body is a bullet strategy either long or short but in the case of condor trades the body comprises usually of two, two bonds identical positions either long or short and of very closely matched durations although not exactly matched durations.

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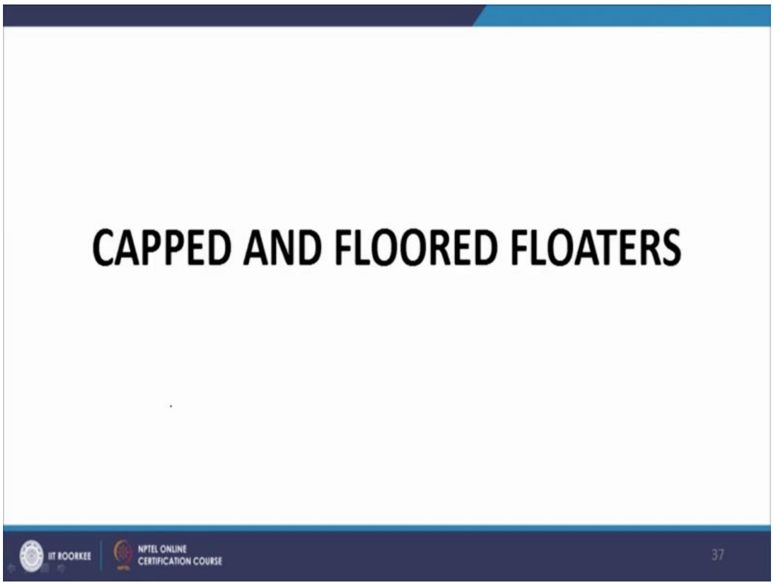

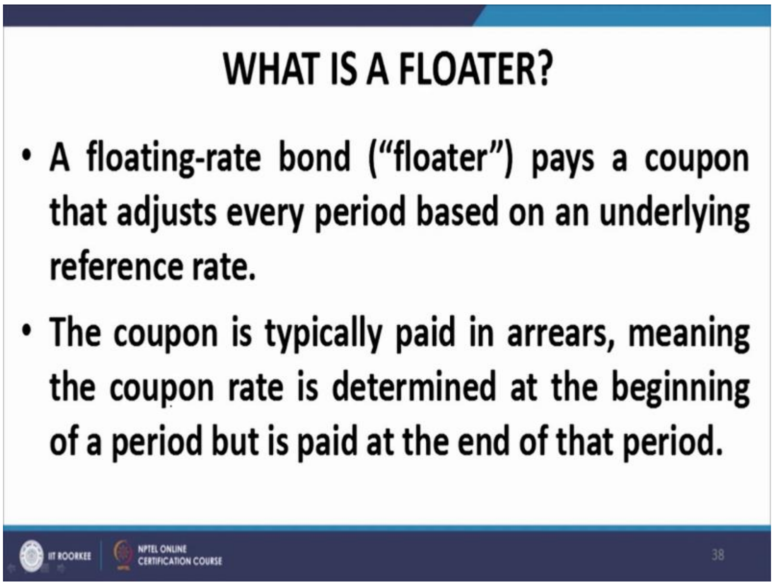

Miscellaneous depth instruments which we will be talking about capped and floored floaters in this section. A floating rate bond, a floater ps a coupon that adjusts every period based on an underlying reference rate so, so far you see what we have been talking about the bonds or the depth instruments that we have been talking about are fixed rate instruments in which the rate of interest is incorporated in the offer document, in the issue document and the manner of payment of the interest is also mentioned in the issue document, it is fixed and the timing is fixed, the frequency is fixed, the amount is also fixed or the percentage is also fixed and specified in the offer document.

However we also have floating rate instruments in which the interest rate is not fixed however the method of computation of the interest, the interest rate is usually tagged to another market variable and the method of computation of the interest given the value or the given the progress or given the evolution of the tagged variable is given in the issue document.

How? What is the relationship? How would we calculate the rate of interest given the value of the tagged variable is given in the offer document, usually floating rate bonds are issued with a reference rate which is usually the London interbank offer or the LIBOR rate. So depending on the evolution of the LIBOR rate for a particular period is fixed the usually the practices that the first rate would be fixed at t equal to 0 it would operate for the first period of 6 months, the payment would be made on the basis of the rate fixed at t equal to 0 at t equal to 6 months then at t equal to 6 months the rate for the period from t equal to 6 months to t equal to 12 months will be fixed, it will be fixed at t equal to 6 months the payment will be made at t equal to 12 months and so on this will be the progression.

The rates are fixed in advance and the payment of interest is made in arrears that is the standard practice in so called as floating rates instruments are concerned but the important thing is why it is called floating is because the rate is fixed at the commencement of each interest period, if it is a 6 month period at the commencement of each 6 month period, if it is a annual frequency bond it will be fixed at the beginning of each year for the that particular year but the actual payment of interest will be made at the end of the year. The coupon is typically paid in arrears meaning that the coupon rate is determined at the beginning of a period what is paired at the end of that period.

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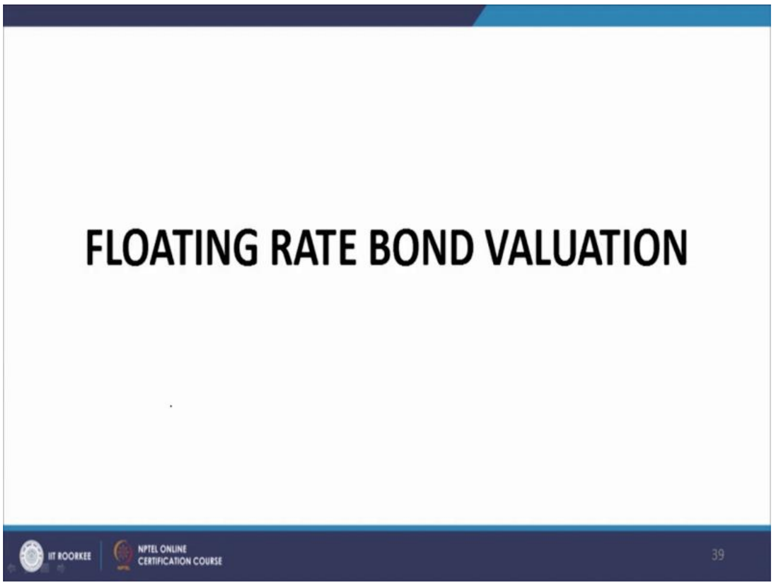

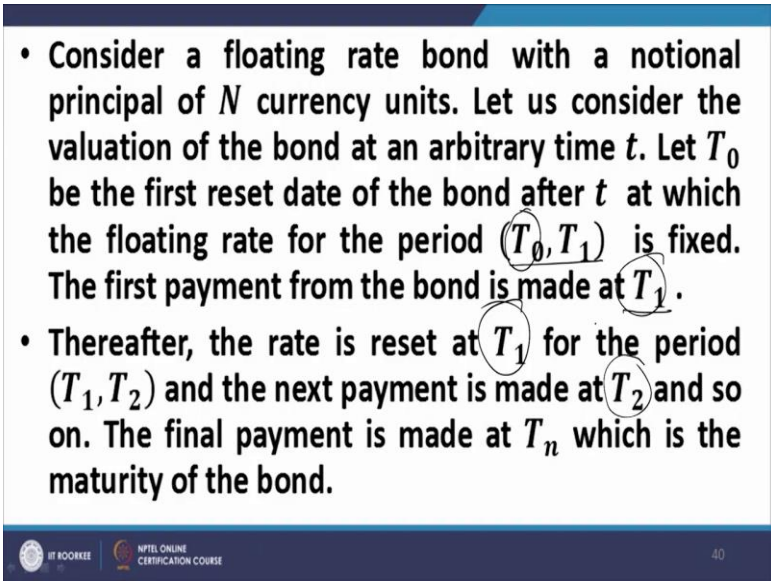

Floating rate bond valuation, now this is interesting, consider a floating rate bond with a notional principle of n currency units, let us consider the valuation of bond at an arbitrary time small t, let T0 be the first reset date of the bond after T at which the floating rate for the period T0 to T1 so please note at T0 we are fixing the interest rate that would operate for the period from T0 to T1 at T1 the actual interest would be paid and then next the interest rate for the next period from T1 to T2 will be fixed at T1 and this process will continue until the life of the bond.

The floating rate for the period T0 T1 is fixed the first payment from the bond is made at T1 so the rate is fixed in advance at T0 the rate is paid the interest is paid in arrears at T1 and this process continues for the next period and every subsequent period up to the life of the bond.

Thereafter the rate is reset at T1 for the period T1 to T2 and the next payment is made at T2 again fixed in advance and paid in arrears but please note the rate is not fixed over the entire life of the instrument this is the difference, the rate is fixed from period to period for each interest rate P, interest period the rate is fixed again and again and again.

However, for the fixed rate bond the rate is fixed once and for all over the entire life of the bond and the next payment is made at T2 and so on the final payment is made at Tn which is the maturity of the bond.

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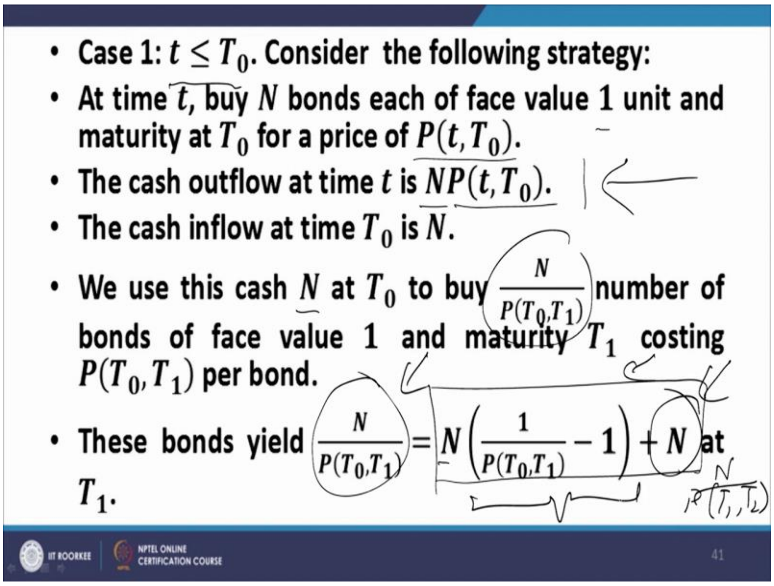

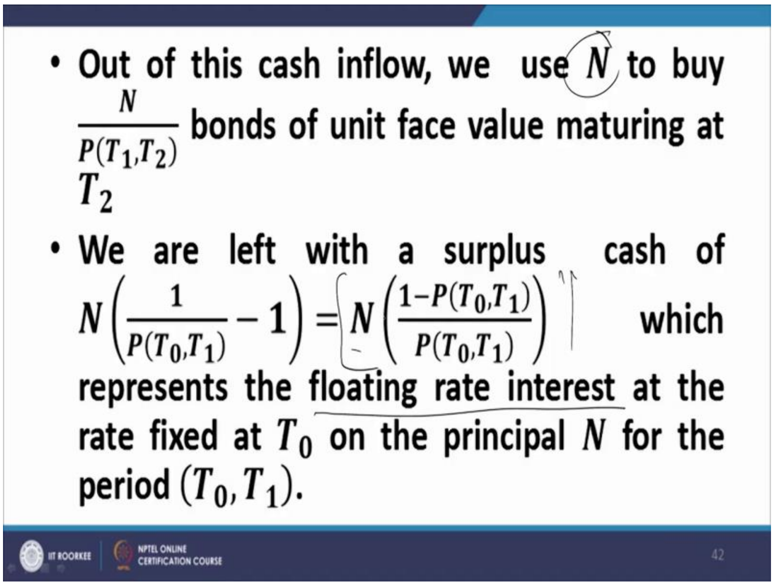

Let us assume the, let us look at the first case the valuation of the bond is at a point which is either earlier than or at the point of the first reset, I repeat the date of valuation of the bond is either earlier than either prior to or on the date of the first reset that is what is meaning of this expression, so consider the following strategy at t equal to T that is the point at which we are doing the valuation by by capital N units where what is N, N is the notional principle of the of the bond by N bonds each of face value 1 unit and maturity at T0 by N bonds each of face value one currency unit which is also its maturity value.

Please note this we are buying N units each of face value one currency units which also happens to be its maturity value that is it is redeemed at face value that is to say and maturity at T0 for a price of P small t comma T0 okay so the cash outflow at time T is N into number of bonds into the price per bond that is N into P t comma T0 the cash in flow at time T0 is what, it is equal to N because each bond is going to give you a cash in flow of one unit, the face value is one unit the redemption is at face value so the redemption value is also one unit and that means for N bonds you are going to get a payment of N.

We can use this cash N which we get from redeeming of this investment, this investment when we redeem this investment at t equal to T0 we get N and we can use the same for buying another set of bonds, how many bonds, N divided by P T0 comma T1, what is P T0 comma T1, P T0 comma T1 is the price of one bond of face value one unit at t equal to T0 which will mature for payment at t equal to T1.

Let me repeat what is P T0 T1, P T0 T1 is the price of one bond of face value one unit that at the point in time t equal to T0 when that bond is redeemable at t equal to T1 at its face value of one unit that is P T0 T1 and N is the amount of money that we have, price of one bond is this much so I can buy these many units of and the bond which will at t equal to T0 which will mature at t equal to T1.

So at t equal to T1 what is the cash flow, at t equal to T1 the cash flow will be equal to N, this much divided by P T0 T1 because this is the number of bonds and each bond has a face value of one so at maturity it will give you one unit one of currency and because you have this number upon the total currency that you are going to get is equal to this much and this can be written in this form. There is an objective of writing it in this form which will be clear in the next slide.

Now out of this cash flow, which cash flow? This cash flow this which can be written in this form N upon P T1 comma T2 bonds of unit phase value maturing at T2 this amount, this N that is there you see let me go back, this N, this N that you have is deployed for a fresh investment, fresh investment of what? N divided by P T1 comma T2 number of bonds, what is P T1 comma T2? P T1 comma T2 is the price of a one bond of face value one unit redeemable at one unit at t equal to T1 when the redemption is going to occur at t equal to T2.

The value of that bond, the price of that bond is P T1 comma T2 and you use this N amount for buying fresh bonds for rolling over the investment in a sense and the the remaining quantity, which is the remaining quantity? This is the remaining quantity and what is this remaining quantity let us look at it carefully, this remaining quantity can be written in this form N into 1 minus P T0 T1 divided by P T0 T1 what is this, one is the redemption value of the instrument P T0 T1 is the purchase price so redemption value minus purchase price divided by purchase price so this is the return on one bond and if you have N units this is the total return, this is the total return for the period T0 to T1.

So this represents the floating rate interest at the rate fixed at t equal to T0 for on the principle N for the period T0 to T1 so this amount as you can see here the redemption value minus initial value divided by initial value, this is the percentage operated on the total investment that is N then you get the total interest which represents the floating rate interest, the interest that was calculated at t equal to T0 for the period T0 to T1 on the principle of N in for the period T0 to T1.

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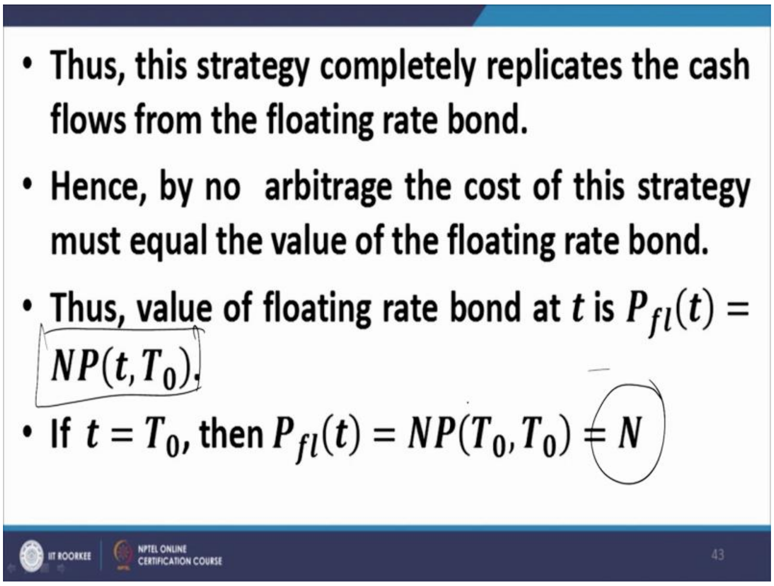

Thus the strategy completely replicates the cash flows from the floating rate bond, as you can see here is that if you are investing in a floating rate bond you would get exactly the same amount of money hence by no arbitrage the cost of the strategy must equal the value of the floating rate bond and what is the cost of the strategy it is equal to N P t comma T0, so this is the value of the floating rate bond at t equal to small t when when the first reset rate is at t equal to T0 so this is the value of the floating rate bond at t equal to small t when the T, small t is less than or equal to T0 that is it is either before or at the reset rate.

And if t is equal to T0 that is the value of the bond at t equal equal to T0 at the first reset date then this turns out to be equal to N which is the face value of the floating rate bond. So very important inference that we have is at the first reset rate the value of the floating rate bond is equal to its face value.

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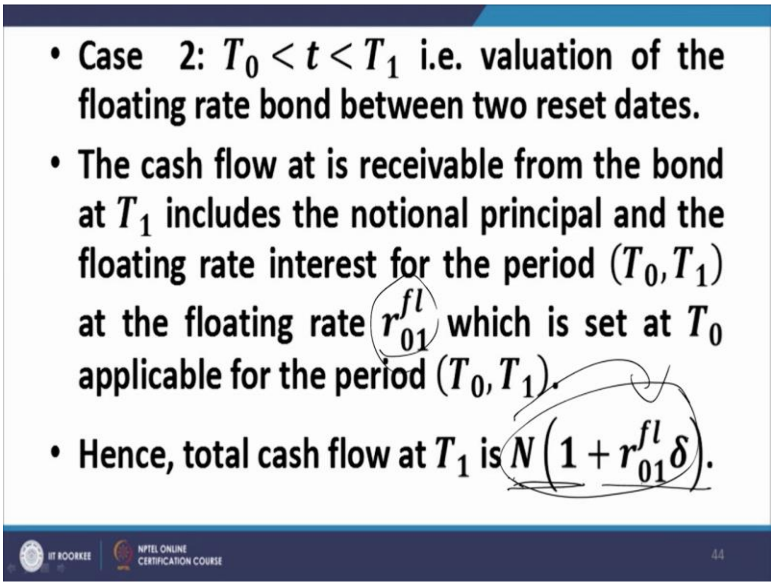

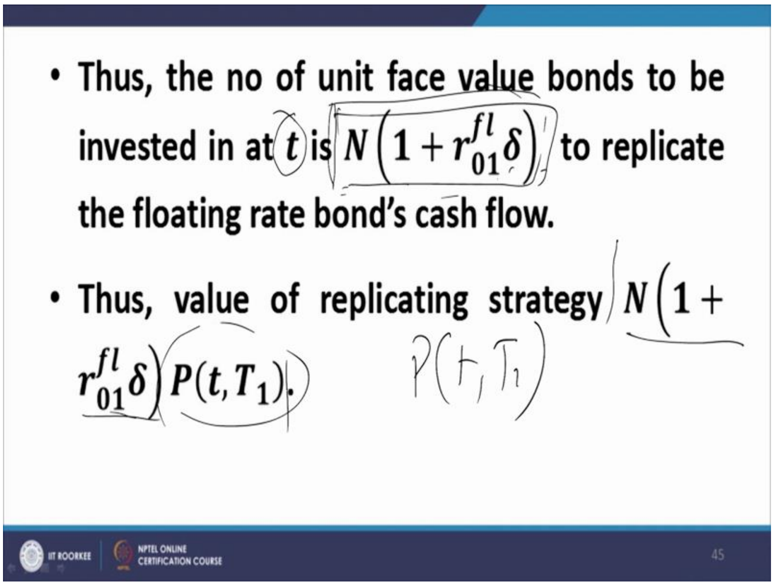

Then we look at the second case T0 less than t less than T1 that is valuation of the floating rate bond between two reset dates, the cash flow receivable from the bond at T1 includes notional principle and the floating rate interest for the period T0 to T1 at the floating rate r01 which is set at t equal to T0 applicable for the period t is to T0 as the total cash flow at T1 is equal to, this is this portion is the in principle and this is the interest N multiplied by this gives you the interest and N is the principal amount.

Thus the number of unit value bonds to be invested at in at t is equal to N into this much, the total cash flow that you are receiving is invested in unit phase value bond, this remember this is the cash flow that you receive at t equal to T1, t equal to T1 and the number of bonds that you need to invest is this amount, to replicate the floating bonds cash flow and the investment is going to occur at small t where what is small t, small t is some point between T0 and T1 and this delta is the day count fraction.

Let me quickly go through it again the cash flow that is receivable at the bond, from the bond at T1 includes the notional principle and the floating rate interest for the period T0 to T1 and that amount in, this is the rate, this rate is fixed at what, this rate is fixed at T0 and this rate applies from T0 to T1 so the total cash flow that you have here is at T1 is equal to N into 1 plus r floating 0 to 1 into delta where delta is the day count fraction.

Therefore the number of units of face value bonds that you need to be in that you need to invest it small t so that you get this cash flow so you are, you see you are replicating the cash flow from the floating rate bond at t equal to T1 and the floating rate bond is giving you a cash flow equal to this amount so the number of unit value, unit maturity value, unit redemption value bonds that you need to buy in order that the yield cash flow of this amount at t equal to, t equal to capital T1 is equal to this amount.

So when the price of this, the cost of investing in this number of bonds at t equal to small t is given by this quantity why, because one bond would cost you P t comma T1 which one, face value one unit, redemption value one unit, price at t equal to small t and the redemption at t equal to T1 so this would be the also the value of the floating rate bond at small t where small t lies between T0 and T1.

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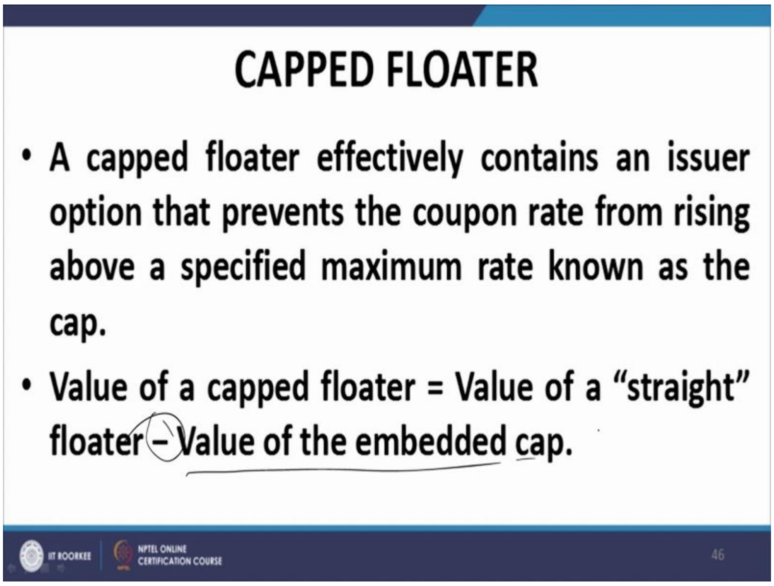

Now we talk about a capped floater, a capped floater effectively contains an issuer option that prevents the coupon rate from rising above a specified maximum rate known as the cap, value of a cap floater is equal to value of the straight floater minus value of the embedded cap, why minus, because the issuer has retained an option or issuer has this prerogative, issuer has the privilege that the interest rate on that floater will not surpass, will not go beyond, the rate of interest will not go beyond a certain rate, will not increase beyond a certain rate it is capped at a certain rate therefore it operates to the benefit of the issuer.

And because it operates to the benefit of the issuer, the issuer pays the price for that and that therefore this is the value of the embedded cap is deducted when we value a capped floater.

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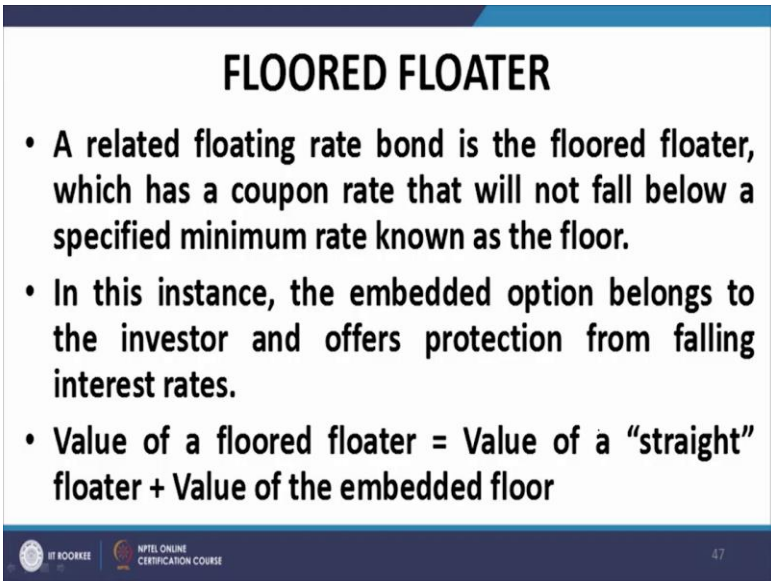

And similarly in the case of a floored floater the value of the floored floater is added to the value of the statement, why, because in this case the investor has the advantage what is the floored floater, floored floater has a downside cut, a downside truncation that the interest rate if the floating rate interest rate falls below the floor the then the investor would still get the floor.

Let me repeat, if the interest rate falls below the floor the investor would still get the floor rate so it operates to the benefit of the investor and therefore the investor pays the price for this and as a result of which when you value a floored floater it is equal to the value of the straight bond without the floating of, without the floor option plus the embedded value of the floater.

Let me repeat, when we talk about the floating rate instrument which is floored we are talking about a bond which has the floating rate plus which also has a truncation that if the floored floating rate falls below the floor then the rate would be, it would be fixed at the floor and not the floating rate.

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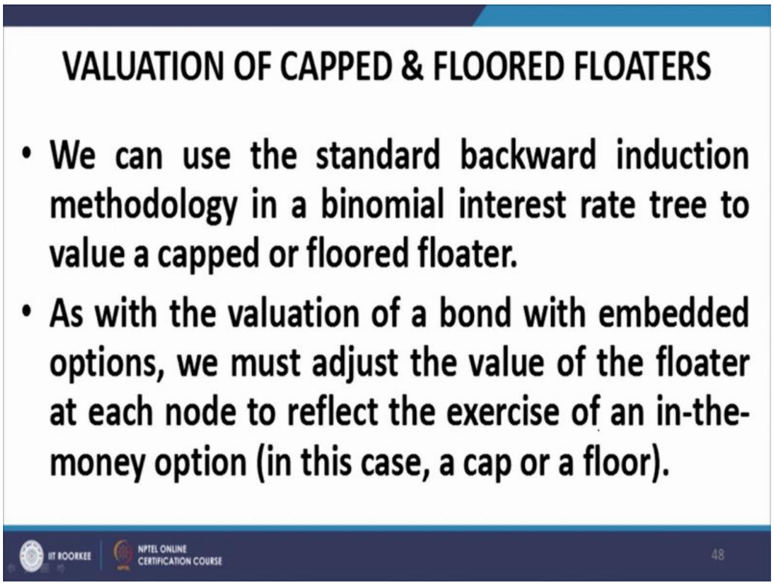

Valuation of a capped and floored floater, you can use the standard backward induction methodology in a binomial interest rate tree to value a capped or floored floater, as with the valuation of a bond with embedded options we must adjust the value of the floater at each node to reflect the exercise of an in the money option.

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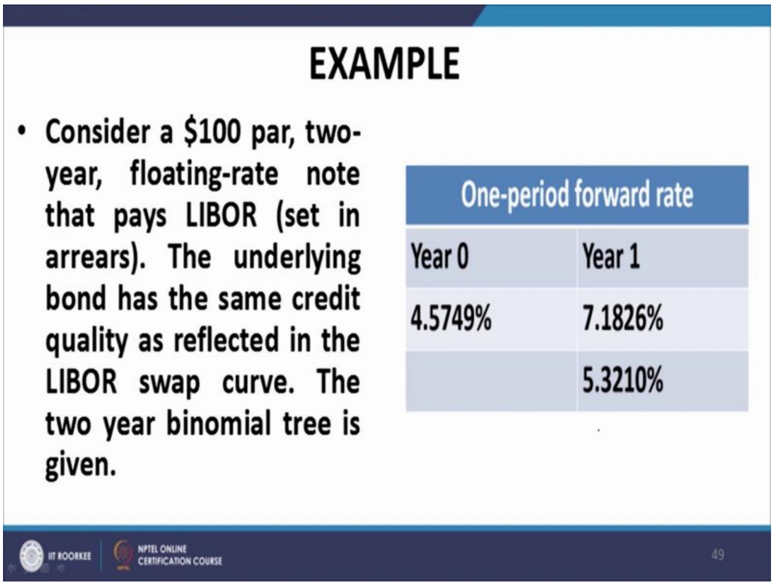

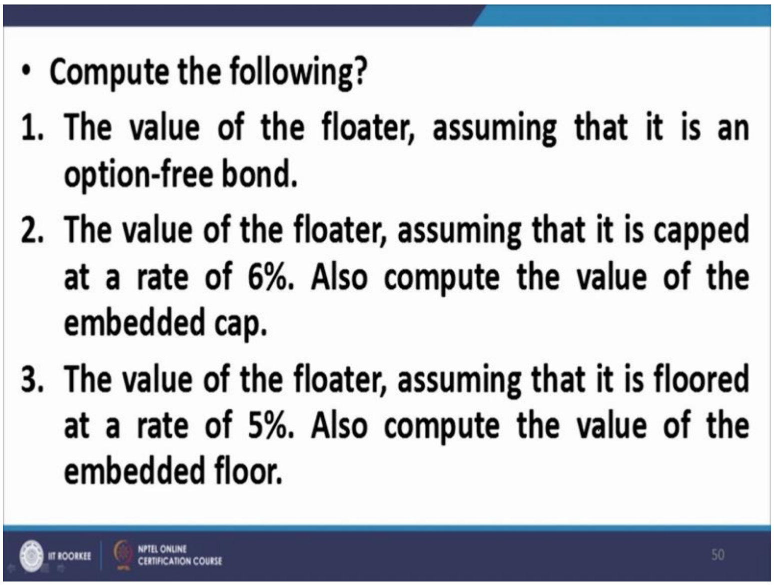

Let us do an example, consider a dollar 100 par, 2 year floating rate note that pays LIBOR set in arrears, the underlying bond has the same credit quality as reflected in the LIBOR swap curve, the two year binomial tree is given, this is the two year binomial tree, compute the value, the value of the floater assuming that it it is an option free bond, the value of the floater assuming that it is capped at a rate of 6 percent also compute the value of the embedded cap and similarly number 3, the value of the floater assuming that it is floored at a rate of 5 percent, also compute the value of the embedded floor.

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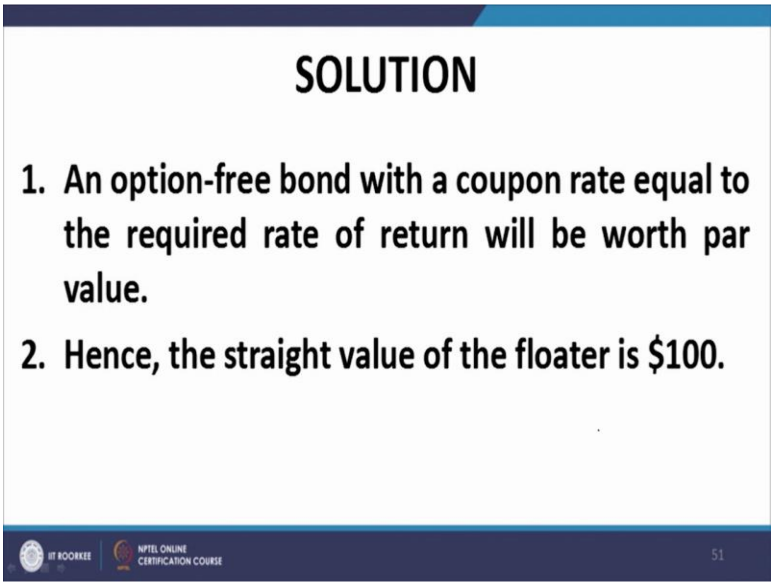

Solution the option free bond with a coupon rate equal to the required rate of return will be worth par value as we have just seen, hence the state value of the floater is dollars 100 right so as far as the state value of the state bond is concerned it is being valued at t equal to 0 and at the commencement of, in fact at the commencement of each interest period the value of the floater will be equal to, will be equal to what, will be equal to its face value, so that is not an issue.

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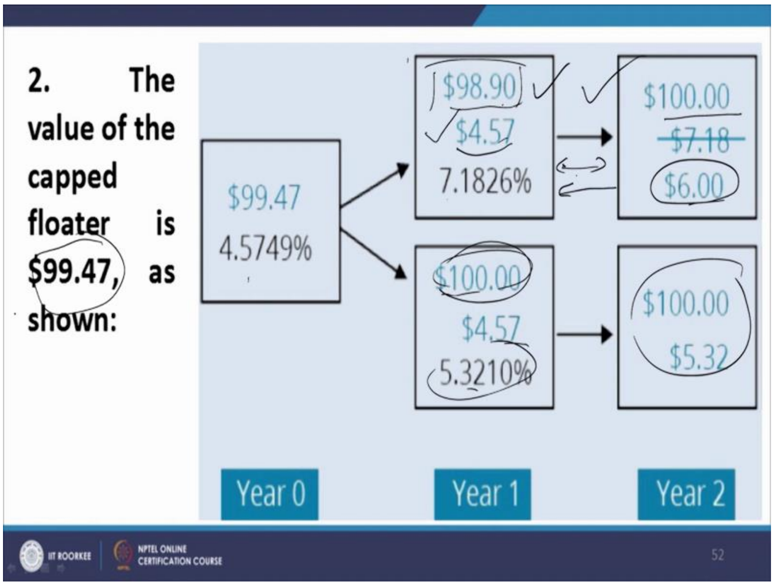

Now we look at the capped floater, the cap is at 6 percent we look at the figures very carefully, the interest rate when we talk about this particular branch, when we talk about this the interest rate is given as 7.1826 percent but the interest rate is capped at 6 percent therefore when we talk about the valuation, calculating the cash flow at this point it would be the principal value, the redemption value that is 100 plus interest on that at 6 percent not at 7.1826 percent that is where the capping comes into play.

The binomial tree interest rate for the calculation of interest at this node is equal to 7.1826 percent but when we look at the cap of 6 percent this interest must be cut down, must be truncated to 6 percent and the investor would be paid interest only at 6 percent so the amount that would be discounted when we move from here to here is 106 and 106 when discounted will give you 98.90.

Now we can look at the next tree, in the second case because the interest rate operating is 5.32 percent which is below the cap if the interest rate turns out to be 5.32 percent it is below the cap so we shall be paid interest rate at this amount 5.32 and therefore the amount that needs to be discounted is 105.32 as you can see here it is 105.32 and then when 105.32 is discounted at 5.32 percent you get exactly a 100.

Here what, let me repeat, in the case of the first node that is the upper node the amount to be discounted 106 discounted at 7.1826 percent and that gives us a figure of 98.90, so repeat once again, the 100 is the face value the 6 is the interest rate which would operate in view of this cap in view of the cap of 6 percent the interest rate which is actually the floating rate interest is 7.18 percent but it will not operate or it will only the 6 percent will operate and therefore I will get a coupon of 6 percent total 106 but the discounting would be done at 7.1826 percent then this value will be 98.90.

And because the rate at given in the first node is 4.57 percent which is below 6 percent the interest rate at this point will be 4.57 percent and then the entire thing 98.90 plus 4.57 will be discounted at 4.5749 percent, similarly for the lower branch of the tree the cap will not operate because the interest rate is how much, interest rate is 5.32 percent and therefore the coupon will be paid at 4, at 5.32 percent and we will have 105.32 discount it at 5.32 percent that gives you 100 then you discount, then you the interest for the period from t equal to 0 to t equal to 1 would be at 4.57 percent and the total amount that would be discounted it would be 104.57 discounted at 4.57 percent we will take the average of these two that would be the value of the capped floater and that would be 99.47.

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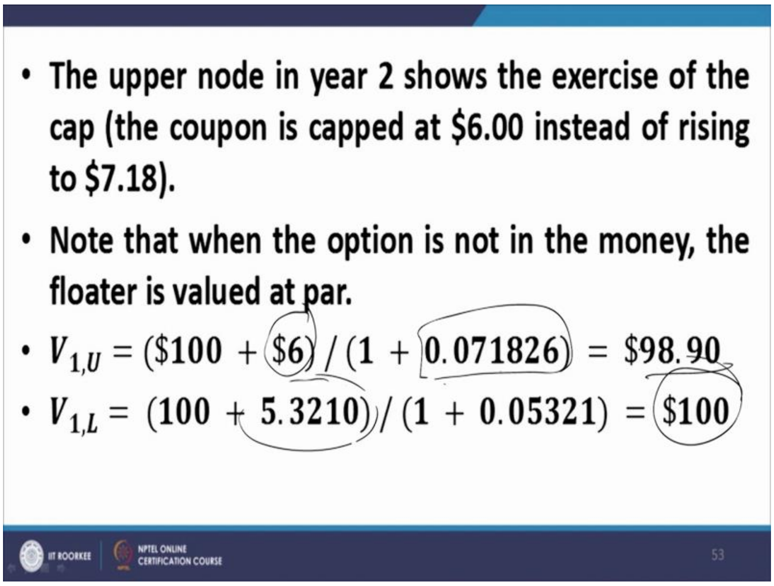

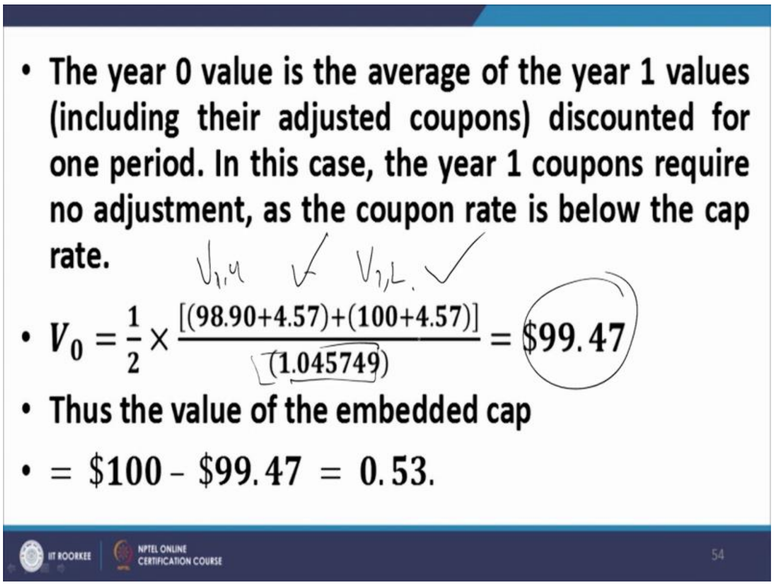

So that is what is given here 100 plus 6, this is the catch, this is where the cap is coming into play and please note the discounting rate is not capped, the discounting rate is the actual rate please note this point very carefully, the discounting rate is not kept it is the actual rate then this gives you 98.90 in this case the cap does not operate because the rate is below the cap rate of 6 percent.

Then therefore when you discount this we have 100 here, so these are the values at the upper node and the lower node at t equal to 1 and then we go to t equal to 0, when we evaluate at t equal to 0 the interest rate is 4.57 percent so this is 4.57 coupon payment, this is 4.57 coupon payment, this is the value at V1u this is V1l and we take the average and we disc, please note the discount rate, the discount rate is also 4.57 percent and that gives us a value of 99.47 so this is how the value of the cap can be calculated.

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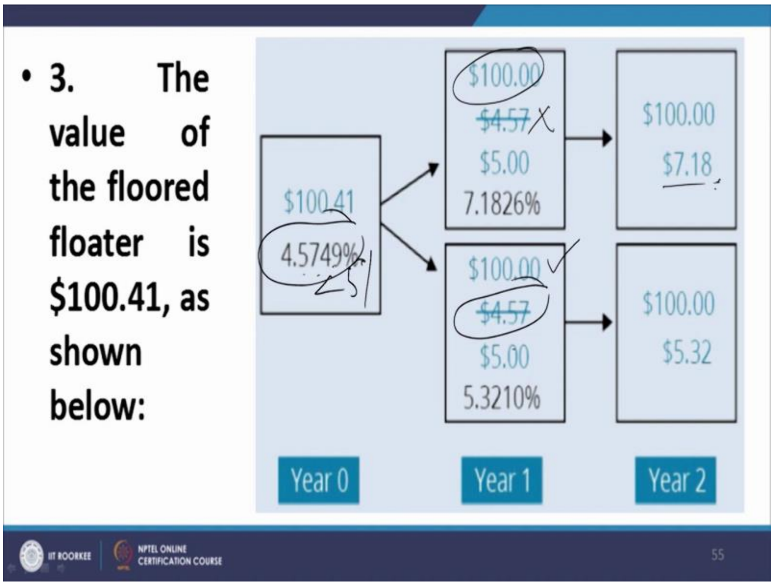

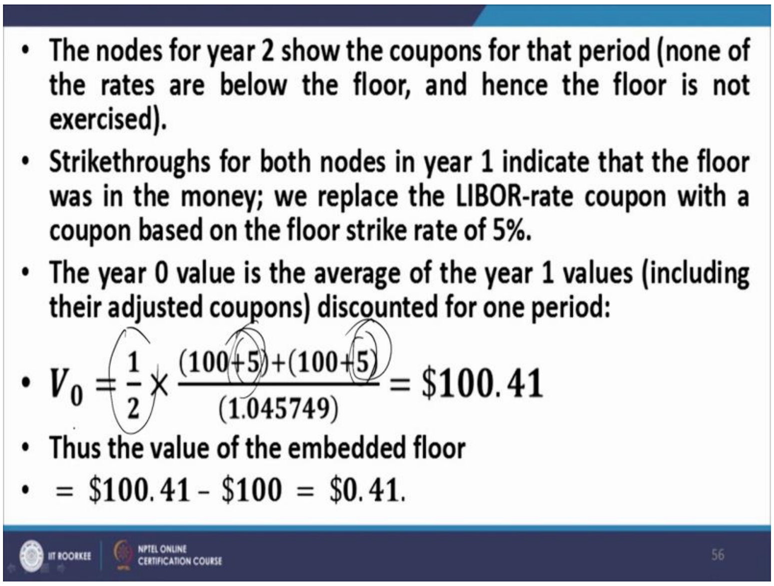

The value of the valuing the floor is pretty much similar its an identical exercise in this case wherever the rate is below the floor we use the floor as the appropriate rate, the floor will operate here in this case because the floor is at 5 percent the rate given is 4.57 percent so instead of using 4.57 percent we shall use a rate of 5 percent and the rest of it will remain the same, the coupon is paid at 7.28, 7.18 percent here so the total payment is 107.18 and the discounting is at 7.18 percent so what we get is 100.

And then when we calculate the coupon rate it is not at 4.57 percent it is at 5 percent which is the floor rate and so we get 100 plus 5 that is 105 and we discount it at 4.57 percent, similarly in this case it is 100 plus 5.32 that 5.32 is above the floor rate so it 105.32, 105.32 is and then it is discounted at 5.32 so that gives us 100 again and then we have the coupon rate for the period 0 to 1 which is not 4.57 which is 5 percent because this is less than 5 percent.

And therefore the total value is 105 we discounted at 4.57 percent to take the average of the two values that is the value of the floored.

You can see here, the floor is operating at this point and at this point, the rest is straightforward, please note we take the averages because we assume that the probability of the up move and the down move of the interest rate is the same.

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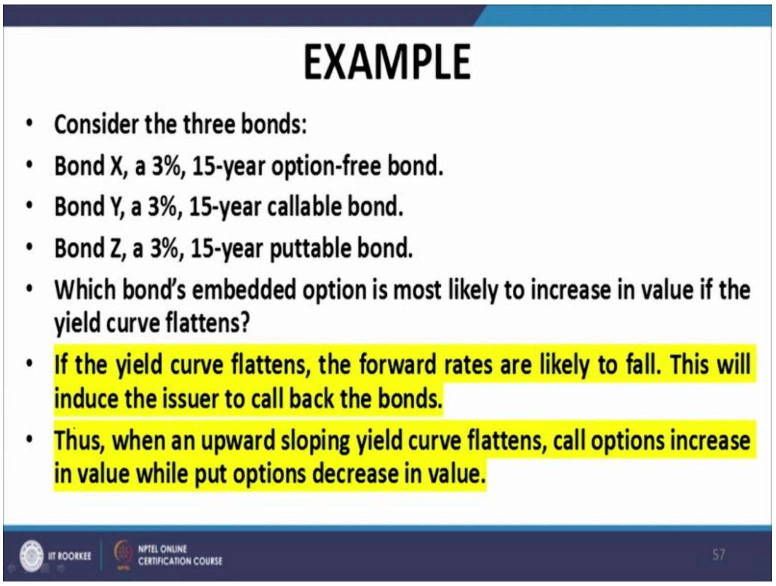

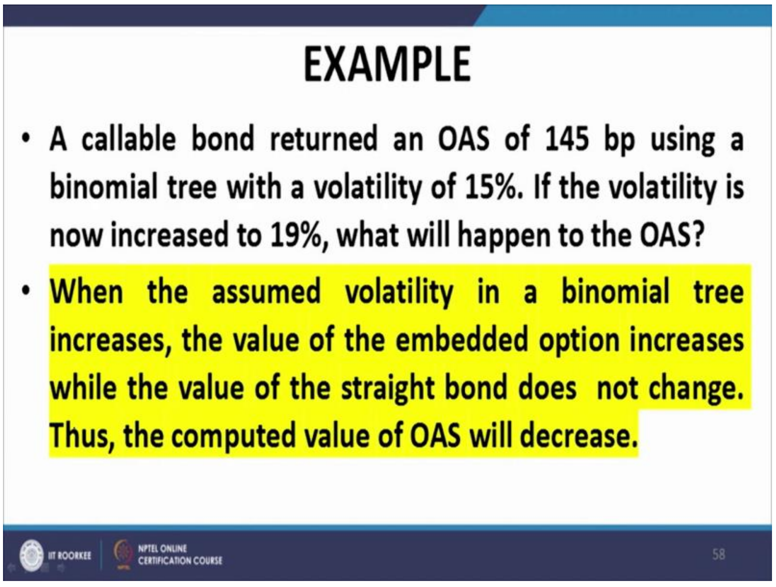

And then we have some examples which I will put in the notes. In the next lecture we will start with derivatives. Thank you.