Quantitative Investment Management
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Lecture: 03
Financial Risk

Welcome back. when talking about financial derivatives, and defined a financial derivative as a financial contract as a contract that, whose value is a function of whose value depends upon the value of another asset which is called the underlying asset. We have financial derivative contracts return on a variety of underlying assets, we have financial derivatives on stocks, or share indices, stock indices, currencies, commodities, and so on.

Then, we moved over to talking about the types of fundamental, types of derivatives. We discuss the forward contract which is a contract, which is negotiated at t equal to 0, but the actual settlement of the contract takes place at t equal to capital T that is at a future date, the terms of the contract are agreed upon a t equal to 0 including the price, the terms of delivery and so on. All the terms that would enable an unambiguous settlement, but the actual settlement the actual payment of the price and delivery of the underlying asset takes place at a future date.

Later discussed about futures which are in essence forward contracts, but with the additional property of them being tradability on appropriate exchanges which are constituted for that purpose. Then we move over to options, options differ from futures and forwards in the sense that, in forward and future contracts, both the parties to the contract have obligations to fulfill.

Whereas in the case of the option contract, one of the parties who buys the option, who is the holder of the option was long in the option, call it whatever you like. But the essence is that the party who is holding the option has a discretion has a right over and above the party, who is short in the option who is written the option to the extent that if the holder of the option decides to exercise the option, then the writer of the option is under obligation to order his leg of the option contract.

Of course, the holder of the option has the additional discretion to let the option lapse, and in the event that he lets the option lapse, the writer of the option goes scot free. But because the two parties are at unequal pedestals, the older of the option or the buyer of the option has to pay an upfront fee, which is usually called the premium or the price of the option to the party

was short in the option the party was written the option to compensate, the party was written the option for the risk that he is ticking under his obligation due to the short position in the option contract.

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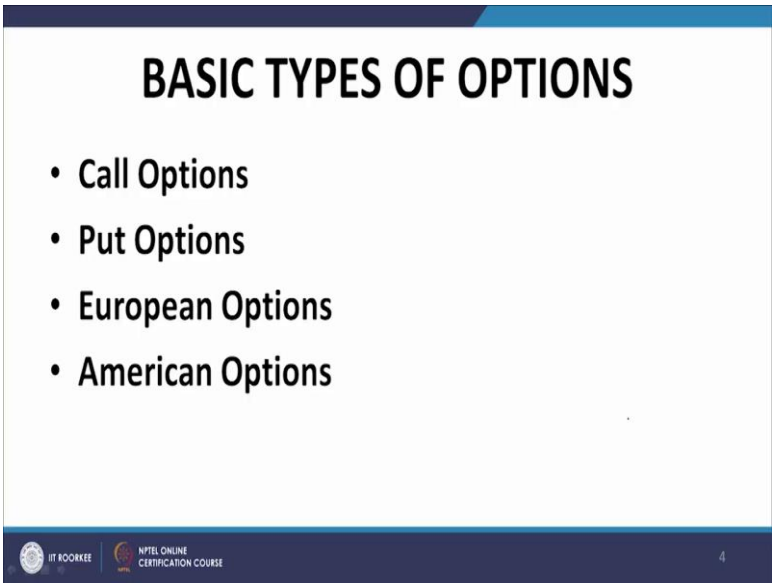

Then, talking about options there are two fundamental types of options we call them call options and put options. Call options give the holder of the option, the buyer of the option or right to buy the underlying asset at a predetermined price at a price which is embedded in the option contract at a particular point in time.

We will come back to the issue of time in a minute. But for the moment, call option is a contract which gives the holder of the option the right to buy the underlying asset at a predetermined price. The put option is the other way around. The put option gives the holder of the option, the buyer of the option, the right to sell the underlying asset at a predetermined price.

Whereas the put option is a right to sell the underlying asset at a predetermined price. Now, depending on the exercise ability of the two options, the option contracts may be classified into European options and American options. European options can be exercised only at a particular date, which is again incorporated in the option contract is a part of the option contract, it is called the maturity date of the option.

So, the European options can only be accessed on the date of maturity of the option contract whereas the other type of options which are called American options can be extended at any time during the life of the option contract up to the date of the maturity of the contract. So,

American options obviously enjoy the additional flexibility in terms of the time of exercise. Whereas, European options are fixed in terms of the time of access to a particular date, that is a part of the option contract, American options can be accessed at any point in time when the option holder decides to do so. And that can be it can be accessed up to the date of maturity of the option contract. Then, brief touch about swaps.

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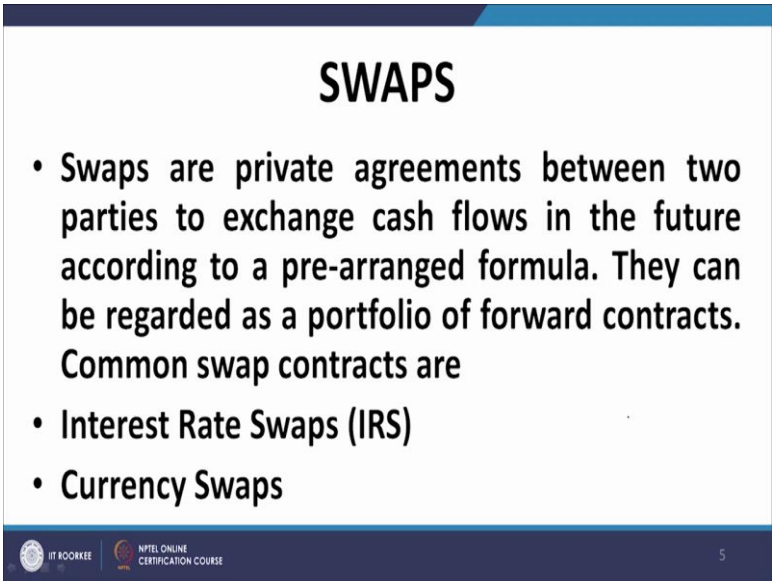

Swaps are private agreements between two parties to exchange cash flows swap in common parlance also means exchange so, that meaning is carried into the finance dictionary. And we define swaps as private agreements between two parties to exchange cash flows in the future according to a predetermined formula prearrange formula, they can be regarded as a portfolio of forward contracts. And we will come back detailly about swaps in next. Common swaps that are usually traded are the interest rate swaps, which essentially entail the exchange of a fixed rate principle with a floating rate principle.

The two parties to the swap may find in their interest to have a fixed rate loan, the other party may prefer a floating rate loan, and they may because of the asymmetries prevailing in the market, the party preferring the fixed rate loan may not be able to get a desired rate in terms of the boring interest rate, when it approaches the market for a loan, it may find that the floating boring if taken up by it, it turns out to be cheaper.

Whereas the other party may find that although it needs floating rate, it could borrow at a fixed rate at competitive rates, and does the two parties can mutually borrow at the first party can borrow at the floating rate and the second party can borrow at a fixed rate and then they

can mutually exchange the principles or extend the interest shims. This is called a plain vanilla swap for a typical interest rate swap. We also have currency swaps where currencies are exchanged. We will come back to these in a later section.

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Now, let us start a new topic we talk about risk and arbitrage to bring to you the meaning or the philosophy or the underlying motion of risk. Let us discuss about a default free bond, let us assume that we are considering investment in a one-year government T-bill or a government bond that is priced at 95 as of today.

Now, we also assume that the T-bill will be redeemed by the issuer that is the government of India at 100 at the end of 1 year from today. If we assume that the government is not likely to default on the bill, then what happens is that the future value of the bond that is the value of at redemption point of the bond of 100 is a certain cash flow and therefore, because it is a certain cash flow, assuming I reiterate assuming that the prepayment by the government or the redemption by the government is absolutely certain is carries a 0 probability of default, and then we can say that the payment is default free and they are absolutely certain and as a result of which this particular bond is classified as a risk free bond.

So, assume that this redemption value is certain that is, there is low probability of default as to the receipt of the redemption proceeds, then because there is no probability of default, because this payment is certain, because its future value is certain, because its redemption value certain we call it risk free asset or a risk-free bond.

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Now, therefore, to formally define a risk-free asset, we define risk free asset is an asset which has precisely determined future value with no possibility of default, I repeat a risk-free asset is an asset which has precisely determined future value with no possibility of default, that is the future value of the asset is absolutely certain.

A risky asset is an asset which is not risk free, and as a result of which an asset with whose future value is not absolutely certain. So, to put in perspective, risk free asset is an asset whose future value a certain risky asset is an asset which is not risk free. That is whose future value is not certain.

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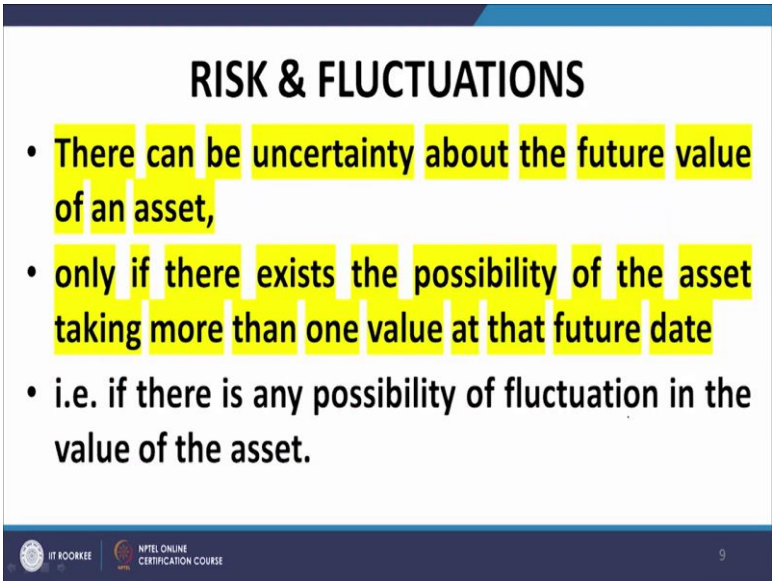

Now, there can be uncertainty about the future value of an asset only if there exists the possibility of the asset taking more than one value at a future date. Otherwise, if the asset can take only one value at a future date, then obviously that value will be certain and then the asset will be risk free.

So, in order that an asset will be not risk free, it is absolutely necessary that the asset should be capable of taking more than one values on the date of maturity or on the desired future date. That is, if there is a possibility of fluctuation in the value of the asset between a pair or more or a spectrum of possible values, the asset could, in other words, the asset could take any one of the of a number of values in contained in the spectrum of possible values that the asset could take.

That in a sense means that the asset would be a random way that the redemption value of the asset would be a random variable, it could have a probability distribution, and it could take any values from a set of given outcomes. Now, we talk about a simple proposition A simple corollary to what we have discussed in the last few minutes.

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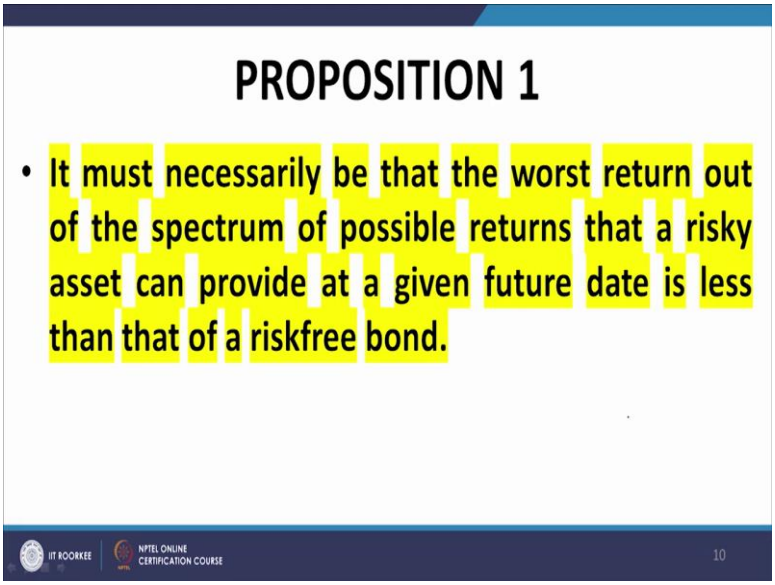

It must necessarily be that the worst return out of a spectrum of possible returns that a risky asset can provide at a given future date is less than that of a risk-free return. It must necessarily be so, that the worst return out of a spectrum of possible returns that a risky asset can provide at a given future date is less than that of a risk-free bond. You have a risky asset and obviously, the risky asset, as I mentioned, just now can take a number of values in a spectrum of possible values, it is a random variable, the final value of the asset or the redemption value of the asset is a random variable.

And as a result of widget could take one out of a spectrum of values, and it also has an associated probability distribution. What we are trying to say here is that the worst possible return is return would depend on the final value that the initial value of the asset is obviously known that is the point that is the value of the asset at which we enter the investment, that is the T equal to 0 value of the investment and the exit value that is the maturity value or the redemption value, as I mentioned is a random variable. So, depending on the value that the asset could take on the date of maturity, the return would also be a random variable.

If the redemption value turns out to be high, you get a higher return, if the redemption value turns at below, you get a lower return. So, basically, as you see, written return is formally defined as the acquisition in the value of the asset per unit time per unit of money, if we are talking about the return per unit of time, and if we are talking about the holding period return and the normalization with respect to time has not taken and it is simply the change in value or the position or deposition in value of the asset over the holding period of the asset per unit of money, of course.

Now, briefly explaining that the return or the worst possible return, please note this word possible, it is not actual returns, it is the possible it is the corresponding to all the values that the asset could take at maturity will have a possible return, will have a spectrum of returns corresponding to every value that the asset could take on maturity. Now, what we are trying to say here is that the worst possible return in the spectrum of returns must be less than the risk-free rate of return, let us see why it is so.

Now suppose the asset could take the risky asset could take one of two possible values, let us keep the exposition simple, the risky asset can take either the value Su that is the upper value, or it could take the value St, which is the lower value, these are the only two possible values that the asset could take at maturity at the end of the redemption period. Let r be the risk-free asset, so, the value if asset was the initial value at the value at which you enter the investment, had you invested in the risk-free asset you would have received an amount as 0 e to the power rt, where you are assuming continuous compounding at the risk-free rate r over the maturity period t.

Now, if the redemption value Su and St are both higher than S0 e to the power rt, what does it mean? It means that if you invest in the risky asset, the worst case scenario, listen to my words carefully, if you invest in the risky asset, the worst case scenario that you could encounter, the worst case scenario that you could encounter would correspond to the generation of return, which is higher than the risk free rate, you could not if you invest in the risky asset, you could not get a return, you cannot get a return which is lower than the risk free asset.

Obviously, this would mean that everybody in this world who is rational would invest in the risky asset and nobody would invest in the risk free asset, because if you are investing in the risky asset, you are certain of getting a return which is higher than the risk free rate or if the worst case scenario happens, and if the worst case scenario does not happen, you it turns out, that the asset returns the value of Su at maturity, then you get an even higher return. So, in any case, whatever the situation may be, whatever state of nature evolves into as on date of maturity, what will happen is that you get a return which is higher than the risk-free rate.

So, everybody would invest in the risky asset, nobody would invest in the risk-free asset, the demand for the risky asset would escalate, would increase and the demand for the risk-free asset would diminish massively, as of today, and consequently, the returns would also change. And we would end up with the risky returns going down, and the return on the risk-

free asset because there is a decline in the initial price at which you are going to enter the investment, the risk-free return will go up. And as equilibrium approaches it must necessarily be that is return lies between the return corresponding to the price of Su and St.

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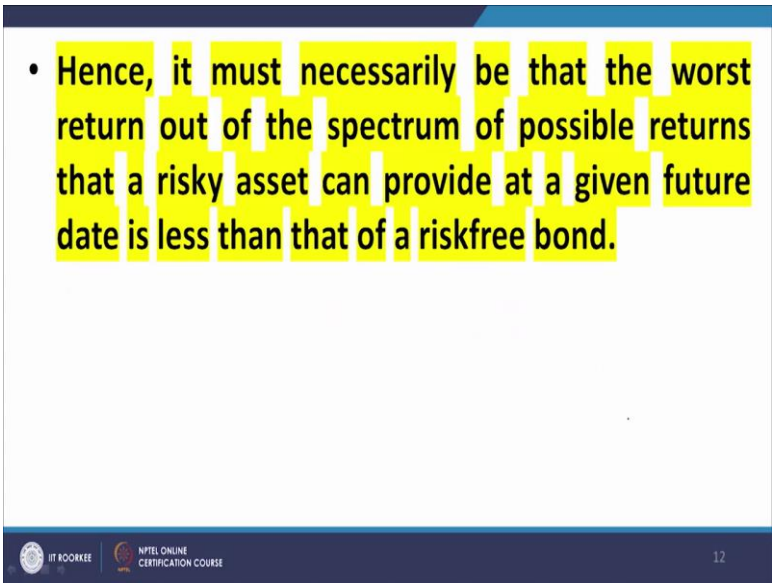

So, this vindicates our theory, that it must necessarily be true that the worst return out of a spectrum of possible returns, the return corresponding to St in our case that a risky asset can provide at a given future date is less than that of a risk free bond, you cannot have a situation where all possible returns in the spectrum of returns corresponding to the spectrum of possible price outcomes of the risky asset give you a return which is higher than which is greater than the risk free written, at least one of the returns must necessarily be lower than the risk free rate.

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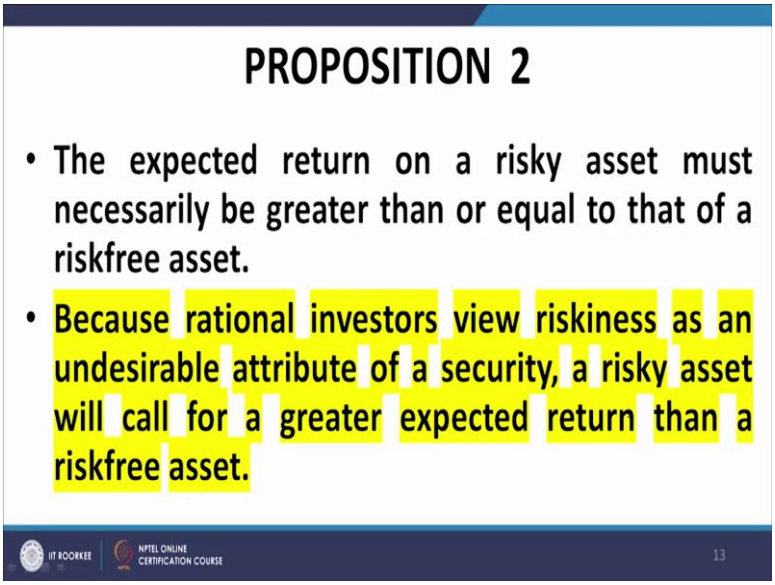

Another simple proposition, the expected return on a risky asset must necessarily be greater than or equal to that of a risk-free asset. Now, I would like to emphasize that because rational investors use riskiness as an undesirable attribute. You see, as I mentioned in the last lecture also, whenever we are evaluating an investment, we invalidate the investment in two columns. First is the expected return that we expect to derive by entering into an investment in that asset and secondly, the riskiness in realizing of that expected return. So, these are two factors, which go into the evaluation of an investment.

Whereas, as far as rational investors are concerned, expected return is taken as a positive attribute, in other words, is the expected return of an asset favorable is the inclination of the investor community towards that asset and on the other hand, as far as riskiness is concerned higher the riskiness of an asset, the lower is the inclination of the investors in investing in that asset. And as a result of which what we find is that investors view expected return as a positive attribute of the investment and the riskiness and realization of that expected return as a negative attribute of that asset of that investment.

Therefore, because the if an asset is risky, it is quite natural that the investor would ask for a compensation for the additional risk that he is taking in entering that investment. That compensation would take the form of an extra risk premium added on to the risk-free rate of return. So, the bottom line is that higher the riskiness of the asset, or higher the riskiness in the realization of returns or expected returns of an asset, greater is the expected return greater the demand for expected return on that asset for the investor community. So, because rational

investors view riskiness as an undesirable attribute of a security or risky asset must call for a greater expected return than a risk-free asset.

The only extreme case the limiting cases, when we have a situation in the risk neutral world where all the investors are in different to risk, all the investors do not care about risk. In that situation, what will happen is that all assets will end up providing the risk-free rate of return due to arbitrage considerations, but we will talk more about it when we talk about pricing of derivatives.

Just to recap, in the case of a risk neutral word where all investors are indifferent to risk they do not care about the riskiness of an investment, if that is if that word prevails, if that word occurs, rather than we are evaluating an investment in that particular word, where all investors are risk neutral, then, because riskiness has no consideration to any investor, no investor will demand and it is premium. And consequently, the all the assets will give you a risk-free rate of return.

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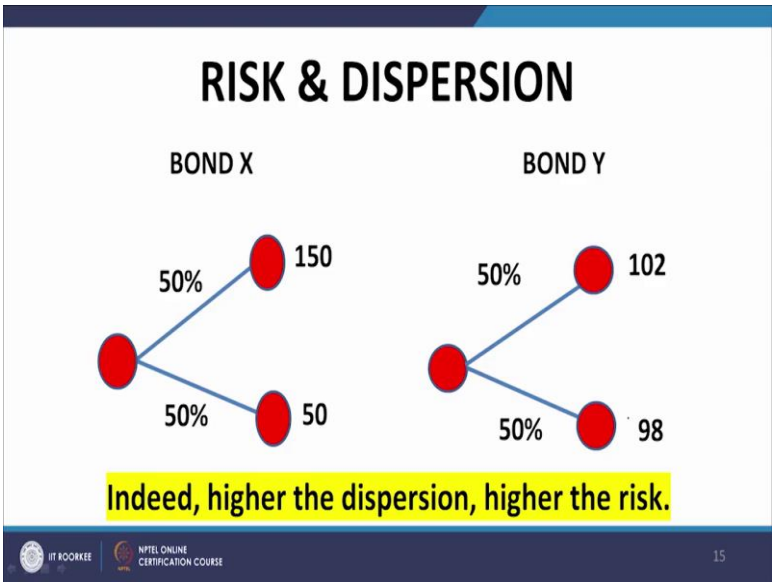

Riskiness and probabilities let us look, let me explain this by virtue of this diagram that you have on the slide. You have two bonds X and Y. The bond X has a 50 percent probability of returning a value of 150 and a 50 percent probability of returning of 50 whereas bond Y has a 50 percent probability of returning a value of 102, and a 50 percent probability of returning a value of 98. You can clearly see first of all that the expected returns on the two instruments have whatever the value of the initial investment, whatever the value of the investment at t equal to 0, the expected returns or the expected prices at maturity are the same.

So, that is one thing the expected returns are the same. But if I asked you the simple question, that which of the two assets is risky, any rational person would say that bond X is riskier, bond Y is less risky and why is that? Because the possible values of the bond Y are clustered together around the expected value, whereas bond X shows a higher dispersion.

So, the outcome is that higher the dispersion, higher the risk. Similarly, when we talk about default probabilities, it is quite simple again, we I illustrate this with an example. Again, I consider two bonds, bond X and bond Y, bond X has a 90 percent probability of giving an output or ending up in a price of 100. Bond X has a 10 percent default probability that is returning a value of 0.

On the other hand, if we look at bond Y, bond has a 95 percent probability of returning 94.7368 and a 5 percent probability of returning zero. Again, we find that the expected returns and the expected price of both the bonds at the maturity of the bonds are identical. However, provided of course, the initial prices are identical, which we are assuming as in the previous case as well.

So, if I asked you again, the question that which is less risky or which is more risky, you will again say that X is riskier than Y, why is that? Because X has a higher default probability, Y has a lower default probability, it is immediately obvious that X is risk content is higher compared to Y. Therefore, risk also depends on the relative probabilities of the various outcomes of the experiment.

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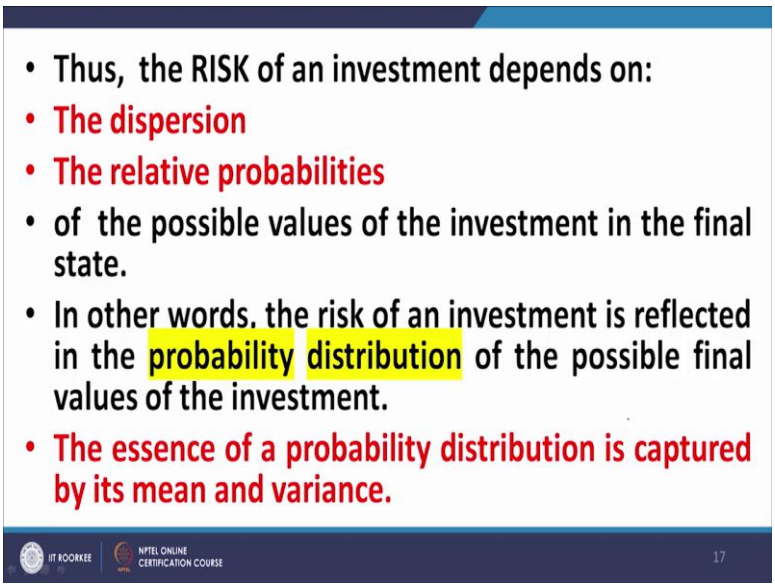

Thus, we can see that the risk of an investment depends on the dispersion that is how widely scattered the values of the asset are at maturity, or the final state of the investment, as well as the relative probabilities of the various values that the asset could take at maturity. In other words, the risk of an investment is reflected in the probability distribution of the possible final value of the investment.

Both these items, the dispersion, and the relative probabilities are captured by the probability distribution. And therefore, we can say that the risk of an investment is reflected in the probability distribution of the possible final values of the investment. And we also know for future reference, that the essence of a probability distribution is captured by first and second moments that is the mean and the variance of the distribution. I repeat the things of any probability distribution is largely captured by the mean and variance of the distribution that is the first moments.

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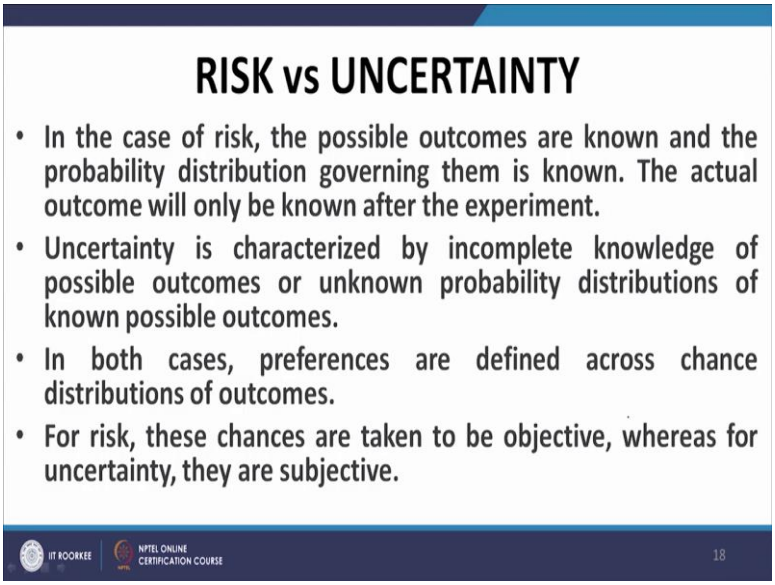

Just to clarify, in economic literature, we tend to differentiate between risk and uncertainty although in common parlance, we tend to use the two terms with a lot of overlap more or less synonymously. But let us clarify the difference at a deeper level at a conceptual level between risk and uncertainty. In the case of risk, the possible outcomes are known and the probability distribution governing them is also known.

The actual outcome of course, will be known only after the experiment is conducted. So, I repeat in the case of risk the possible outcomes are known and the probability distribution

governing them is also known, the actual outcome will depend on the will be determined or will be obtained only on the completion of the experiment.

Uncertainty on the other hand, is characterized by incomplete knowledge of possible outcomes or unknown probability distributions of known possible outcomes. In both cases preferences are defined across chance distributions of outcomes. For risk, these chances are taken to be objective, this is the important part, this is the fundamental part for risk these chances are taken to be objective.

Risk is in some sense, a more objective measure of uncertainty, it improves upon uncertainty by capturing more information about the possible outcomes and therefore relative distributions. So, the chances are taken to be objectives whereas in the case of uncertainty, they are subjective estimates.

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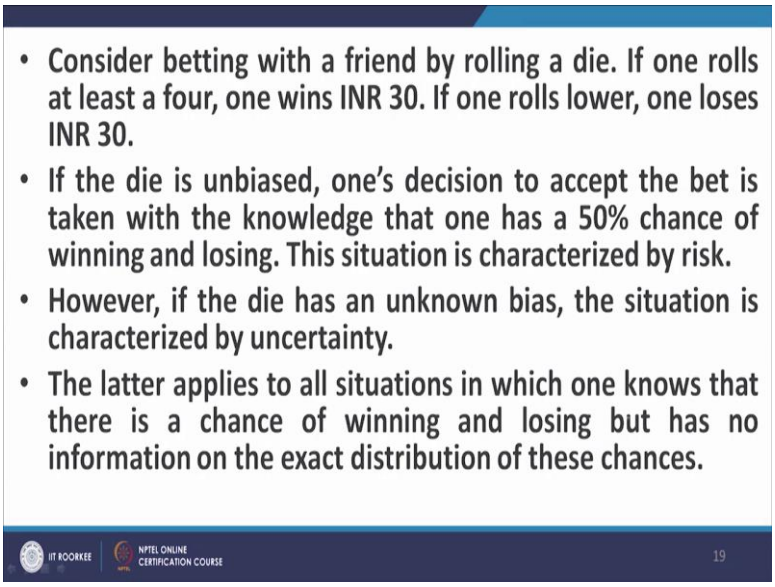

A typical example here about the differential between risk and uncertainty considered betting with a friend by rolling a dice. Now, if you roll the dice, and if you play the game, that if your return, and outcome which is 4 or more than 4, and the dice is in unbiased, then naturally you are playing the game with full knowledge that the probability of winning the game is 1 by 2. That is if you return 4, 5 or 6, and your probability of losing the game is also 1 by 2. That is, if you return 1, 2, or 3. Now suppose the dice instead of being unbiased, it is a biased dice.

And therefore, the probabilities of getting 1, 2, 3, 4, 5 and 6 on the dice are not equal, not identical and they are unknown, that would epitomize if you play that game, same game with a biased dice where one of the faces as a greater probability of turning up, then as it turns out,

it is a case of uncertainty. It is not a case of risk, because the probability distributions of outcomes are not known with any degree of accuracy.

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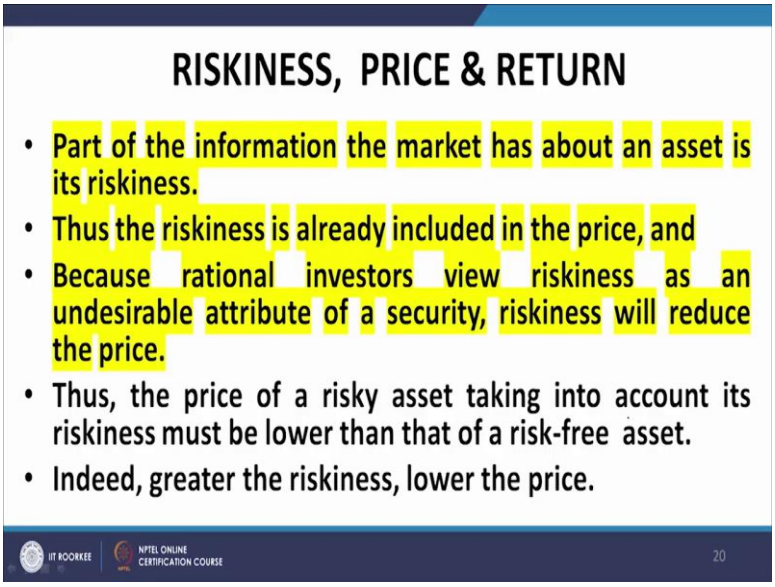

Now riskiness, price and return, riskiness price and return, the part of the information that the market has about an asset is its riskiness that the risk, therefore, because the market evaluates as I mentioned, again, and I will be mentioning again and again, evaluation of investment is based on two parameters expected return and risk. So, risk of being an evaluation parameter whenever any individual is going to enter into the investment, market also evaluates in various securities, various investment opportunities on a two-dimensional framework comprising of expected return and risk.

And in other words, what I am trying to say is that risk is evaluated by the market and because risk forms a evaluation parameter as far as market participants are concerned, it follows that riskiness of the investment is incorporated in the price at which the investment is being traded. Therefore, now, because rational investors have touched upon this earlier also, because rational investors view riskiness as an undesirable attribute expected return is a desirable attribute higher the expected return of a security higher is the demand for that security, on the other hand higher is the riskiness of a security lower would be the demand for that asset.

So, because rational investors view riskiness, as an undesirable attribute of a security riskiness will reduce the price, that is the price of a risky asset, taking into account just riskiness must be lower than that of a risk-free asset, because riskiness is an undesirable

attribute and when you take up something which has incremental undesirable attribute, then it obviously you would be willing to pay a less price for that.

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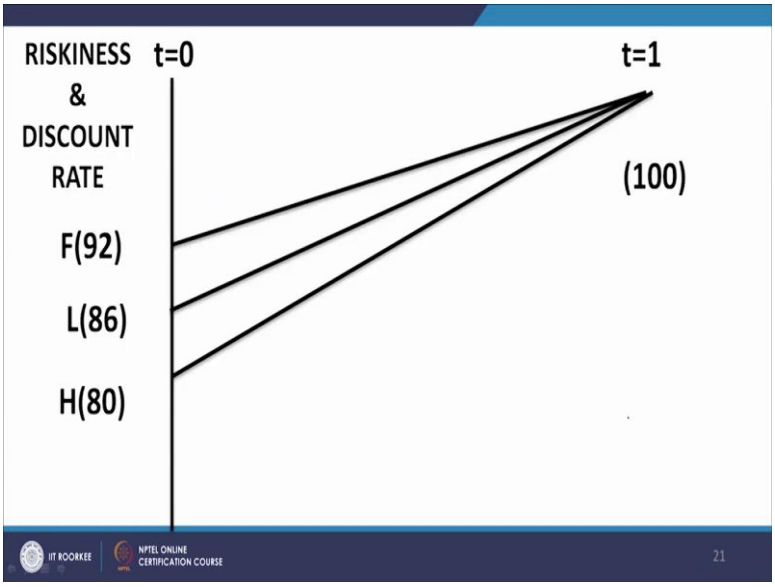

Thus greater the riskiness lower the price, this is diagrammatically shown in this diagram that we have on the slide, we have three assets are risk free asset at called F, low risk asset called L and high risk asset called H all these three assets are going to return an amount of 100 at maturity in one year's time. Now, because F is a risk free asset, it has the lowest content of risk, and as a result of which it would be selling at the highest price that is 92 in this case.

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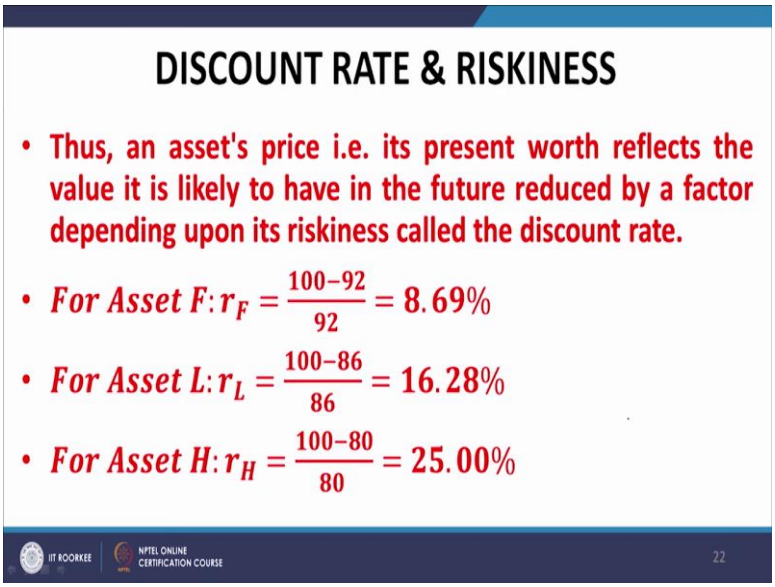

Now a discount rate and risk rate, now this is very important fundamental. You see, how do we work out the asset price as of today, if we know the payoff of the asset at maturity, let us say the payoff of the asset at maturity is 100. And I want to work out the asset price as of today, the traditional approach is to discount the payoff using an appropriate rate. Now, that when I use the word appropriate rate, what is that appropriate rate, that appropriate rate is called a discount rate and that consists of three components in general.

Number one is the risk free rate, number two is the connection or the additional premium for inflation and number three, most important in our current context is the risk premium. So, higher the riskiness of an asset higher is the risk premium incorporated or forming part of the discount rate at which the final payoff will be discounted, and therefore, it would naturally return a lower value.

As we can see here, in the case of asset F, the discount rate is 8.69 percent. In the case of asset L, we find that the discount rate is 16.28 percent. And in the case of asset H, it is 25 percent. So, the relationship between discount rate and riskiness is one of higher the riskiness higher the discount rate. We shall continue from here in the next lecture. Thank you.