Quantitative Investment Management
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Lecture 29
Fixed Income Portfolio Strategies - 1

Welcome back. So towards the end of the last lecture, I was talking about the process that should be adopted for calculating the cardinals of a portfolio of bonds, not a single bond, but a portfolio bonds. Let us take the simplest example when you have got a portfolio comprising of two bonds, A and B. Now, there can be two approaches to calculating. For example, let us talk about the YTM.

The first is that you work out the YTM of the bond A individually and you work out the YTM of B individually, and then you take a weighted average of the YTMs of A and B using the market value of the bonds as the weights. That is one typical example, which is very often used in practice.

The other approach is that we work out the cash flows from the portfolio, the combination of the two bonds, we work out the cash flows from the portfolio at different points in time, and using those cash flows, not using the YTMs of the individual bonds, using the cash flows of the portfolio as such, we work out the YTM of that portfolio.

Obviously, the second method holds merit and with the advancement in computer technology and computing power accessible, it is more appropriate that we use this second method for calculating the cardinals of a bond portfolio.

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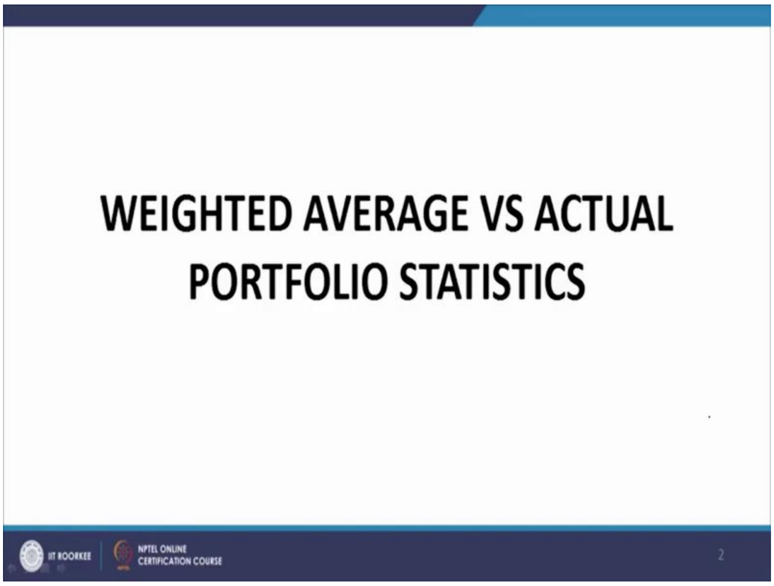

So that is what we were talking about at the end of the last lecture whether we should go for a weighted average calculation of the cardinals or we should work out actual portfolio statistics based on the cash flows emanating from the bond

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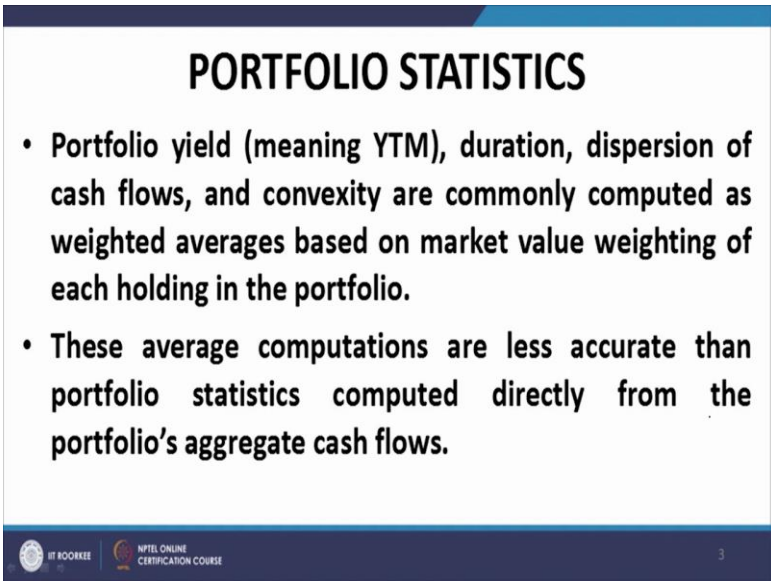

So let me recap this for you. Portfolio yield, meaning YTM, duration, dispersion of cash flows, and convexity are commonly computed as weighted averages based on market value weighting of the holdings, each holding in the portfolio. These average computations are less accurate. That is the important point.

They are less accurate than portfolio statistics or portfolio cardinals that are computed directly from the portfolios aggregate cash flows. We work out each cash flow, aggregate those cash flows and use that stream of cash flows for calculating the various cardinals of the world like YTM, duration, convexity and dispersion.

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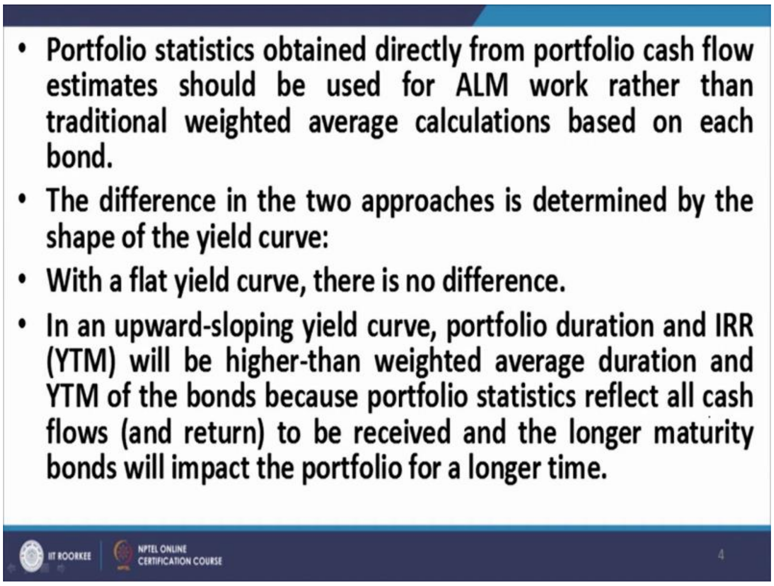

So portfolio statistics obtained directly from portfolio cash flow estimates should be used for asset liability management work rather than traditional weighted average calculations based on each bond. The difference in the two approaches is determined by the shape of the yield curve, the spot yield curve.

With a flat spot yield curve there is no difference, in an upward sloping yield curve, portfolio duration and IRR will be higher than the weighted average duration. I will show that with an example as well. And YTM of the bonds, because portfolio statistics reflect all cash flows and return to be received and the longer maturity bonds will impact the portfolio for a longer time. I will illustrate this with an example right now.

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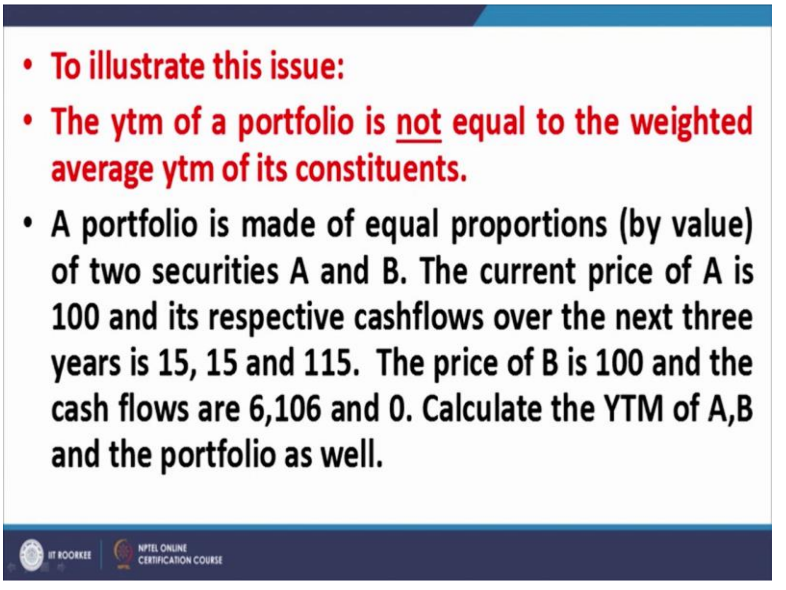

We consider a portfolio comprising of equal proportions by value of two securities, A and B. The current price of a is 100 and its respective cash flows over the next three years are 15, 15 and 115. That translates to a YTM of 15 percent. the price of B is 100 and the cash flows are 6, 106 and 0. So that turns out to be a YTM of 6 percent. Calculate the YTM of A comma B and the portfolio as well.

As far as the YTM of A and B are concerned, it is pretty straight forward. YTM of A is 15 percent, YTM of B is 6 percent, and the weighted average YTM, because the weights are equal, the weighted average YTM turns out to be 10.5 percent. Let us now work out the YTM based on the portfolio cash flows.

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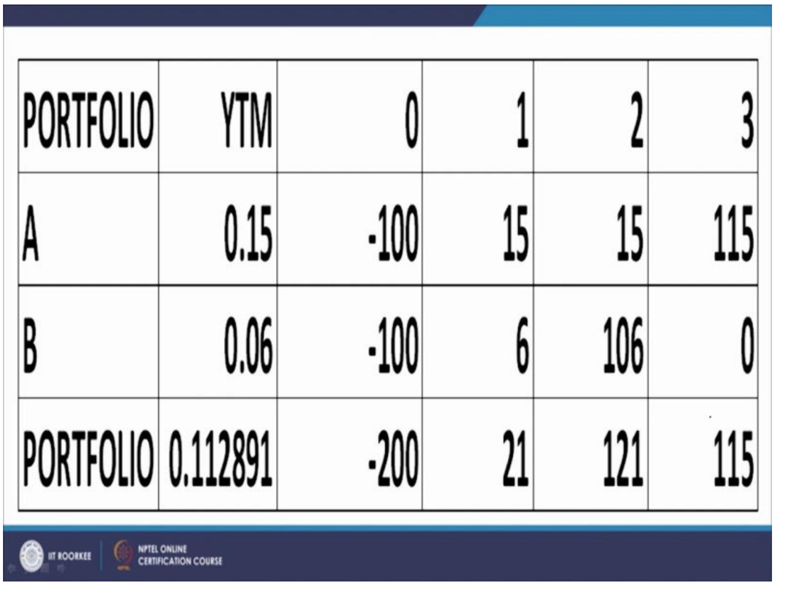

PORTFOLIO	YTM	0	1	2	3
A	0.15	-100	15	15	115
B	0.06	-100	6	106	0
PORTFOLIO	0.112891	-200	21	121	115

The cash flows are tabulated in this slide. You can see here, there is an initial cash outflow of 200 and the cash flow at t equal to 1 is 21, the cash flow at t equal to 2 is 121 and the cash flow at t equal to 3 is 115. And when you work out the IRR of the stream of, YTM of the stream of cash flows, it turns out to be 11.29 percent.

So as you can see here, the and, and please note this point there. This is an upward sloping yield curve. The two year YTM is 6 percent and the three year YTM is 15 percent. So this is clearly an upward sloping yield curve, and the average, the weighted average YTM turns out to be less, being 10.5 percent than the portfolio YTM, which is 11.29 percent.

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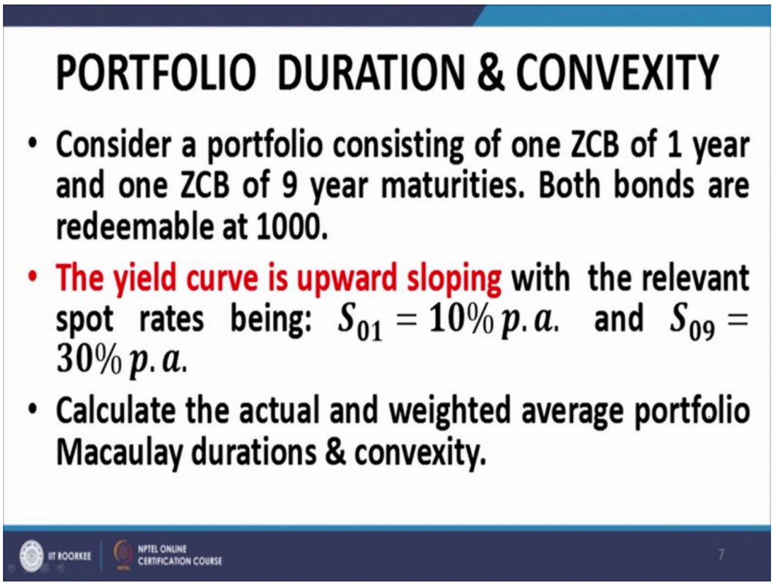

Portfolio duration and convexity examples. So let us look, go through these as well. We talked about the YTM of the portfolio, how the weighted average YTM does not reflect the correct picture, it does not equal the actual YTM of the portfolio of bonds. We shall see that the same thing holds in the context of duration and convexity in the event that the spot yield curve is upward sloping or its not a flat spot yield curve.

Consider a portfolio consisting of one zero-coupon bond of 1 year and one zero coupon bond of 9 year maturities. So you have two bonds, let us call them bond A, that is of 1 year maturity, and bond B, that is of 9 year maturity. Both the bonds are redeemable at 1,000. The maturity value is 1,000. The yield curve is upward sloping.

In this problem, it is explicitly given, in the previous problem it was more of an implication, it was an implied value. We had the three year YTM as 15 percent and the two year YTM as 6 percent. In this case the figures are explicitly given. It is an upward sloping yield curve and the relevant spot rates are S 01 is 10 percent and S 09 is 30 percent. S 01 is 10 percent, S 09 is 30 percent. Calculate the actual and weighted average portfolio, Macaulay duration and convexity.

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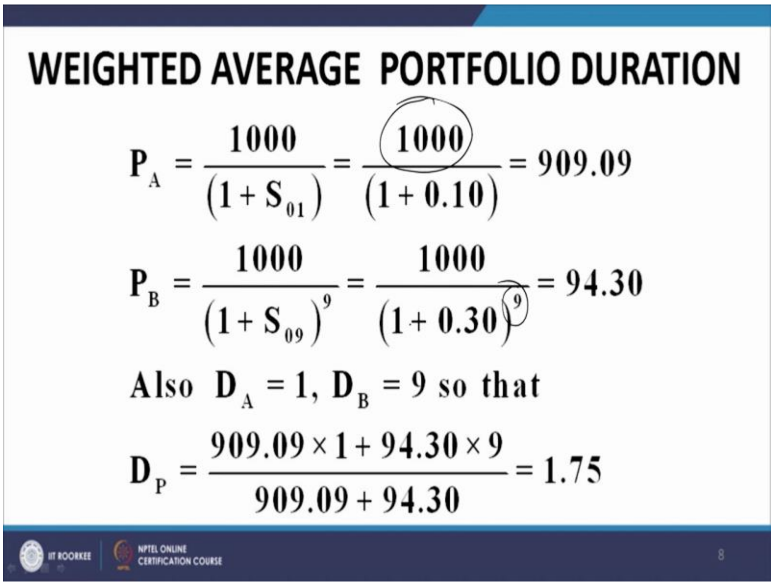

So for working out the weighted average portfolio duration is quite straightforward. We work out the duration of A and B. Both are zero coupon bonds, so the duration equals their maturity, and the maturity of A is 1 year, the majority of B is 9 years. So the duration of A is 1 year and the duration of B is 9 years.

As to their current market prices, we can discount the phase, redemption value of 1,000 at the respective spot rates. In the case of A, it, the spot rate is 10 percent, in the case of B, the spot rate is 30 percent. The maturity of A is 1 year, the maturity of B is 9 years, as you can see in this figure here.

And that translates to a current market price for A as 909.09 and 94.30 for B. So the current market price of A is 909.09, the current market price of B is 94.30. The duration of A is 1, the duration of B is 9, and that translates to a weighted average duration of 1.75 for the portfolio with market value as the respective weights.

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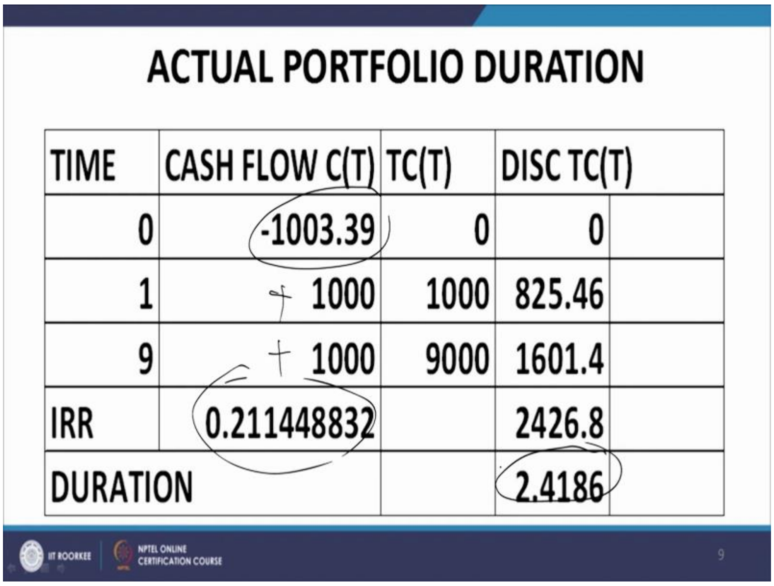

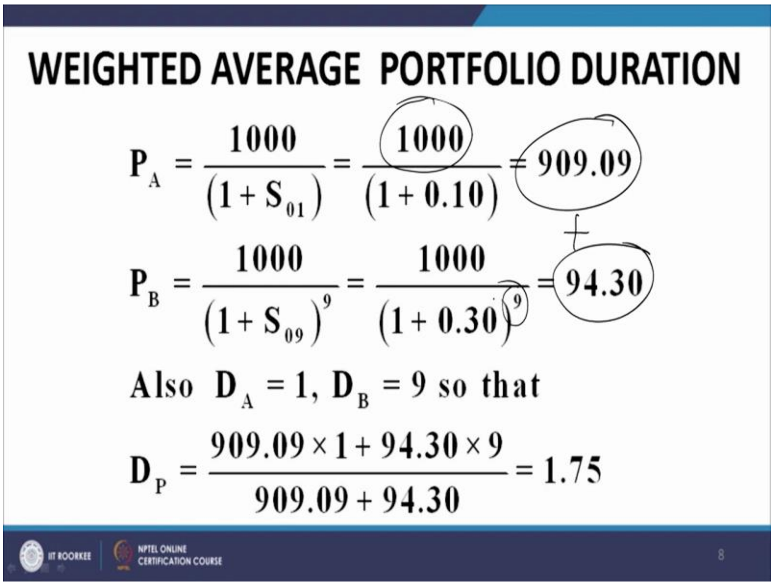

Now, this is the actual portfolio duration working. The cash flow at t equal to 0 is equal to the sum of these two prices, is equal to 909.09 plus 94.30, and that is 1,003.39. That is the figure that is given here. t equal to 1, there is a redemption of the bond A, that is plus 1,000, and t equal to 9, there is a redemption of the bond B, that is again plus 1,000. These are the two figures of reflecting cash inflows from an investment in these bonds. And t equal to 0 is the price that we pay for constituting this portfolio of two bonds, 1,003.39.

When we work out the IRR of the stream of cash flows, it turns out to be 21.14 percent, that is given in this figure here. IRR is equal to 21.14 percent. That is the portfolio IRR, please note this point. And using this portfolio IRR, if you calculate the duration, the duration turns out to be 2.4186 years, 2.4186 years. How do we compute the duration? It is a pretty much the use of the standard formula. And the important thing is that the rate that we use for calculating duration is this IRR of 21.14 percent.

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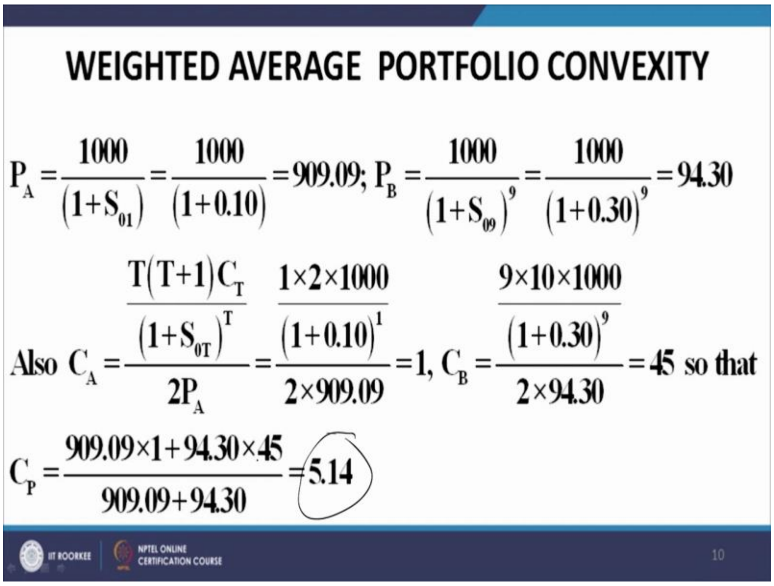

Similarly, talking about the portfolio convexity, the for, convexity of A it turns out to be 1. The convexity of B turns out to be 45. The market prices, we have already worked them out. The market price of A is 909.09, the market price of B is 94.30, and therefore the weighted average convexity is 5.14.

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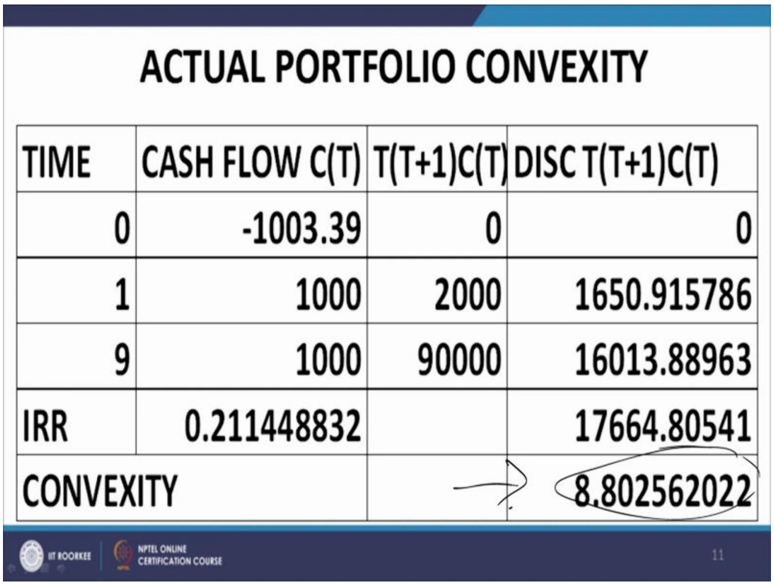

TIME	CASH FLOW C(T)	T(T+1)C(T)	DISC T(T+1)C(T)
0	-1003.39	0	0
1	1000	2000	1650.915786
9	1000	90000	16013.88963
IRR	0.211448832		17664.80541
CONVEXITY			8.802562022

And then we work out the actual portfolio convexity, that is the convexity worked out on the basis of the portfolio cash flows using the IRR that we worked out earlier of 21.14 percent, and we find that the convexity turns out to be 8.80 years square. So please note, this is an upward sloping yield curve, and what we find is that the weighted average duration and convexity are lower and the actual portfolio convexity calculated from the portfolio statistics from the portfolio cash flows turns out to be higher.

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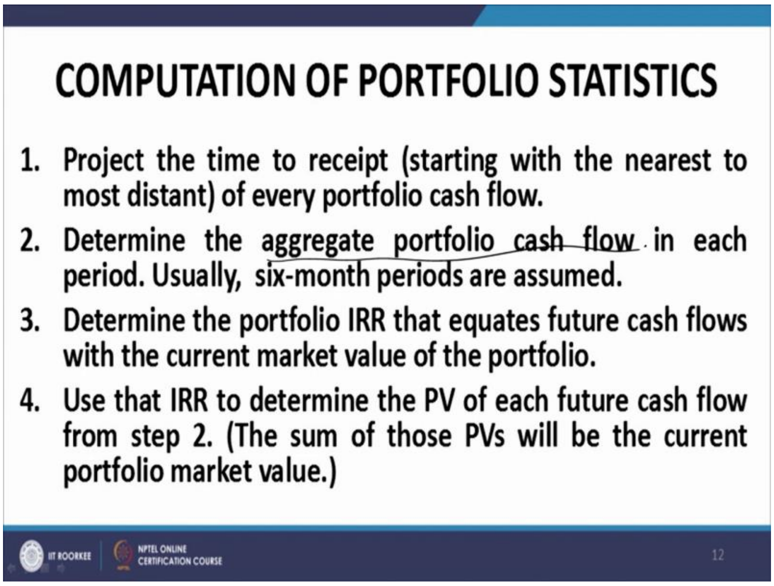

This is the process, this particular slide and the next two slides outline the process, step wise, that needs to be followed for calculating the convexity or duration or YTM, for that matter, of bond portfolio. So let me go through this step by step. Project the time to receipt, starting with the nearest to the most distant of every portfolio cash flow.

Every constituent, every bond in the portfolio you work out or compute, the cash flows arising out of the bond at the respective function time, at which the cash flow arises. Let me repeat, for each bond, let us say you have two bonds, A and B, for bonds, starting with bond A, t equal to 1, 2, 3, 4, whatever this t may be, it need, not necessary that this may be yearly cash flows.

The cash flows may be half yearly, the cash flows may be quarterly for that matter, the cash flow may be at the end of 10 years or 9 years, whatever the case may be, whatever is the unit of time, that is not the point. The point is, in, in terms of that unit of time, t equal to 1, t equal to 2, t equal to 3 and you work out the corresponding cash flows at each of these time points.

For each of these bond portfolios, you want constituents of the portfolio. Determine the aggregate portfolio cash flow. Then you add the total cash flows at each point in time, t equal to 1, bond A this much, bond B this much and total portfolio this much. At t equal to 2, the same thing, cash flow from bond A, cash flow from bond B and then the aggregate cash flow. And similarly, we work out for all the time points at which there is going to be a cash flow from either of the two bonds.

Because if A is giving a cash flow, B is not giving a cash flow, you impute a 0 cash flow to that, and work out the aggregate cash flow of the portfolio. So that is what the second step says. Determine the aggregate portfolio cash flow in each period. Usually, six-month periods are assumed. Usually, not necessarily. So it would depend from case to case. Determine the portfolio IRR that equates future cash flows with the current market value of the portfolio.

This is, in some sense, this is the YTM of the portfolio item of the combination of bonds that connect, that comprise the portfolio. So determine the portfolio IRR that equates future cash flows with the current market value of the portfolio. Use that IRR to determine the present value of each future cash flow from Step 2. Now, you have the aggregate cash flow that from Step 2. This is, the aggregate portfolio cash flow, and this, using this (carry) aggregate portfolio cash flow, we work out the present value of this aggregate portfolio cash flow.

At what rate? At the rate that is calculated in step number 3. So the cash flows or the aggregate cash flows at each point in time during the life of the portfolio at which there a cash flow arises, and then we work out the IRR on, of the stream of cash flows, including t equal to 0, that would usually be an outflow being the price paid for constructing the portfolio. Then the next step is, once you work out the IRR, using this IRR, using the projected cash flows, they have projected aggregate portfolio cash flows, we work out the present value of each of these product projected aggregate cash flows from the portfolio.

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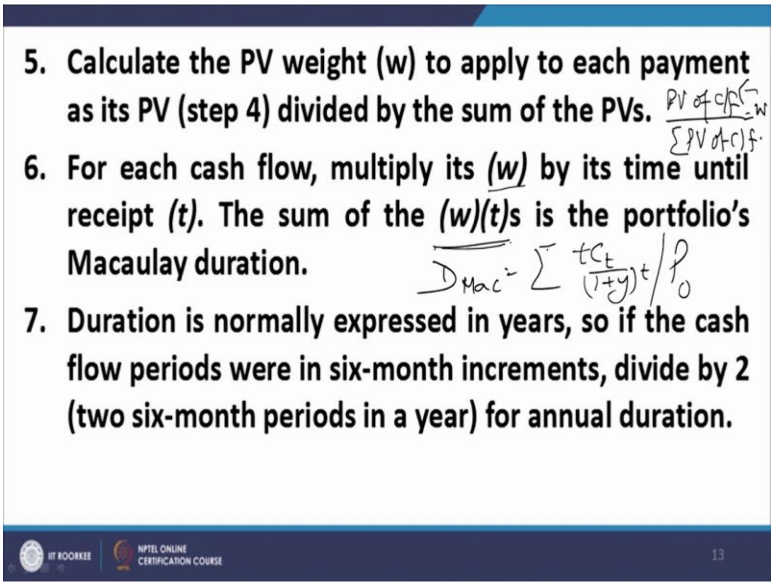

So these portfolio, these present values will now constitute the weights, as you can see here. Calculate the present value weight to apply to each payment as its present value which is calculated in Step 4, divided by the current, divided by the market price which will be equal to the sum of these present values.

So let me repeat. Step 1, arrange the cash flows in terms of their timings for each bond. Step 2, calculate the aggregate cash flows at each point in time at which there occurs the cash flow from any of the constituents of the portfolio where there is no cash flow from a particular constituent but there is a cash flow from another constituent, take, impute the cash flow as 0 and work out the aggregate cash flow.

Step 3, calculate the IRR of the stream with the initial price at being the outflow. And Step 4, work out the present value of each of these cash flows. At what rate? At the IRR. So these are the four steps. And Step 5, calculate the weights. What are those weights? The present value of the cash flow worked out at the IRR, divided by the sum of the present values which will represent the market price of the instrument.

For each cash flow, multiply this weight y. This weight y is what? This weight y is the present value, present value of cash flow divided by sum of market price or you can say sum of present value of cash flows. So present value of a particular cash flow divided by the aggregate of all cash flows occurring during the life of the instrument, excluding the t equal to 0 cash flow in this particular case.

So for each cash flow, multiply its weight, this is w, this, the figure that we have calculated is w, for the that particular cash flow which is represented in the numerator. So for each cash flow, multiply its w by its time, until received. So whatever time, at whatever time this particular cash flow is received, multiply that time by w. w into t. This.

So this is how the portfolio's Macaulay duration is calculated. Pretty straight forward. The formula as you know, we have discussed it a number of time, is t into C t divided by 1 plus y to the power t, the whole thing divided by P 0. This is D Mac. Duration is normally expressed in years. So if the cash flow periods were in six-month increments, divide by 2 for annual duration. So that is the convention in the event that we are using the unit of time as six-month periods instead of 1-year period.

Similarly, if it is a quarterly cash flow bond or a portfolio of bonds giving you quarterly cash flows, then you have to adjust the duration calculation accordingly. Use that quarterly period for calculating the duration, and multiply it by 4. I am sorry, divide it by 4 to get the annual duration.

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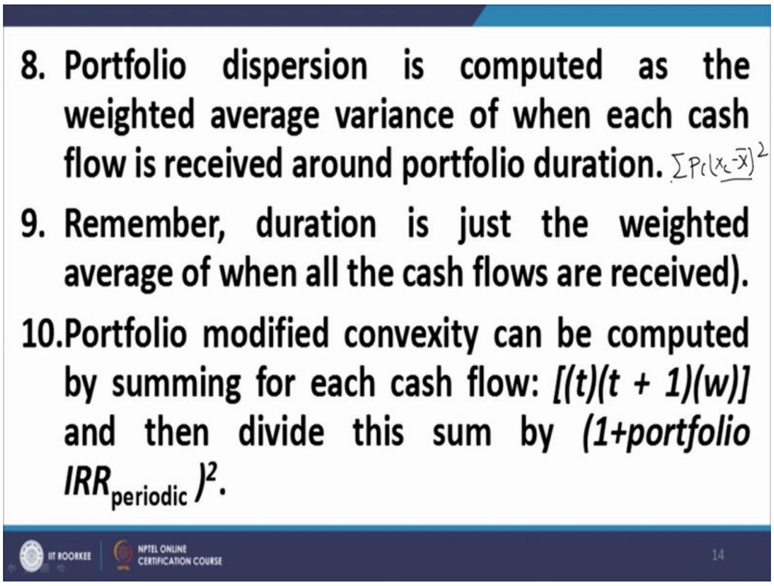

Portfolio dispersion is computed as the weighted average variance of when each cash flow is received around the portfolio duration. It is something very similar to the conventional definition of variance. And what is the conventional definition of variance? We write it as summation of P i into x i minus x bar whole square.

So in, in this case, x bar is nothing but the duration, x i is the timing of that particular cash flow, and P i is the weight that we calculated earlier in the preceding step, that was w. So that is the one to one correspondence between the calculation of dispersion in the context of bond portfolios and this calculation of variance, which is conventionally done in statistics.

P i corresponds to w i, the weight of that particular cash flow, x i represents the timing of that cash flows starting from t equal to 0 onwards, and x bar represents the duration of that, of that bond. So that, this quantity x minus x bar basically represents that what is the timing of that cash flow with the reference to the duration of the bond, which is a measure of dispersion using this formula.

Remember, duration is just the weighted average of when all the cash flows are received. We have discussed this just now. Portfolio modified convexity can be computed by summing for each cash flow t into t plus 1 into w and divided by 1 plus IRR whole squared. So that is the formula that again, we have discussed in a lot of detail when we talked about the definition of convexity. It is pretty much the same thing, a repetition, rather.

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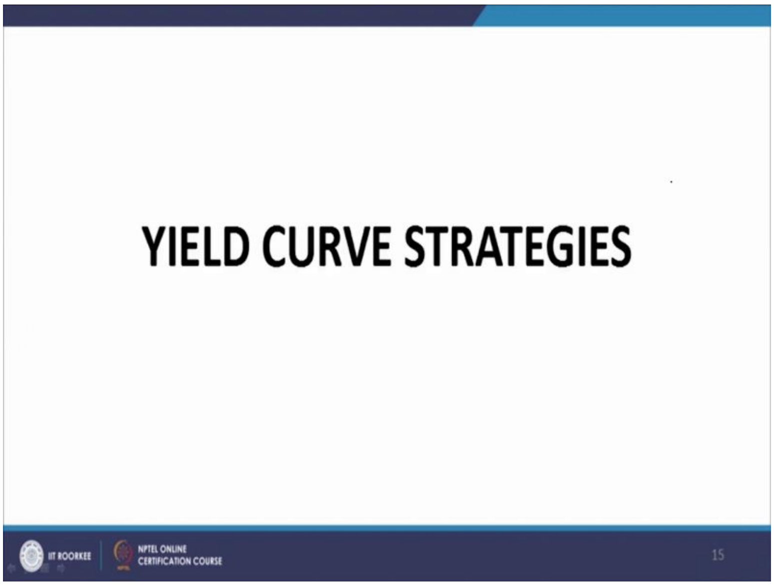

Now we talk about yield curve strategies.

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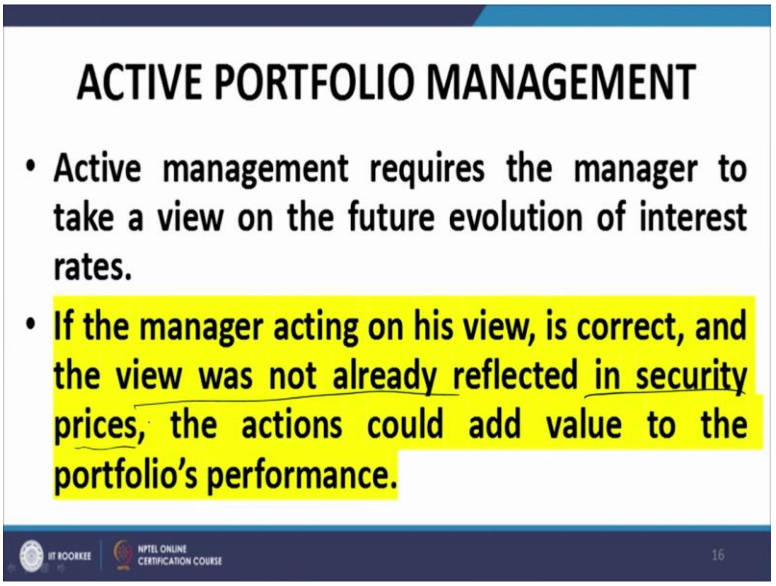

Active portfolio management is pretty much a well-known term. So let us quickly run through it. Active portfolio requires the manager to take a view on the future evolution of interest rates. If the manager acting on his view is correct and the view is not already reflected, this is the catch, the view is not already reflected in the security prices.

That means, the markets perspective or the market's perception is at variance with the analyst perspective or analyst perception, or rather, portfolio manager's perception of the future, evolution of the prices of his portfolio or the constituents of the portfolio is different from that of the market.

Then, and his perception turns out to be correct, then obviously, he is going to make profits out of, out of encashing his perception because the, the market perception loses out because it does not turn out to be correct, his perception turns out to be correct, and that is how he makes a profit.

That is the entire game of portfolio management, the perception of a particular analyst, the perception of a particular portfolio manager vis-à-vis the markets perception, because if the market perception coincides with the portfolio manager's perception, then that means what?

That means the portfolio managers perception is already encoded into the prices, and then therefore, you cannot extract any profit in that situation. It is only when the portfolio manager's perception is at variance with the market's perception and the portfolio manager's perception turns out to be correct, that he turns out, that he would be making a profit.

Because he, acting in consonance with his perception would construct a strategy that would benefit him if his perception turns out to be true, and consequently, if his perception actually turns out to be true, that strategy would give him high returns from his efforts.

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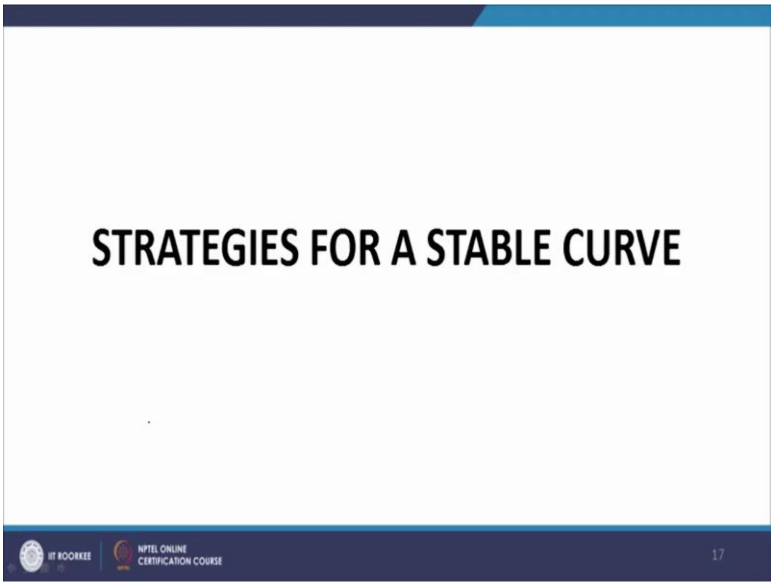

Strategies for a stable yield curve, if you assume that the yield curve is stable.

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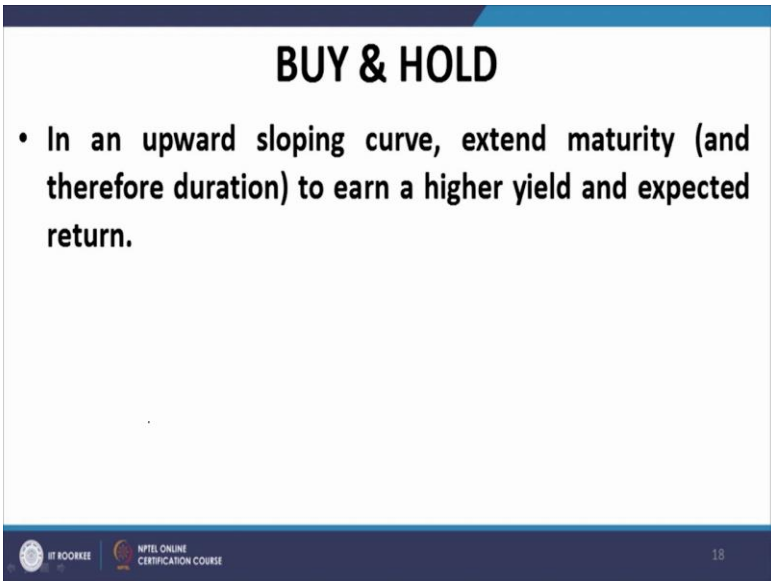

In an upward sloping yield curve, extend maturity, and therefore duration, toward higher yield and expected return. I will illustrate this with an example, it is pretty straightforward and it is very interesting as well.

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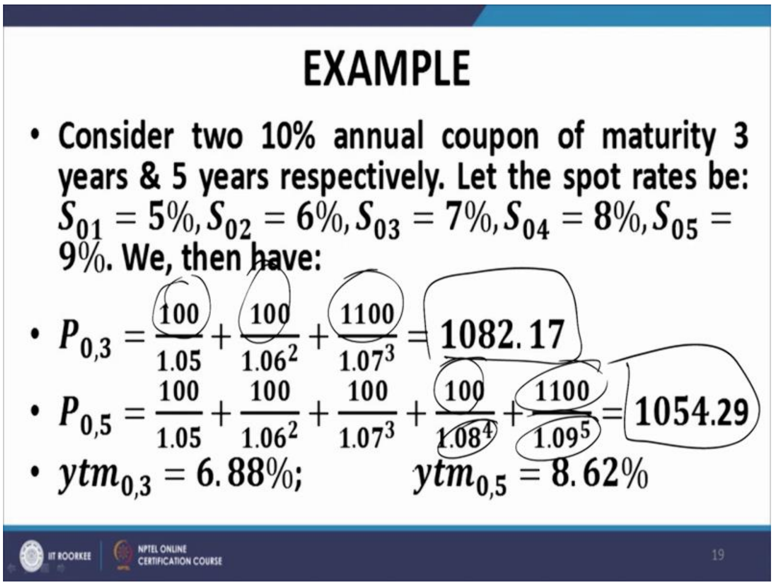

Now, in this example, what you have is an upward sloping yield curve. We have S 01 is equal to 5 percent, S 02 is equal to 6 percent, S 03 is 7 percent, S 04 is 8 percent and S 05 is 9 percent. So clearly, it is an upward sloping yield curve. The spot rates are increasing with the maturity of the underlying instrument.

Now we have got two 10 percent annual coupon bonds. One is of maturity 3 years, and the second is of maturity 5 years. A, let us say, bond A is of maturity 3 years, and bond B is of maturity 5 years. The price of pond A at t equal to 0 is equal to 1,082.17. And the price of bond B at t equal to 0 turns out to be 1,054.29.

And when we work out the YTM of these bonds, using these prices, when we work out the YTM of this bonds using these cash flows, this cash flow t equal to 1 this is t equal to 2, this is t equal to 3 and this is t equal to 4, this is t equal to 5, using this cash flows for, respectively for bonds A and for bond B, if we work out the YTMs, we find that the YTM of bond A turns out to be 6.88 percent, and the YTM of 1b turns out to be 8.62 percent. This is naturally so because of these figures.

You see, if the cash flow that is going to be, that is going to be realized at t equal to 4 years in the case of bond B is giving you a return of 8 percent, and the cash flow of 1,100 that is going to arise at t equal to 5 years is giving you a return of 9 percent, so naturally the YTM which is, in some sense, the weighted return is, I repeat that in some sense, the weighted return in the case of bond B would naturally be higher in the case of an upward sloping yield curve because the later, the later year cash flows would be giving you a higher return in the case of an upward sloping yield curve.

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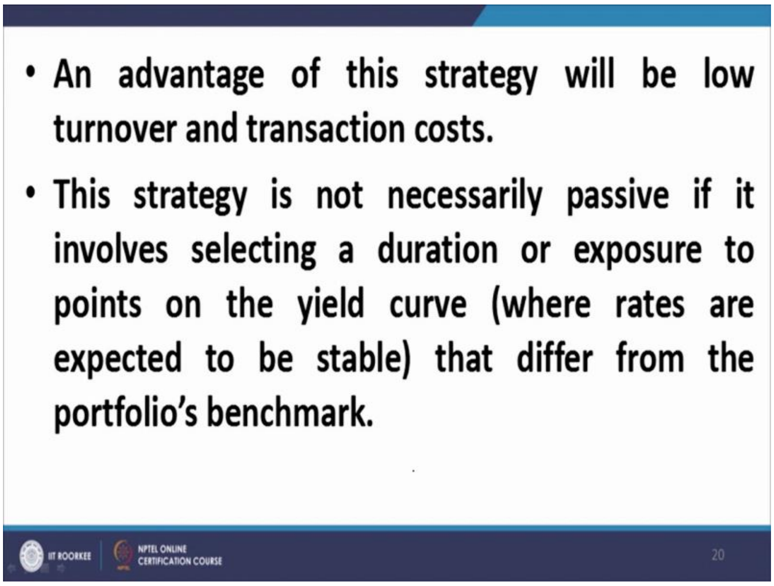

An advantage of this strategy will be low turnover and transaction cost. It is a buy and hold strategy, approximately. And therefore, the, naturally the transaction cost would be very, very low, insignificant, literally. And the turnover would also be very small. This strategy is not necessarily passive if it involves selecting a duration or exposure to points on the yield curve where rates are expected to be stable.

That differs from the portfolio's benchmark. So if you are also managing the duration and convexity, then naturally you will have to take care that as time progresses there is not, there does not arise significant divergence between what your benchmark is and what is the progression of your portfolio.

So in that event, you may have to rebalance from time to time. Otherwise, if that constraint is not there, if the constraint of matching of duration or managing of duration, convexity et cetera is not there, then it is pretty much a buy and hold strategy, and quite a simple strategy as well.

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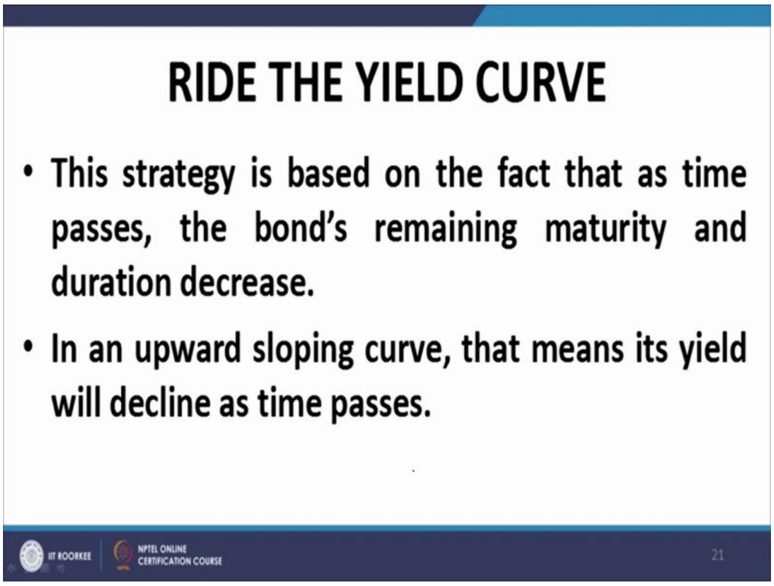

Ride the yield curve. This is the second strategy, if you have a stable yield curve. This is also very interesting. Let us just read through it and then again, I will illustrate this with an example. This strategy is based on the fact that as time passes, the bond's remaining maturity and duration decrease. In an upward sloping curve, that means its yield will decline as time passes and therefore its price will increase. That is the basic.

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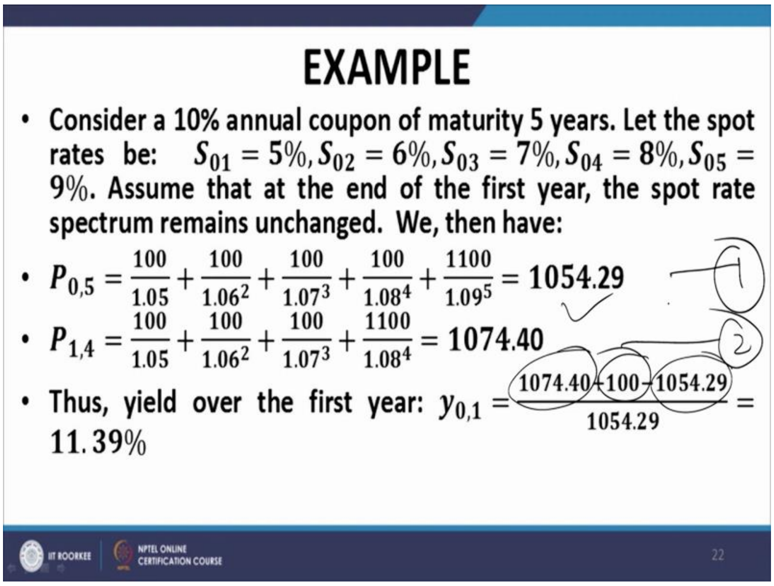

For example, let me first give you a typical example. Let us say you have a bond investment of over two years. Now at t equal to 0 when you value the bond, you will be using S 01 and S 02 for valuing the cash flows occurring at t equal to 1 and occurring at t equal to 2. That will give you the current market price of the bond.

Now, as time passes you, you buy that bond at t equal to 0 and you hold on to it up to t equal to 1. Now, at t equal to 1, assuming that the spectrum of spot rates that were there at t equal to 0 continue to be holding forth, when you liquidate the investment at t equal to 1, what will happen?

You will liquidate the investment at a price which will be determined on the basis of S 01 because the maturity of the, of the bond at t equal to 1 is only 1 year. It was a 2 year bond, you, you bought it at t equal to 0, at t equal to 1 the remaining maturity is only 1 year. So when you value the bond or when the market value of the bond at which you are going to sell the bond, it will be equal to the final cash flow divided by the spot rate corresponding to a 1 year maturity.

And as it is an upward sloping curve, the 1 year maturity rate would be the lowest rate that would be there on the curve. And therefore, the corresponding price at which you are going to liquidate the bond in the market would be higher than the price that would be computed if it was a flat yield curve. So that is illustrated by an example, let us also go through this example.

Consider a 10 percent annual bond of maturity 5 years. Let the spot rates be S 01 is 5, S 02 is 6, S 03 is 7, S 04 is 8 and S 05 is 9 percent. All of these are per annum, of course. Assume that at the end of the first year, the spot rates spectrum remains unchanged. That means these rates hold at t equal to 1 as well.

What does it mean? That means that the price of this bond at which you enter the investment, that is the t equal to 0 price of this bond as worked out as per equation number 1 here, is 1,054.29. And when you work out the price at t equal to 1, that is, at the end of 1 year, after receiving the coupon for the first year, the price is worked out as per equation number 2, and it turns out to be 1,074.40.

The important thing that you need to note here is that when you work out the price at t equal to 1 year, you are using what rates? You are using the rates S 01, S 02, S 03 an S 04 because the remaining life of the bond is 4 years now. So the rates that would be relevant for valuing the bond at t equal to 1 year it would be S 01 to S 04. And that is precisely what is shown in this example, equation number 2.

So what is the yield for you? The yield for you is 1,074.40, the price at which you sell the bond plus the coupon payment that you receive at t equal to 1 year minus the entry value of the investment, that is, 1,054.29, that is, this figure, and divided by the entry value of the investment that turns out to be 11.39 percent, which is significantly higher than the 1 year rate of 5 percent. Indeed, it is higher than S 05 as well. It turns out to be 11.39 percent.

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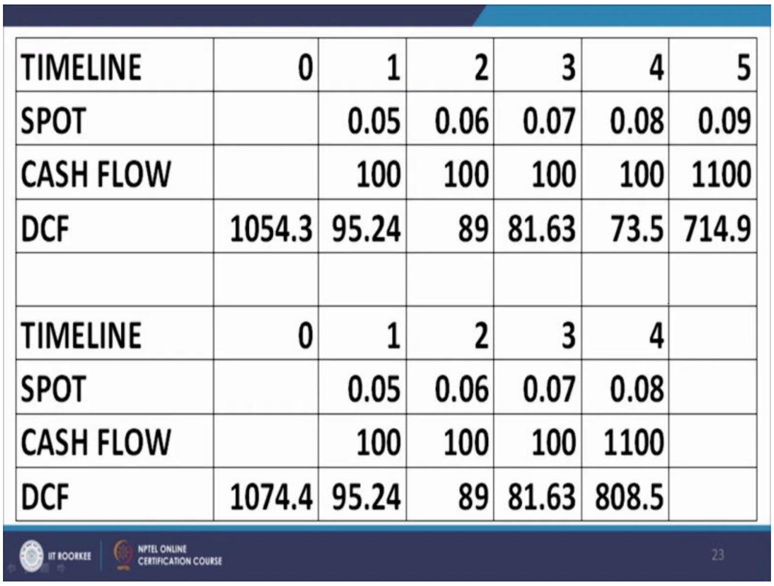

TIMELINE	0	1	2	3	4	5
SPOT		0.05	0.06	0.07	0.08	0.09
CASH FLOW		100	100	100	100	1100
DCF	1054.3	95.24	89	81.63	73.5	714.9
TIMELINE	0	1	2	3	4	
SPOT		0.05	0.06	0.07	0.08	
CASH FLOW		100	100	100	1100	
DCF	1074.4	95.24	89	81.63	808.5	

This is the excel calculation of those figures that I mentioned just now.

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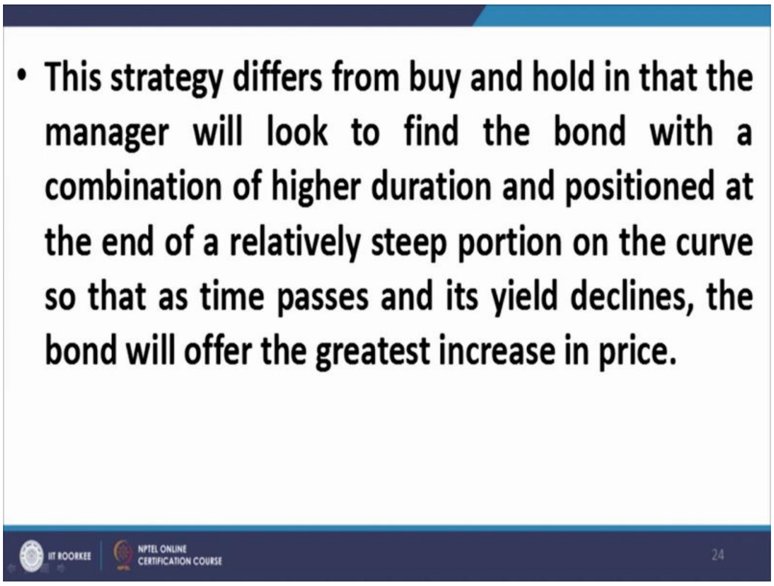

This strategy differs from the buy and hold strategy in that the manager will look to find the bond with a combination of higher duration and positioned at the end of a relatively steep proportion of the curve. Greater the steepness of the curve, greater would be the returns realized from the strategy. So that, as time passes and its yield declines, the bond will offer the greater increase in price, the greatest increase in price.

Greater is the steepness, greater would be the, the slope of the curve, and correspondingly greater would be the return that would be realized by doing the strategy because the rate of decline of the interest rates would be higher as the maturity decreases, and as a result of which, when you compute the bond's price at the end of a holding period, you will be computing at relatively lower rates, and relatively lower rates means higher price. So that is the philosophy behind the strategy.

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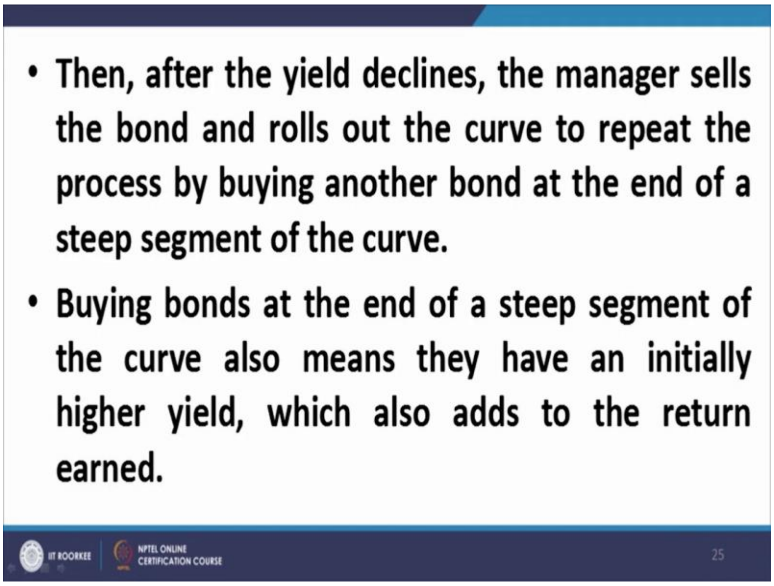

Then, after the yield declines, the manager sells the bonds and rolls out the curve to repeat the process by buying another bond at the end of a steep segment of the curve. Buying bonds at the end of a steep segment of the curve also means that they have an initially higher yield which also adds to the return earned. That is another important point. Higher the yield, lower the price at entry.

So this, greater the steepness as far as the far distance rates are concerned, higher would be the distance rates, and lower, because they would be contributing at the point of entry of the investment. Therefore, the entry price would be lower, the exit price would be higher, and the returns would be higher. So I shall continue in the next lecturer. Thank you.