Quantitative Investment Management
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Lecture 28
Yield Shifts and Immunization

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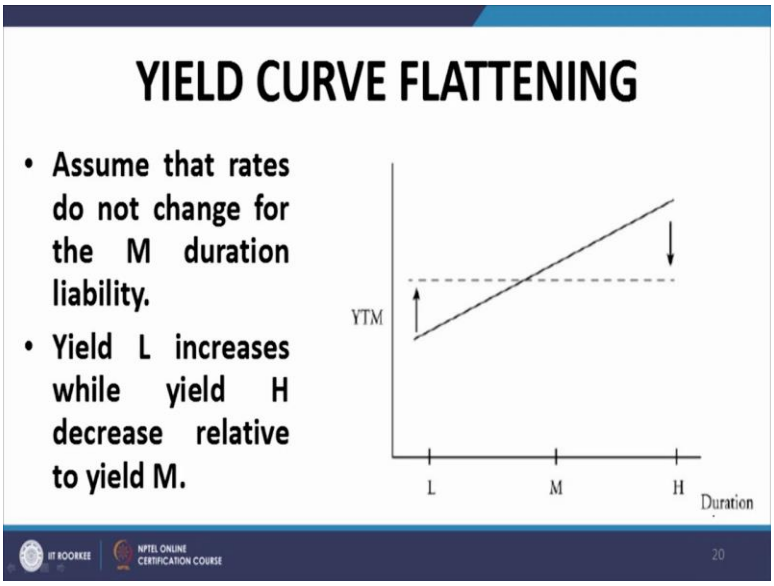

So let us continue from where we left off before the break. I have talked about yield curve steepening, yield curve flattening is pretty much the inverse of that. What happens here is that the near term rates increase and the long term rates decrease, and the slope of the yield curve tends to decline, whereas the intermediate term rates of course do not change, or change marginally.

So in this case, we assume that the rates do not change for the M duration liability that is the bullet liability, and yield L increases and the yield H decreases relative to the yield M, that is the near term rates increase and the long term rates decrease. So in this case, as a, in contra distinction to the yield curve steepening case, the bond with the lower duration the low maturity bond because the rates have increased, will decrease in value but that decrease in my value will be small.

Whereas the increase in value of the high maturity, high duration bond H will be much more. Why? Because its duration is larger, and therefore if the impact of a equal change in the interest rates will impact this high maturity bond much more significantly. The net result would be that the present value of the assets would increase.

I repeat, the low maturity, low duration bond price will decrease but will decrease by a small amount because it has a small duration, whereas the high maturity, high duration bond H will increase in price because the interest rates have declined and the increase will be quite significant because the maturity or the duration of the bond is higher.

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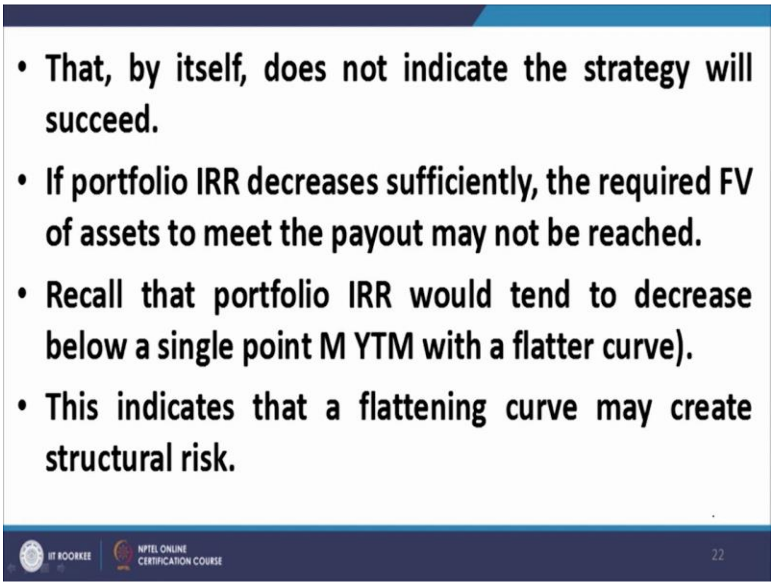

As far as the present value of liabilities is concerned, it will not change because we are assuming that the intermediate rates are unchanged by the flattening of the curve. So the portfolio market value will increase because the increase in value of the longer duration bond will exceed the decrease in value of the shorter duration bond.

Present value of liabilities will remain unchanged with no change in yield of M, and therefore the present value of assets will now be above present value of liabilities assuming that the two were equal to start with before the change, before the shift. The impact of the shift will be that the present value of assets will now be above the present value of liabilities.

But just as in the case of the steepening of the curve, this does not of itself mandate or guarantee conclusively that the strategy would definitely succeed. The issue of the IRR of the portfolio, the impact on the IRR of the portfolio corresponding to the flattening of the curve needs to be explored before we can conclude that the strategy has succeeded in reality or not. So that is the issue that is highlighted on the slide.

That, by itself, does not indicate that the strategy will succeed. If portfolio IRR decreases sufficiently, the required future value of assets to meet the payout may not be reached. If due to this flattening of the curve, because you see what is happening is long term interest rates are declining.

And if the impact is significant enough, then what will happen is the overall portfolio IRR will be reduced to such an extent that notwithstanding the fact that PV of A is higher than PV of L, the future value of A may not be adequate to meet the future value of L and we may end up with the future value of liabilities being higher because they would be growing at a higher rate relative to future value of assets, which will be growing at a lower rate.

Therefore, notwithstanding the fact that future value of assets is a, present value of assets, I am sorry, is higher, it, the future value of assets will turn out to be lower than the future value of liabilities. So recall that portfolio IRR would tend to decrease below a single point M YTM with the flatter curve. This indicates that a flattening curve may create structural risk.

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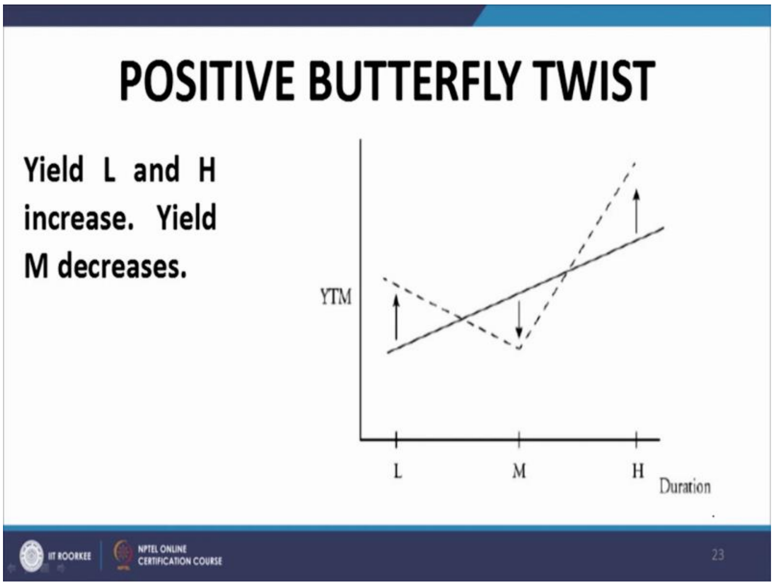

Then we have the positive butterfly spread. Yields L and H increase and yield M decreases. Now clearly, because yield L and H both are increasing, the prices of both the bonds, L bond and H bond will decrease. The price of L bond would decrease less, the price of H bond will decrease more, but, because the duration of H is more, duration of L is less.

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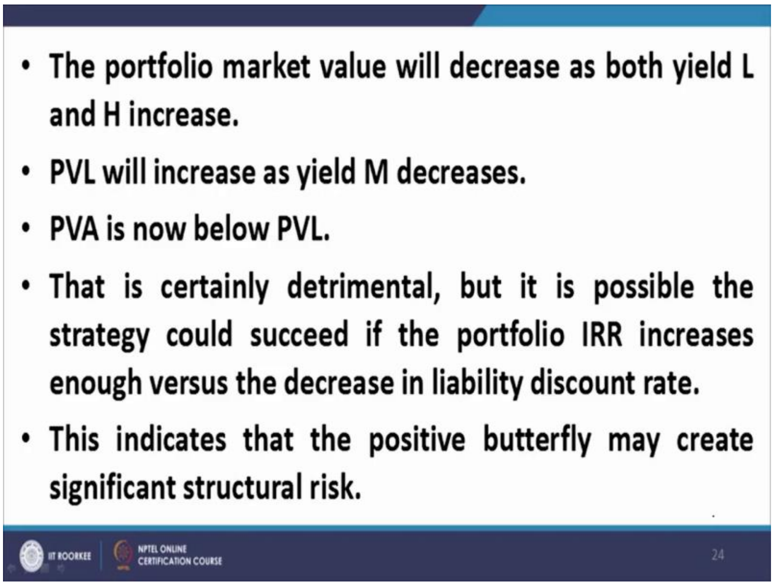

But notwithstanding the fact the decree, the prices of both of them will decline. And because the yield of M has decreased, the price of M will increase and as a result of which t equal to 0 subsequent to the positive butterfly twist what happens is that the present value of liabilities becomes significantly more than the present value of assets.

So that is what is given here. The portfolio market value will decrease as both yield and H increase, as both yield L and H increase. Present value of liabilities will increase as yield M decreases. Present value of assets is now much below present value of liabilities. That is certainly detrimental, but it is not absolutely conclusively establishing the failure of the strategy.

It is possible that the strategy could still succeed if the portfolio IRR increases sufficiently enough such that, notwithstanding the fact that present value of assets has declined below present value of liabilities, the portfolio IRR of assets has increased sufficiently that at the time of maturity of the liability, the value of assets grows sufficiently rapidly to meet the payout as on the date of maturity of the liability which would be growing at a rapidly at a relatively lower IRR. So this indicates that the positive butterfly may create significant structural risk.

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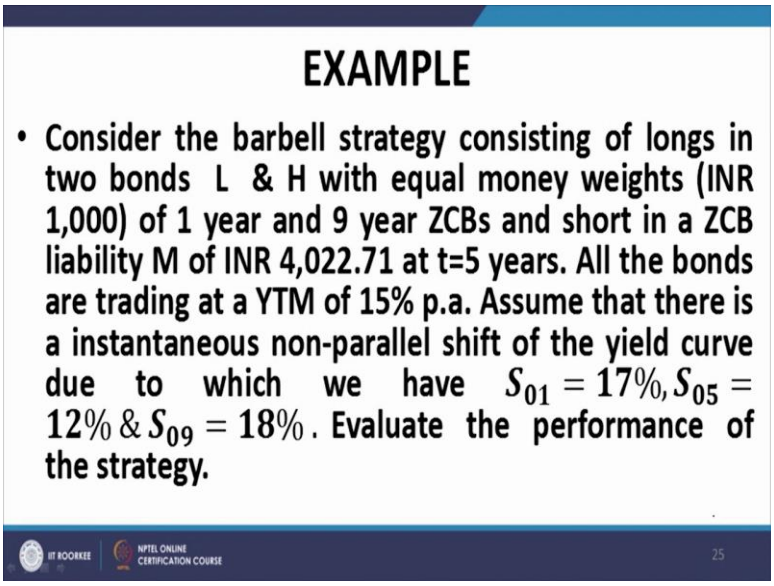

This is an example of the butterfly spread. Let us look at it. Butterfly spray, twist, I am sorry. Consider the barbell strategy consisting of longs in two bonds L and H with equal money weights INR 1,000 of 1 year and 9 year zero coupon bonds and short in a zero coupon liability M of INR 4,022.71 at t equal to 5 years. All the bonds are trading at a YTM of 15 percent.

Assume that there is an instantaneous non-parallel shift of the yield curve due to which you have S 01 has gone from 15 percent to 17 percent, S 05 has declined to 12 percent from 15 percent, and S 09 has increased from 15 percent to 18 percent. So both S 01 and S 09 have increased, L and H have both increased. L has increased by 2 percent from 15 to 17, H has increased from 15 to 18 percent, whereas M has decreased from 15 to 12 percent.

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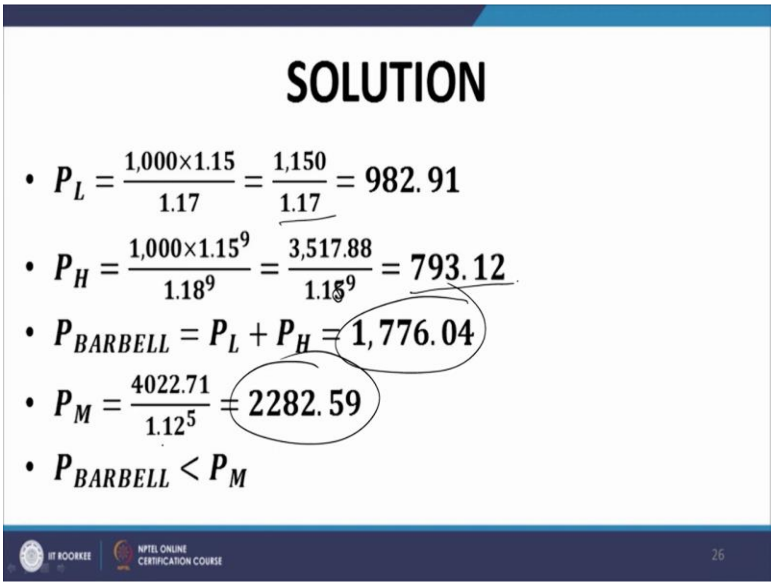

Now, let us look at the solution. The present value of the L bond now becomes 982.91 at the rate of 17 percent, you can see here. The present value of the H bond which has a maturity value of 3,517.88 at the end of 9 years at a YTM of 15 percent is now evaluated at the rate of 18 percent and that turns out to be, this should be 18, and it turns out to be 793.12, 793.12.

So the barbell strategy has a present value that is t equal to 0 value after the shift has taken place of 1,776.04. You can see that it is far below the present value of the liabilities which has increased from 2,000 at 15 percent to 2,282.59 at 12 percent. So the present value of the assets has gone far below the present value of the liabilities. The P BARBELL is less than P liability.

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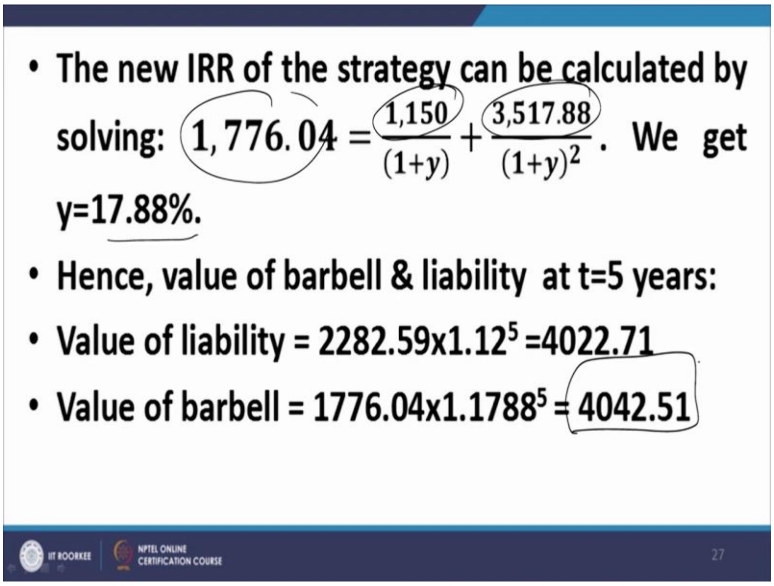

The new IRR of the strategy works out to 17.88 percent computed in the manner that I explained before the break. This is the t equal to 1 cash flow from the 1 year maturity bond. This is the t equal to 9 cash flow from the 9 year maturity bond, and this is the current price of the bond.

When you solve this equation as a quadratic, what you get is y is equal to 17.88 percent. And using this growth rate, what happens is that the growth, that the future value of the barbell on the date of maturity of the strategy works out to 4,042.51. This is against the future value of the liability which is 4,022.71.

So not understanding the fact that PVA has gone far below PVL due to the unfavorable changes in interest rates, what happens is that the IRR has changed so much favorably that as on the date of maturity of the liability, we still end up with a little bit of surplus after the payout of the liability.

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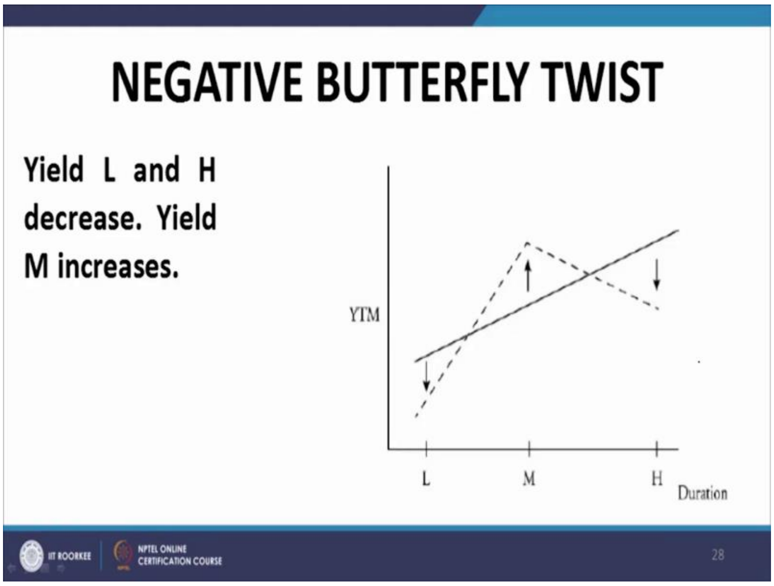

Negative butterfly spread is pretty much similar, so I will not devote time to it.

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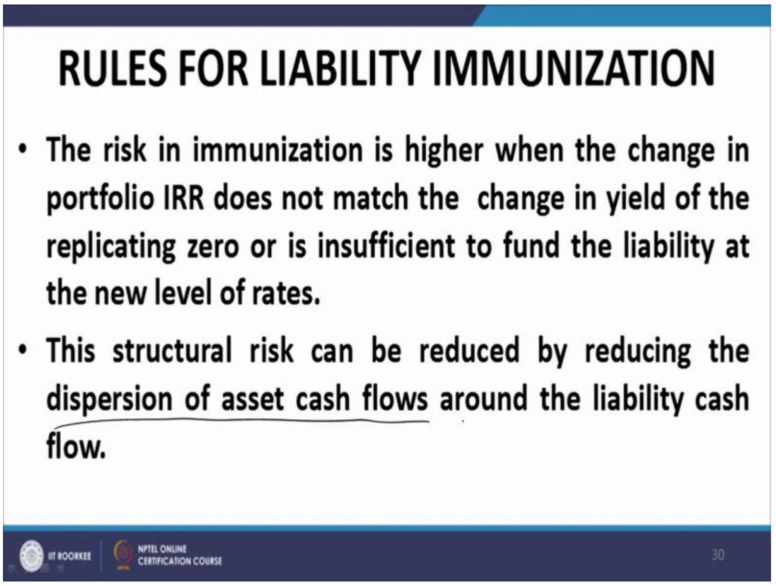

Now, we talk about rules for liability immunization. This is, in some sense, the summary of what I have been talking about in the last couple of lectures. So let us quickly read through it. The risk in immunization is higher when the change in portfolio IRR does not match the change in yield of the replicating zero or is insufficient to fund the liability at the new level of rates. So that is important.

The IRR, the portfolio IRR, the impact on the portfolio IRR of the shift in the yield curve, whatsoever shift it may be in the yield curve, needs to be evaluated with care, needs to be considered with care because it may so happen that the present value of assets may go up compared to the present value of liabilities, but the change in IRR may be so unfavorable that the, at the maturity rate of the liability, the payout date of the liability, you may not have adequate funds because the assets have not grown at the rate that you would desire because the IRR has gone down due to the unfavorable shift in the yield curve.

The structural shift can be reduced by reducing the dispersion of asset cash flows around the liability cash flows. The lesser is the dispersion of the asset cash flows around the liabilities cash flows, the lesser is the chance of a failure of the strategy on account of twisting, or on account of non-parallel shifts of the yield curve.

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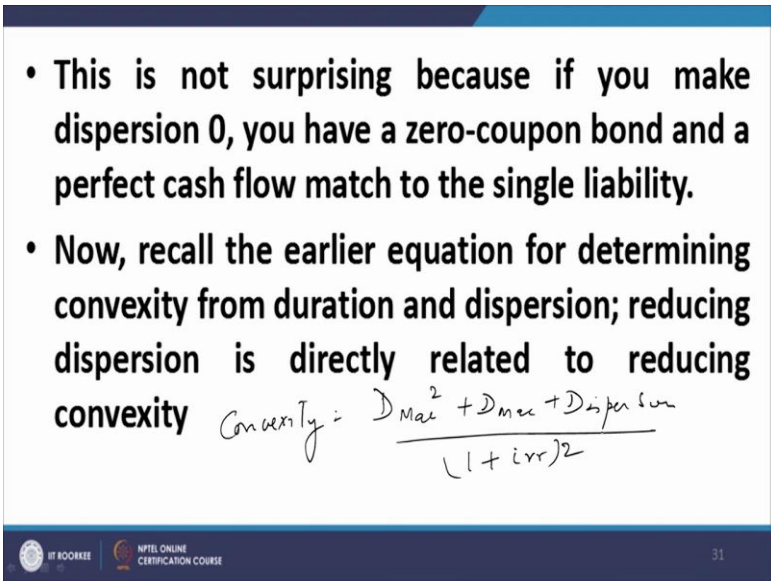

This is not surprising because if you make dispersion 0, you have a zero coupon bond and a perfect cash flow match to the single liability. So dispersion 0 implies if you have a strategy with a dispersion 0 that can only be when you match the liability, the single coupon liability, the zero coupon liability, I am sorry, with a zero coupon bond of the same maturity and par value.

So putting it the other way around, if you have a perfect cash flow match of the liability then you can say that the dispersion is 0. And if the dispersion is 0, then the risk is almost negligible if the default risk is not considered. Now, recall the earlier equation for determining convexity from duration and dispersion, reducing dispersion is directly related to reducing convexity.

You would recall what was that equation. Convexity is equal to D Mac square plus D Mac plus dispersion. So you can, you can you can see here that convexity and dispersion are directly related, and therefore a bond with a higher dispersion will have higher convexity and a bond with a lower dispersion will have lower convexity. So that is the, in a, in a sense that is the price we pay when we try to increase the convexity because if the convexity is higher, then the dispersion will be higher.

And if the dispersion is higher the variability of cash flows will be higher and because, if the variability of cash flows is higher, the, the immunization strategy may, may not be that foolproof, may not perform that precisely because at the end of the day if there is a non-parallel shift, then the increased convexity may, or increase dispersion may mean that this strategy will not perform optimally. So that is the issue.

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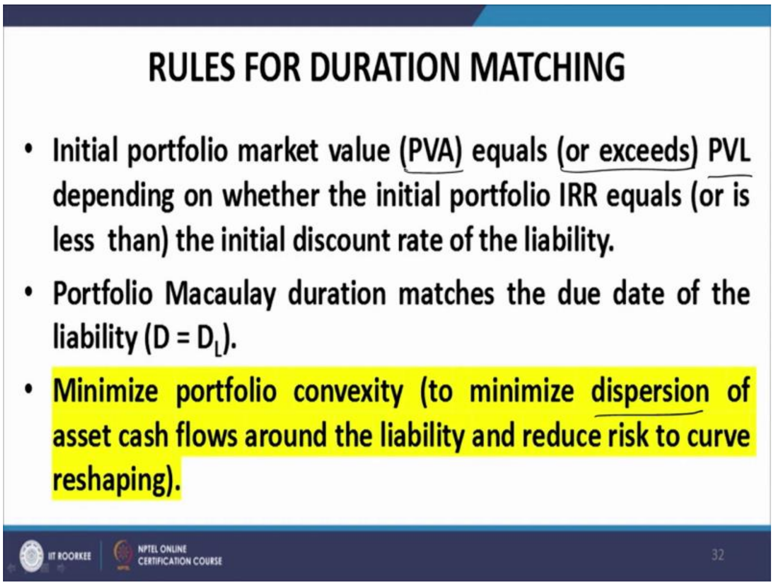

Rules for duration matching. Initial portfolio market value, present value of assets equals. So these are the rules when we try to immunize a liability using a portfolio of assets, portfolio bonds. Initial portfolio market value, present value of assets equals or exceeds, or exceeds the present value of liabilities depending on whether the initial portfolio IRR, that is the IRR before any shift, IRR at the time of planning, the initial portfolio IRR equals or is less than the initial discount rate for the liability.

Now, if it so happens that the initial portfolio IRR is less than the discount rate for the liability, then naturally you have to provide for the lesser rate of growth of the assets compared to the liability to ensure the same payout value of the assets and liabilities. That means what?

That means if the higher of the liability is higher, and the IRR of the assets is lower, assets are growing at a lower rate, liability is growing at a higher rate, then we need to start with a higher level of assets to compensate for the reduced growth rate of the assets so that we can end up with the same value of assets and liabilities as on the date of maturity of the liability that is to be redeemed.

So initial portfolio market value present value of assets that is equals or exceeds, equals or exceeds depending on the IRR of the portfolio of a service the liability of YTM, exceeds present value of liabilities depending on whether the initial portfolio IRR equals to or is less than the initial discount rate of the liability.

Portfolios Macaulay duration matches the due date of the liability because the Macaulay, you see, this is essentially duration matching. The duration of the, the Macaulay duration of the liability is equal to its maturity. So essentially, what we are talking about is duration matching of assets and liabilities.

Minimize portfolio convexity to minimize dispersion. So that if there is a, a non-parallel shift in interest rates and that is not heavily detrimental to our strategy, our strategy does not completely fail in the event of a steepening or a curvature change or a twist of the yield press, yield curve, not yield price curve.

So minimize portfolio convexity to minimize dispersion of asset cash flows around the liability and reduce risk to curve reshaping. That is important. The lesser is the convexity, lesser will be the dispersion, lesser the dispersion, lesser the risk arising out of reshaping of the curve, arising out of steepening, curvature, or, or twist effects of the yield curve, spot yield curve.

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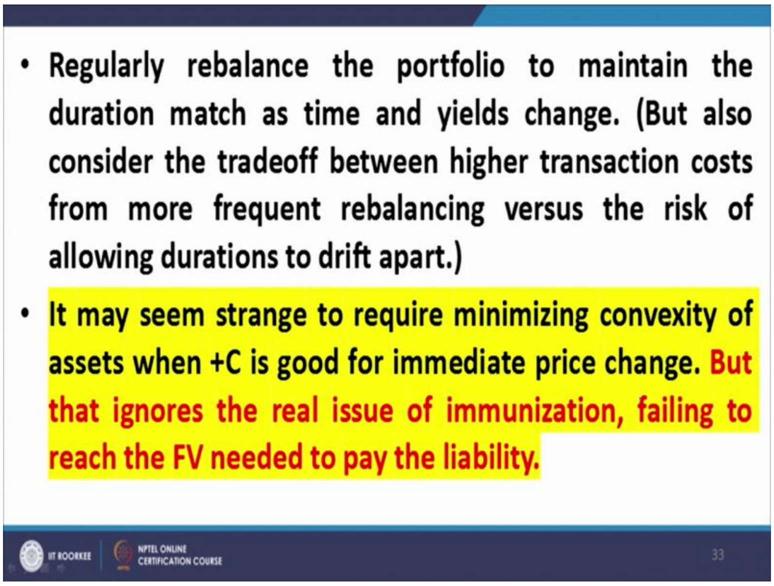

Regularly rebalance the portfolio to maintain the duration match as time and yields change, but also consider the tradeoff between higher transaction costs from more frequent rebalancing versus the risk of allowing durations to drift apart. This has been explained earlier.

The rate of decline of duration as the bond bonds approach their maturity is, is less than the rate of declines of their maturity in the case of coupon bonds. And this has to be considered and you have to adopt a active strategy to be, to remain immunized over a sustained period of time.

It may seem strange to require minimum convection, this is important. That is why I have highlighted in yellow and red. It is, it may seem strange to require minimizing convexity of assets when plus C is good for immediate price change. Remember, higher the convexity, higher is the positive price change, positive highlighted, positive emphasized.

So higher the convexity, higher is the positive pricing. In other words, given two bonds of identical duration, if there is a decline in, in interest rates, the bond with the higher convexity will show a higher price increase compared to the bond with a lower convexity. And if there is an increase in interest rates, the bond with the higher convexity will show a lesser price, price decline compared to the price decline of the bond with the lower convexity.

So both ways, convexity is good, convexity is beneficial. Not withstanding this, the problem is firstly convex, higher convexity means higher dispersion, higher dispersion means higher risk to reshaping of the yield curve. The second is that higher the convexity of a bond, market forces will operate in such a way that it will generate a lesser YTM, it would be trading at a lesser YTM.

But that ignores the real issue of humanization. What is the real issue of immunization? Following, failing to reach the future value needed to pay the liability. So that is the thing. Higher convexity implies higher variability, higher dispersion. We do not want that. But nonetheless, higher convexity gives us this higher positive correction as well. So it is a two edged sword, you may say.

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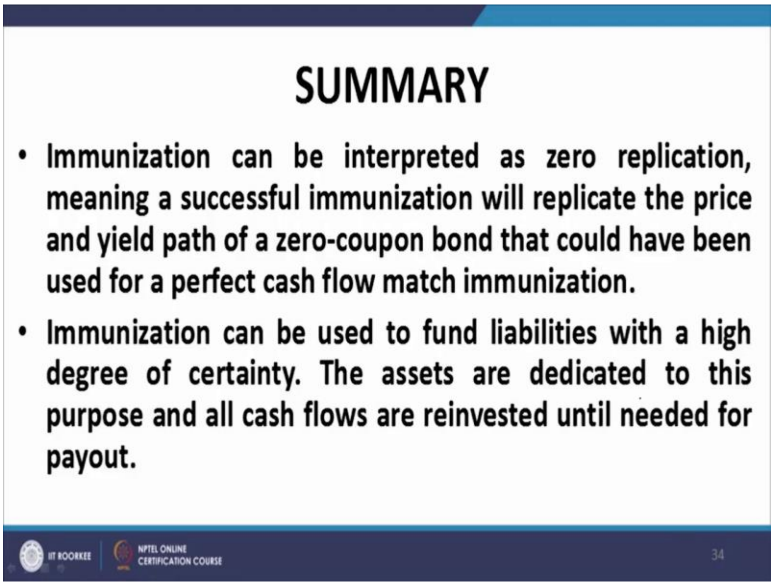

Summary. Immunization can be interpreted as zero replication. Meaning, a successful immunization will replicate the price and yield path of a zero-coupon bond that should have been used for a perfect cash flow match immunization. Immunization can be used to fund liabilities with a high degree of certainty. That is the purpose of immunization. What does it mean? It means that if you are immunizing a liability, you are protecting yourself from interest rate changes.

Interest rate changes will not impact the immunizing portfolio or at most, impact the immunizing portfolio only to the extent that the impact to liability. As a result of which, you do not feel, you do not end up being short of funds on the date of maturity of the liability. Immunization can be used to fund liabilities with a degree of certainty. The assets are dedicated to the purpose and all cash flows are reinvested until needed for payout.

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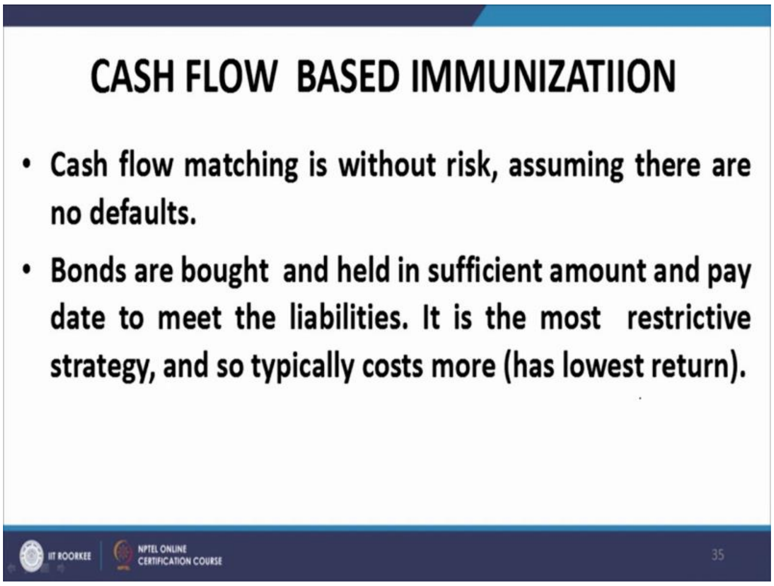

Cash flow based immunization, cash flow matching is without risk assuming there are no defaults. I mentioned that. Bonds are bought and held in sufficient amounts and pay, pay date to meet the liabilities. It is the most restrictive strategy and it typically costs more as lowest return.

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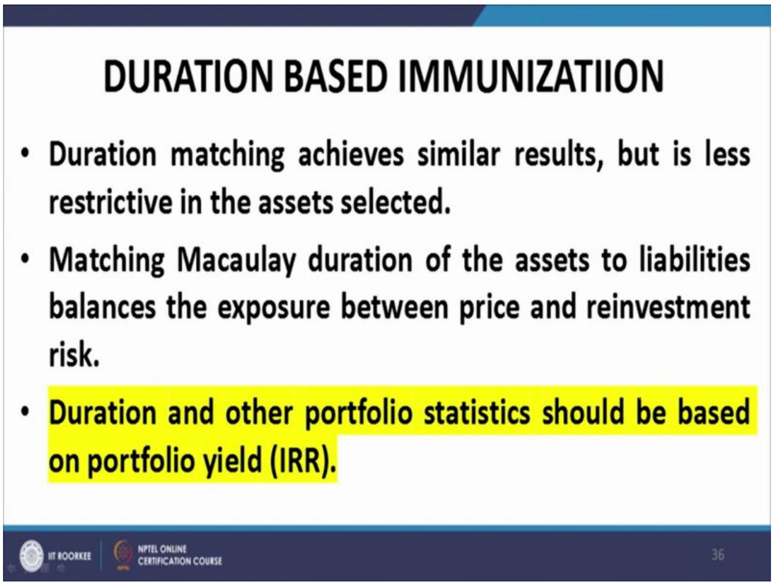

Duration based immunization. Duration matching achieves similar results, but is less restrictive. It is more flexible in terms of the assets that may be selected. You match Macaulay duration of the assets and liabilities, and the thereby you balance the exposure between price and reinvestment risk. Duration and other portfolio strategy, this is important. I will come back to it in a few minutes.

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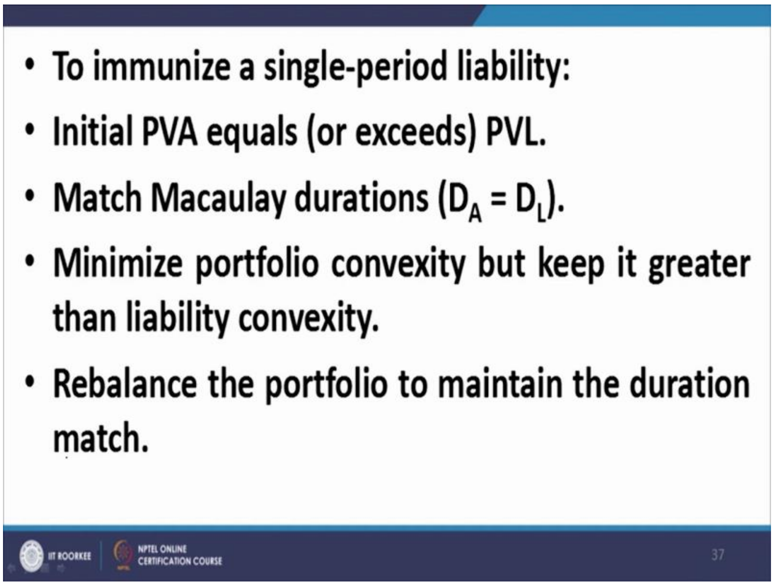

To immunize a single period liability, this is the summary of what I have been talking about, this is the gist of what I have been talking about. To immunize a single-period liability, initial PVA equals or exceeds PVL. Exceeds when? When the portfolio IRR or the immunizing portfolio IRR is less than the growth rate of the liability, then you have to compensate for this lesser growth rate by taking an initial position in present value of assets which is higher than the present value of liabilities. Match Macaulay duration, minimize portfolio convexity but keep it positive, keep it greater than the liability convexity.

The net convexity of the combination, of the immunized combination should be positive. In other words, the convexity of the assets should exceed the convexity of the liabilities. Lesser the convexity, subject to this condition of being positive for the immune system portfolio in totality, lesser would be the variability, lesser would be the dispersion. Higher the convexity, higher the variation, but higher is also the positive impact on the changes in prices. Rebalance the portfolio to match the duration, match.

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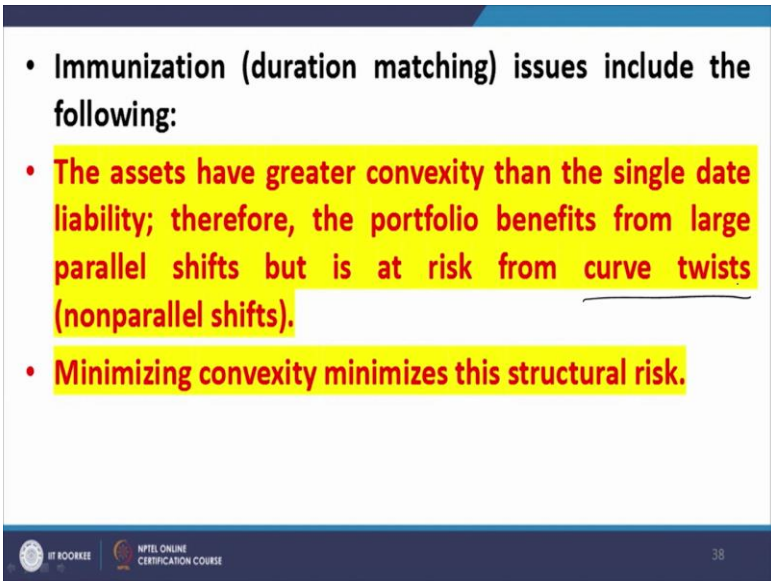

Immunization duration matching issues include the following. The assets have greater convexity than the liability. Therefore, the portfolio benefits from large parallel shifts but is at risk from curve twists. I repeat, at the cost of repeating, if the, if, if a portfolio has higher convexity, it is exposed to structural risk, it is exposed to non-parallel shifts. However, if, as far as parallel shifts are concerned, it would outperform a portfolio with al lower convexity. Immunizing, minimizing convexity minimizes the structural risk.

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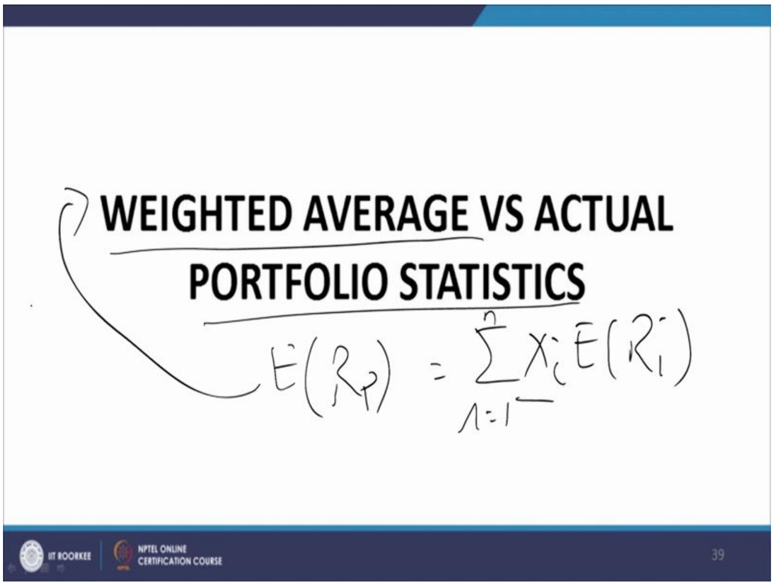

Now, I take up an important point which I have mentioned a couple of times during today's lecture. And let us say you are given two bonds, Bond A and Bond B, and you are constructing a portfolio comprising of Bond A and Bond B. Now, there are two ways. Let us take the concrete case of YTM. There are two ways in which you can work out the YTM of this combination.

Let me repeat, you construct a portfolio, let us call it P. The Portfolio P consists of a bond A and a bond B, a certain amount of investment in Bond A and certain amount of investment in Bond B, let us say X A in A and X B in B. Now the point is, suppose I want to work out the YTM of this portfolio P. There are two ways in which I can do it.

I can simply work out the weighted average of the YTMs of A and B, that is if X A and X B are the, are the weights or converting them into weights by divided by X A plus X B, that is a total investment, then you get W A into YTM of A plus W B into YTM of B. This is one way of doing it.

The other way of doing it, you work out the cash flows from the portfolio as a combination, and then from the cash flows of the combination and the today's market price of the combination, you work out ab initio, the YTM of the portfolio as if it comprised of a single asset.

So the, the difference is very significant, very important, and it, it becomes manifestly relevant when you are talking about immunizing a large portfolio or immunizing a liability of a large magnitude. So this is what is, what do what we mean by weighted average statistics or weighted average values of the various parameters and the portfolio statistics.

When I use the word portfolio statistics what I am doing is I am working out the statistics or parameters from the values of the cash flows that arise from the portfolio, treating them as a single asset. However, when I talk about the weighted average value of those parameters, I am simply taking a weighted average of those parameters just like we do weighted average return, weighted average expected return of a portfolio R P is equal to what, or E of R P is equal to summation X i E of R i.

This is called weighted average, where X i is the weight in, in a security i, sum over i equal to 1 to n. This is this particular thing. However, if I start from scratch, I treat, I aggregate all the cash flows at different points of time in the portfolio and then from there I work out the statistics, I work out the YTM or I work out the convexity or duration or whatever the case may be, dispersion, that is called the portfolio statistic.

And obviously, the second method is the correct method, the second method is the, is the appropriate method. Weighted average will only give you some kind of an approximation because these parameters do not scale in this way, do not, do not merge in this way when we talk about portfolio statistics. Portfolio strategies must be, for, for accurate work portfolio statistics must be considered treating the portfolio as a single asset, working out the aggregate cash flows at different percent time, and then working out the statistics from the stream of cash flows.

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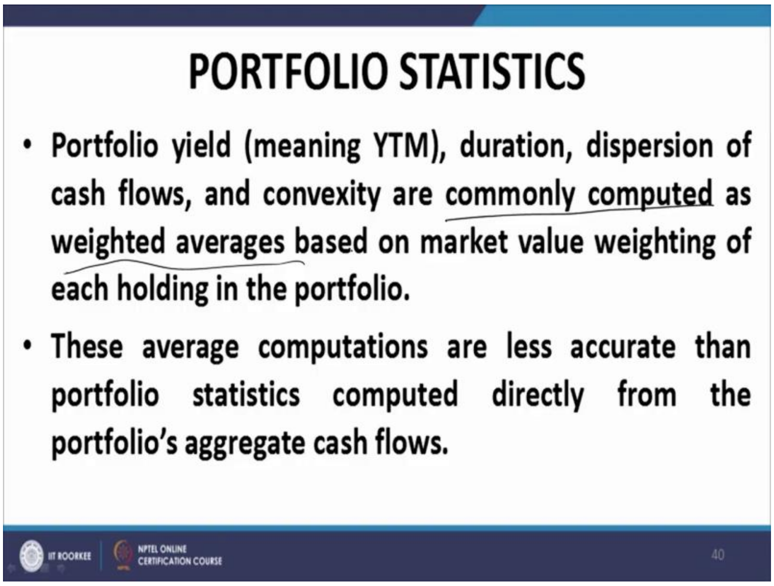

So, the portfolio yield meaning YTM duration dispersion of cash flows and convexity are commonly computed, commonly computed as weighted averages. Now this is the problem, this is not correct. This is not correct based on market value rating of each holding in the portfolio, just like we work out the expected return.

For expected return, it is correct. It can be mathematically established that expected return scales according to the weighted average of various constituents of a portfolio, but variances, standard deviations, convexities, durations YTMs, they do not scale in that way. So if you are doing it, you are making an approximation. These average computations are less accurate than portfolio statistics computed directly from the portfolio aggregate cash flows.

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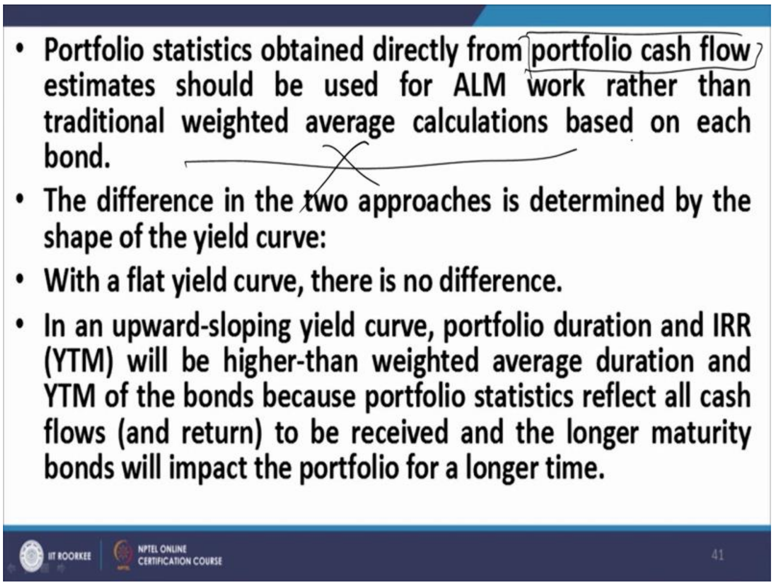

Portfolio statistics obtained directly from portfolio cash flow estimates should be used for asset liability management work, rather than traditional weighted average calculation. This should not be used. Directly from portfolio cash flows. That is what I have been emphasizing. This is fundamental, this is very important.

The difference in the two approaches is determined by the shape of the yield curve. With a flat yield curve, there may not be any difference. For an upward sloping yield curve, portfolio duration and IRR will be higher than weighted average duration and YTM of the bonds because portfolio statistics reflect all cash flows and return to be received. And the longer maturity bonds will impact the portfolio for a longer time.

So if the yield price curve is flat, you can afford to make this approximation but not, certainly not if the yield price curve is, I am sorry, if the yield curve is flat, you can make this approximation, you can afford to make this, but if the yield curve is upward sloping, this spot yield curve is upward sloping, then you cannot do that.

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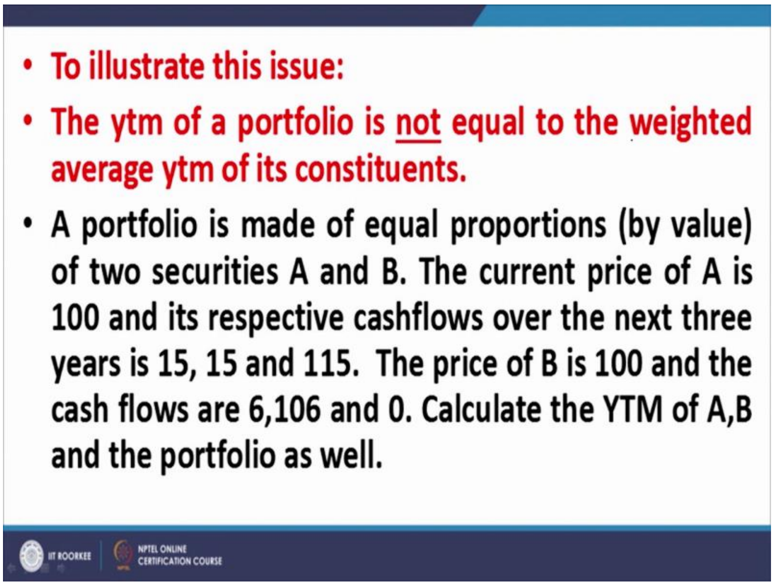

Let us take the example, a typical example. The YTM of a portfolio is not equal to the weighted average YTM of its constituents, you can see here in this example. A portfolio is made of equal proportions by value of two securities A and B. Current price of A is 100 and its respective cash flows over the next three years is 15, 15 and 115.

Clearly, its YTM is 15 percent. The price of B is 100 and the cash flows are 6 and 106 and 0. Clearly, the YTM of B is 6 percent. So the weighted average YTM is 6 because there are equal weights. So 6 plus 15 upon 2. That is 10.5 percent. Calculate the YTM of A, B and the portfolio as well.

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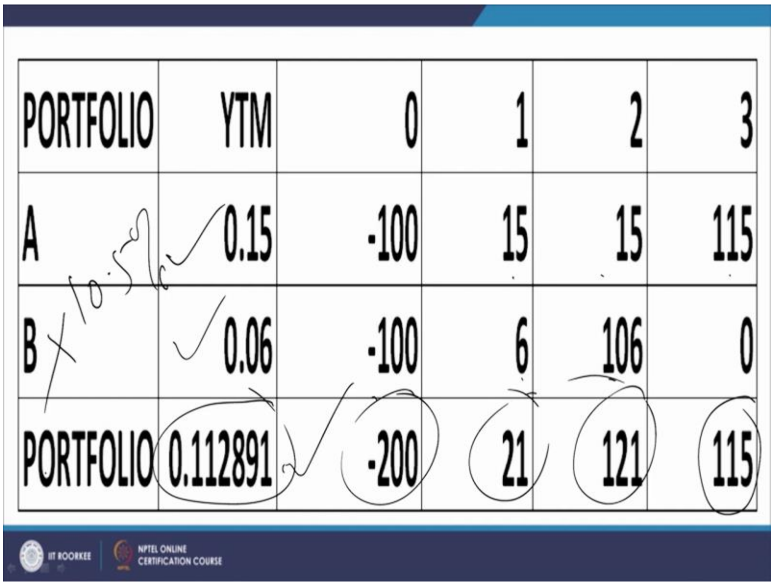

PORTFOLIO	YTM	0	1	2	3
A	0.15	-100	15	15	115
B	0.06	-100	6	106	0
PORTFOLIO	0.112891	-200	21	121	115

So the port, A's YTM is 15 percent, B's YM is 6 percent, and the weighted YTM is 10.5 percent. But what is the portfolio of YTM, what are the cash flows? The cash flow is given to us as tabulated here. Minus 100, 15, 15 and 115. Minus 100, 6, 106 and 0. We work out the aggregate cash flows. At t equal to 0, it is minus 200, t equal to 1, it is 21, t equal to 2, it is 121, t equal to 3, it is 115.

So clearly, if you work out this IRR or YTM of the stream, you get 11.29 percent, approximately. So look at the gigantic gap between the two values, 10.5 percent versus 11.29. That is the problem. This is the correct value, this is the wrong value. So we should not use weighted average statistics, weighted average parameter values for, for valuation of YTM and duration convexity or dispersion. We should use them or we should work it out on the basis of the cash flows from the portfolio.

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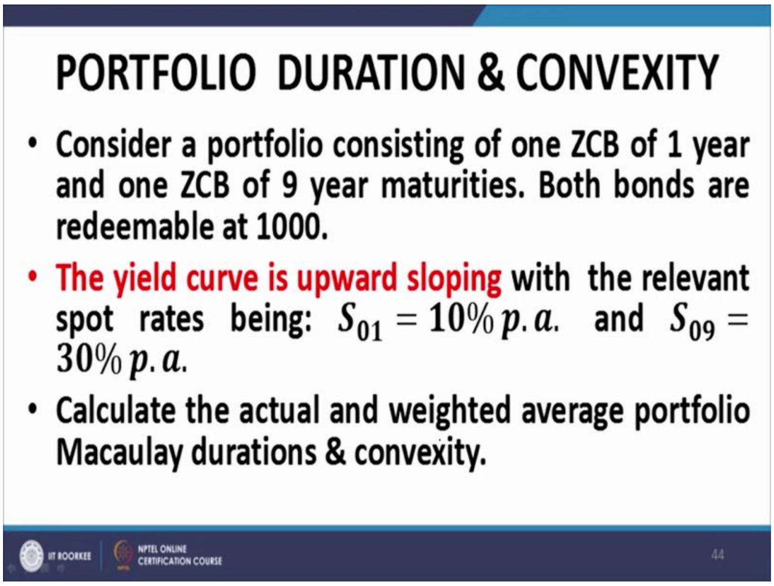

I shall continue from here in the next lecture. Thank you.