**Quantitative Investment Management**
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**Lecture 27**
**The Barbell Strategy - 2**

Welcome back. So in the last lecture, we were discussing how a bullet liability, that is a liability occurring at one point in time, can be immunized or can be managed effectively by using a barbell strategy. The barbell strategy, as its name suggest, consists of two bonds, one bond a short duration, a short maturity bond, and as the other one being a long maturity bond.

Now, the, the motivation behind the use of the barbell strategy is that this particular strategy would match or this particular combination, the barbell combination would match exactly in terms of duration the bullet liability, but it would have a higher convexity than the bullet liability.

As a result of which because the convexity correction is always positive, irrespective of whether the interest rates go up or the interest rates go down, the correction due to convexity is always positive. In other words, there is always an increase in price corresponding to the convexity correction.

The point is duration always operates opposite to the direction of the movement of the, of the yields or the interest rates, whereas convexity, irrespective of the movements, gives always a positive outcome. So it is, it is a small correction, nevertheless it is a correction which is positive, which increases the impact of duration if the duration results in an increase in prices or the interest rates have gone down.

And if the interest rates have gone up and the prices have declined, the convexity will reduce the amount of the decline in prices. Then, in the, towards the end of the last lecture I talked about the impact of an increase in yields, where the parallel increase in yields in the situation where we are immunizing bullet liability by use of a Barbell strategy. Let us continue from there, let us now look at how our decrease in the, parallel decrease in the yield curve impacts this particular combination.

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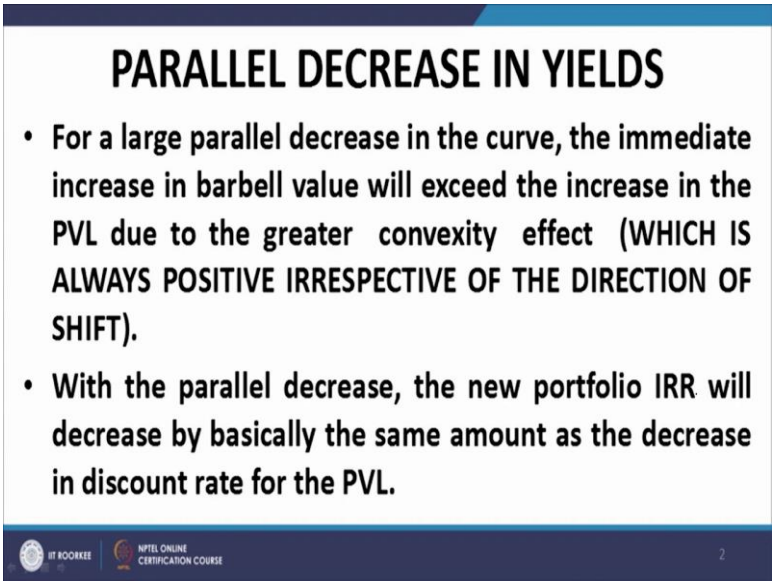


So we now talk about parallel decrease in yields. For a large parallel decrease, decrease in the yield curve the immediate increase in the barbell value will exceed the increase in the present value of liabilities due to the greater convexity effect. So that is the important point. The duration is matched.

Please note, whenever we do this kind of an exercise, immunize a bullet liability with a barbell strategy, we exactly match the durations of the two combination or the two assets, combined asset portfolio and the liability, that is the budget liability. We exactly match the duration of the bullet liability with the barbell combination of bonds or assets.

Now the point is, because the durations are matched up to the level of the first order, up to the extent of the first order, if there is a decrease in yields the increase in price will be equal on, on account of the duration effect. But, because of the convexity effect, the convexity of the barbell is more.

Therefore, there would be a greater increase in prices of the assets that is the barbell combination compared to the increase in price of the convexity because, of the liability I am sorry, because the convexity of the liability is lesser, the convexity of the barbell is more. So the convexity effect will ensure or will impact this combination in such a way that the barbells increase in price corresponding to a given decline in yields will be higher than the increase in price of the bullet liability.

So that is what we are trying to say in this paragraph. For a large parallel decrease in the yield curve, the immediate increase in barbell value will exceed the increase in the present value of

liabilities due to the greater convexity effect. Recall, the barbell has a higher convexity than the bullet liability.

And please note, the second point to recall, convexity effect is always positive, it always results in an increase in price the convexity correction always decrease and increase in price irrespective of the direction of the shift of the yield curve. With the parallel decrease, the new portfolio IRR, that is the IRR of the barbell combination will decrease by basically the same amount as the decrease in the discount rate for the present value of liabilities.

So because the shifts are parallel across the entire spectrum of spot rates, and therefore it is approximately true that the portfolio IRR or the barbell IRR will decrease by the same amount as the decrease in the IRR or the YTM of the liability.

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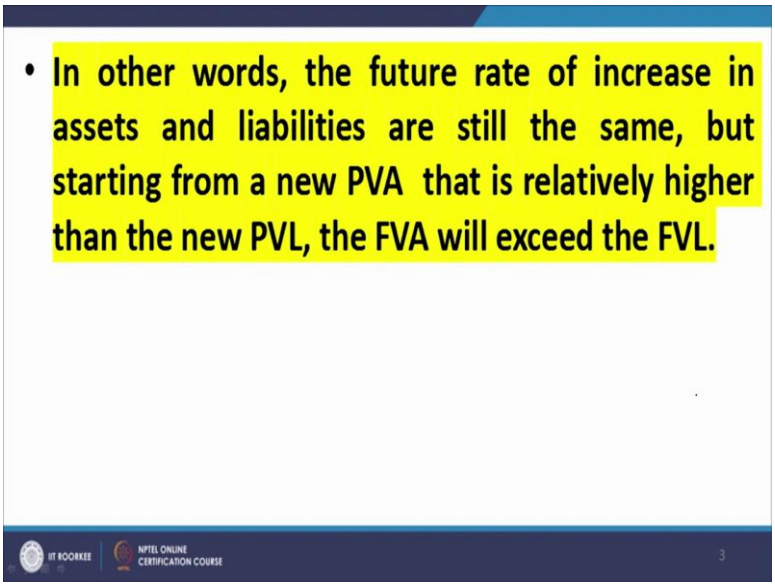


In other words, the future rate of increase in assets and liabilities are still the same because the IRRs are the same, but starting from a new present value of assets that is relatively higher than the new present value of liabilities. Therefore, the future value of assets will exceed the future value of liabilities. And why is the present value of assets higher than the present value of liabilities? Because the barbell has a higher convexity and convexity is positive.

And the rate of increase, the rate of change, with the passage of time, is the same because the IRR, the shift is parallel. So the changes in IRR is approximately equal and therefore what happens is that at the end of the day, the increases is at the same rate and therefore the future value of assets will be more than the future value of liabilities because the future value of assets is starting from a higher point due to the higher convexity.

So that is the story behind the decrease or a parallel decrease in the yield curve, how the, how a parallel decrease in the yield curve impacts the strategy of immunizing a single liability with a barbell combination of bonds.

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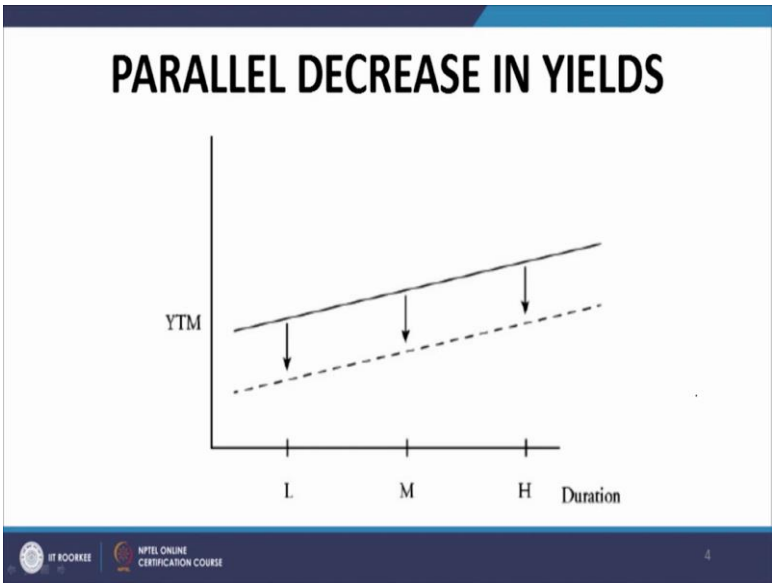


And this is the, this is the pictorial representation of a parallel decrease in yields. We have this medium. M represents the liability, this bullet liability. And we have L and H, the low maturity bond and the high maturity bond comprising the assets, which are doing the immunization work. I repeat, the durations of the combination are exactly matched, and the convexity of the barbell is higher than the convexity of the bullet liability.

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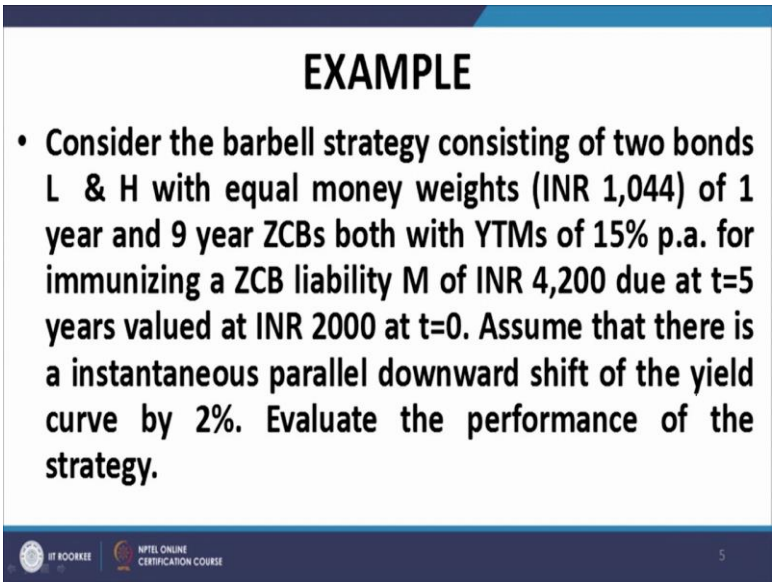

Let us do an example now. Consider the barbell strategy consisting of two bonds L and H with equal money weights. L is a low maturity bond, H is a high maturity bond, with equal money weights of INR 1,044. How this 1,044 figure has been arrived at, I will show in the solution, of 1 year and 9 years zero coupon bonds. L is a 1 year zero coupon bond of initial value 1,044. Initial value, please note that, not the final value. And H is a 9 year zero coupon bond with initial value 1,044 and YTM of 12, 15 percent. YTM of both the bonds L and H is 15 percent.

And the liability to be immunized as a zero coupon bond M of INR 4,200 which is due at t equal to 5 years. And it is valued at INR 2,000. Now please note, in this example, we do not have the IRR of the liability, we do not have the YTM of the liability, the growth rate of the liability. What we are given is that at the end of 5 years, the value of the liability will be 4,200 and the current value of the liability is 2,000. This will enable us to determine the YTM of the liability. Assume that there is a instantaneous parallel downward shift of the yield curve by 2 percent. Evaluate the performance of this strategy. So that is the question. Let us see how we solve it.

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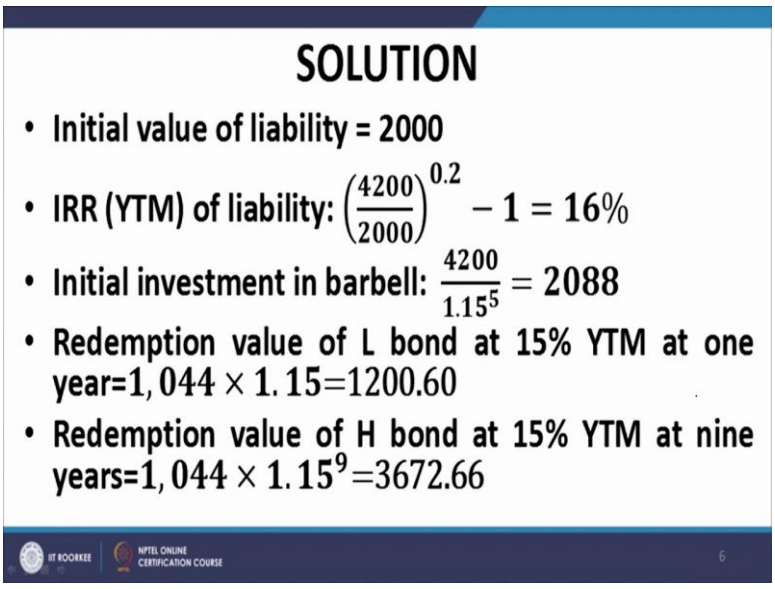


Initial value of the liability is 2,000 and the value of the liability at t equal to 5, the maturity value of the liability is 4,200. So this corresponds to a growth rate or an IRR of 16 percent. So the liability is growing at the rate of 16 percent. Its present value is 2,000 and its value at maturity is 4,200.

Initial value in the bar, initial investment in the barbell, you need 4,200 at the end of 5 years, and the YTM of the bonds comprising the barbell is 15 percent. Both the bonds have a YTM of 15 percent. So your initial investment is obtained by discounting 4,200 by 15 percent for 5 years. And that turns out to be 2,088.

So we are investing half of this 2,088 in a short maturity bond, in a low maturity bond L, single year maturity bond, YTM 15 percent, and 1,044 in a 9 year maturity bond, a long maturity bond, again with a YTM of 15 percent. So that is about the barbell strategy.

Now, the redemption value of the 1, of the 1 year bond at YTM of 15 percent at the end of 1 year will be equal to 1,200.60. The redemption value of the high majority bond or the long maturity bond at 15 percent YTM over a period of 9 years would turn out to be 3672.66. So these are the redemption values of the two bonds that constitute the barbell.

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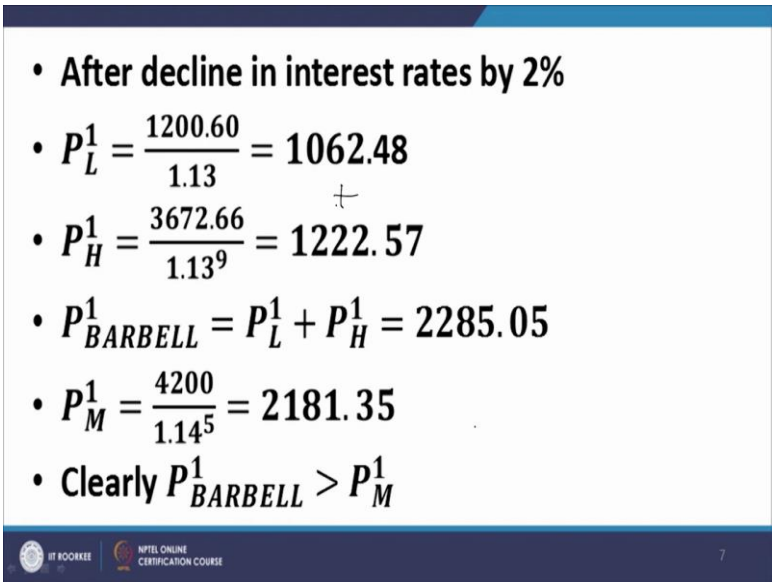

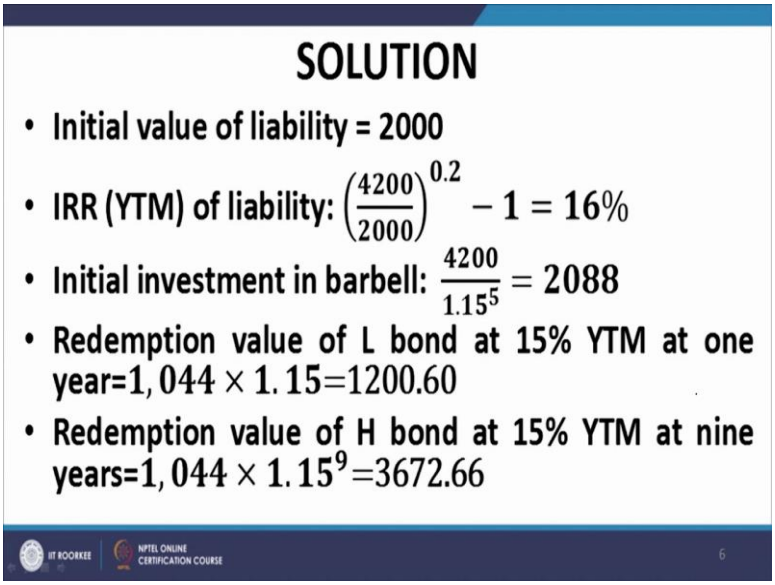


After the decline of interest rates to, by 2 percent what happens to the price of the short maturity bond or the low maturity bond, L? It turns out to be 1,062.48, dividing the maturity value of 1,200.60 by the interest rate of 1.13, discount factor of 1.13. So we get 1062.48.

And similarly, the present value of the long maturity bond or the high maturity bond H, which is a 9 year bond, so it will, have to be discounted for 9 years. The YTM has gone down from 15 percent by 2 percent, and therefore its current value is 1,222.57 after the shift in interest rates.

So the total value of the barbell turns out to be the sum of this plus this, and that gives us 2,285.05. So this is the t equal to 0 value of the barbell after the shift in interest rates by 2 percent has taken place, the decline in interest rates by 2 percent is taken place. Please recall, the earlier value of the barbell was 2,088, and now it has gone to 2,285 because of the decline in interest rates by 2 percent. The value has increased as it should be.

As far as the t equal to 0 value of the liability is concerned after the shift in the interest rates, decline in the interest rates by 2 percent, it turns out to be 2,181.35. So clearly, the t equal to 0 value of the barbell after the shift in interest rates is greater than the t equal to 0 value of the liability.

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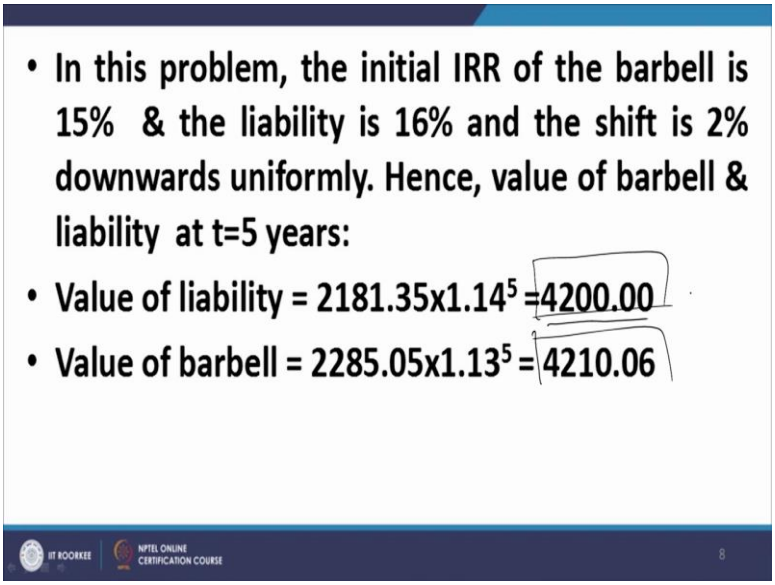


Now let us see what happens at the point of maturity of the liability. Now, assuming that the initial IRR of the barbell was 15 percent, that is given to us, that is the YTM of the two bonds. And because both the bonds have items identical, so the curve is flat and therefore we can use the weighted average YTM as such. I will come back to this issue of weighted average YTMs and so on later on in today's lecture. It is a very important point and needs to be discussed in detail.

But anyway, the initial IRR of the barbell is 15 percent and the liability is 16 percent. And the shift is 2 percent downwards across the entire spectrum of interest rates. And therefore, the new IRR will be 2 percent less in both the cases, and therefore the growth rate of the liability will be 14 percent, and that will lead to a value of 4,200 at the end of 5 years, which is the amount that needs to be redeemed. And the value of the barbell turns out to be 4,210.06.

So in this scenario, in this particular situation what we find is that the IRRs remain, or a change by the same amount and the future value of the assets that is this value, 4,210, is more than adequate to meet the future value of the liability, that is 4,200. So this is how this example needs to be attempted. And this is the illustration that depicts the dynamics of the strategy, the barbell strategy for immunizing a bullet liability when there is a decline, a parallel decline in the yield curve.

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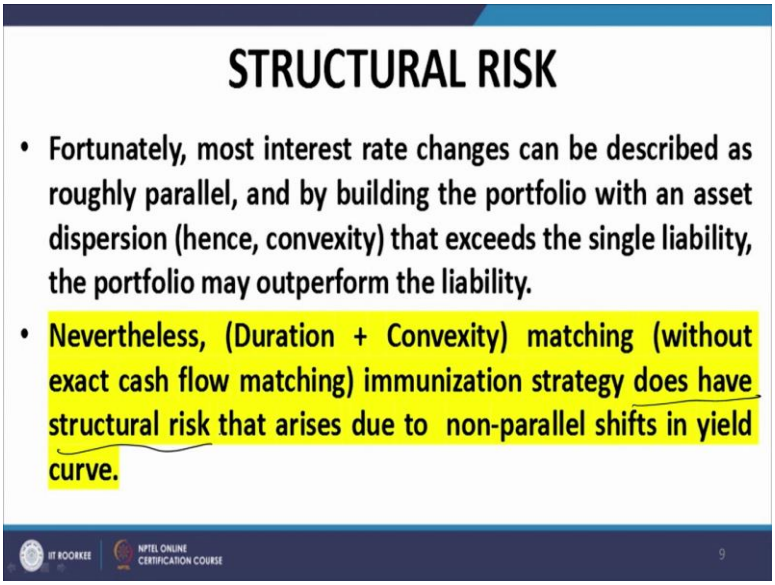


Now we talk about structural risk. So far, what I have discussed is about parallel shifts in the yield curve. Because you see, basically the, the duration convexity model is compatible with parallel shifts in the interest rates or in parallel shifts in the yield curve. That is the entire yield curve shifting upwards or downwards by the same amount. All the spotlights change by the same amount across the entire spectrum of short rates.

So, but that need not necessarily be the case. Empirically, it has been observed that that is not necessarily the case. Very often it happens that the yield curve under, undergoes non parallel shifts like steepening or like changes in curvature and so on, or twisting or whatever the case maybe. We will again talk about it gradually, as we progress in this lecture.

But for the moment, there are situations, for the moment we need to know that there are situations where the duration convexity model may not be the perfect fit because of the yield curve shifts not being parallel. This duration convexity model is appropriate for yield shifts which are parallel. So that we need to keep at the back of our mind.

So fortunately however, most interested changes can be described as roughly parallel and by building the portfolio with an asset dispersion, dispersion hence convexity. The two are related. In fact, very closely related. The dispersion of cash flows and the convexity of the bond are very much closely related. That exceeds the single liability, the portfolio may outperform the liability.

So if we have the portfolio that is the barbell portfolio or the immunizing portfolio with an asset dispersion that exceeds the single liability, the portfolio may outperform the liability.

Because you see, the point is the convexity correction is always positive. And secondly, the convexity of the portfolio, of the dispersed portfolio is higher than the convexity of the single bullet liability.

So because of the higher convexity, the correction due to convexity is higher, and because the convexity, correction is always positive, so it would naturally happen that the portfolio would do better than the bullet liability in terms of increase in price or the degree, or for that matter, the decrease in price would be lesser for the portfolio compared to the decrease in price of the bullet liability if there is an increase in yields.

Nevertheless, the duration plus convexity matching without exact cash flow matching, cash flow matching is a very rigid sort of strategy, which normally practitioners are not able to implement for immunizing their, their bond portfolios, their liability portfolios because bonds of adequate maturity and appropriate coupon size and so on may not really be accessible in the market.

So, and even if they are accessible in combination or the strategy of immunizing with perfect cash flow, matching may turn out to be extremely costly. So duration matching is the next best strategy, and that is what we are talking about right now. So nevertheless, duration plus convexity matching without exact cash flow matching immunization strategy does have structural risk, does have structural risk, this is important, that arises due to non parallel shifts in the yield curve.

As far as parallel shifts in the yield curve are concerned, as I mentioned, these strategies of having a barbell combination to immunize a bullet portfolio is pretty much adequate, it is pretty much satisfactory if the shifts are reasonable. But if the shifts are non parallel, that is, there is a steepening or a twist or a change in curvature for that matter, then we have a problem.

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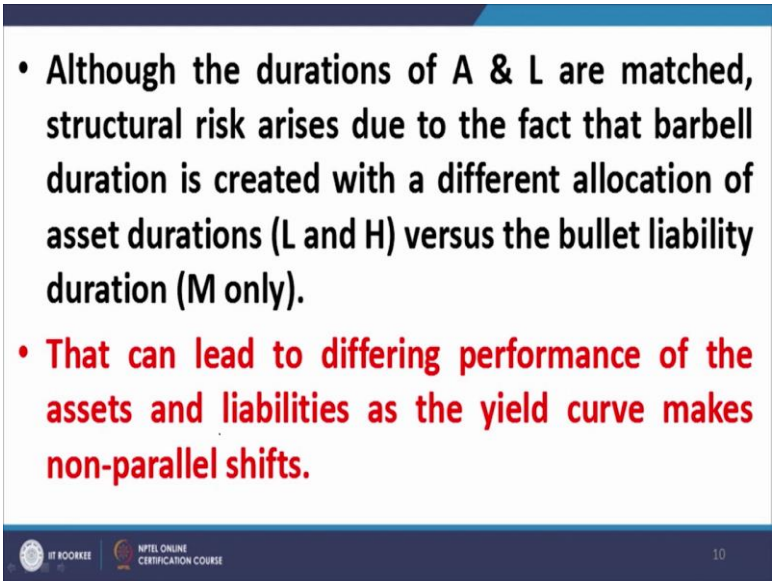


Although the durations of assets and liabilities are matched, structural risk arises from the fact that barbell duration is created with a different allocation of asset durations, L and H versus the bullet liability duration that is M only. So this, because we are having, in the case of the barbell strategy, we are having two instruments, we are having two bonds to implement the barbell strategy, these, there may not be exact synergy between the movements or the implica, impact of the movements on the prices of the port, of the bullet portfolio and the barbell portfolio.

I repeat, because the barbell durations and convexities are created out of two assets, in, whereas the bullet is created by a single asset and the assets obviously do not match, the bullet has a maturity and a duration of M, the barbell has a maturity and duration of L and H. So there are two assets in the barbell. And as a result of this, if there is a non parallel shift in the yield curve, if there is a twist in the yield curve, if there is a change in curvature of the yield curve, then the impact on the prices may not be exactly the same. That is the problem. So that can lead to differing performance of the assets and liabilities a the yield curve makes non-parallel shifts.

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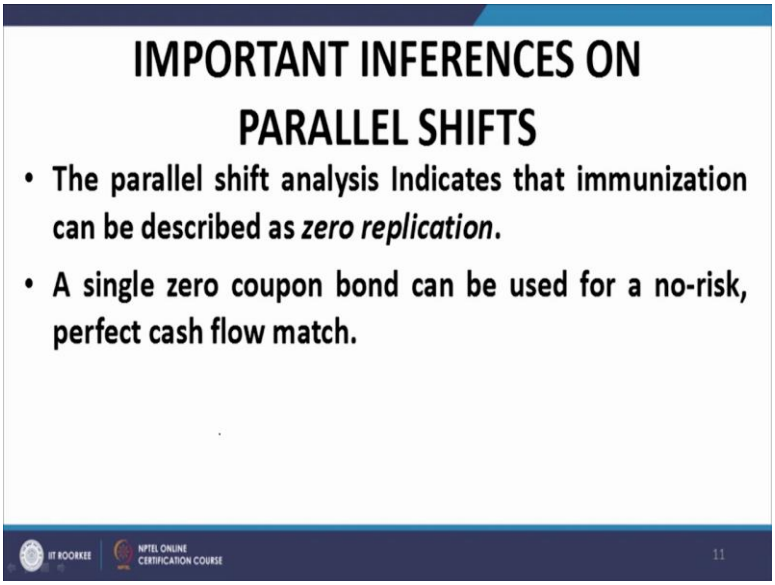


Important inferences on parallel shifts. The parallel shift analysis indicates that immunization can be described as zero replication. What is zero replication? Basically, what we are trying to say is that we can replicate the liability by using a bond of the same maturity, using a zero coupon bond of the same maturity as the liability and the same maturity value as the liability. A single zero coupon bond can be used for no-risk, perfect cash flow match with the liability. That is why it is called zero replication because it is being replicated by a zero coupon bond of the same maturity, same par value as the liability.

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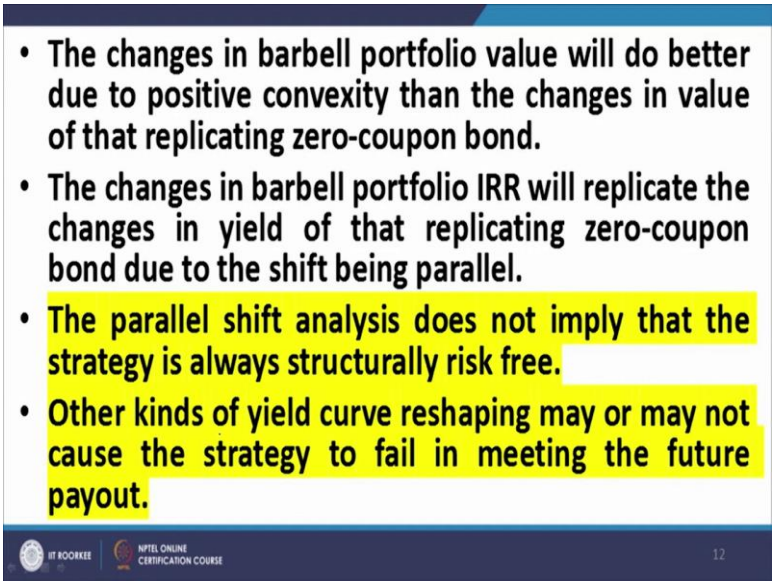


The changes in the barbell portfolio value will do better due to positive convexity than the changes in the value of that replicating zero coupon bond which is nothing but an, a replica, a

perfect replica of the lab, zero coupon liability, or the bullet liability. The changes in the barbell portfolio IRR will replicate the changes in the yield of that replicating zero coupon bond due to the shift being parallel.

This, this point will remain valid only if the shifts are parallel, as you should see in the examples that follow. The parallel shift analysis does not imply that the strategy is always structurally risk free. It is structurally risk-free only if the shifts are parallel. Other kinds of field curves reshaping may or may not cause the strategy to fail in meeting the future payout. So that is the point. So long as the shifts are parallel, the strategy is good, the strategy is appropriate, the strategy does well.

But if the shifts happen to be non, non-parallel, if there is a change in curvatures, twisting or steepening as the case may be, then we may not necessarily will, but we may end up with a problem. The future value of assets that we have out of the barbell portfolio may not be adequate, may not turn out to be adequate to meet the liability at the point of maturity of the liability. So that is the situation that may happen due to the structural non-parallel shift of the yield curve, and this is called the structural risk.

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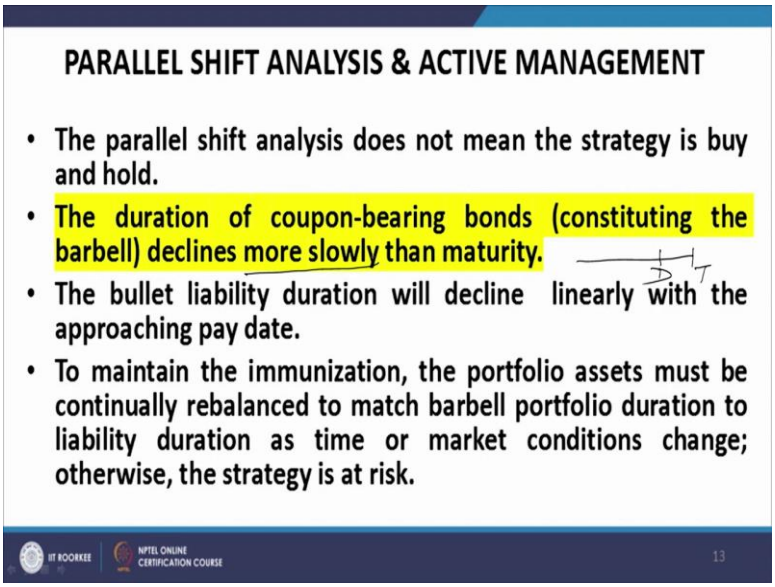


Parallel shift analysis and active management. Parallel shifted, the, the strategy that I have talked about of immunizing a bullet liability by using a portfolio of a short maturity, short duration bond and a long maturity, long duration bond, the barbell strategy is not necessarily a passive strategy, is not necessarily a passive strategy. Let us see why it is so.

It is so because the duration of the coupon bearing bonds constituting the barbell declines more slowly than maturity. Why is it so? Let me explain this in a minute. Let us assume that the maturity of the, of a, of a single bond is capital T. It is a coupon paying bond. So the majority of the bond is capital T with, and obviously, the duration of the bond is less than capital T because it is a coupon paying bond.

So since it is a coupon paying bond, its duration must necessarily be less than its maturity. Let us see it, and let us assume that its duration is D. Now, that means what? That means this duration D, 0 to D has to be amortized because the point is, the duration at maturity of the bond will be 0. The maturity will be 0 and the duration will be 0 when the bond reaches its maturity point, when, if the bond reaches its maturity date.

And therefore, this duration D has to be amortized over a period T. And T is larger than D. And what does it mean? It means that the rate of amortization of D has to be less than the rate of amortization of T. So that is the point. The duration of coupon bearing bonds constituting the barbell declines more slowly.

That is the point, declines more slowly because the duration is less than capital T, less than the maturity, and it has to be amortized over capital T, and therefore it follows that the duration will be, the rate of decline of the duration as the bond approaches its maturity will be lesser than the rate of decline of its maturity.

The bullet liability duration will decline linearly with the approaching pay date, because it is a zero coupon bond. So as far as the duration of the bullet liability is concerned, it is equal to its maturity, and therefore both of them will decline at the same rate. That is linearly with the passage of time.

To maintain the immunization, the portfolio assets must continually be rebalanced to match barbell portfolio duration to liability duration. So because the duration is a dynamic concept, it changes with the passage of time, the strategy that we have talked about is not necessarily a passive strategy.

You have to look at the look at the durations matching of the asset, the barbell asset and the and the bullet liability at periodical intervals, and see that the gap between them does not become significantly large because if the gap becomes significantly large, the immunization will break down.

So that is precisely what is mentioned in this paragraph. To maintain the immunization, the portfolio assets must be continually rebalanced to match barbell portfolio duration to liability duration, as time or market conditions change. Otherwise, the strategy is at risk.

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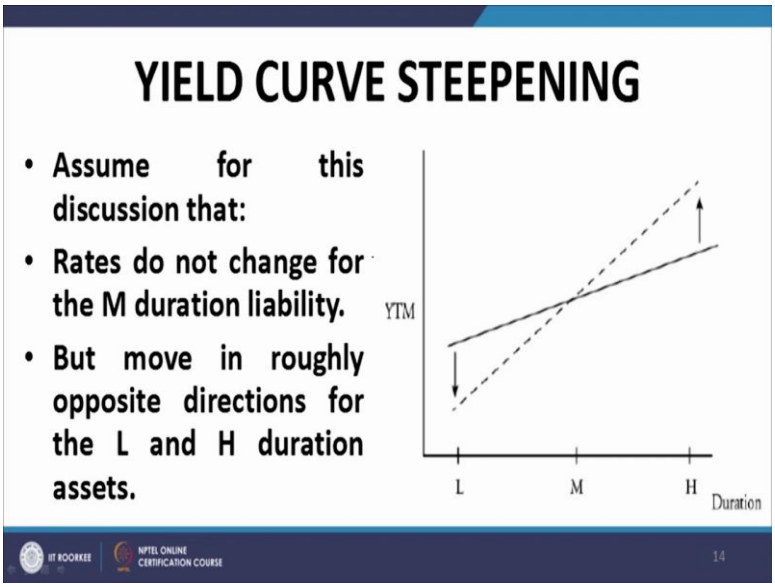


Now, we talk about yield curve steepening. Yield curve steepening means the short side rates are going down, and the long side rates are going up. The long maturity rates go up, the short maturity rates go down, and the intermediate maturity rates remain more or less unchanged.

So assume for the discussion that rates do not change for the M duration, the liability duration rates do not change, the YTM for the liability does not change, but move in roughly opposite directions for the L and H duration assets. L is the low duration, low maturity assets of the barbell, and H is the high maturity high duration asset.

So the rates relating to L are declining and the rates relating to H are increasing. So that is what is called an increase in steepening, increase in steepness or the stiffening of the yield curve.

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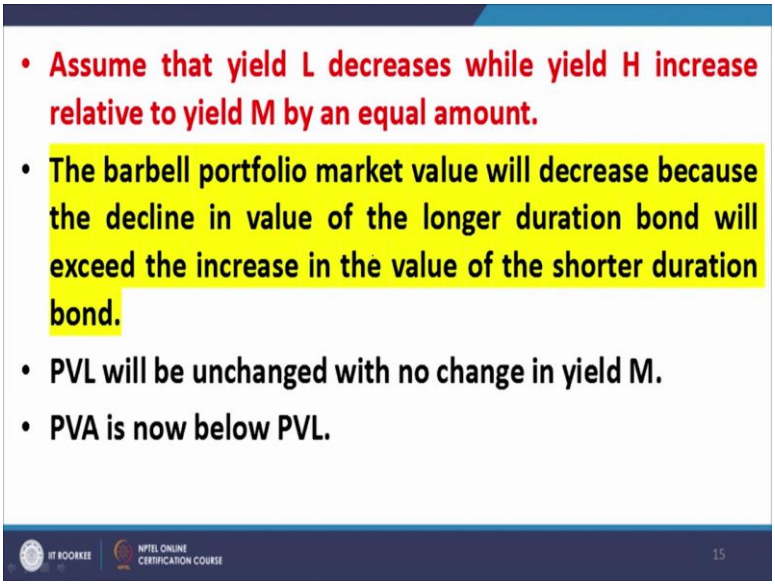


So what happens in this case? Assume that the yield L decreases while yield H increases relative to yield M by an equal amount. For the purpose of illustration, we assume that the YTM of L decreases and the YTM of H increases by the same amount whereas the YTM of M remains unchanged. Now, what will happen? Because the yield of L has decreased, the, the low duration low maturity part of the barbell or the bond constituting the verbal has decreased, therefore its price will increase.

It will increase in tandem with the duration of this particular bond but the duration of this bond is very small, so the increase in price of this particular one that is the L bond that is the short maturity bond, low maturity bond will increase, no doubt, because decline of, decline in interest rates, but will increase by a small amount.

As far as the H bond is concerned, the high maturity or the long maturity bond is concerned, because it will have a high duration, it will have a long duration, therefore the change in YTM by the same amount or the increase in YTM by the same amount will mean that the price of this bond will decrease will by a significant amount, certainly by more than the increase in price of the bond L.

Let me repeat, the L bonds price will increase because of a decrease in YTM, but that increase will be small because the duration of this bond is small. The decrease in price of the high maturity bond, high duration bond will be much larger. Why? Because duration is larger, and therefore corresponding to the same magnitude of change in YTM, as for L the price will decline much more.

What will happen to the combination? The present value of the assets will decline, which is the aggregate value of both the bonds will decline, naturally, because the decline of the low match, of the high maturity bond exceeds the increase in price of the low maturity bond. So that is one thing. As far as the present value of liabilities is concerned, there will be no change because there is no change in the YTM in this case.

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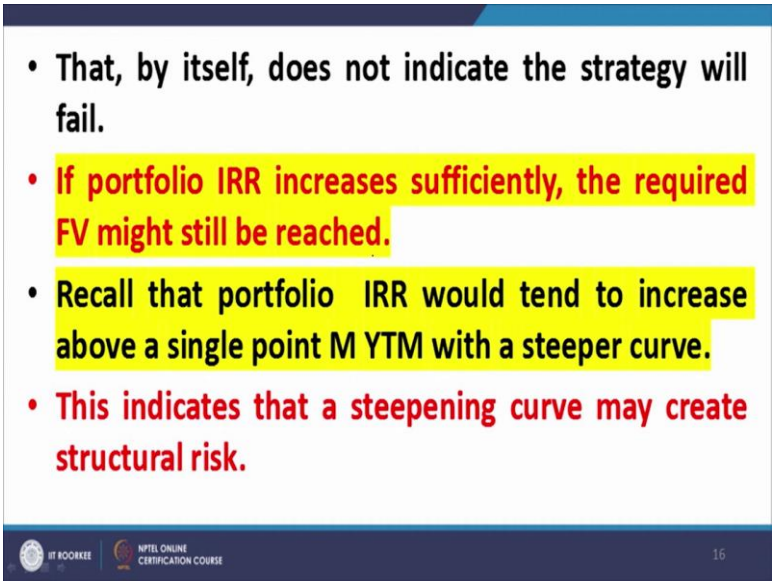


However, this will not conclusively establish that the strategy will fail. At prima facie, it may seem that because the present value of the assets has gone, gone down below the present value of liabilities or it has declined because of the decline in value of the high maturity bond, significant decline in value of the high maturity bond. So that does not conclusively establish the failure of the strategy.

Why? Because the issue of IRR also comes into play, because of this change in interest rates, the IRR of the barbell will change, IRR of the low maturity bond and the high maturity bond will change, and it will so happen that the portfolio IRR will tend to increase above a single point M YTM with a steeper curve. So that is important.

And, and if it so happens that the portfolio IRR has increased sufficiently that it can cover the decline in the present value of assets over due to the stiffening of the curve adequately, then the strategy may still succeed. Let me explain it again. You see, the first thing is that the present value of assets has gone down, the present value of liabilities has remained unchanged.

So from here or from equal values of assets and liabilities, now assets is below liabilities at t equal to 0 after the steepening of the curve. But as we move forward in time, both of these things, the present value, the value of the assets and the value of the liabilities will grow at their respective IRRs.

The point is, if after the shift of the yield curve, the IRR of the asset side has increased sufficiently so that at the point in, of maturity of the liabilities, the, the value generation or the growth in the value of assets is adequate to compensate for its gap at t equal to 0, then the, then the strategy may still succeed.

So that is the, that is a caveat that it cannot be conclusively said that because there is this kind of a change in pattern or a change in the structure of the yield curve or spot yield curve, that the strategy will fail. It need not necessarily be so, as I illustrate in the example that follows.

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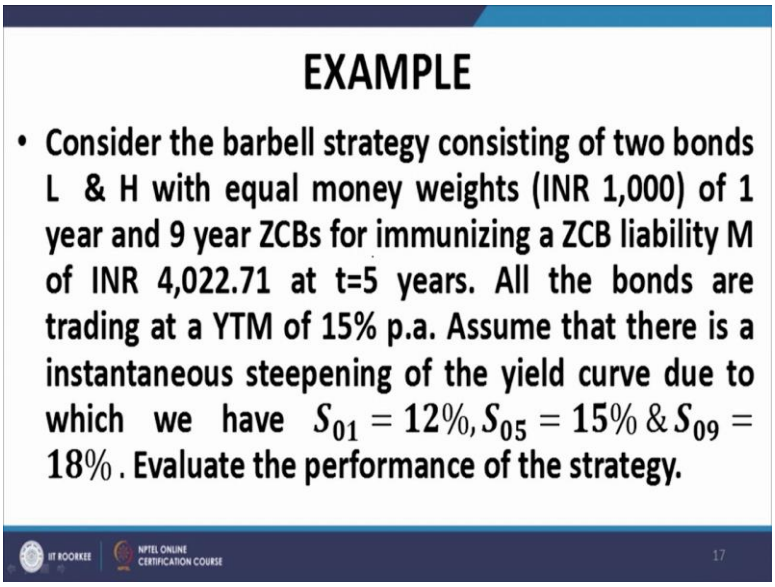


Let us look at this example. Consider the barbell strategy comprising of two bonds L and H with equal money weights INR 1,000 of L of maturity 1 year and H, maturity 9 year. Both are zero coupon bonds. They are used for immunizing a zero coupon bond liability M of value 4,022.71 at t equal to 5 years. All the bonds are trading at a YTM of 15 percent.

Assume that there is an instant in steepening of the yield curve due to which the near end spot rate that is S 01 has declined from 15 percent to 12 percent, and S 05 has remained unchanged at 15 percent, and S 09 has increased from 15 percent to 18 percent. So that is the new situation. Let us look at the solution here.

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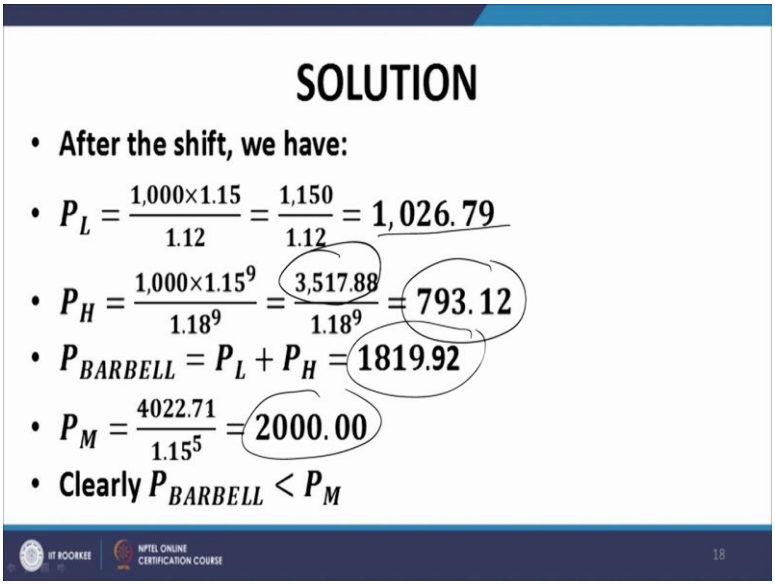


This is the present value of the low maturity bond or the low coupon bond. The, the low coupon bond has a maturity value of 1,150 because it is trading at 1,000. And it has a YTM of 15 percent. That means, its redemption value at t equal to 1 year has to be 1,150. So what will be the present value of this particular instrument? When the spot rate has changed from 15 percent to 12 percent, this, the curve has steepened, it will be equal to 1026.79.

What about the high maturity bond, what about the high coupon bond? I am sorry, high duration bond. Yes. So in this case what happens is the maturity value of this bond at t equal to 9 years with the initial value of 1,000 and a YTM of 15 percent turns out to be 3,517.88. This is the maturity value of the 9 year zero coupon bond trading at 1,000 and having a YTM of 15 percent.

Now, the YTM has changed to 18 percent for a 9 year maturity bond. And therefore its present value will turn out to be 793.12. So the present value of the barbell law is 1,819.92. And what is the present value of the liability, present value of M? The present value of M is 2,000, as you can see here. The rate is remaining unchanged at 15 percent and it has a maturity value of 4,022.71.

So clearly, due to the steepening of the curve, the present value of assets, that is the barbell assets has gone below the present value of the bullet liability. Present value of the barbell assets is 1,819.92, present value of the bullet liability is 2,000.

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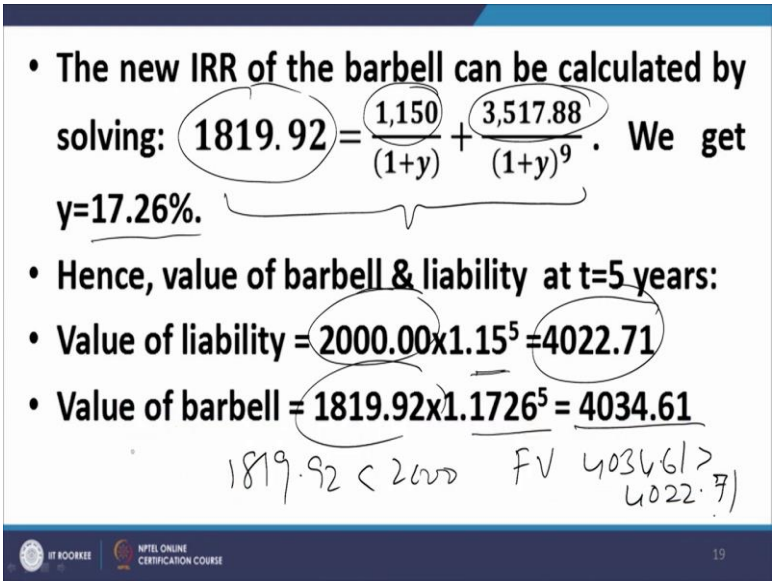


Now, we work out the new IRR of the barbell strategy. The new, the equation for the new IRR of the barbell strategy is given by this. And when you solve this as a quadratic or use excel or some other software for solving this equation, you find that the on the YTM or the IRR of this particular cash flow stream turns out to be 17.26 percent.

This equation itself is self explanatory. This is the t equal to 0 price of the instrument, and these are the, this is the redemption value of the 1 year bond, and this is the redemption value of the 9 year bond. So these are the cash flow that will emanate from this barbell strategy, and when you solve this equation for the IRR you get it as 17.26 percent.

Therefore, the value of the barbell and liability at t equal to 5 years will turn out to be, liability value is 4,022.71. Please note, the 15 percent is unchanged because of the data given in the problem. And in this case, we are using a growth rate of 17.26 percent since the portfolio value will grow at the portfolio IRR. And we end up with 4,034.61.

So therefore, notwithstanding the fact that this quantity is lower than this quantity, this is lesser, 1,819, that is the present value of assets, is less than the present value of the liability. The future value of assets, that is 4,034, is, is greater than the future value of the liability that we have to repay. So the strategy would still succeed, notwithstanding the fact that present value of the assets has become lower than the present value of liabilities. I shall continue from here in the next lecture. Thank you.