Quantitative Investment Management
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Lecture 26
The Barbell Strategy - 1

So let us continue. We are talking about immunization of a single liability. Now, the fundamental philosophy when we talk about immunization of reliability is to match the duration of the asset and the liability.

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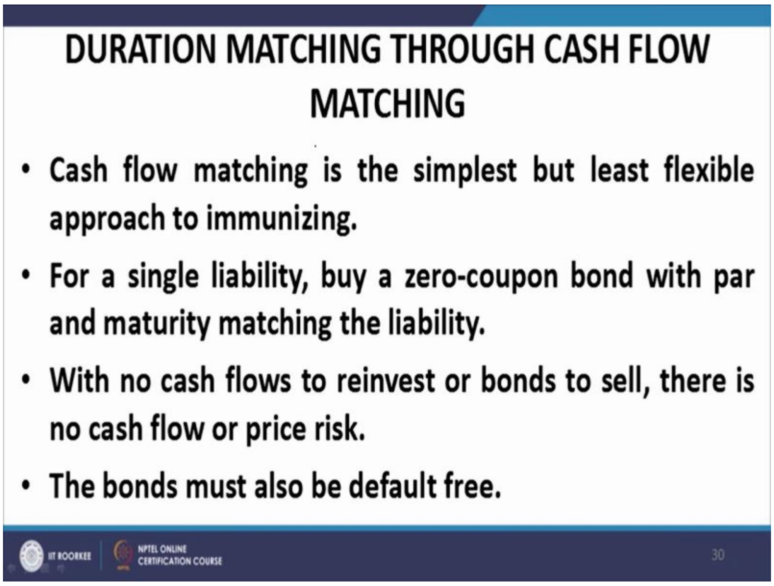

Now obviously, the straightforward or the most elementary or the obvious method is to match the timing and the magnitude of the cash flows of the asset and liability that would naturally match or that would, by immediate implication, match the duration of the asset and the liability. So this is the first approach.

Obviously, it is the least flexible approach because what you are doing here is for a single liability, you are buying a zero coupon bond with the same par value and maturity matching as the liability. I repeat, for the cash flow matching what we are doing here is we are matching the magnitude and the timing of the cash flows.

Therefore, if you have a single liability which to match you match it by a zero coupon bond, by investment in a zero coupon bond which matures on the same date as the liability and whose value at maturity is the same way as the value of the liability. Now, the other important thing is because this is a zero coupon bond, there will be no coupon payments during the life of the bond.

And because there have been no coupon payments during the life of the bond there is no question of reinvestment so there is no question of reinvestment risk. The maturity of the bond coincides with the maturity of the liability and therefore the redemption value is also known up front and as such our interest rate risk is totally eliminated in this situation. But the issue is of flexibility, issue of availability and the cost of immunizing using this strategy.

So let me quickly read out for you. Cash flow matching is the simplest but the least flexible approach to immunizing for a single liability, buy a zero coupon bond with par and maturity matching the liability. So the final cash flow, the redemption cash flow must be equal to the value of the liability and it should occur at the same point in time at which you have to make payment against the liability.

With no cash flows to reinvest, there is no coupon payment so no cash flow to your invest naturally or bonds to sell because you are redeeming, you are getting the redemption proceeds, the maturity of the bond coincides with the life of the liability and therefore on, on the date that you have to redeem the liability you will get the redemption proceeds on the company which are known upfront which is part of contract of issue and therefore no uncertainty there as well. We assume of course that the bonds are default free.

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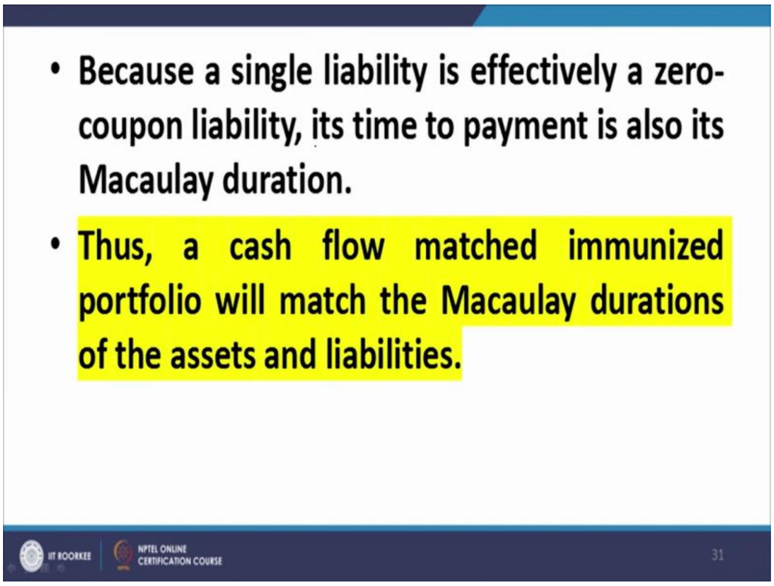

Because the single liability is effectively a zero coupon liability, its time to payment is also its Macaulay duration. So the fact is that you are, by using the strategy, by matching the timing and the magnitude of cash flows, you are obviously matching the Macaulay duration of the asset side and the liability side. Thus, a cash flow match immunized portfolio will match the Macaulay duration of the assets and liabilities.

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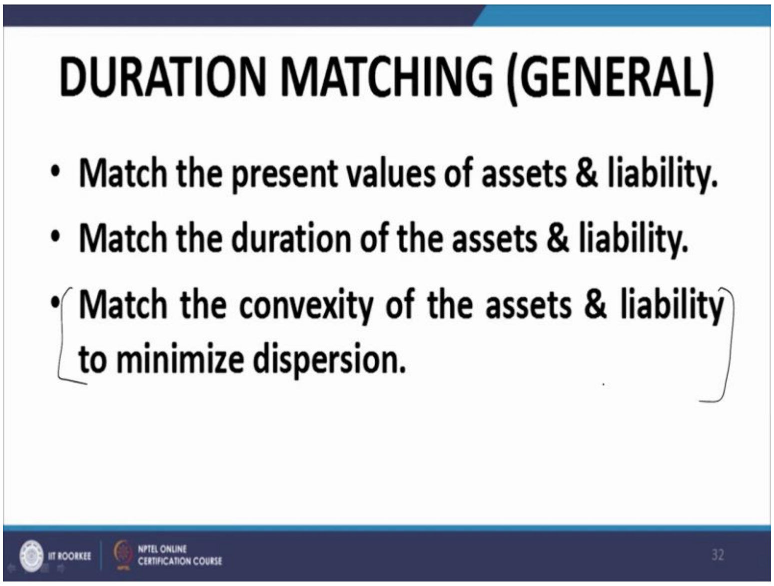

Duration matching in general with new donation matching without the necessity of cash flow matching, how does that operate? That, that is very interesting. Now, what, what would be the constituents of the strategy? Match the present values of assets and liability. The present value of assets and liability at the respective rates, at the respective IRRs rather must be equal at the t equal to zero point.

Match the duration of the asset and liabilities, match the convexity of the assets and liabilities to minimize dispersion. This issue, the point number three that we talk, that is, is subject to what I will be talking about in the later part of this lecture. You see, convexity is a two-way sword. Higher the convexity, better it is in the sense that your cash flows would be higher corresponding to a change in interest rates because the d y appears not as d y but as d y squared, is the second order correction.

And therefore it is always positive, and therefore whatever be the changes in the interest rates they manifest themselves as an increase in the convex, price due to the convexity correction. So that is a positive feature but attached to this positive feature is the negative feature that higher the convexity, greater is the variability of the price around the YTM of the one.

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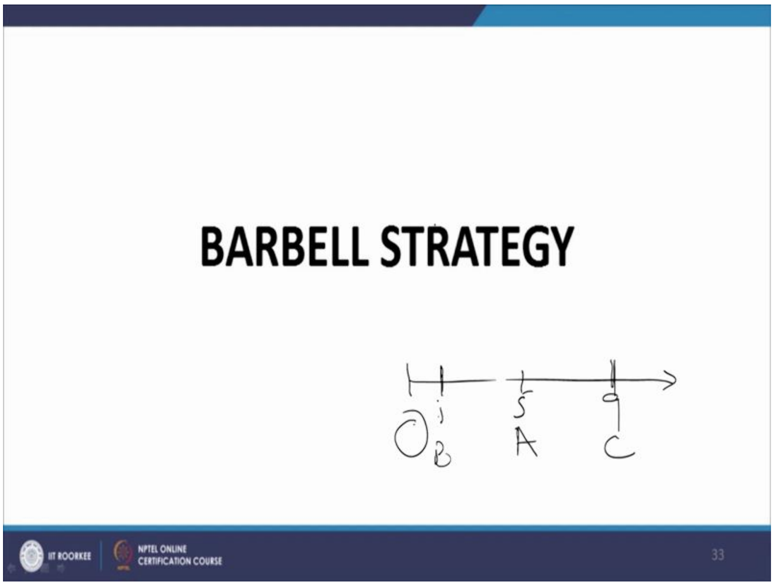

So now let us look at one strategy which manages or which approaches the issue of convexity, by using which we can discuss in more detail the issue of convexity because duration matching is rather straightforward rather elementary. You simply match the direction of asset and liability, the first order correction. Therefore, by matching the assets and liabilities for small changes in interest rates, small parallel shifts in the spot yield curve.

You will have no problem, the portfolio will be reasonably well immunized. But the issue is when the changes are significant, the convexity correction becomes prominent. That is where we need to take, look at alternative strategies rather than purely duration matching or purely cash flow matching for that matter. So this barbell strategy is another strategy. What do we do in a barbell strategy?

Suppose we have a t, this is t equal to zero, this is let us say the timeline, this is t equal to five we have a liability with us, a zero coupon liability which is going to mature at t equal to 5 for payment that we have to, we want to immunize ourselves against. What we do is that we constitute an immunizing portfolio or constitute a portfolio at t equal to 0 which consists of two parts.

Part 1 that is a zero coupon bond that matures at t equal to 1 and second, Part 2 that is the, the zero coupon bond with materials at t equal to 9. Now as we shall see just now, as we shall see just now, the effective duration of this combination of bonds B, let us call it bond B, let us call this bond C, and let us call this bond A. The portfolio comprising of bonds B and C has the same duration as portfolio A provided, of course, they are 50 percent A and B and 50 percent C.

Then the duration of B plus C will be the same as the duration. But when we talk about convexity the convexity of B and C, the portfolio comprising of B and C is far more than the convexity of A. This strategy of having a bond with, of a shorter duration and another bond of a longer duration to manage liability or to immunize the liability of intermediate duration by matching durations but with higher convexity is called a barbell strategy. Let us look at it in greater detail.

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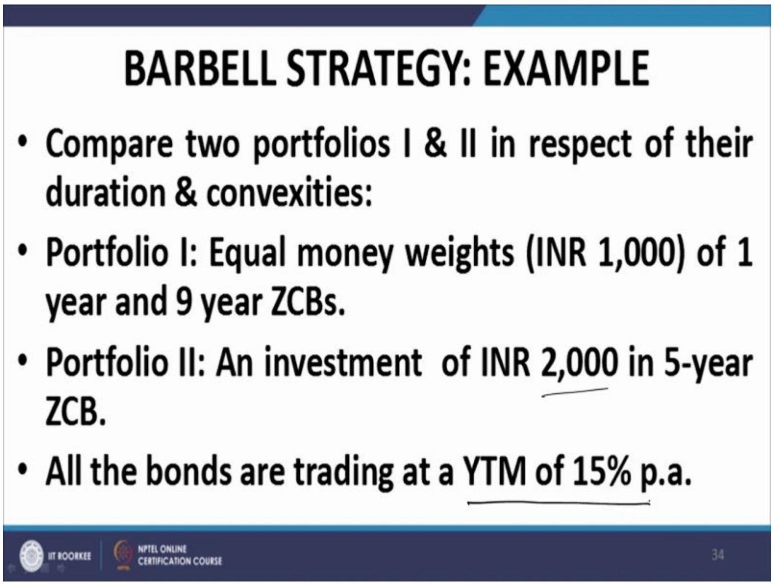

Barbell strategy, by an example I will illustrate this. Compare two portfolios, I and II in respect of their duration and convexities, Compare two portfolios, I and II in respect of their duration and convexities. Portfolio I, equal money weights INR 1,000 of 1 year and 9 year zero coupon bonds.

Equal money weights that is you invest 1,000, in t equal to 0, investment is 1,000, please note this, in bond I, and t equal to 0 investment in bond II, that is of maturity 9 years is also 1,000. Both are zero coupon bonds. The maturity of first bond is 1 year, the maturity of second bond is 9 years. The two together from portfolio I.

And portfolio II is involves an investment of INR 2,000 in a 5 year bond. Again, this is also a zero coupon bond. So let me repeat quickly. Portfolio I comprises of equal money investment in 1 year zero coupon bond and a 9 zero coupon bond. Portfolio II consists, involves an investment equal to the aggregate investment of Portfolio I, but it is in a single bond of maturity 5 years, and that is also zero coupon bond. All the bonds are trading at a YTM of 15 percent.

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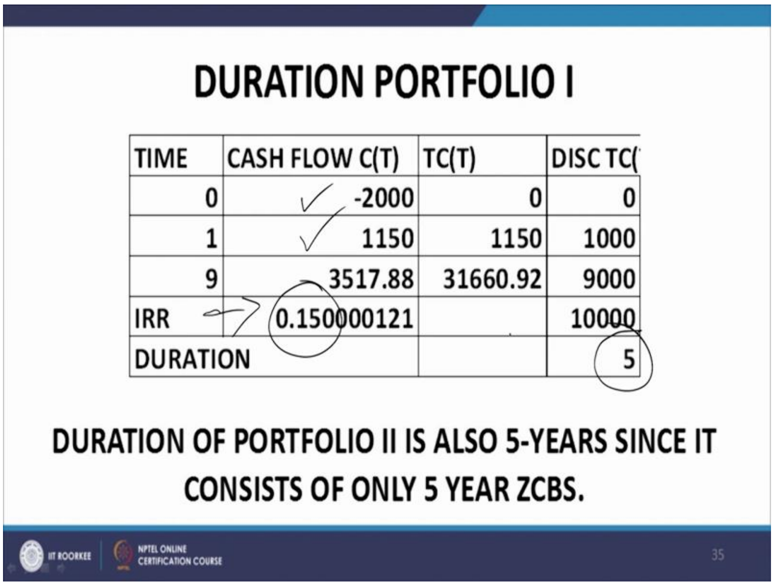

So let us work out the duration of Portfolio I. When you work out the duration of Portfolio 1, what you find is, is quite straightforward. It turns out to be 5 years or both of them are zero coupon bonds. Therefore, you can use the weighted average duration but it is better to work from the basis.

Let us work out the cash flows. At t equal to 0, the cash flow is minus 2,000. At t equal to 1, the first one will mature, which is a 1 year bond. And because it is at a YTM of 15 percent, the cash flow would be 1150. If the initial price is 1,000 and maturity is 1 year and YTM is 15 percent, naturally the maturity value of the bond will be 1150.

Similarly, if the initial price is 1,000, maturity is 9 years and the YTM is 15 percent, then the maturity value of the bond will be 3517.88. So that is how these figures are obtained. Minus 2,000 is the initial investment, 1150 is the redemption value of a bond of maturity 1 year, zero coupon bond trading at a YTM of 15 percent and initial value 1,000.

Similarly, 3517.88 is the redemption value of a 9 year zero coupon bond trading at a YTM of 15 percent and having an initial value t equal to 0 value of 1,000. Then we work out TC(T) and working out TC(T) is quite straight forward, and on that basis we work, then we work out the discounted TC(T). Discount rate is given at 15 percent because all the bonds are, both the bonds are trading at YTM of 15 percent. And therefore, the YTM of the portfolio will also be 15 percent. That is precisely what is given here.

This is, portfolio IRR or portfolio YTM is 15 percent. And on solving this expression for the duration what we find is the duration is 5 years. Now, the duration of Portfolio II is also 5 years. Why? Because it is a zero coupon bond of maturity 5 years. So the duration is 5 years. And duration equals maturity for a zero component bond.

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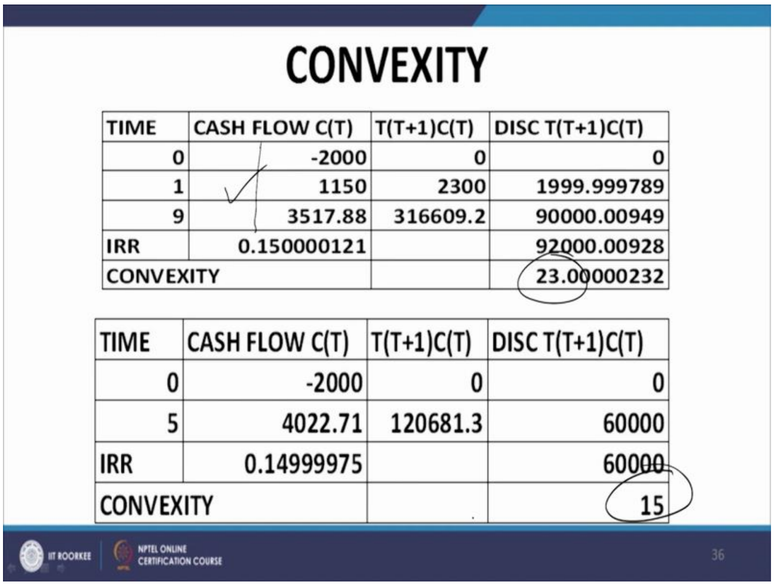

Now let us look at convexity. So as far as the duration is concerned, the Portfolio I and the Portfolio II are perfectly matched, but what about the convexity. Let us look at the convexity. We have got these figures from the previous slide, so no issue regarding these figures. We work out the convexity on the basis of these figures only.

The portfolio IRR or the portfolio of YTM is given as 15 percent and we end up with the convexity for bond, for Portfolio 1 as 23. Convexity for Portfolio I is 23, and when you do, work out the convexity for Portfolio II, which is again quite straightforward, we find that the convexity is 15.

So here lies the catch, here lies the relevance of the barbell strategy. A barbell strategy notwithstanding the fact that the duration of this barbell strategy is the same as the duration of the liability of intermediate value or intermediate timing, the convexity of the barbell strategy is much, much higher than the convexity of the single or the bullet liability, as it is called.

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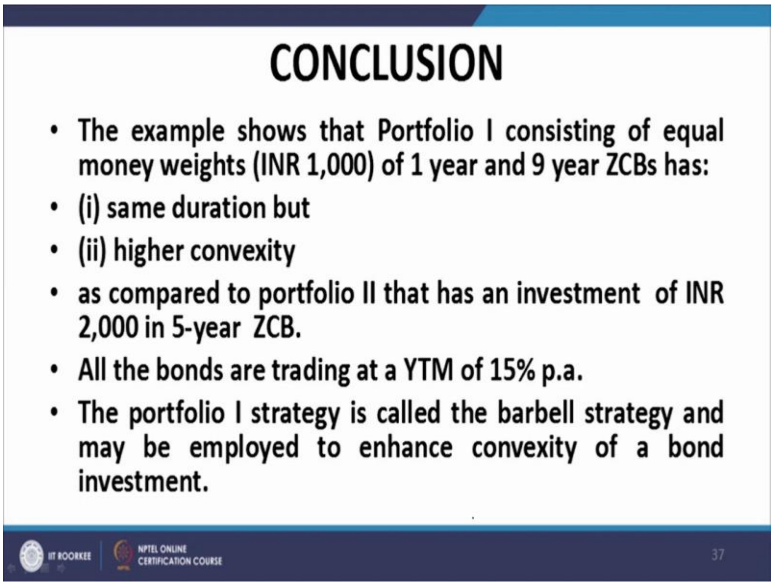

So what is the conclusion? This example shows that the Portfolio I consisting of equal money weights of 1 year and 9 year zero coupon bonds has the same duration but has higher convexity as compared to Portfolio II that has an investment equal to that of investment in Portfolio I but in a single bullet 5 year zero coupon bond. All the bonds are trading at 15 percent YTM. The bond I strategy is called the barbell strategy and may be employed to enhance the convexity of a bond investment.

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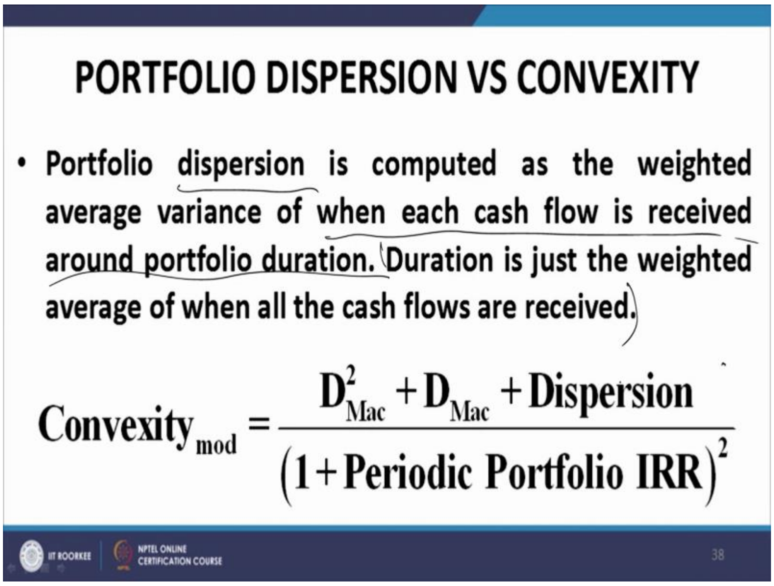

Portfolio dispersion. Now, this is another term which we have not encountered as yet but which is very relevant when we talk about portfolio management. This basically talks about the variability. It is a measure of the variability of cash flows emanating from a bond portfolio. How do we define it?

Portfolio dispersion is computed at the weighted average variance of when each cash flow, variance of what? Variance of time. What time? When each cash flow is received around portfolio duration. So just like duration is a measure of time, convexity is a measure of time squared, dispersion is a measure of time.

What is that measure? When each cash flow is received around portfolio duration. I repeat, when each cash flow is received around portfolio duration. Duration is just the weighted average of when all cash flows are received. So this is, this is duration, this is the what you call abridged definition of duration.

But as far as dispersion is concerned, let me repeat, dispersion is computed as the weighted average variance of when each cash flow is received around portfolio duration. The relationship between convexity and dispersion is given in this particular equation. And you can see here, higher the dispersion of cash flows, higher is the convexity and vice versa.

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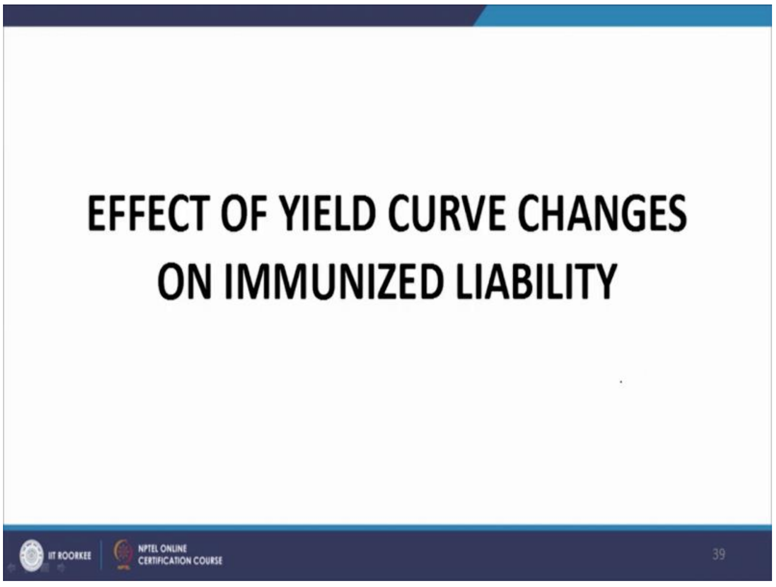

Effect of field curve changes on immunized liability. Let us explore the various, the, how the immunized liability, immunized portfolio, how the immunized portfolio behaves when there is a change in the yield curve. The change in the yield curve as we mentioned, may be modeled in terms of three possible changes.

Number 1, the level shift, a parallel shift, you must say, a steepening where the short term rates decrease and the long term rates increase, the curve becomes more steep or the change in curvature where the short term rates and the long term rates increase but the intermediate rates remain more or less unchanged.

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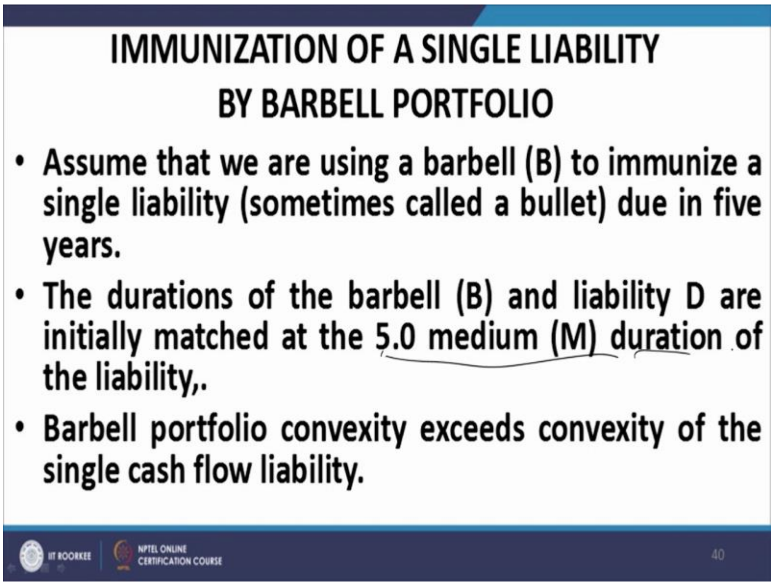

So immunization of a single liability by barbell portfolio. Assume that we are using a barbell B to immunize a single liability sometimes called a bullet liability due in 5 years, which we are carrying forward the example that we did a few minutes back. The duration of the barbell B and liability D are initially matched at the 5 year medium duration.

As you saw the duration of the barbell was 5 years and that is also the duration of the liability. Barbell portfolio convexity exceeds convexity of the single cash flow liability. In our example at a YTM of 15 percent for all the bonds, it turned up to be 23 for the barbell portfolio, convexity was 23 for the barbell portfolio and 15 for the bullet liability.

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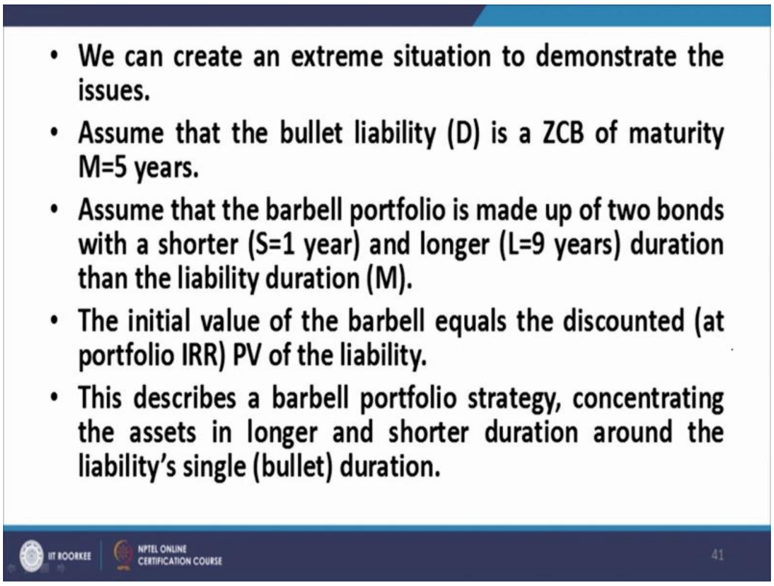

We can create an extreme situation to demonstrate the issues assume that the bullet liability D is a zero coupon bond of maturity M equal to 5 years. Assume that the barbell portfolio is made up of two bonds with a shorter S equal to 1 year and a longer L equal to 9 year duration than the liability duration which is M. M is in between. M is equal to 5, S is equal to 1 year, L is equal to 9 years.

The initial value of the barbell equals the discounted at portfolio IRR present value of the liability. This describes a barbell portfolio strategy concentrating the assets in longer and shorter duration around the liabilities single bullet duration. So that is, that is the basic definition of how we use the barbell strategy for immunizing a portfolio.

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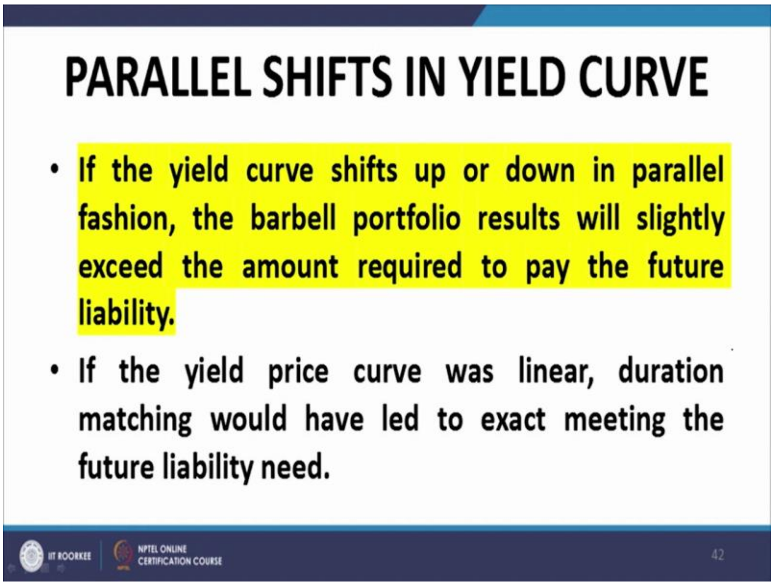

What is the impact of various shifts? Let us start with the simplest, let us start with the parallel shift in the yield curve. Let us assume that there is an increase in yields if the yield curve shifts up or down in parallel fashion, the barbell portfolio results will slightly exceed the amount required to pay the future liability.

If the yield price curve were linear, duration matching would have been adequate and would have resulted in exact matching of the immunization results. But the yield curve is not linear, that is why the issue of convexity comes into play. And convexity is such that, such a parameter that for the barbell, it is more, for the bullet liability, it is less.

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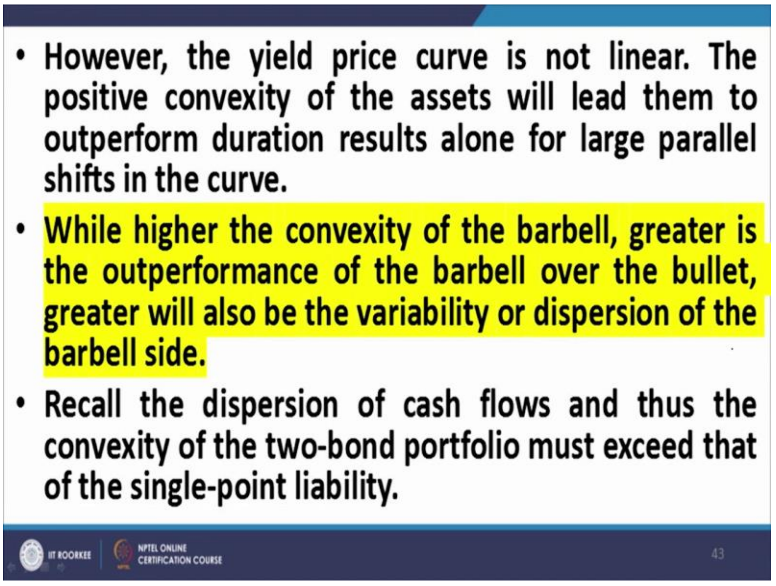

However, the yield price curve is not linear, the positive convexity of the assets will lead them to outperform the positive net convexity, that is positive net convexity or the higher convexity, you may say, higher convexity of the assets compared to that of the liabilities may lead them to outperform duration results alone for large parallel shifts in the curve.

While higher the convexity of the barbell, greater is the outperformance, higher the convexity greater is the outperformance of the barbell over the bullet liability, greater will also be the variability or dispersion of the barbell side. Let me read these two points again.

However, the yield price curve is not linear. The positive convexity of the assets that you can, will lead them to outperform duration results alone for larger parallel shifts in the curve. So if the curve has large parallel shifts, duration alone will not be adequate and convexity correction will be necessary.

And because this convexity correction is positive, the performance by incorporating the convexity correction would, would be better would be superior, and the price change would be more compared to the price change as determined or as ascertained on the basis of the pure duration model.

So the outcome of what I have said just now is also that if you have this barbell strategy and you have this bullet strategy, the barbell convexity is higher, the bullet convexity is lower, therefore the pos, the convexity correction which is positive, please note this, the convexity correction which is positive for the barbell would be higher than the convexity correction for the bullet.

And the net result would be an outperformance by the barbell compared to the bullet. Recall the disperse, however the higher convexity also implies higher dispersion of cash flows. Recall the dispersion of cash flows and thus the convexity of the two bond portfolio must exceed that of a single point liability.

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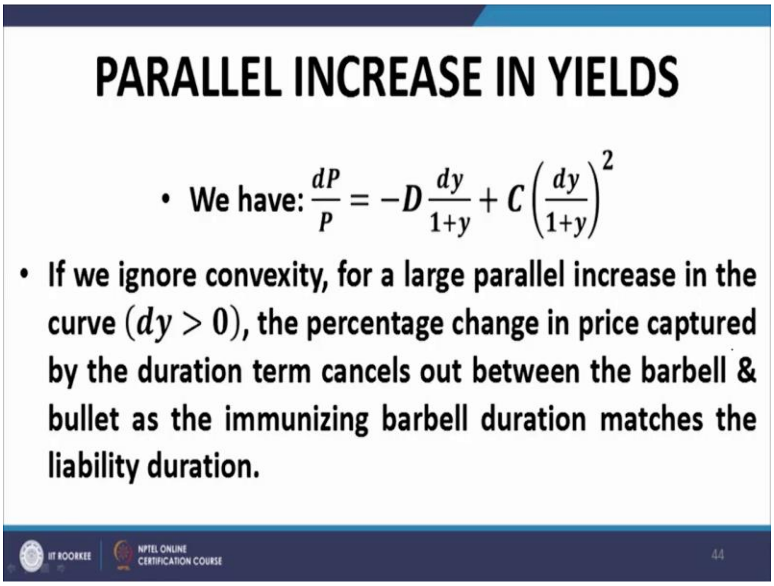

Parallel increase in yields, now we talk about the specific case of an increase in yield. If we ignore convexity for a large parallel increase in the yield curve, in the spot yield curve, the percentage change in price captured by the duration term will cancel out between the barbell and the bullet because both the barbell and the bullet have same duration as we saw just now, when I took up the example to illustrate the barbell strategy.

So if you have a liability and you have a barbell to cover that liability against him, or to immunize that liability, you will match the durations of the two. And by matching the duration, the net zero duration becomes zero, or the, you may also say that the price change corresponding to the duration term will be neutralized, will be equalized between the liability and the barbell assets. However, what happens in the case of convexity?

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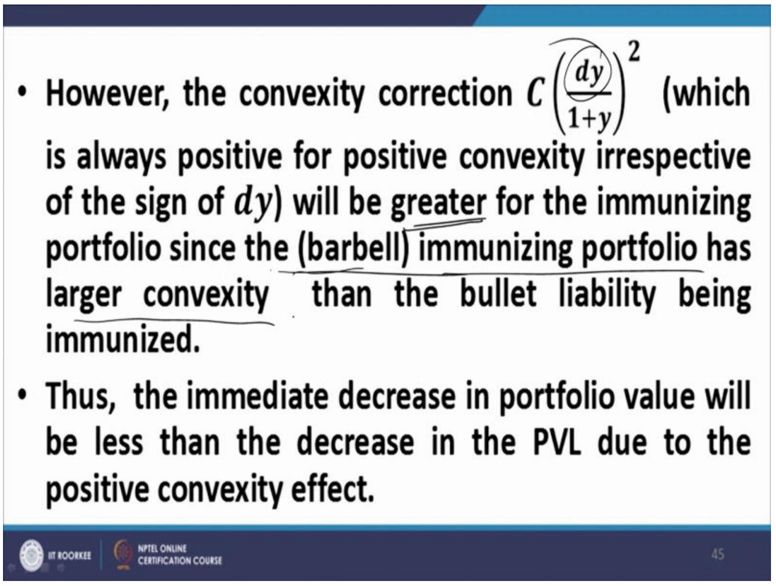

The convexity correction is positive. The convexity correction is always positive irrespective of whether there is an increase in interest rate or decrease in interest rate, the convexity correction is positive. So however, the convexity correction, which is always positive for positive convexity, irrespective of the sign of d y, will be greater for the immunizing portfolio since the barbell immunizing portfolio, first of all, it will be greater for the immunizing portfolio since the barbell immunizing portfolio has higher convexity, has larger convexity.

So because the barbell has larger convexity, its, it will outperform the bullet in terms of the price change due, irrespective of the direction of the price change, or the direction of the interest rate change rather. So if the interest rates increase, what happens? The bullets value will decrease, the barbells value will decrease, but the barbells value will decrease lesser because the convexity correction in the case of barbell which operates positively is higher compared to the correction due to the, in the bullet. Thus, immediate decrease in portfolio value will be less than the decrease in the present value of the liability due to the positive convexity effect.

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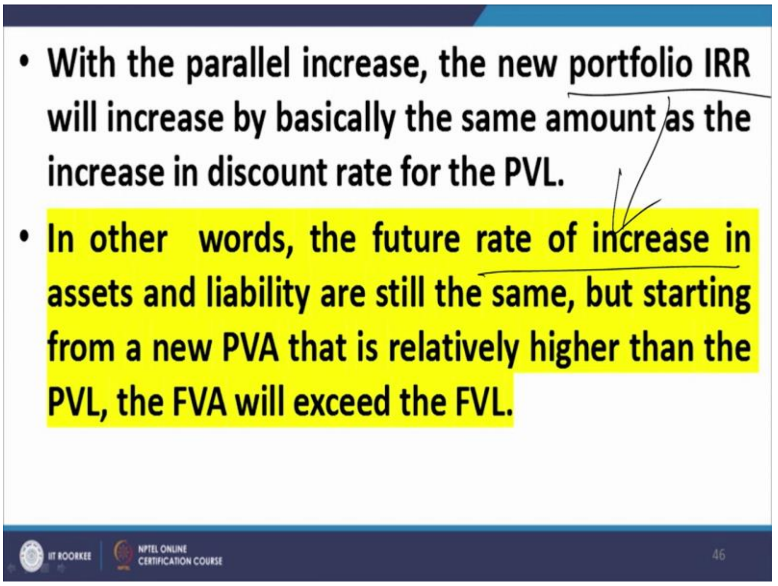

With the parallel increase, the new portfolio IRR will increase basically by the same amount as the increase in the discount rate for the present value of liability. So because this is a parallel shift, all interest rates are increasing by the same amount. This, this would imply that the new portfolio IRR will increase by basically the same amount as the increase in the IRR or discount rate or YTM of the liability.

In other words, the future rate of increase in assets and liability is still the same. Why? Because the rate of increase here, when we talk about this rate of increase, this is simply the portfolio IRR. In other words, the future rate of increase in the assets and liabilities are still the same, but starting from a new present value of assets which is relatively higher than the present value of liabilities, the future value of assets will exceed the future value of liability. Let us understand this with an example.

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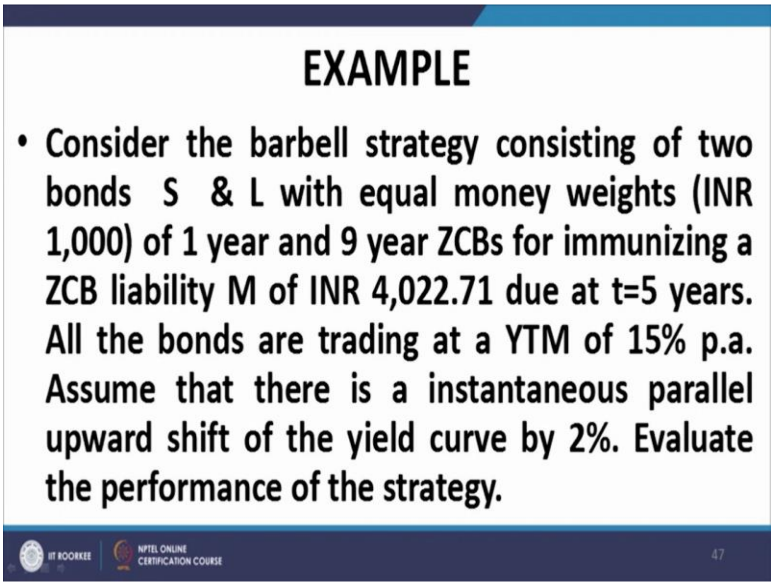

Consider the barbell strategy consisting of two bonds S and L with equal money weights INR 1,000 of 1 year and 9 year zero coupon bonds for immunizing a zero coupon liability M of INR 4,022.71 due at t equal to 5 years. Assume that there is a instantaneous parallel upward shift of the yield curve by 2 percent. Evaluate the performance of the strategy.

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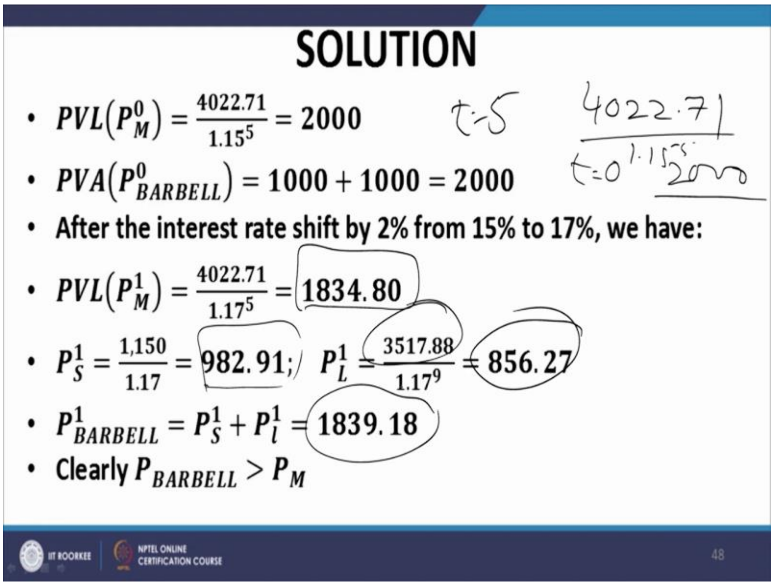

Now, what is the t equal to 0 value of the liability? It is given that t equal to 5 value, that is the value at t equal to 5 is equal to 4,022.71. And it is also given that the YTM of this liability is 15 percent. So this is a zero coupon liability. So there are no intermediate coupons and that means the today's value of this liability, t equal to 0 value of this liability turns out to be 1,000.

So to immunize this liability of 2,000 at t equal to 0, which will develop into a liability of 4,022.71 at t equal to 5 years, what we do? We take, or we make use of the barbell strategy. How do we use this barbell strategy? We invest equal amounts, 1,000 in a 1 year zero coupon bond and 1,000 in a 9 year zero coupon bond. 1,000 in 1 year zero coupon bond and 1,000 in 9 year zero coupon bond both trading at a YTM of 15 percent. It is given that all the bonds are trading at YTM of 15 percent.

Now, after the interest rates shift by 2 percent, from 15 percent to 17 percent, there is an upper shift of the YTM, of the YTM curve or the spot yield curve, the present value of liabilities will reduce. Naturally, it will reduce. By how much? It will reduce to 1,834.80 because now the YTM required will be 17 percent. And as a result of it, the present value of the liability will decrease.

What about the present value of the shorter bond, that is the 1 year bond? It will decrease from 1,150 which was the maturity value because it was a 1 year bond trading at 15 percent YTM and having today's value of 1,0000. So naturally, the value at t equal to 1 year would be 1,150. So 1,150 divided by 1.17, that gives you 982.91.

Similarly, today's value of the 9 year bond, the bond L which, at the YTM of 17 percent when the maturity value is given by 3517.88 which is the redemption value corresponding to a YTM of 15, zero coupon bond t equal to 0, value 1,000, will now be 856.27. So what is the present value of the barbell? The present value of the barbell is 1,839.18. And what is the present value of the liability, the present value of the liability is 1,834.80. So clearly, the present value of the barbell is more than the present value of the liability.

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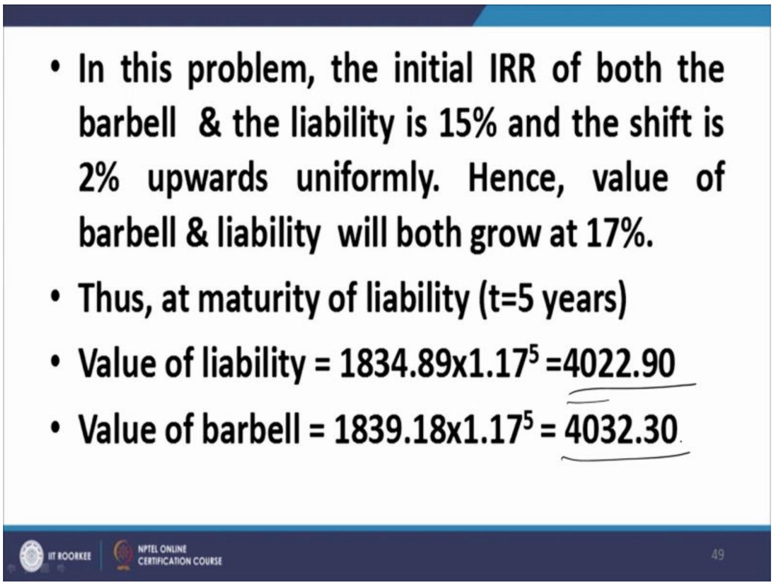

In this problem, the initial IRR of both the barbell and the liability is 15 percent and the shift is 2 percent upwards uniformly, hence the value of the barbarian liability will both grow to 7, grow at 17 percent. Thus, at maturity, value of liability t equal to 5 years, that will be equal to 1,834.89 into 7, at the rate of 17 percent. That becomes 4.2, 4,022.90, whereas the value of the barbell grows to 4,032.30. Thus clearly, the barbell has adequate assets to meet the liability when it materializes at t equal to 5 years.

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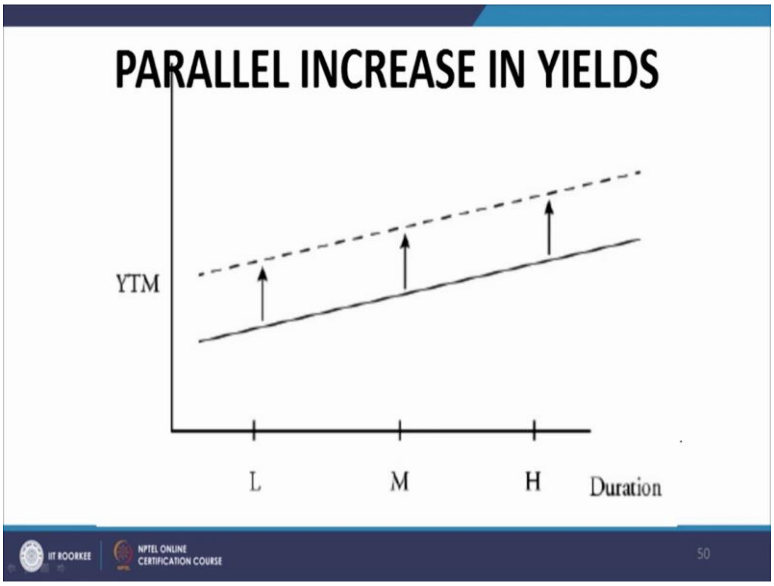

This is a example of a parallel upshift in the yields.

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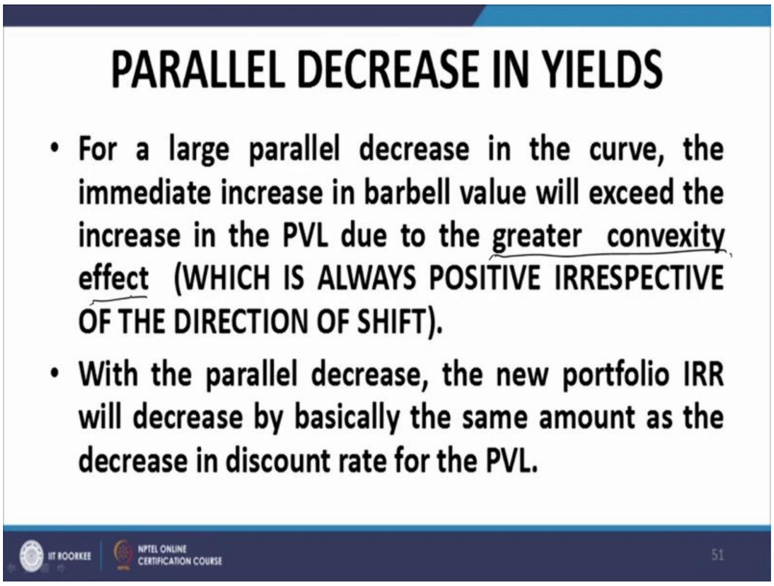

Now, we talk about a parallel downshift, parallel decrease in yields. For a large parallel decrease in the curve, the intermediate increase in the barbell value, the immediate increase, I am sorry. For a parallel decrease in the spot yield curve, the immediate increase in the barbell value will exceed the increase in the present value of liabilities due to the greater convexity of the barbell.

Let me repeat. This is very important. For a, for a large parallel decrease in the yield curve, spot yield curve, the immediate increase in the barbell value will exceed the increase in the present value of liabilities. Why? Because the barbells convexity is higher than the convexity of the bullet liability.

So, and because the convexity correction is always positive irrespective of the direction of the shift even if the direction of the shift is low is negative, even if the interest rates are going down, the convexity correction is positive, and because the convexity of the barbell is higher the increase in price of the barbell will be more than the increase in price of the bullet liability. With the parallel decrease, the new portfolio IRR will decrease by basically the same amount as a decrease in the discount rate for the present value of liabilities.

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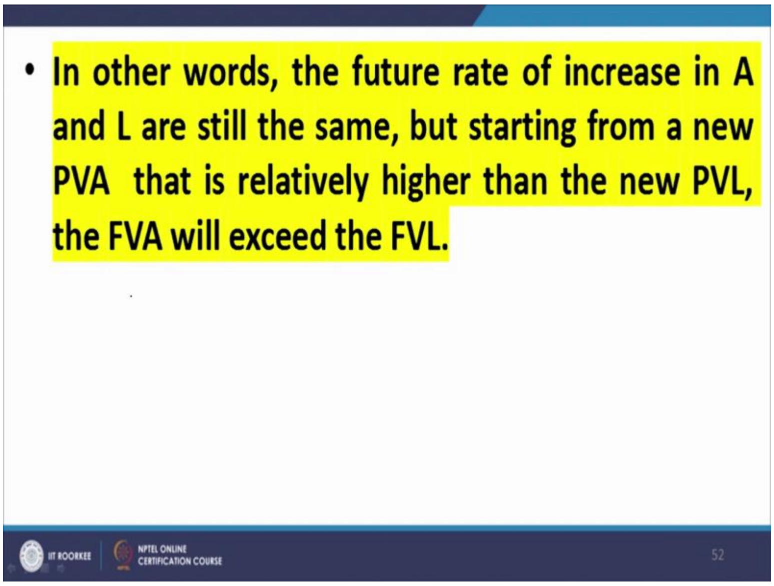
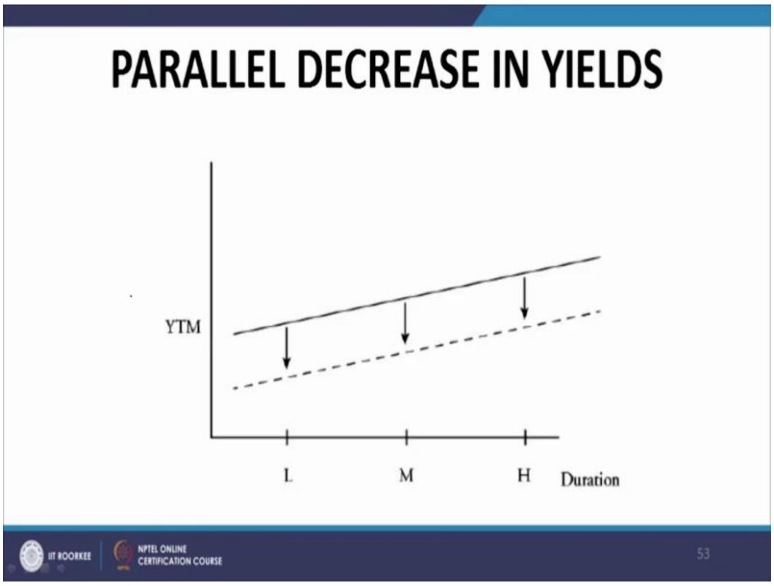


In other words, the future rate of increase in A and L are still the same, but starting from a new PVA which is relatively higher than the new PVL, the future value of assets will exceed the future value of liabilities. Let me repeat. In other words, the future rate of increase in A and L are still the same, but starting from a new present value of assets that is relatively higher than a new present value of liabilities, the future value of assets will be, will exceed the future value of liabilities.

(Refer Slide Time: 30:51)



This is the diagram. This shows a decline in the years, parallel decline in the yields.

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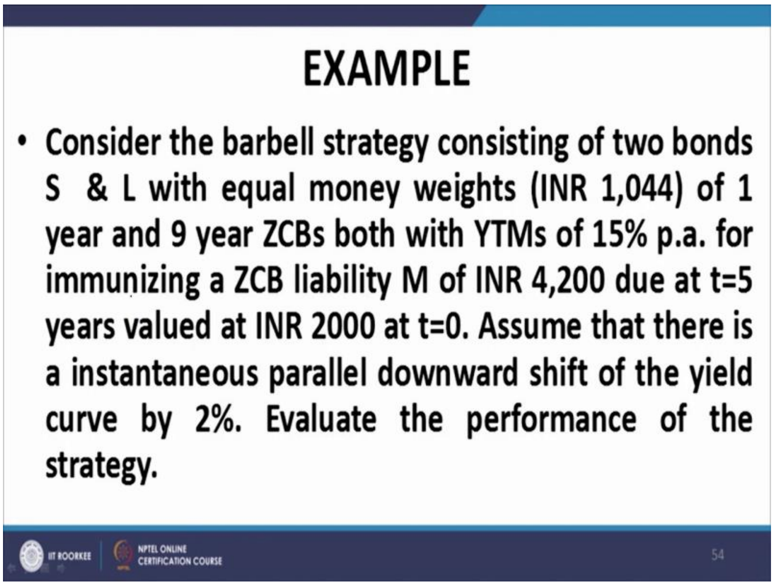

This is an example which I will take in the next lecture. Thank you.