Quantitative Investment Management
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Lecture 24
Immunizing a Single Liability

Welcome back, so let us continue with the example that we are discussing that we had initiated towards the end of the last lecture, let me read out the example once again very quickly, it goes as follows, a fixed income analyst wishes to forecast the expected return of a bond portfolio for the next year. He gathers the following information and assumes that no reinvestment of coupons cash flow takes place. The average coupon rate of the portfolio is 3 percent per annum. The coupon frequency is semi-annual. The horizon analysis that is the holding period, projected holding period that is 1 year. And the average bond price of portfolio T equal to 0 that is right now is 101.

5 double 0. The average life of the bonds constituting the portfolio is 7 years. The YTM to a price of 101.500 that is the current price, the t equal to 0 price is 2.

76 percent per annum. We assume that the initial yield curve is upward sloping and the 6 year yield is at 2.56 percent. Please note the 7 year yield that is the t equal equal to 0 yield corresponding to the price of 101.500 is 2.

76 percent. At the end of 1 year the bond will have a remaining life of 6 years and it is projected that at that point in time the YTM for a 6 year bond will be at 2.56 percent. It is also given that at the YTM of 2.56 percent that is at the end of 1 year, the price of the portfolio will be equal to 102.

419. So, let me repeat this, this is the cardinal content of this question, this problem, t equal to 0, YTM is 2.76 percent and the price is 101.500. and the life is 7 years of the bond portfolio. And at t equal to 1, the remaining life that is of the bond portfolio will be 6 years and because the yield curve is upward sloping, the YTM has changed from 2.

76 percent to 2.56 percent. and evaluated at 2.56 YTM, the 6 year bond carries a price of 102.419, this is given in the problem.

The analyst also projects that a general decline in interest rates of 50 basis points is likely, a decline in credit spreads is expected as well. Combining the credit spread decline for the portion of the portfolio in credit risky bonds with the general 50 basis points decline, the analyst projects a 54 basis points So, by apportioning or by distributing the decline in the credit spreads over the credit risky bonds, you see the entire portfolio may not comprise of credit risky bonds, it may comprise of some credit risky bonds and some government bonds as well. for example, and the basic thing is if you distribute the impact of the credit spread over the entire portfolio, do some kind of an averaging over the entire portfolio of the change in credit spread, it turns out to be approximately 4 basis points assessed or apportioned over the entire portfolio. Added to this is the 50 basis point general decline, so we end up with a total decline 54 basis points in the across the portfolio. The expected yield and spread change of portfolio is therefore minus 0.

54 percent as I explained just now 54 basis points. The modified duration is given as 5.60 and the modified convexity of the portfolio is given as 28. To assess credit losses, the analyst estimates 0.071 percent of the portfolio's bond value will default and 50 percent of any defaulting value will be recovered, 85 percent will not be recovered.

on the average if a particular bond defaults, then 15 percent will be recovered, 85 percent will be the default amount, it will not be recovered. Then as far as the currency impact is concerned, the portfolio has invested 40 percent in foreign denominated bonds and the investor expects the foreign currencies to appreciate by 3.925 percent. Now we start with the income or the current yield, this is quite straight forward, this is elementary, the annual coupon payments, what is the annual coupon payment? The annual coupon payment is 3.00 over the current market price which is 101.

50, so that gives us annual coupon yield of 2.956 percent. So this is straight forward, you divide the annual coupon payment by the current market you get the income yield or the current yield and that turns out to be 2.956 percent. Now as far as the annual coupon yield, this is the per annum yield, please note this 2.

956 percent. If you want to convert it to a periodic return, you can for example the half yearly return or the return over a 6 month period will be equal to 2.956 divided by 2, that is equal to 1.478 percent. Please note, we have not considered reinvestment of coupons in solving this problem. We have assumed that the cash flow or the coupon payment at t equal to 6 months will not be reinvested.

If we assume that the reinvestment does take place and for example, if we assume a reinvestment rate of 2 percent per annum. then the cash flow of 1.5 that will be that will take place at t equal to 6 months will be reinvested for the next 6 months and we will get at the 2 percent per annum or 6 or 1 percent for the 6 month rate or the half year rate. So, we will get 1.5 into 1 percent that is 0.

015 percent of the return as reinvestment income. If we assume a reinvestment rate of 2 percent per annum and apply 1 percent over the amount of coupon payment that occurs at

t equal to 6 months which will be reinvested from t equal to 6 months to t equal to 12 months or 1 year that is 6 month period. The roll down yield relates to a bond pricing model assuming that the yield curve is unchanged. So assuming that the yield curve is unchanged what will happen? The bond is priced today at t equal to 0 as a 7 year bond at a YTM which corresponds to a 7 year interest rate and which is given as 2.76 percent at which the value of the bond has been estimated or has been arrived at 101.

50 units of money. At the end of 1 year, the bond will have a remaining tenure of 6 years and assuming that the yield curve, spot yield curve does not change, we shall use YTM rates corresponding to a 6 year maturity, which is given in the problem as 2.56 percent and at this rate of YTM of 2.56 percent, the value of the bond increases to 102.

419. So, 102.419 is the projected price at t equal to 1 year assuming that the yield curve is unchanged and the bond is now a 6 year bond is started as a 7 year bond at t equal to 0. Now, it is a t equal to 1 and therefore, the 6 years remain to maturity. And therefore, the 6 years spot rate becomes relevant for the YTM of this bond, which is given as 2.56 percent and using this 2.56 percent, we arrive at the price at t equal to 1 year of 102.

419. The roll down yield will be given by the projected price at t equal to 1 year at the end of the holding period or at the end of the investment horizon which is 1 year in our problem and that is 102.419 minus the initial price which is 101.50 divided by the initial price that is 101.50, so that turns out to be 0.

905 percent. The rolling yield is the aggregate of the income yield or the current yield and the roll down yield the aggregate is sometimes called the rolling yield and that turns out to be 3.86 percent. Now as I mentioned just now if you have a flat yield curve then and you have a bond coating at a premium then the premium gets amortized over the life of the bond until its maturity in a regular in a continuous manner and as a result of it the price of the bond at t equal to 0 would be higher than the price of the bond at t equal to 1 year will be higher than the price of the bond at t equal to 2 years. And at the end, at the maturity the bond will reach its par value. This is dictated by the considerations of arbitrage free pricing.

In the case of a bond quoting at discount, the discount is amortized just like the premium is amortized in the previous case. and the amount of discount gradually diminishes with the remaining maturity of the bond. However, this is not necessarily so when we have a sloping yield curve, for example, as I mentioned in the case of the upward sloping yield curve, what will happen is that the T equal to 0 price will be based on a longer spectrum of interest rates S01, S02, S03, S04 up to S0 capital T where capital T is the maturity whereas if the investment horizon is H years then the price of the bond worked out or the projected price of the bond worked out at H years will be based on the remaining spot rates that is starting from t equal to H the rate for a 1 year deposit that is S01, S02, S03, S04 up to S0 t minus H. Let me repeat for the pricing of the bond at t equal to 0 you have spot rates varying from S01, S02, S02 up to H0T, capital T. Now when H years have passed, where H is the holding period, the remaining life of the bond is T minus H years.

Therefore, when you assuming that the yield curve does not change, the relevant rates will be S01, S02, S03 up to H0T minus H. Now because we are considering here an upward sloping yield curve, the price or the YTM at on the basis of the first price that is the t equal to 0 price will be higher because it includes rates from t equal to H minus t to t equal to capital T which are higher compared to the rates H01, H02, H03 because you have an upward sloping yield curve whereas the price at t equal to H would comprise of the rates H01, H02 up to H0, t minus H that will be lower and thus the YTM would be lower and the price would be higher. So, let me read out what I have explained just now. with a sloped yield curve, this may not always be true. This is because the bond price PH, PH means price at the end of the holding period, price at the end of the investment horizon will be worked out at a lesser term to maturity T T equal to H to T equal to capital T that is the period is T minus H then its price at T equal to 0 which is calculated with the term to maturity of T equal to capital T.

If the curve is upward sloping the shorter end rates are lower thus the rate for maturity of T minus H are lower than the rates for maturity of T equal to capital T. Hence, the price calculated at exit at t equal to h will be at a lower rate and hence higher. Now, expected prices as we have discussed, the analyst has projected a decline in interest rates of 0.54 percent, a decline of 0.

54 percent in interest rates. The duration and convexity are given as 5.60 into 28 and we can simply apply the duration convexity model which is embedded in this particular equation, let us call it equation number 1 and using equation number 1 where M D is the modified duration plus 1 by 2 into effective convexity into delta y square that is on computation turns out to be 3.065 percent. So, this is the price change, this is the positive price change due to the decline in interest rates. As far as the credit losses are concerned, the analyst estimates that 0.

071 percent of the portfolio's bond value will default and 15 percent of any defaulting value will be recovered, which means 85 percent of defaulting bond. value will be lost and this implies that the expected credit losses are 0.071 percent into 0.85 that is equal to 0.

06 percent, this is straight forward. Similarly, the expected gains or losses versus the

investor currency, the 40 percent of the investment in the bonds is in foreign denominated bonds and the expected increase is 3.925 percent and therefore, the effective increase over measured over the entire value of this portfolio is 3.925 percent into 0.

40 that is 1.570 percent. So, here is the summary of the return components, the yield income or the annual coupon over the current bond price is given as 2.956 percent. The roll down yield we have worked it out as 0.905 percent and the price change due to investor yield change predictions is 3.065 percent and as far as credit losses are concerned it is minus 0.

06 percent and as far as the currency gains are concerned that is 1.57 percent. The aggregate expected total return for the year is 8.395 percent. When you algebraically add all these quantities taking care of the minus signs wherever relevant, you end up with the value of 8.

395 percent. So, this is the example, this is the illustration of what could be the possible return components when you make an investment in a bond portfolio. Let us now talk about a new topic. Suppose you are given, you have a liability that is going to materialize at the end of And that is represented by a bond, let us say it is a 5 year 0 coupon bond that is a liability for you that may be you have to redeem let us say. And we need to immunize ourselves against any changes in interest rates in the intervening period between t equal to 0 and t equal to 5 years when the liability is to be redeemed. Now, for this before we get into the nitty-gritty of this particular problem which is very fundamental problem in the context of fixed income portfolio management, let us recall some features of the duration convexity model specifically and in general bond analysis that I have discussed in the previous lectures.

The duration-convexity model is given by the equation that is equation number 1 here, right at the top of your slide. In terms of the McAuley's duration and convexity, we have the left-hand expression that is this expression and in terms of modified duration and modified convexity which is usually used in the CFA examinations, we have the second part of the equation which I have represented by the arrow. Now, the duration model works well on the works on the YTM and that is it assumes a single rate for all maturities. The duration model works on the YTM that is it assumes a single rate for all maturities. y that is it ignores the term structure of interest rates, it assumes a flat spot rate curve, I have already explained this point a number of times.

There are two fundamental assumptions in duration, number one that the term structure is flat that is all the cash flows are discounted at the YTM rate. that is number 1, all cash flows from the bond are discounted at the YTM rate. And number 2, the shift corresponding to an interest rate change is a parallel shift in the yield curve, that is there is no asymmetry in so far as the shift in the yield curve is concerned. The D y that we consider in the duration model, the D y that you see here, the D y that you see here or here is uniform.

across all maturities. In other words, the term structure or the yield spot yield curve shifts parallel to itself by an amount dy. So, that is the second fundamental assumption, two important assumptions in the duration convexity model. We work on the basis of YTM that is we assume a flat yield curve and then the shift is also flat that is the shift is parallel to the yield spot yield curve in the original place. It assumes a straight line yield price curve in the locality of the given yield if you use the only duration that is if you use only this term. Then, what you are not considering the convexity or the curvature of the yield price curve that is how the behavior of the price corresponding to different yields that is captured by the yield price curve.

Please note the difference between yield price curve and the spot yield curve. The yield price curve is the relationship between price and changes in yield or changes in yield along the X axis and price along the Y axis. Whereas, the spot yield curve gives you the YTMs corresponding to different maturities, different maturities along the X axis and the corresponding YTMs along the Y axis. So, they are fundamentally different objects. Now, it assumes a straight line yield price curve in the locality of the given yield.

This is because we arrived at the definition of duration using the concept of Newtonian differentiation and in the Newtonian differentiation framework, we assume that in the infinitesimal neighborhood, infinitesimal proximity of the point at which we are working out the derivative The curve or the functional relationship y equal to f x is assumed to be a straight line within that very narrow point, narrow proximity both to the left and right of the point under reference. So, convexity accounts for the curvature of the yield price curve not the spot yield curve. So, convexity basically has nothing to do with the spot yield curve, nothing to do with the term structure of interest rates, it accounts for the curvature that is embedded or that is there in the yield price curve. The term structure is unattended, it is not captured in the Macaulay duration or modified duration framework. As I mentioned we assume a flat term structure, we assume a constant value of Y which is the YTM and if you assume a constant value of Y obviously you are ignoring the functional relationship between the spot between the maturities and the corresponding spot rates.

Now, one very important point that we have here is the convexity correction is always positive. If you look at this equation, let me call it equation number 1 on this slide, this is dy squared. Therefore, irrespective of whether dy is positive or negative, this correction so long as the convexity is positive and in fact, the straight bonds always have a positive convexity. So, so long as the convexity of the underlying portfolio is positive, so long as this is greater than 0. this will always be greater than 0 and therefore, the convexity correction into the price change or the correction due to the curvature will always be positive.

So, this is very important. The convexity correction is always positive so long as the convexity of the bond is positive irrespective of the direction of change of rates that is irrespective of the sign of d y. irrespective of whether rates are going up d y is positive or the rates are going down d y is negative, the convexity correction is always positive. The ramifications of this particular attribute shall become apparent as we move along in today's lecture. Convexity of straight bonds is invariably positive. Convexity of callable bonds may be negative at points where the option is at or near the money.

You see the reason behind this as I explained in the earlier lectures also is because as the bond approaches As the price of the bond approaches the excise price, when the interest rates become so low that the price of the bond approaches the excise price, the change in the price corresponding to a further change in interest rates is going to be less and a point in time will come when changes in interest rates will not be accompanied by any change in price. Why? Because as soon as the price goes above the exercise price, the issuer will exercise the call option and call back the bond. So, in no way can the investor sell the bond in the market at a price higher than the exercise So, what will happen? As the market interest rates decline, the price goes up, price keeps on going up until the point in time that the call option becomes in the money or near the money until when the exercise price is reached and once the exercise price is reached, any further decrease in interest rates will not result in a further increase in price because the investor will, I am sorry, because the issuer will exercise the option. So, if the liability duration and asset duration is matched, this is a first order matching. If the liability duration, please note this point, duration and asset duration is matched, this is a first order matching and will operate reasonably well for small shifts of the yield curve.

Let me repeat, this is another important point. If liability duration and asset duration is matched, that is the net duration is 0, this is a first order matching, duration is a first order approximation. So, this is a first order matching and will operate reasonably well for small parallel shifts of the yield curve. Larger the convexity of the assets or liability, greater will be the second order correction term. Naturally, larger is the convexity, greater is the magnitude of the second order correction. And please note this point, this convexity correction is always positive.

So, if what happens, if the assets convexity is higher, if the liabilities convexity is lower, irrespective of the direction of movement of the interest rates. the correction due to convexity of the assets would be higher and the correction due to the convexity of the liabilities will be lower. Please note both of them will be positive and as a result of which at least at first sight the benefit will accrue to the investor or to the party who is doing the portfolio management exercise. The second order correction term is positive always for state bond. As I mentioned, state bonds have a positive convexity and dy square is always positive irrespective of the sign of dy.

Therefore, whether the interest rates increase or the interest rates decline, dy square will always be positive. If c is positive, the convexity term in totality is positive and therefore, and furthermore if the asset side convexity is higher naturally the positive increase in price or the increase in price due to the convexity correction will be more for the asset side compared to the liability side. Hence higher the convexity of assets over liabilities greater will be the positive correction in asset price over liability price that is the asset price will move more favorably than the liability price, this is what I explained just now. However, greater convexity as I mentioned there is nothing called free lunch in this Greater convexity means higher prices, higher corrections in prices and if the asset side has more convexity, the insulator or the immunizer has the opportunity to gain. but the important catch is given in this paragraph, the catch is greater convexity also implies greater dispersion or variability of the cash flows corresponding to interest rate changes.

This I explained with an example, a detailed example when I talked about the concept of duration. Now, we talk about immunization. Again, this is more of a revision stuff, I have talked about immunization in an earlier lecture, so let us quickly run through it. Immunization is a fixed income management process in which the portfolio is managed to minimize the variability.

To minimize the variability, here lies the objective of immunization. It is not to maximize the return, it is to minimize the variability. Please note this point. When you are talking about immunization, you are talking about protection, you are talking about the protection against what? Against interest rate risk, changes in interest rates. When you are talking about immunizing a portfolio what you are interested in is minimizing the variability the change in the price of that portfolio if the interest rates show changes ratio deviations from projections. So, immunization is a fixed income management process in which the portfolio is managed to minimize the variability of the rate of return and over a specified time period.

That means now if a portfolio is immunized, the impact of interest rates on the portfolio is minimized, it is eliminated or at least minimized and that means what? That means the future value of the portfolio can be confidently predicted. Interest rates are a random variable. Interest rates are stochastic. So, interest rates change. If interest rates change, but your portfolio is so constructed that the impact of interest rates is eliminated, is minimized, then you can confidently predict the future value of the portfolio.

And therefore, you can work out if enough funds are invested initially, then known future liability can be funded. The goal of the immunized portfolio is to earn the initial portfolio IRR. This is important. The significance of this term will be clear as we progress along today's lecture. But you see the objective is that If you are immunized, if you are protected, if you are cut off from the rest of the world, the portfolio is cut off from the rest of the world, what will the portfolio earn? The portfolio will earn its initial portfolio IRR that the cash flows that are going to be generated from the portfolio will not change due to changes in the environment represented by changes in interest rates and as a result of which the initially projected or initially estimated interest rates will be earned notwithstanding the fact that the market interest rates.

So, the goal of the immunized portfolio is to run the initial portfolio RR, not the average YTM of the bond. Please note these are not identical quantities. I will talk more about it in later. It would be a digression if I talk about it at the moment.

So, I will talk about them at a later point in time. Note that the portfolio IRR need not equal the average YTM of the bonds constituting the bonds. This is the problem with the YTM. This is the drawback of the concept of YTM. I briefly touched upon this when I talked about the YTM. When you talk about the YTM of a portfolio, it is not equal to to the weighted average YTM of the bonds.

The YTM of the portfolio needs to be calculated on the basis of the aggregate cash flows emanating from the portfolio at the respective times and then using those cash flows when we equate the discounted value of those cash flows to the current market price, whatever is the discount rate. That will be the YTM of the portfolio. It need not be equal to the weighted average YTM of the constituents of the portfolio. Earning the IRR means the portfolio will grow to a sufficient future value to fund the liability. So, I will continue from here in the next lecturer.