Quantitative Investment Management
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Lecture 24
Modeling of Fixed Income Returns

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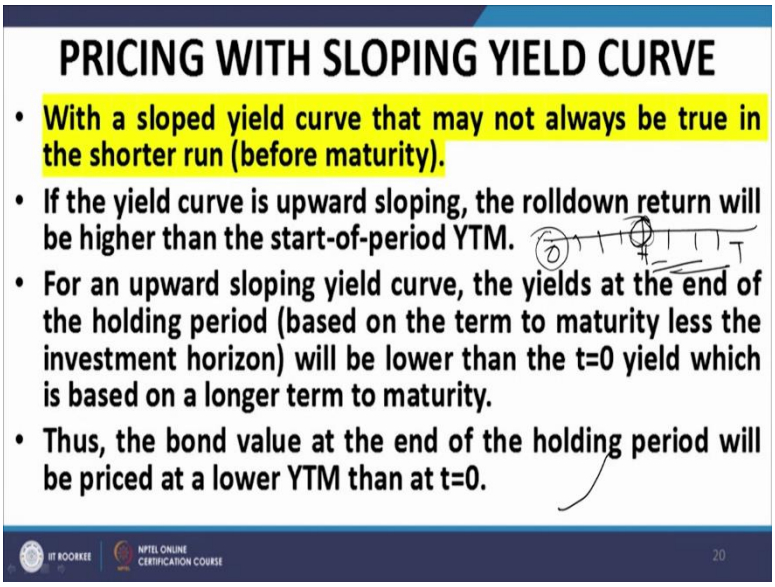

So, in last lecturer, we were talking about the pricing of bonds with reference to yield curve which is upward sloping way where this spot yields are increasing with maturity. In this case the simple or straightforward approach that we encountered by solving the previous example would not really hold and what we will find is that with a sloped yield curve, it may not always be true that the price gradually decreases as we move towards the maturity.

Now, if the yield curve is upward sloping, the rolldown yield and the rolldown return will be higher than the start of the period YTM, I illustrate this with an example, bear with me for the moment. Why is that, I also explained before the break but let us read it out again.

For an upward sloping yield curve, the yields at the end of the holding period based on the term to maturity less the investment horizon will be lower than the T equal to 0 yield which is based on a longer term to maturity because the longer the term to maturity the higher will be the spot rates that would come into the picture that would be involved when we do the pricing.

For example, if you do the pricing of a one year bond you are only having s01, if you do the pricing of a two year bond you are having s01 and s02 and please note s02 is greater than s01 because you are talking about an upward sloping yield curve. So, the point is when you at t

equal to 0 suppose you have a 10 year bond, you will be using the 10 year rates with s01 less than s02, less than s03 and up to s010.

But at the end of one year, it will now become a 9 year bond and therefore you will use assuming that the rates are unchanged you will use s01, s02, s03 up to s09 and you will ignore s010, it will not come into the picture and it so happens that s010 is the highest rate therefore the price when you work out the price at the end of one year, it will be based on the lower interest rates that is the issue. So, that is what is meant by in this particular paragraph.

For an upward sloping yield curve, the yields at the end of the holding period based on term to maturity less the investment horizon. End of the holding period, what is remaining? Remaining is the maturity minus the investment horizon that left, let me draw this time line, this is 0 this is H and this is T.

Now when you are evaluating the price of the bond at T equal to 0, you are looking at all the cash flows that occur up to capital T and you will be using the spotlights corresponding to each of these cash flows, timing of this cash flows. But at the end of the holding period, at the end of the investment horizon you are at this point.

Now at this point, the maturity of the bond has now become T minus H years. So, you will be using s01, s02, s03 up to s0 T minus H and it so happens that s01, s02, s03 T minus H gradually increase and therefore when you were here at T equal to 0, you had more discounting periods and the discounting periods which are ignored when you are at T equal to H are those discounting periods which happened at a higher discount rates, higher interest rates and as a result of which the YTM's corresponding to the bond at this point will be lesser than the YTM corresponding to this point, that is the point.

So, for an upward sloping yield curve, this is very important this is very fundamental. For an upward sloping yield curve because upward sloping yield curves are the order of the day that is not that is called a normal yield curve where the spot rates increase as maturity increases. For a upward sloping yield curve, the yields at the end of the holding period based on term to maturity minus investment horizon will be lower than the T equal to 0 yield. Why? Because T equals 0 yield also has the spot rates corresponding from T minus H onwards up to capital T and all of these are more than the rates which are earlier.

So, will be lower than t equal to 0 yield which is based on a longer term to maturity. Thus, the bond value at the end of the holding period will be higher at a lower because it is at a

lower YTM then at t equal to 0. The bond value at the end of the holding period will be priced at a lower YTM then the YTM at T equal to 0.

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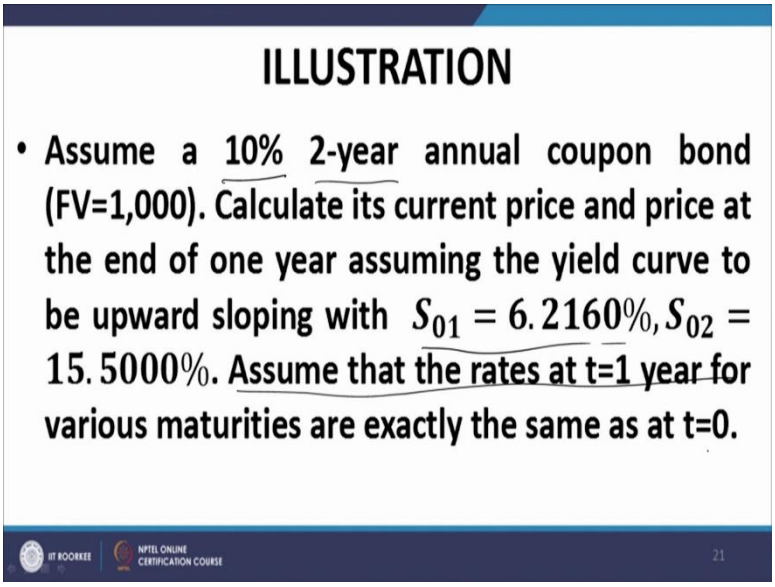

Let us do this illustration, I think this will clarify what I have been trying to say. Assume 10 percent, assume a 10 percent 2 year annual coupon bond face value is 1000. Calculate the current price and the price at the end of one year assuming that the yield curve is upward sloping with s01 equal to 6.21 percent and s02 equal to 15.50 percent. So, clearly it is an upward sloping curve s01 is much smaller than s02. So, it is a steep upward sloping curve.

Assume that the rates at t equal to 1, this is important this is also I mentioned it several times. We assume that the yield curve remains unchanged, the rates do not change at t equal to 1 or at the end of the investment horizon, we are faced with the same yield curve which we faced at t equal to 0. Assume that the rates at t equal to 1 year for various maturities are exactly the same at t equal to 0. Let us see how we solve this problem.

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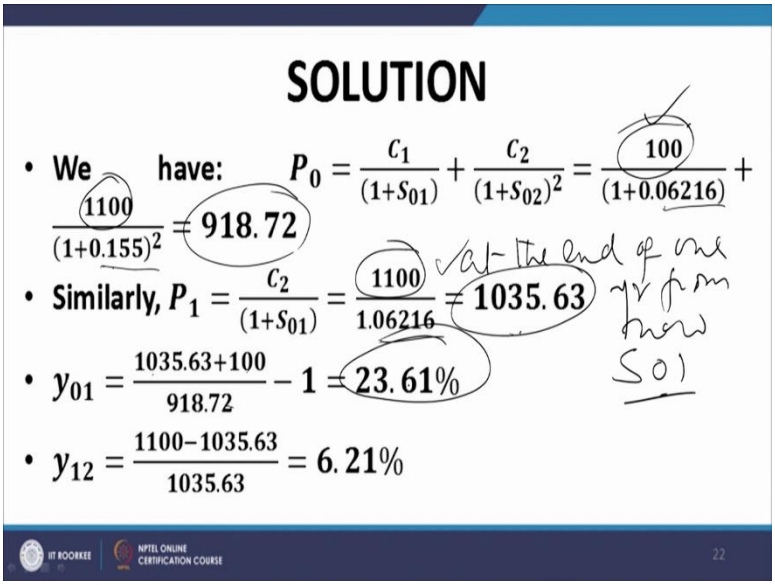

The price at t, t equal to 0 that is equal to c1 upon 1 plus s0 plus c2 upon 1 plus s02 square. The coupon rate is 10 percent, face value is 1000. So, this is hundred and the 1 year spot rate is equal to 6.21 percent. And at the end of the second year what will you get? You will get the coupon and you will also get the redemption of the principal. So, the total cash flow will be 1100 and it will be discounted at s02 which is 15.50 percent. The value that we arrive at is 918.72. So, far no issues no problem is quite straight forward.

Now at P1, when you reach P1, what will you get? You will get, assuming that the coupon at t equal to 1 has already been paid off. So, the only cash flow that remains from the bond is the 1100, that will be paid at t equal to 1 year from now t equal to 2 from t equal to 0 and now you have already crossed 1 year. So, there is 1 year left up to the maturity of the bond.

So, the cash flow is 1100 assuming. So, the pricing is after the payment of the coupon and now we have got this amount is going to be received at the end of 1 year from now and therefore you will discount it at s01 and we assume that the interest rates have not changed. So, s01 is nothing but 6.21 percent so this gives us a value of 1035.63.

This is the value at t equal to 1. If the yield curve has not changed, replace the same yield curve but we are left with only 1 year to maturity so the relevant spot rate will be s01 and s01 happens to be the lowest rate that is 6.21 percent and when we work out the return, what return we get? 1035 is the price at which the bond will sell in the market plus 100 is the coupon that I would have received. So, 1035 minus plus 100 and what would be my investment? My investment is 918 at t equal to 0. So, that gives me 23.61 percent return. This is much much more than the spot rate of 6.21 percent, please note this important feature.

Had it been a flat yield curve, you would have got a return of equal to s01 that is the spot rate but because it is an upward sloping yield curve and we are assuming that the spot rates do not change over the life of the investments, we continue with the same yield curve, it means that the yield is turning out to be much much more that is 23.61 percent and the yield of between the period t equal to 1 and t equal to 2 turns out to be 6.21 percent.

So, this is the anomaly that we encounter when we are faced with an upward sloping yield curve and which distinguishes itself from a situation where you have a flat yield curve.

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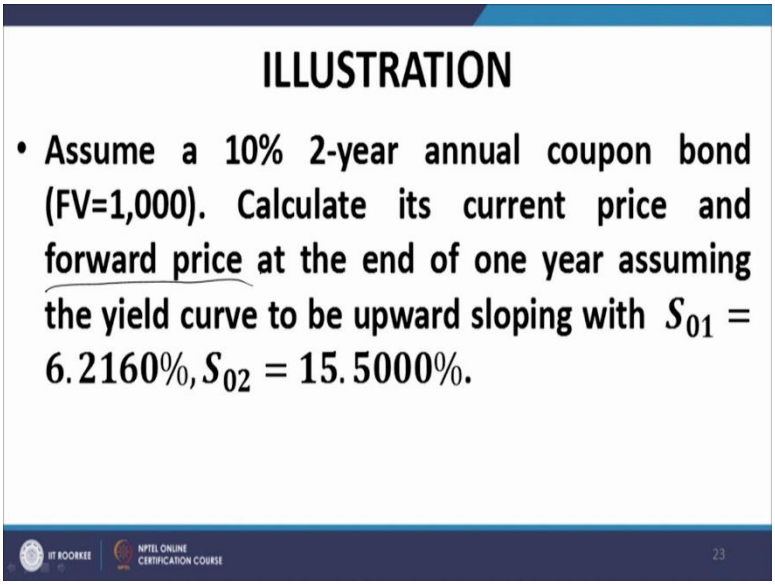

Now let us look at another situation, assume a 10 percent 2 year annual coupon bond. Calculate its current price and forward price, please note the words are forward price. This is different, this is a different problem at the end of 1 year and calculate the interest rate and calculate the forward yields.

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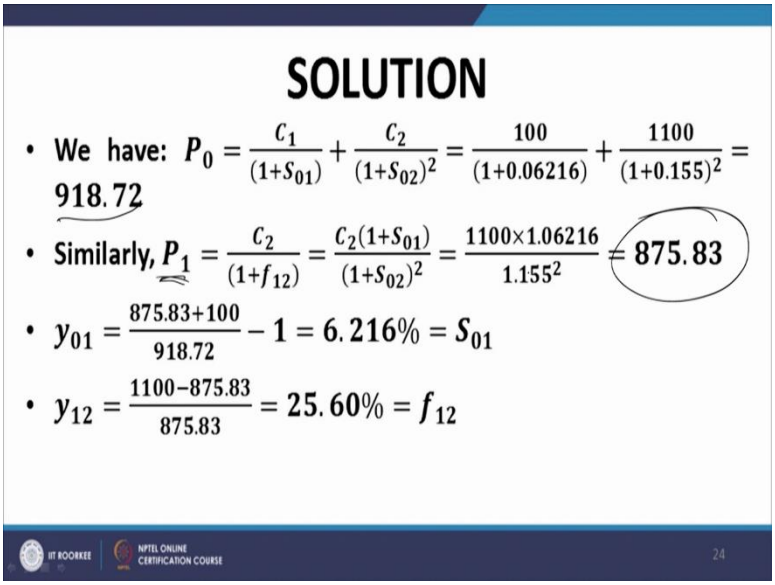

Now if you look at this problem as for the first part is concerned, it is same no issues. As far as the second part is concerned, the price at the end of only the forward price, please note this forward price at the end, this is the forward price at the end of 1 year will be given by c2 divided by 1 plus f12 and that is given as 875 plus a point 83 where we use the arbitrage free methodology or arbitrage free relationship for calculating the forward rate and we find that the return over t equal to 0 to t equal to 1 is equal to the spot rate and the return over t equal to 1 to t equal to 2, the forward return the forward yield over t equal to 1 to t equal to 2 is equal to the forward rate as mentioned earlier also to in today's lecture.

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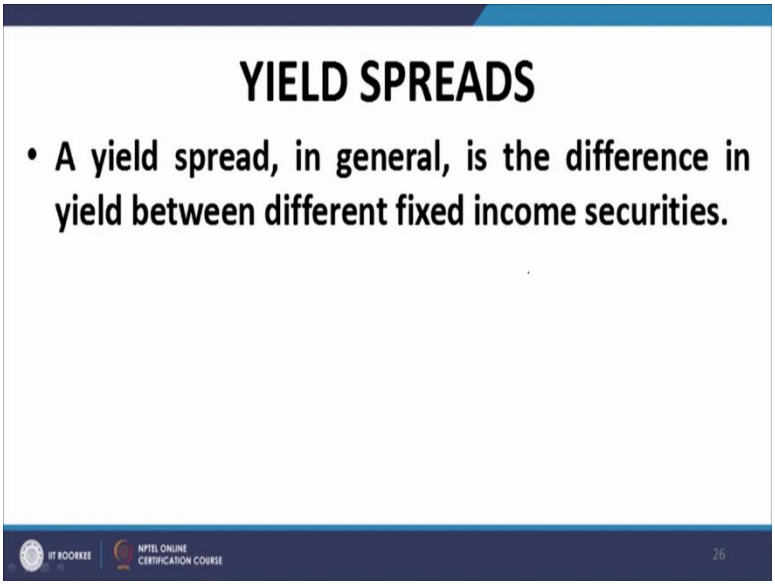

Now let us talk about yield spreads. Again, this is more of a revision more of a recapitulation before we get into bond portfolio management, fixed income securities portfolio management. A yield spread in general is the difference between the yield between two fixed income securities. Let me repeat, a yield spread in general is the difference in yields between different fixed income securities.

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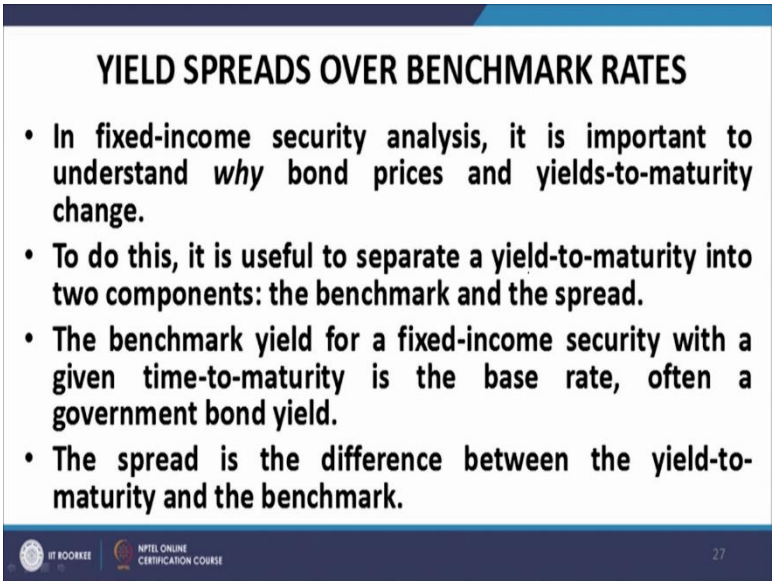

Yield spreads over benchmark rates, yield spreads over bench markets. So, let us do it quickly, in fixed income security analysis it is important to understand why bond prices and yields to maturity change. To do this, it is useful to separate a yield to maturity into two parts. Number 1, the benchmark and number 2, the spread over the benchmark.

So, the YTM of any particular risky bond can be split up into two parts. The benchmark yield and the excess over the benchmark yield which is called the spread. The benchmark yield for a fixed income security with a given term time to maturity is the base rate often a government bond yield, often the government bond yield.

The spread is the difference between the yield to maturity and the benchmark rate, whatever that benchmark rate you choose, if it is a fixed rate bond usually we use the government bond yield and if it is a floating red bond with use libre that is usually the case not necessarily so of course and the difference between the yield YTM on the fixed income security or the risky security and the government corresponding maturity government bonds is the yield spread over the benchmark.

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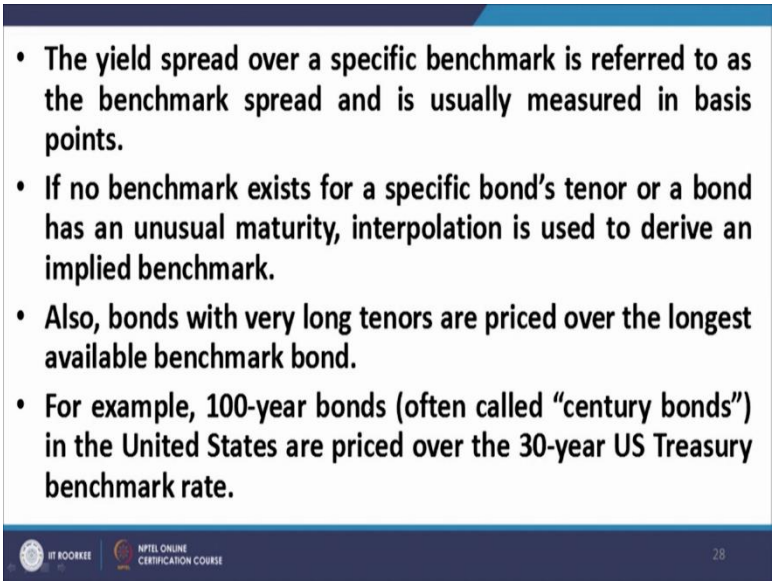

The yield spread over a specific benchmark is referred to as a benchmark spread and is usually measured in basis points. One basis point is equal to 0.01 percent, that is 1 upon 100 percent. If no benchmark exists for a specific bond standard or a bond has an unusual maturity, interpolation is used to derive its implied benchmark. Also, bonds with very long tenors are priced over the longest available benchmark bond. For example, 100 year bonds often called century bonds in the United States are priced over the 30 year US treasury benchmark rate.

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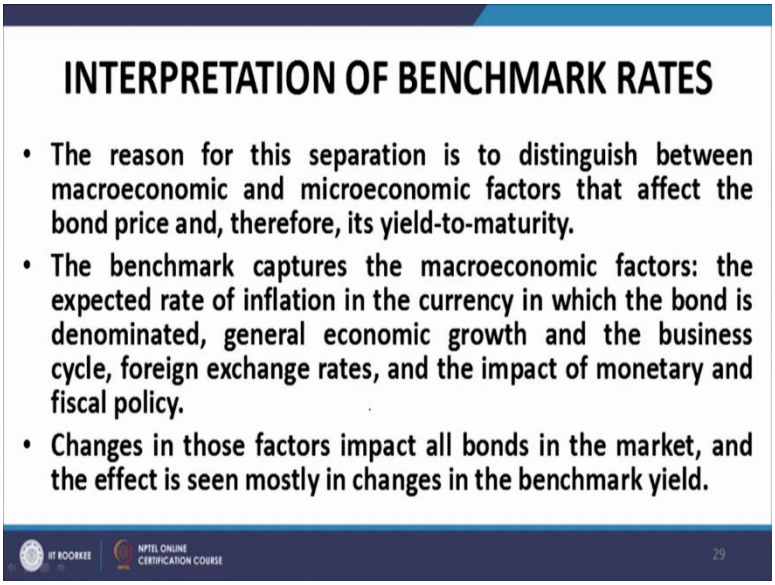

Interpretation of the benchmark rates, this is interesting, the reason for the separation is to distinguish that is the benchmark and the spread over the benchmark. The reason for the separation between the benchmark rate and the spread over the benchmark rate is to

distinguish between macroeconomic and micro economic factors that affect the bond price and therefore its YTM.

The benchmark captures the macroeconomic factors, the expected rate of inflation in the currency in which the bond is denominated, general economic growth and the business cycle, foreign exchange rates and the impact of monetary and fiscal policy. So, all these macroeconomic issues are captured by the benchmark rate.

Let me repeat, the benchmark captures the macroeconomic factors that is the expected rate of inflation in the currency in which the bond is denominated. General economic growth and the business cycle, foreign exchange rates and the impact of monetary and fiscal policies. Changes in these factors impact all bonds in the market and the effect is usually seen as changes in the bench market.

So, if there are changes in the macroeconomic factors, in the factors that influence the whole economy of a nation that would manifest itself as changes in the benchmark rate.

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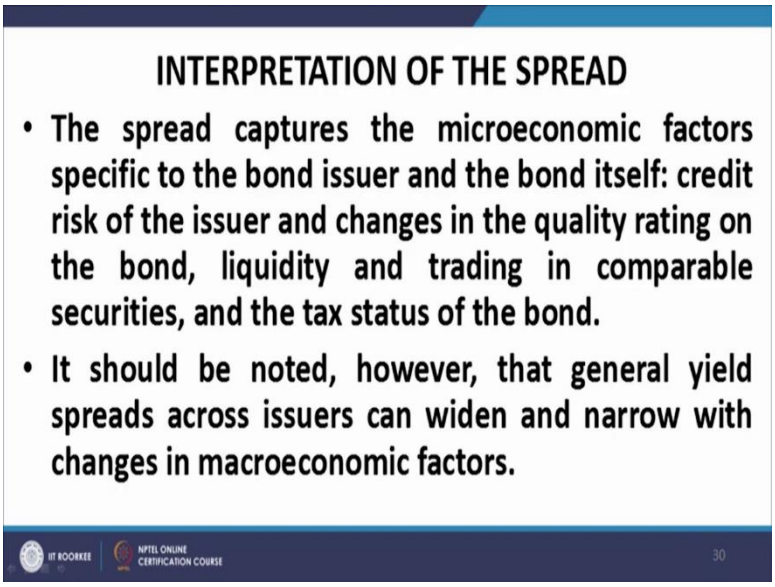

As far as the spread is concerned, the interpretation of the spread is rather elementary. It is rather obvious, it is the microeconomic factors, the factors that relate to the specific instrument that we are talking about which are captured in the spread. So let me quickly read it out, the spread captures the micro economic factors specific to the bond issuer and the bond itself. Like the credit risk of the issuer, changes in the quality rating on the bond, liquidity and trading in comparable securities and the tax status of the one. All these things contribute to the level of spread, extent of spread of the risky bond over the benchmark bond.

It should be noted however, that general yield spreads across issuers can widen or narrow with changes in macroeconomic factors.

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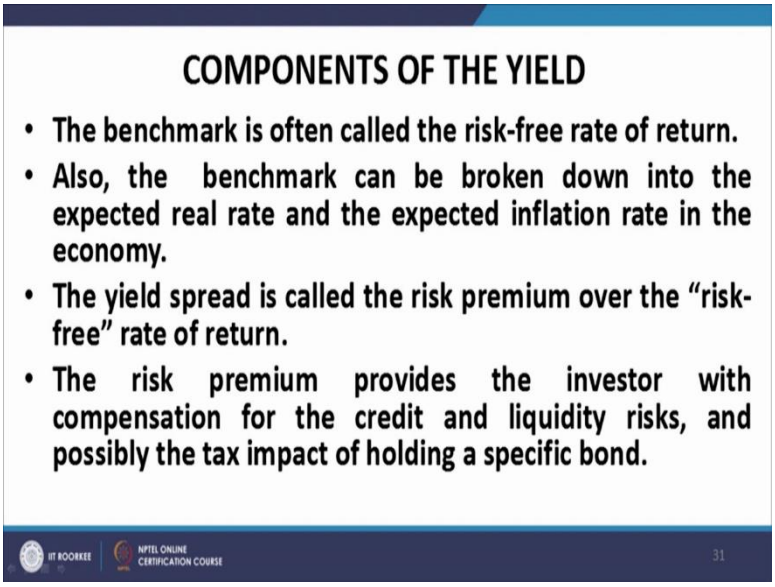

Components of the yield, it is usual to segregate the yield into the two major components and let us talk about them. The benchmark is often called the risk free rate of return. The benchmark rate is often called the risk-free rate of return also the benchmark can be broken down into the expected real rate and the expected inflation rate in the economy.

So, basically the benchmark rate captures two factors, number one the real rate of growth or the real rate of increase or the real rate of increase in the economy and number two, the expected inflation rate over and above the real rate. So, in a sense one can say that the benchmark rate is the price that one pays for the sacrifice of the utilization or the enjoyment or the use of a commodity for a given period. That is the basic rate and then you add to that the inflation premium you get the benchmark rate and you add to that the risk premium, you get the rate for the risky security. So, that risk premium is basically the yield spread.

Also, the benchmark can be broken down into the expected real rate and the expected inflation rate in the economy. The yield spread is called the risk premium over the risk free rate of return. The risk free premium provides the investor with compensation for the credit and liquidity risk and possibility of tax impact of holding a specific bond. So, as far as the benchmark rate is concerned it captures the basic rate, the basic sacrifice of enjoyment, postponement of enjoyment rather postponement of utilization of the money and also the inflation, the bench market captures these two, contributors to the time value of money and the risk is captured by the spread.

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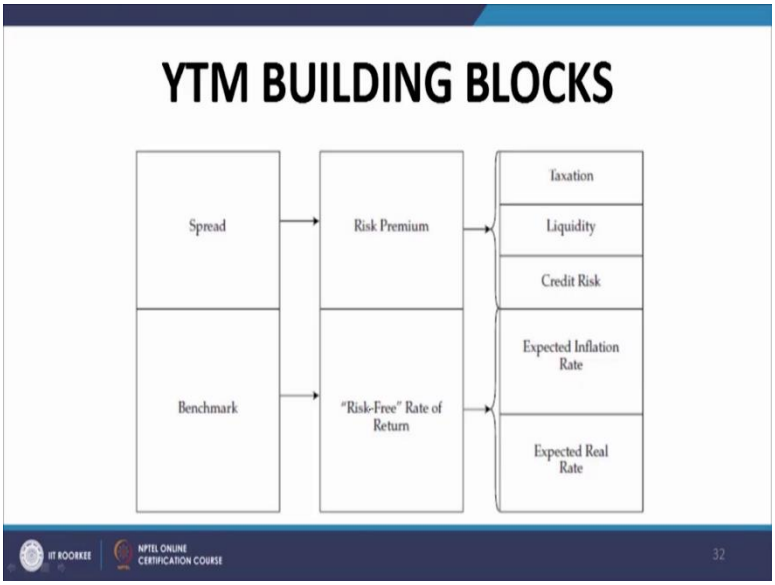

So, these are the YTM building blocks you can see here, this is the spread and the spread is the risk premium and you can see as far as the benchmark goes, this is the risk-free rate of return and the risk-free rate of return is determined by expected inflation and expected real rate. And as far as the risk premium is concerned, it is captured by or it is determined by taxability, liquidity and credit risk of the instrument.

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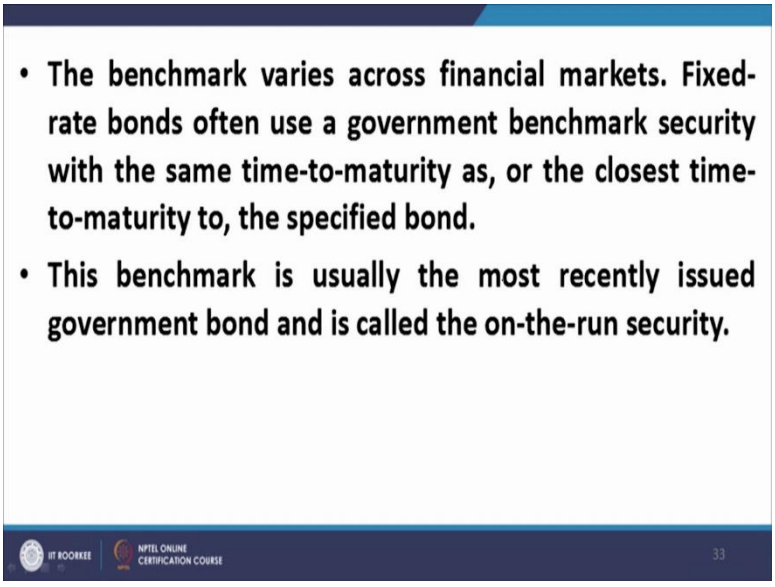

The benchmark varies across financial markets, fixed rate bonds often use a government benchmark security with the same time to maturity as or the closest time to maturity to the specific bond. The benchmark is usually the most recently issued government bond and is called on the run security.

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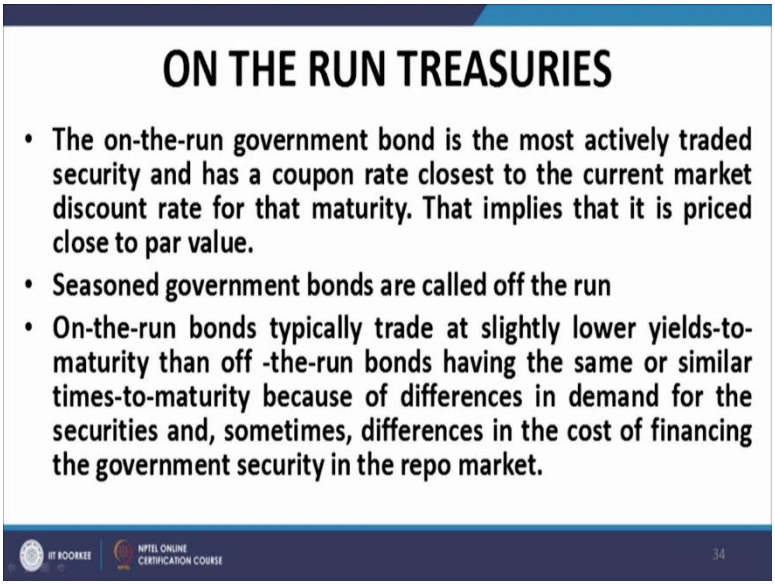

On the run treasuries, this is a term which is very often encountered. So, let me read out the definition of on the run treasuries, as far as benchmark experts are concerned as far as benchmarks are concerned, so the on the run government bond is the most actively traded security and government security and has a coupon rate closest to the current market discount rate for that maturity that implies that it is priced close to its par value.

Seasoned government bonds are called off the run, on the run bonds typically trade at slightly lower yields to maturity than of the run bonds having the same or similar times to maturity because of differences in demand for the securities and sometimes differences in the cost of financing the government security in the repo market.

So, on the run securities are securities which are most actively stated securities and essentially they are securities which are issued latest by the government, by the appropriation authority.

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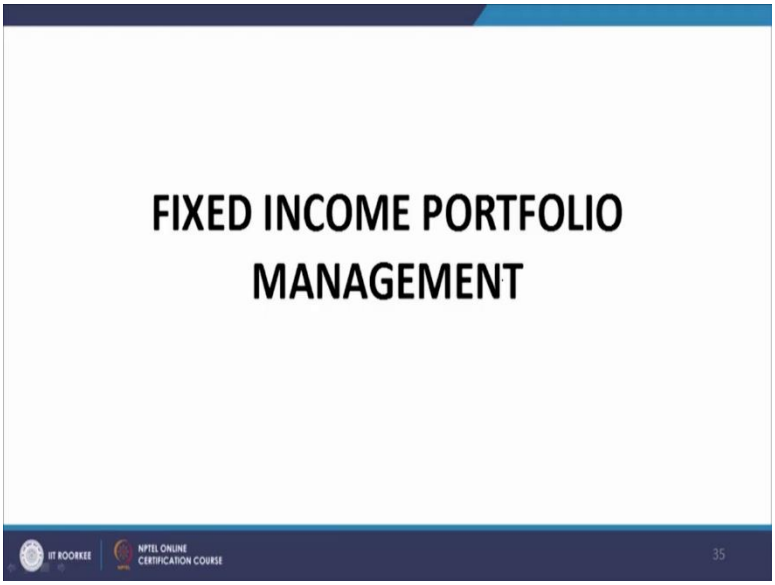

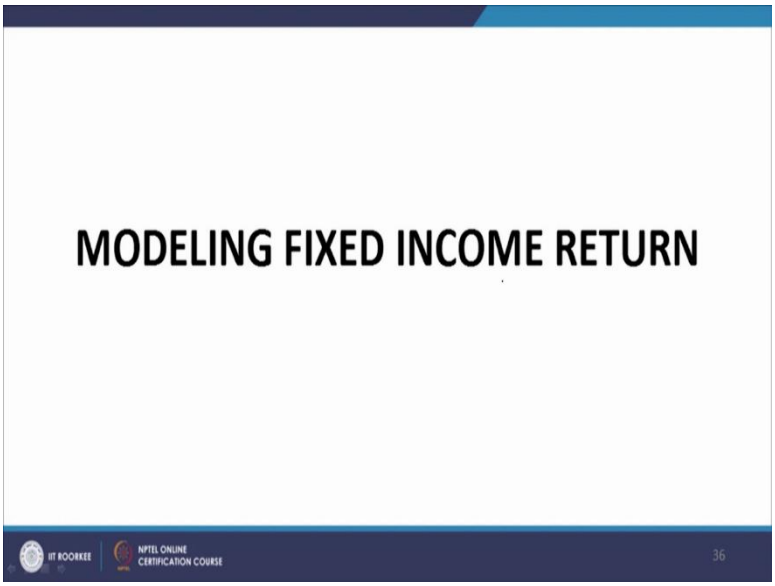

Now we talk about fixed income portfolio management. Before we talk about 4 fixed income portfolio management, let us see how income is derived out of an investment in a fixed income security. So, we start with the modelling of fixed income return.

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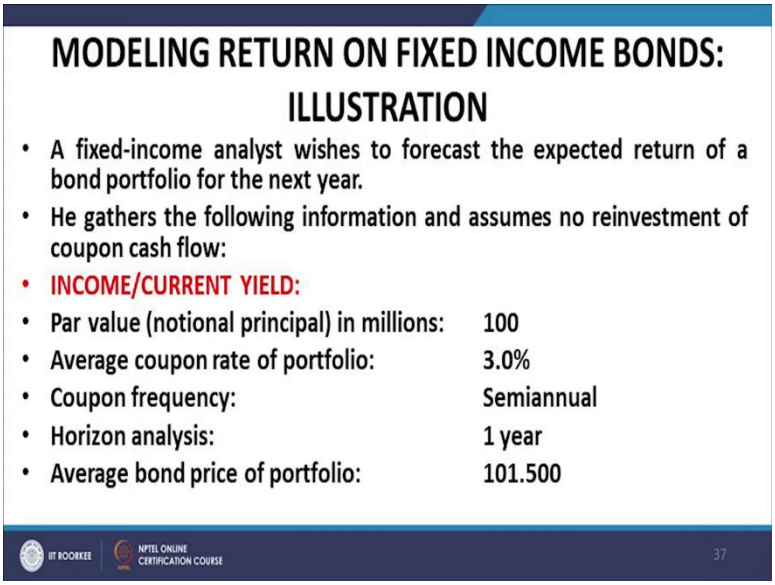

I illustrate this within with a comprehensive example. So, let us start with this example, a fixed income analyst wishes to forecast the expected return on a bond portfolio for the next year. He gathers the following information and assumes no reinvestment this is interesting and we will encounter this later on when I solve this example.

So, he gathers the following information and assumes no reinvestment of the coupon cash flows. Average coupon rate of portfolio is 3 percent, coupon frequency is semi-annual, horizon analysis is 1 year the holding period is 1 year and average bond price of portfolio that is P0 that is equal to 101.500.

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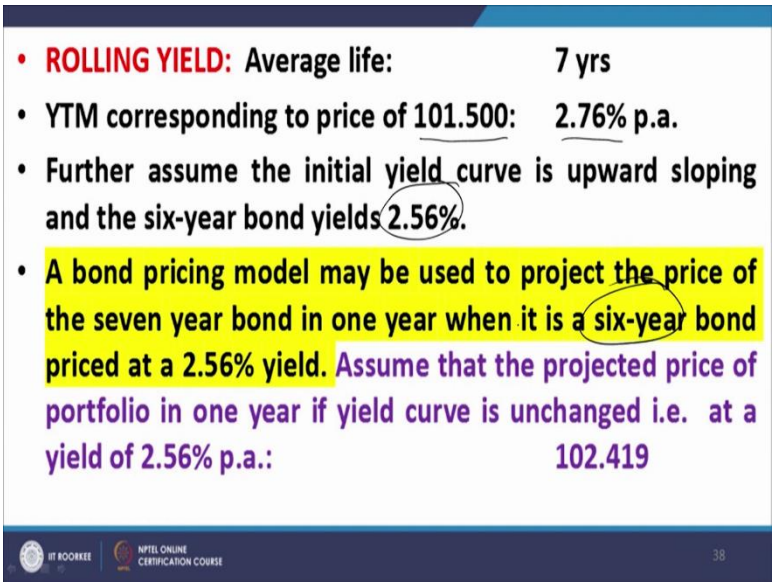

Continuing the average life of the bonds across the investment portfolio is 7 years, the YTM corresponding to the price of 101.500 is 2.76 percent per annum, this is given and further assume that the initial curve is upward sloping and the 6 year bond yield is 2.56 percent. Please note the life of the bond, the life of the investment portfolio is 7 years, you are holding the bond for 1 year and at the end of the 1 year when you are going to dispose of the bond, you will be placed by the same yield curve but with the maturity of only 6 years because the bond has a remaining life of 6 years, 6 year has already elapsed.

So, naturally the yields would change. When you talked about the 7 year investment that is the at t equal to 0, the YTM was 2.76 percent. Now when you have moved 1 year further, you face the same yield curve but because the life of the investment is reduced from 7 year to 6 year and it is an upward sloping yield curve and if we press a lower YTM and that lower YTM is 2.56 percent.

A bond pricing model may be used to project the price of the 7 year bond to in 1 year when it is a 6 year bond, this is important when it is the 6 year bond then priced at 2.56 percent yield. Assume that the projected price of the portfolio in one year, if yield curve is unchanged, i e at a YTM of 2.56 percent turns out to be 102.419. So, this is the price at t equal to 1 when faced with the same yield curve using the fact that the bond has a remaining life of only 6 years and therefore, we need the 6 year maturity rate and not the 7 year maturity rate.

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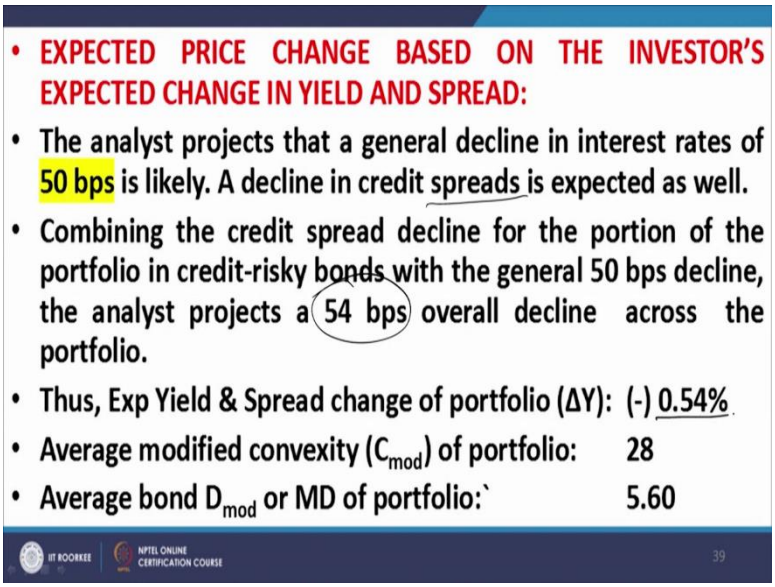

The analyst projects that a general decline in interest rates of 50 basis points is likely a decline in credit spreads is expected as well. Combining the credit spread decline for the portion of the portfolio in credit risky bonds with the general 50 percent, 50 basis point

decline, the analyst projects of 54 basis point. So, the 50 basis points is the general decline across all instruments, then there is also a decline specific to certain risky securities in the portfolio and that when adjusted or when accommodated over the entire portfolio can be considered as a further 4 basis point decline.

So, the total anticipated decline on account of the general effect and the effect due to a decline in credit risk or decline in credit spreads rather is 54 basis points. 50 basis point for the general decline and 4 basis points for the decline in credit spreads which is obtained by apportioning the impact on the credit risky securities in the portfolio over the entire portfolio.

The expected yield and spread change of the portfolio delta y is minus 54 basis points that is what I mentioned just now. The modified convexity is given as 28 and the modified duration is given as 5.60.

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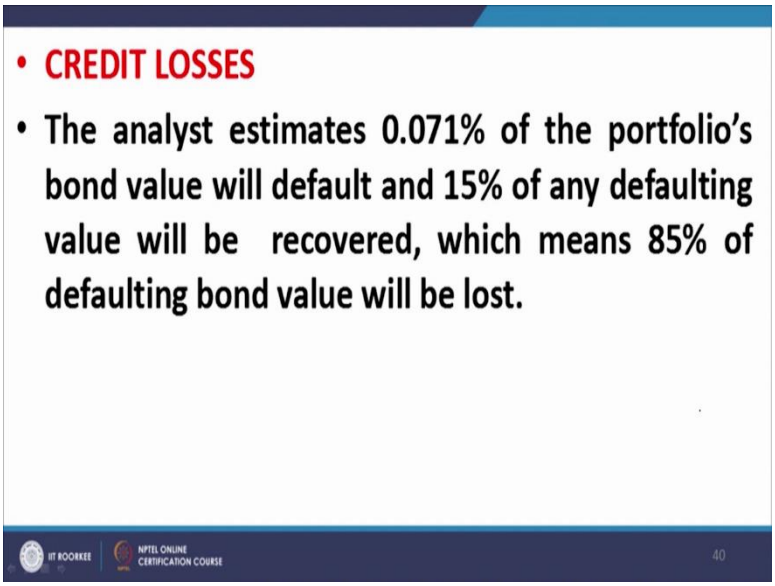

There is further information as far as credit losses are concerned, the analyst estimates 0.071 percent of the portfolio bond value will default and 15 percent of any defaulting value will be recovered so 85 percent will be lost. If any bond issuer defaults 85 percent will be lost, 15 percent will be recovered and the fraction in terms of money value of defaults is equal to 0.071 percent.

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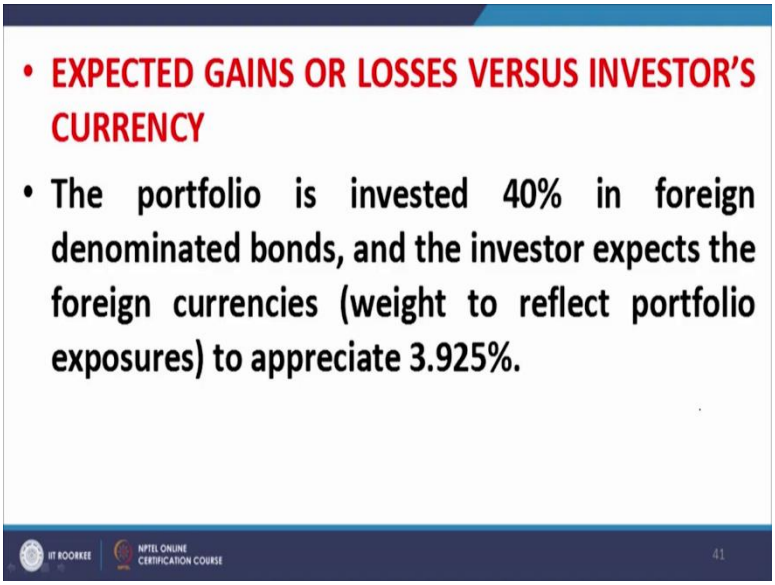

Then expected gains or losses versus investor currency, the portfolio has invested 40 percent in foreign denominated bonds and the investor expects the foreign currencies wait to reflect portfolio exposures to appreciate 3.925 percent. So, this 3.925 percent is going to impact only 40 percent. Please note this point, it is not the overall increase, overall return, overall increase in value, it is the increase in value for 40 percent of the assets which are in foreign denominated bonds.

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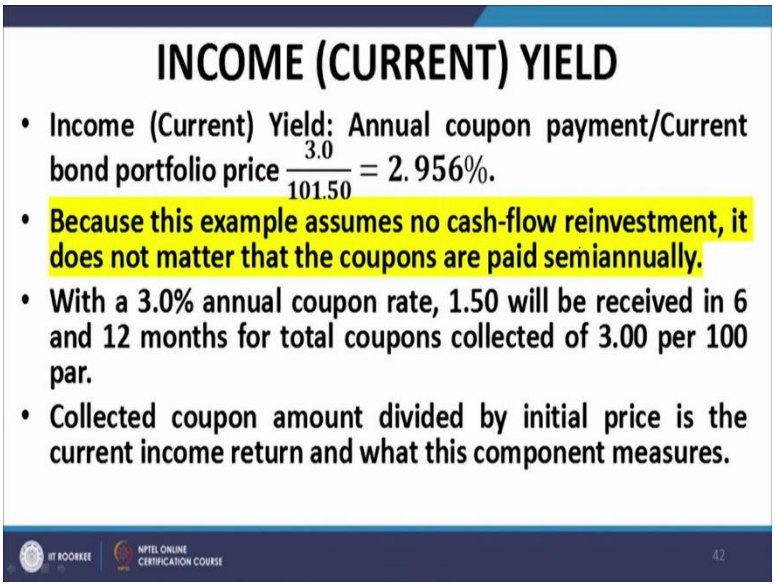

So, as far as the income yield is concerned or the current yield is concerned, it is straightforward the total amount that you receive for the year is how much, you see you receive 1.5 or t equal to 6 months and you receive another 1.5 t equal to 1 year and that gives

you the total coupon payments for the year is 3.00, the current market price of the portfolio is 101.50. So, the current yield or the income yield as you may call it is 3 divided by 101.50 that is 2.956 percent.

So, because this example assumes no cash flow reinvestment, it does not matter that the coupons are paid semi-annually, there is no adjustment for reinvestment here with a 3 percent annual coupon note rate 1.50 will be received in 6 months and 1.50 at the end of 12 months. Total coupon payments collected 3, the current market price is 101.50, so the yield turns out to be 2.956.

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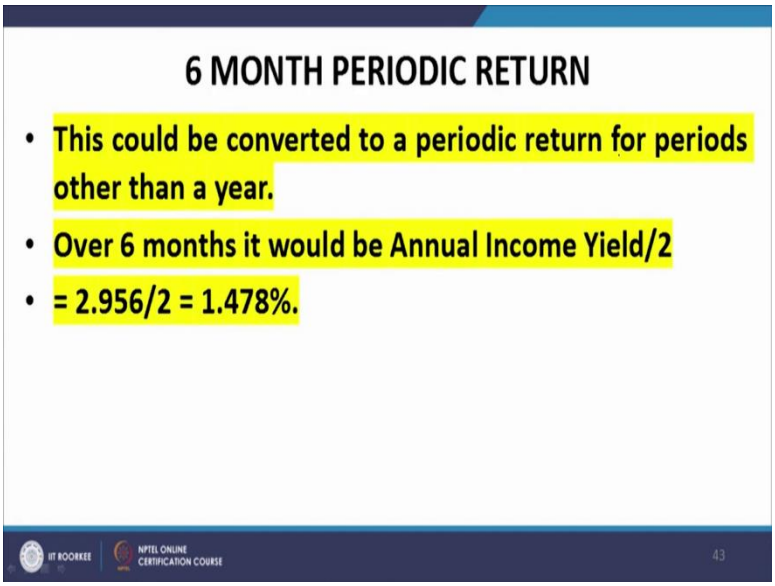

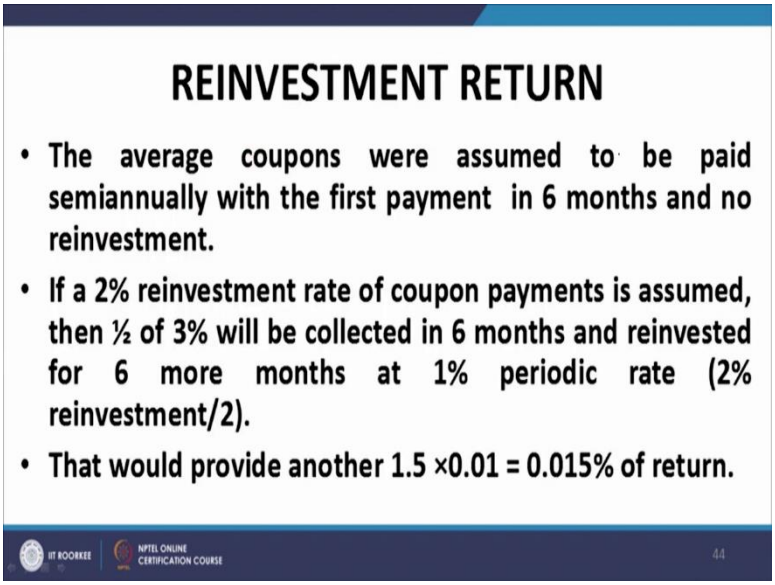

If you want to work out the 6 month periodic return, you simply divide it by 2, you get the return as 1.478 percent. As far as reinvestment return is concerned, please note we assume no

reinvestment, so no reinvestment return will be there. However, if there were, let us assume for example, for the purpose of illustration that if there is a 2 percent, if the coupon payment at t equal to 6 months is reinvested at the rate of 2 percent per annum that is 1 percent for the half year, the coupon at t equal to 6 months will be reinvested for only 6 months because 1 year is the holding period, 1 year is the investment horizon and you what you will get is 1.5 into 1 percent that is 0.015 percent of return.

This would be an add-on, had you reinvested your coupon received at t equal to 6 months at the rate of 2 percent per annum for the next 6 months that is up to the end of the holding period. Had you done that, you would have got this additional return, 1.5 is the coupon payment, the interest rate, periodic interest rate is 1 percent. So, 1.5 into 0.01 that gives you 0.015 percent of return.

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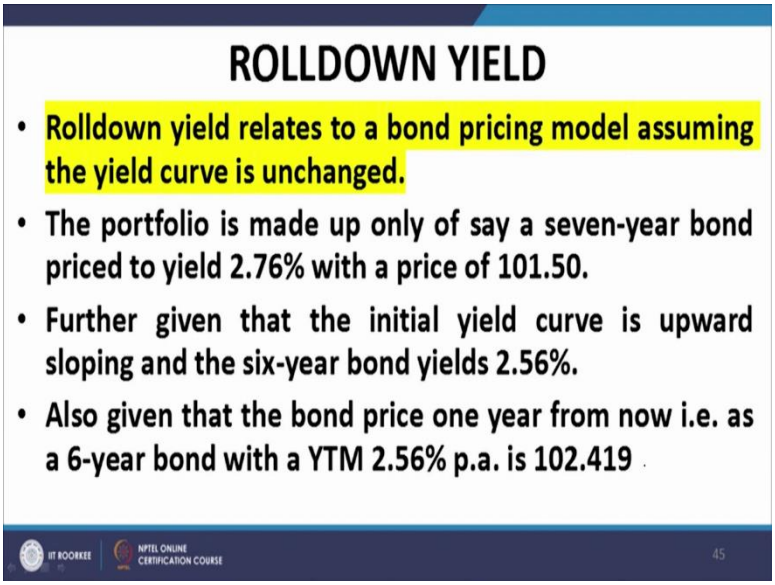

Roll down yield, this is the most important part. Roll down yield relates to a bond pricing model assuming that the yield curve is unchanged. Just as we did in the example. So, the portfolio let us assume that the portfolio is made up of only a 7-year bond priced at a yield of 2.76 percent at t equal to 0 and the price at t equal to 0 is 101.50. So, let me repeat at t equal to 0, the maturity of the bond is 7 years, the YTM is 2.76 percent and the price is 101.50.

It is also given that the at t equal to 1 that is at the end of the investment horizon, when the invention horizon is over then the yield that we face for the 6 year bond or the YTM for the 6 year bond is 2.56 percent and corresponding to this 2.56 percent the market price anticipated is 102.419.

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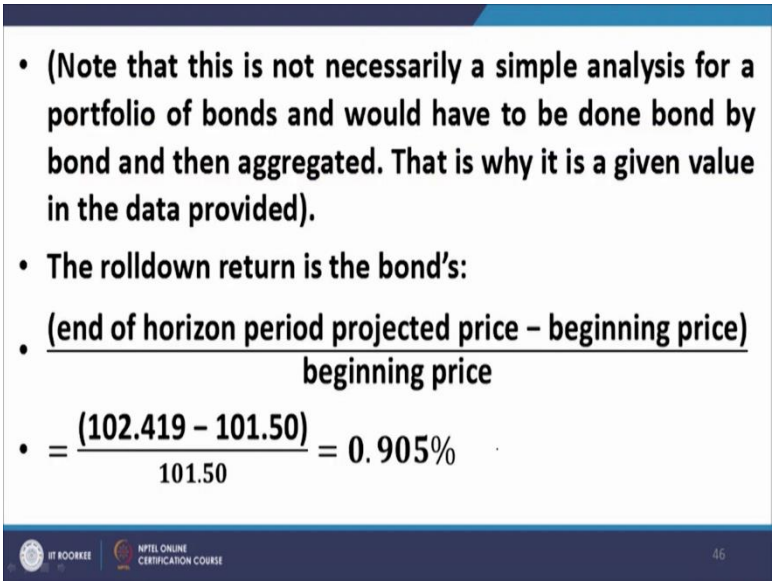

So, what is the yield? What is the rollover yield, it is given by the end of horizon period projected price minus beginning price divided by beginning price that is 102.419 minus 101.50 divided by 101.50 that is 0.905 percent. So, this is the way we work out the roll down yield or the yield that corresponds to the slope of the yield curve.

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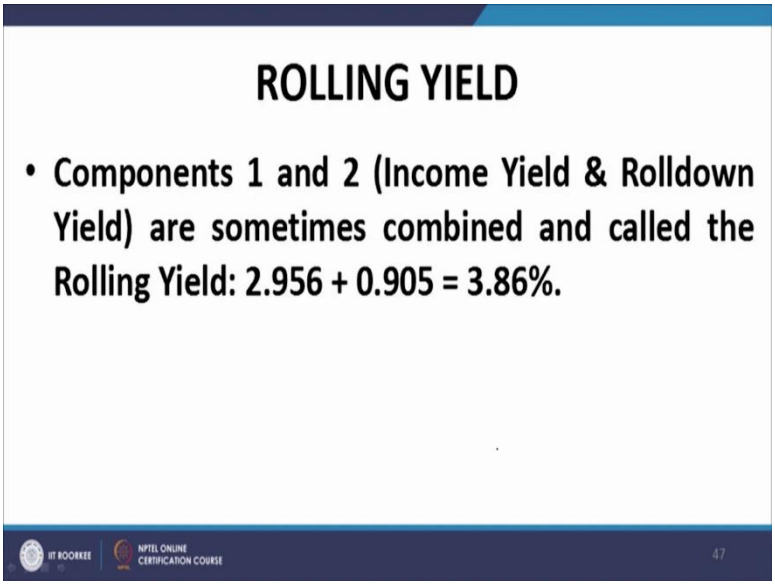

The components 1 and 2, that is the income yield or the current yield and the roll down yield are sometimes aggregated and they are called rolling yield. So, in our case it happens to be 2.956 plus 0.905 that is equal to 3.86 percent.

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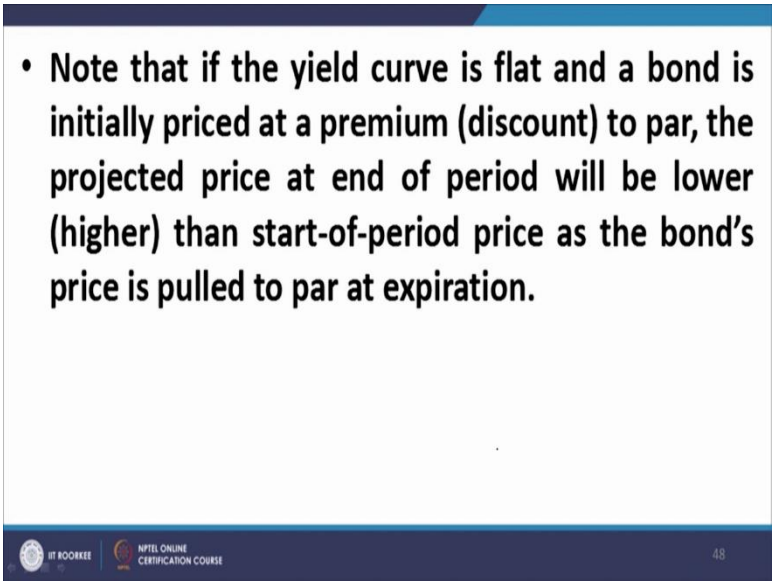

Note that if the yield curve is flat and a bond is initially priced at a premium discount, we have discussed this point, the projected price at the end of 1 year will be lower than the start of period price as the bond prices pull to par to expiration.

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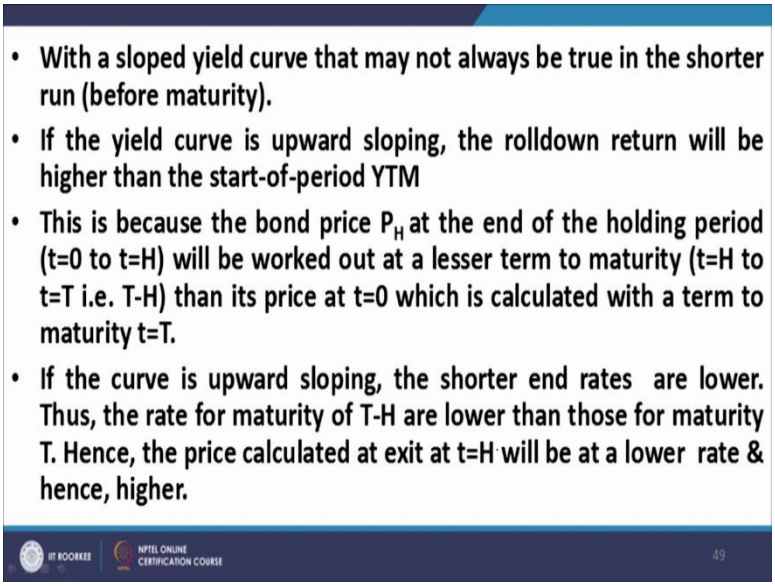

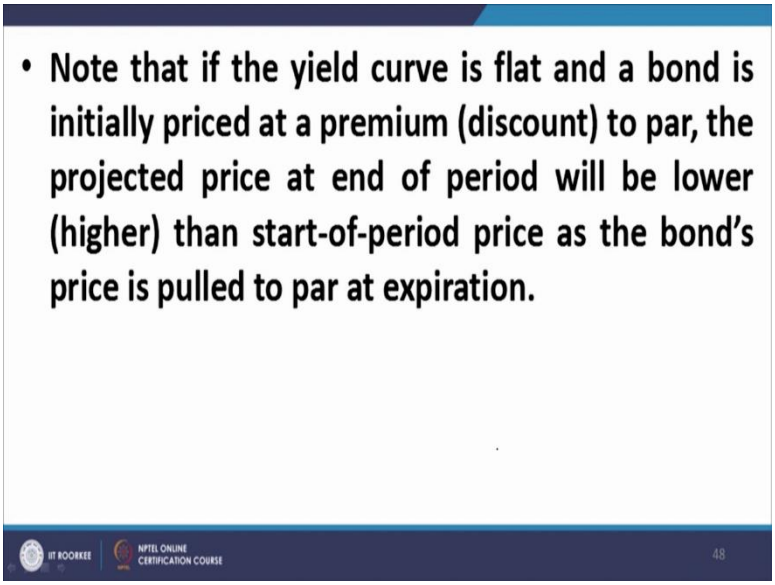

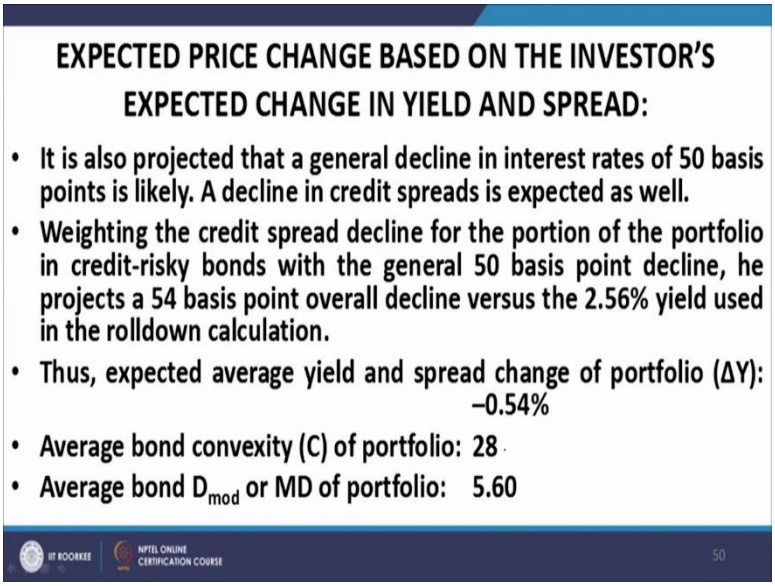

With the slope, let me reiterate this, I have already explained it in a lot of detail. But it is so important, let us read through it once more before we conclude. With a sloped yield curve that may not always be true in the shorter run that is before maturity, what may not always be true? This particular point that we have on this slide, the yield curve is not flat, if it is an upward sloping yield curve, the pulling down or the amortization of premium oblique discount may not be as accurately reflected in this particular statement.

If the yield curve is upward sloping the roll down return will be higher than the start of period YTM, the roll down yield will be higher than the start of period right and this is because this is the catch, this is the important part. This is because the bond price P H at the end of the holding period, that is holding period is t equal to 0 to t equal to H will be worked out at a lesser term to maturity that is t equal to H to t equal to capital T that is t minus H years.

At the end of the holding period at the end of the investment horizon the remaining life of the bond will be less than the price at t equal to 0 where you are where the bond price will be worked out over the entire period from t equal to 0 to t equal to capital T. Because the curve is upward sloping as per our assumption, the shorter end rates are lower. Thus the rate for maturity of t minus H will be lower than the rates for maturity of t equal to capital T. Thus, the price calculated at exit at t equal to H will be at a lower rate and hence the price will be higher. So, from here will be continuing in the next lecture. Thank you.