Quantitative Investment Management
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Lecture 23
One Sided Duration

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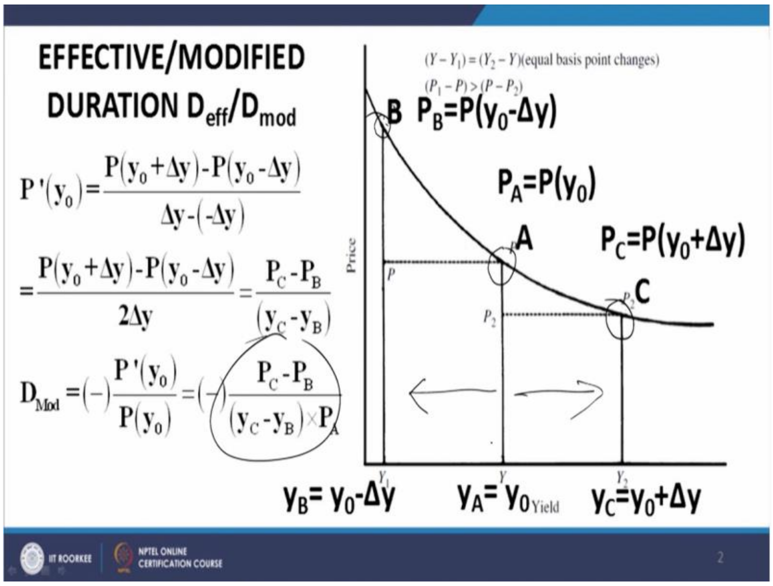

Welcome back. So, towards the end of the last lecture, I was talking about one sided duration. To get more insight into this concept, let us revisit the concept of modified duration or what we call the effective duration and try to understand how we worked it out. We took a given point YTM, we took a given YTM to start with, let us call it point A and let us call the YTM as YA, that is equal to Y0 and that is the price at this YTM of Y0 or YA, you may call it either is given by Pa and that is represented by this point which I am marking now on this particular diagram.

And now, we increase the YTM from Y0 by a small amount to Y0 plus delta Y and the corresponding point we have on this yield price curve is given by the point C here, the price is given by PC and the point is represented by the point C. Similarly, what we do know, finally, is that we reduced the YTM by the same amount that we increase, that is by the amount of delta Y.

So, instead of Y0 plus delta Y, we now have a YTM of Y0 minus delta Y, and the corresponding price is represented by PB and the point on the curve is represented by B. We are taking an increase in YTM and a decrease in YTM. And then, we are working out the corresponding prices that the two YTMs, that is Y0 plus delta Y and Y0 minus delta Y. The process and to that extent is symmetric, we are taking one plus side increase in YTM and one minus side or a decrease in YTM.

Now, the results that we get here, the result that is represented here in this left-hand panel works quite well, works adequately well when you have bonds without any salient or special features like embedded options, where the flow price is not truncated, has not inhibited, has not cut short, cut out by special option features that may be embedded in the bond. If that is not the case, then this model works out very well.

However, in the case of bonds that have embedded options in them, what will happen is, if the, let us talk about a callable bond, which has, where the issuer, and you have the option to call back the bond, if the interest rates fall below a certain level and the price increases above the exercise price. What will happen in that case is if the interest rates decline, and the decline results in a price which is higher than the exercise price, and the issuer will exercise the option and as a result of it, the cash flow will be truncated, the cash flow that is projected purely on the basis of a decline in YTM will not materialize because the insurer will realize the bond, the issuer will call back the bond at a lower price.

So, in that situation, the payoffs do not turn out to be symmetric. And in that situation, we define the new concept that I briefly talked about in the last lecture. That is the concept of one-sided duration, where we look at the changes in price in one direction. In the normal modified duration, we are looking at changes of price in both direction, a decrease in YTM or an increase in YTM.

However, in special situations, particularly dealing with bonds with embedded options, we prefer or to get an appropriate or a more accurate measure of the sensitivity of price to changes in YTM, we use one sided duration, corresponding to the either the increase in YTMs or the decrease in YTMs. The reason as I mentioned in such situations, their payoffs get truncated on account of the exercise of the option and the payoffs obviously do not become symmetric with reference to the changes in YTM.

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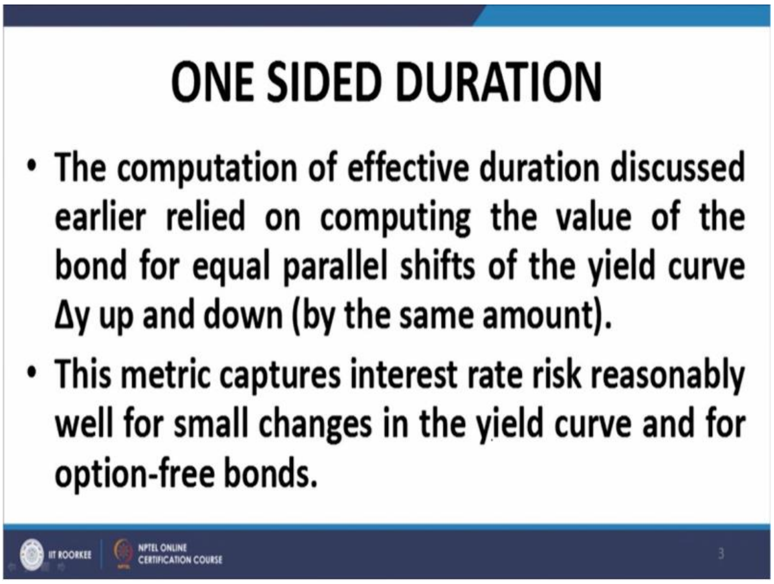

So, the computation of effective duration discussed earlier relied on computing the value of the bond for equal parallel shifts of the yield curve, delta Y up and down that is what I mentioned just now. First, we talk about an increase in delta Y and then a we talk about a decrease, an increase in Y and then we talk about an increase in, decrease in Y by the same amount of delta Y.

This metric captures interest rate risk reasonably well for small changes in the yield curve, and for option free bonds. So as far as option free bonds are concerned, this is the appropriate measure. Because in a sense, it takes cognizance of both the possibility as the YTM increasing and the YTM decreasing with equal chances.

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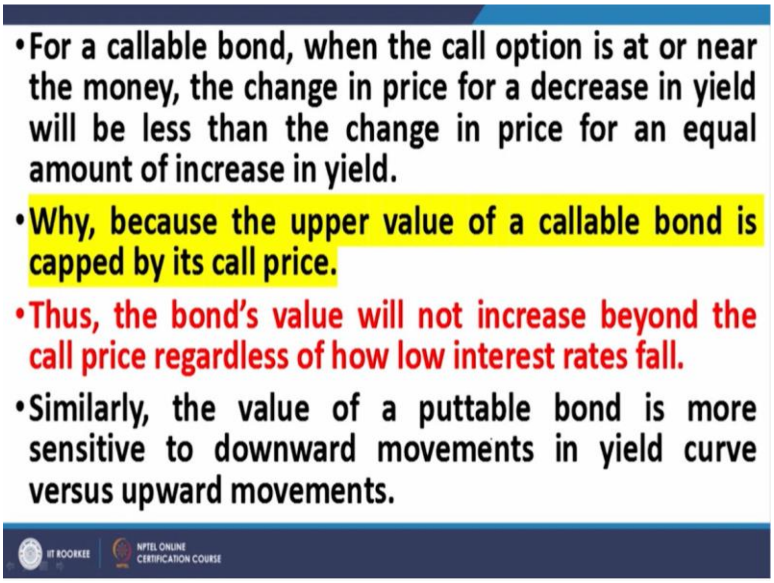

For a callable bond when the call option is at or near the money, the change in price for a decrease in yield will be less than the change in price for an equal increase in yield. That is the important thing. Recall what is a callable option, a callable option is an option, is a bond that has an option, which enables the issuer of the bond to recall back the debt, if the interest rates decline below a certain level and the price increases beyond the exercise price.

So, as soon as the price will increase about the exercise price, the issuer of the bond will call back the bond and pay only the exercise price. That is the payoff is truncated by the exercise price due to the exercise of the option. Why is, why is it that for a callable bond, when the call option is in, is at or near the money, the change in price for a decrease in yield will be less than the change in price for an equal increase in yield?

The reason is, because the upper value of a callable bond is capped by its call price, as soon as the price on the basis of a decrease in yield increases above the exercise price, the issuer exercises the bond, the option embedded in the bond and as a result of which the cash flow gets truncated. Thus, the bonds value will not increase beyond the call price regardless of how low the interest rates fall, because the issuer has to pay only the exercise price when retiring the debt, when recalling debt.

Similarly, the value of a puttable bond is more sensitive to downward movements in yield curve versus upward movements. Why is that? Because a puttable bond has the option with the investor to return back the bond if interest rates rise, because the interest rates that will yield fall in the price of the bond. And consequently, if the fall in the price of the bond, price of the bond falls below the exercise price, then the investor in the bond can exercise the put option and retire the bond or return the bond back to the issuer and recover his investment.

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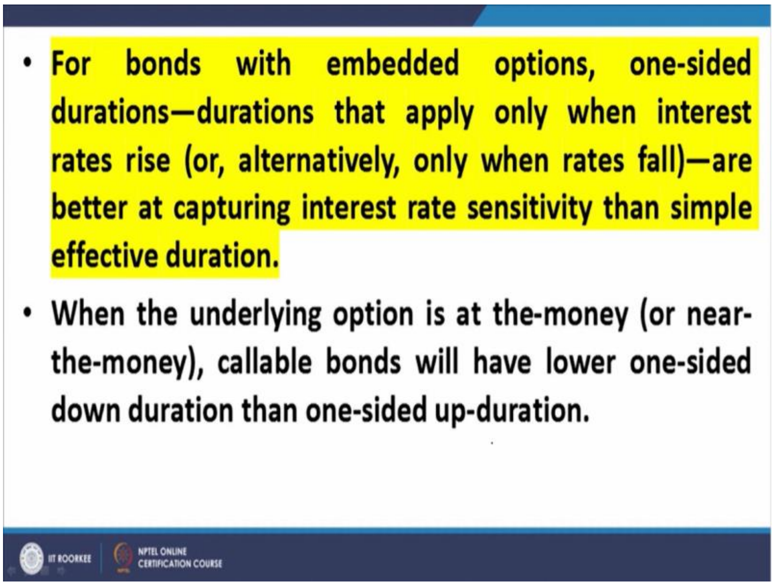

For bonds with embedded options, one sided durations, durations that apply only when interest rates rise or alternatively only when interest rates fall are better at capturing interest rate sensitivity, then special, then simple effective duration. When the underlying option is at the money or near the money, callable bonds will have lower one side duration, down duration rather than one sided up duration.

Because if there is a decrease in interest rates, the price increase will not be commensurate with a decrease in interest rates, it will be cut down, it will be capped, it will be truncated by the exercise price of the embedded option in the callable bond.

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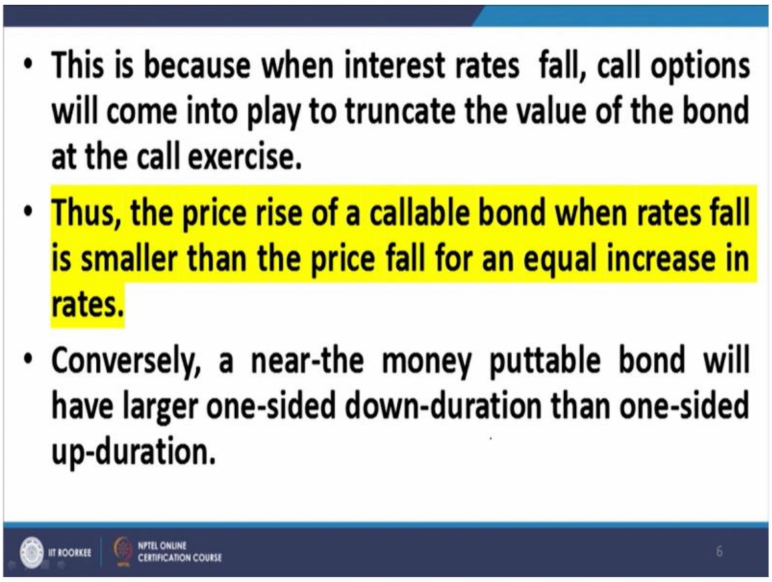

This is because when interest rates fall, call options will come into play to truncate the value of the bond at the call exercise. Does the price rise of a callable bond when rates fall is smaller than the price fall for an equal increase in rates because of the exercise rate, exercise price that we have, in the callable bond as I mentioned just now. Conversely, near the money puttable bond will have larger one-sided down duration than one sided up duration.

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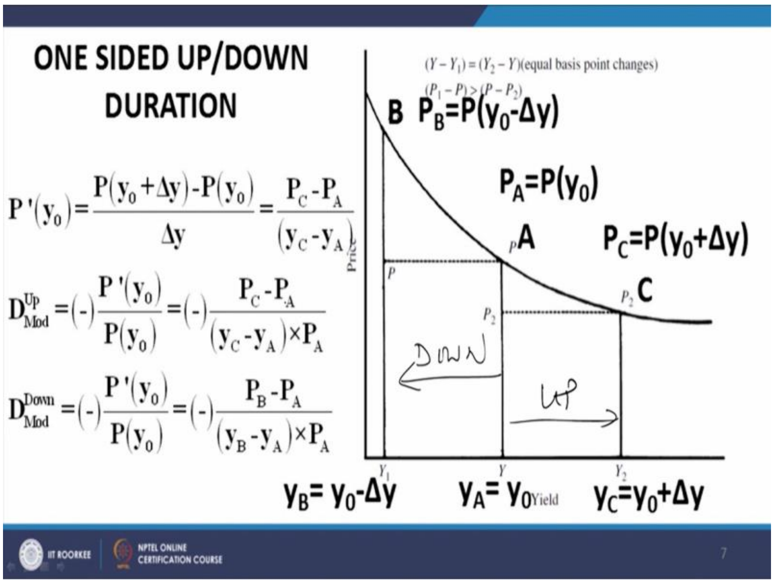

So, this is how one sided up or down duration will be calculated. If you look at the procedure, it is pretty much similar to what we did for the effective or the modified duration. The difference is that in this case, we consider only one side movement of the exercise of the YTM.

For example, while we are calculating the down duration, we are only considering this movement, this side movement, and when we are considering the up duration, we are considering this movement, this is up and this is down. So, when we work out the down duration what happens? We consider only the PB minus PA that is we assume that the YTM has declined from Y0 to Y0 minus delta Y. And when we consider the up duration, we are considering PC minus PA that means we are considering the up movement of the YTM, YTM increasing from Y0 to Y0 plus delta Y.

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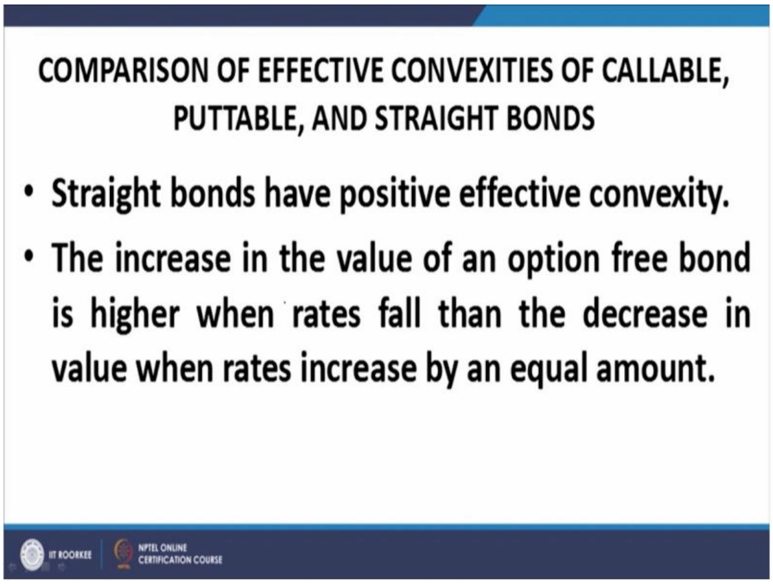

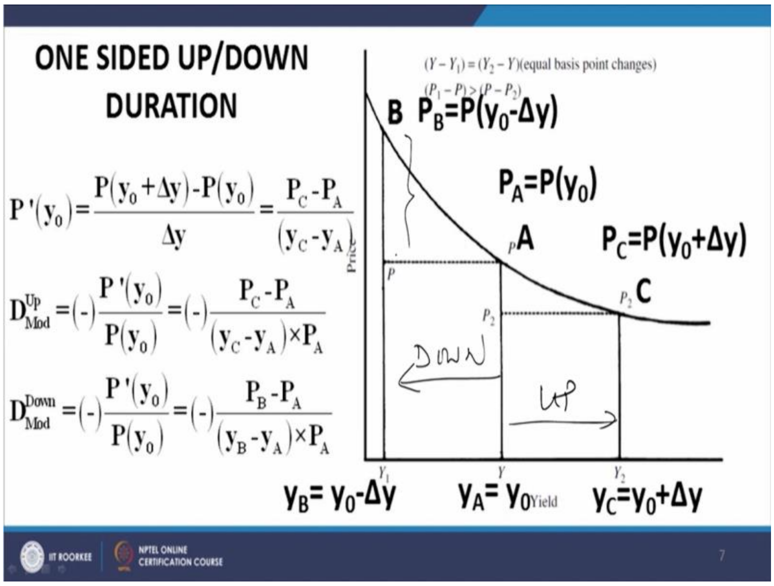

Comparison of effective convexity is of callable, puttable and straight bonds. As far as straight bonds are concerned, we all know that the straight bonds are positive effective convexity, the curve, yield price curve has a convex nature and therefore and we also know that the second derivative of the price with respect to YTM is positive and therefore the convexity of straight bonds is invariably positive.

The consequence of this positive convexity is that the increase in the value of an option free bond is higher, when the rates fall than the decrease in the value, when the rates increase by an equal amount, that you can see here in this figure itself. When there is a decline in the value of YTM by delta Y, we move from the point A to the point B and the change in price is given by this expression. That is PB minus PA.

And if there is a increase in YTM by delta Y that is by the same amount. The change in price is given by PC minus PA and clearly the magnitude of PB minus PA is greater than PC minus PA. That is if there is an increase in YTM by delta Y, the change in price is less than decrease in YTM by delta Y, the change in price is relatively more. This is the nature of a convex curve.

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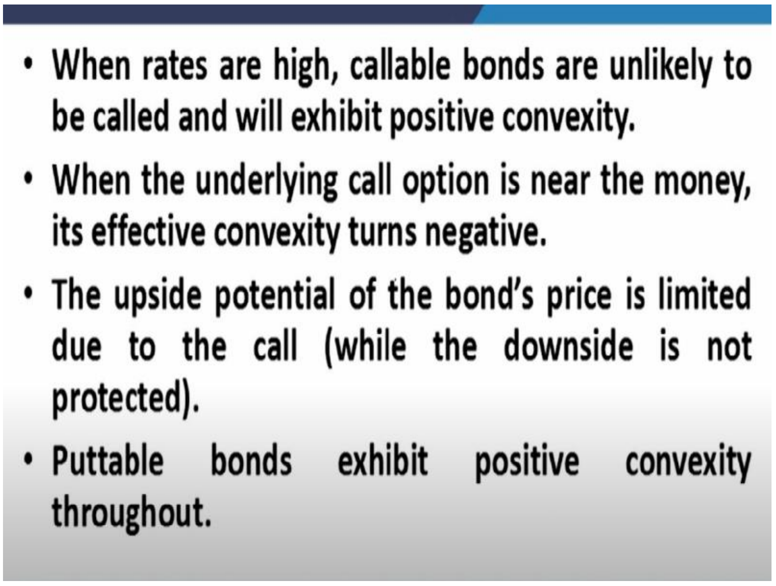

When rates are high, callable bonds are unlikely to be called and will exhibit positive convexity. When rates are high, the prices will be low, and if the prices are low, it is unlikely that the prices reach the level of exercise price which will provoke the issuer. To exercise the call option, would be exercised only when the market price exceeds the exercise price. And if the market price are low, the market interest rates are high and the prices are low, the probability or the possibility of the prices reaching the exercise price will be lower and correspondingly the possibility of exercise of the option would be less.

When the underlying call option is near the money, the effective convexity turns negative. This is for a small region which is around the, which is close to the exercise price around which are close to that region. The convexity will turn out to be negative because of the truncation of the price by the exercise of the call option. So, a small region around the point at which that truncation takes place starting slightly before we read that the exercise price, the convexity would turn out to be negative.

And then because the, because as the interest rates go further down the value of the bond will remain at its exercise price, it will not increase further the convexity will become 0. So, to start with the convexity, will be a positive at large interest rates as the interest rates decline, I am talking about a callable bond, as the interest rates decline and the price approaches the exercise price, the convexity may turn out to be negative for a small region and finally the convexity will be 0, because the price change would be 0 corresponding to a decline in, further decline in interest rates.

The upside potential of the bond's price is limited due to the call while the downside is not protected. As I mentioned as the interest rates decline, you approach the exercise price and once you approach the exercise price, you exceed the exercise price. The option will be exercised and the cash flow gets truncated to the exercise price.

Puttable bonds exhibit positive convexity throughout. So, this is a special feature of callable bonds that, let me repeat this. At high interest rates, the possibility of exercise of the call option is very small. The call option would be out of the money for most of the region and as a result of which it would not disturb the convexity of which will remain positive, which will be remained close to the convexity of this straight bond.

However, se we have, as the interest rates declined and the price approaches the exercise price, the convexity may start changing sign and it may become negative for a small region close to the point at which the exercise price is reached. And once after the exercise price, further decline in interest rates are not going to affect the cash flow, it is going to remain at the exercise price. And as a result of which, the yield price curve would then become a straight line and the convexity would be 0.

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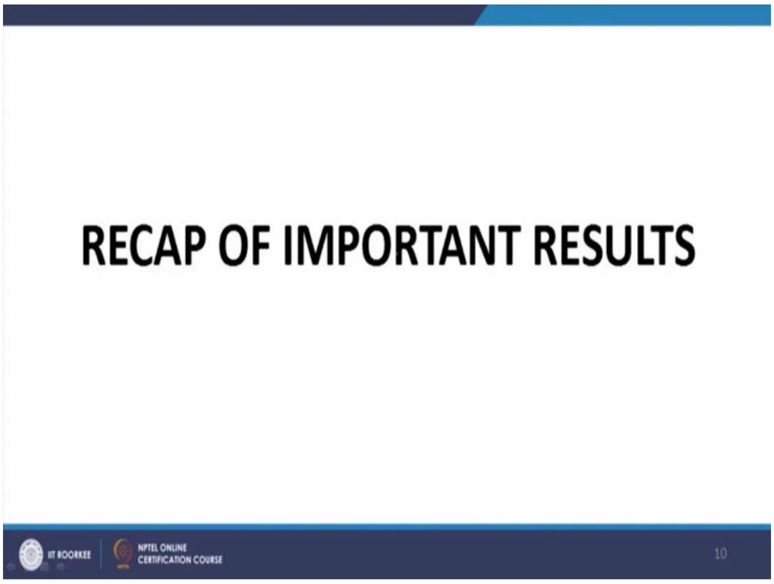

Now we recall, recap some of the important results, before we move over to fixed income portfolio management. Let us quickly recap through the important results that I have talked about in the last few lectures, before we get to the next topic.

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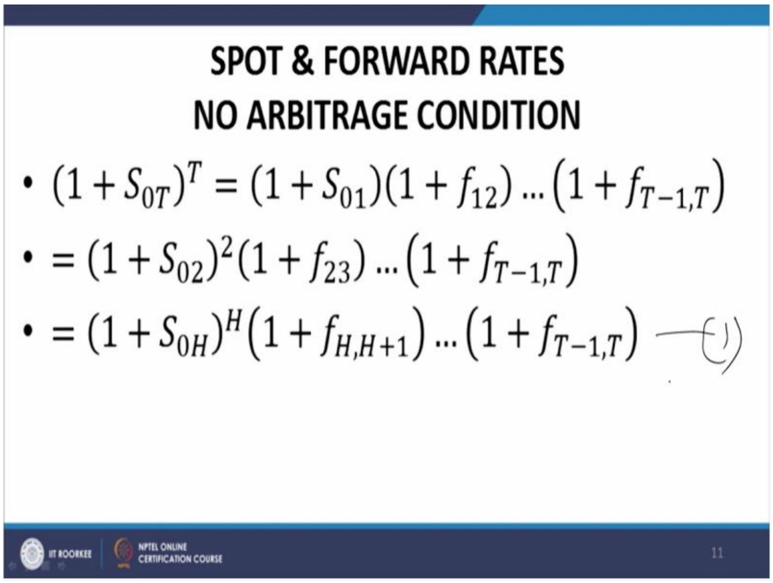

As we know the arbitrage free relationship between spot and forward rates, I believe all of us know what we mean by spot rates. Spot rates are rates which are current rates, which are rates in relation to a deposit that is initiated right now, obviously these rates vary with the maturity and that is what is called the term structure of interest rates.

Forward rates are rates that relate to an investment, that relate to a loan that is going to be initiated at some time in the future, while the interest rate on the terms of the loan are fixed at t equal to 0 that is right now. But the actual loan disbursement and of course, the repayment naturally would be occur at a future, would be occurring at a future date. So that is what is called a forward rate.

By considerations of arbitrage, arbitrage free relationship between spot rates and forward rates can be written as it is shown in equation number 1 on the slide. 1 plus S0t to the power t can be written in many forms. Each of these forms are equivalent, if we consider the issue of arbitrage repricing.

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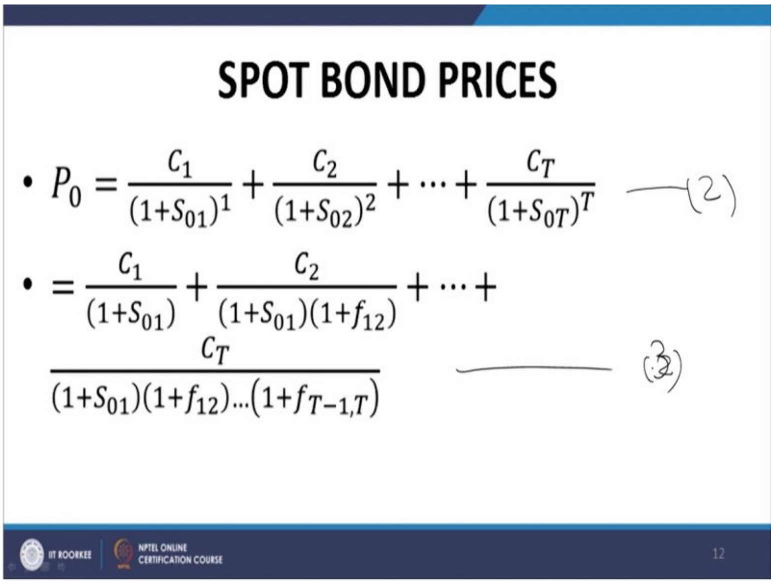

The spot bond prices or the bond prices, the arbitrage free bond prices are calculated or obtained by discounting all future cash flows from the bond at the appropriate risk adjusted spot rates, that is precisely what is represented by equation number 2 here. Let me repeat, the current price of the bond, the current trading price of the bond will be obtained as the discounted value of all future cash flows from the bond discounted at the appropriate risk adjusted spot rates, corresponding to the timing that which the cash flows take place.

You can see here the cash flow at t equal to 1 is discounted at a S01, the cash flow at t equal to 2 that is C2 is discounted at S02 and so on. And this can be written in terms of the forward rates also using the relationship that I showed in the previous slide, using the arbitrage free relationship between the spot rates and the forward rates in the form of equation number 2 here on this equation number 3 here on this slide.

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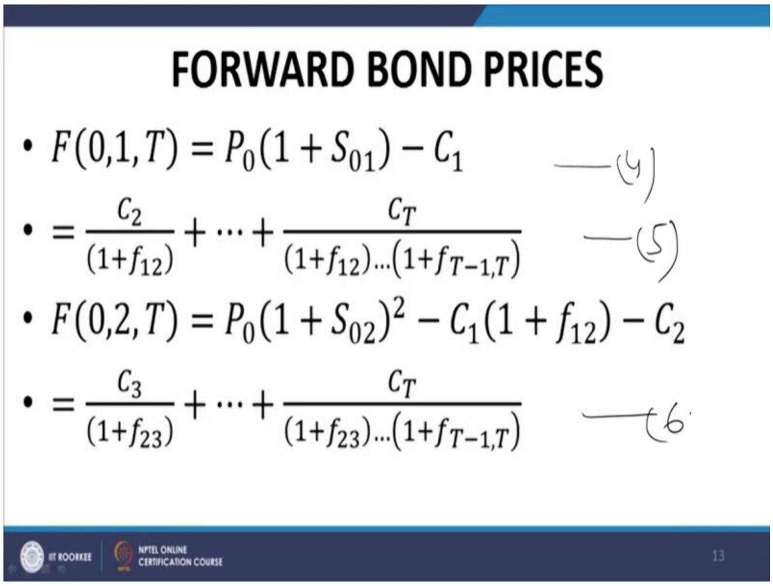

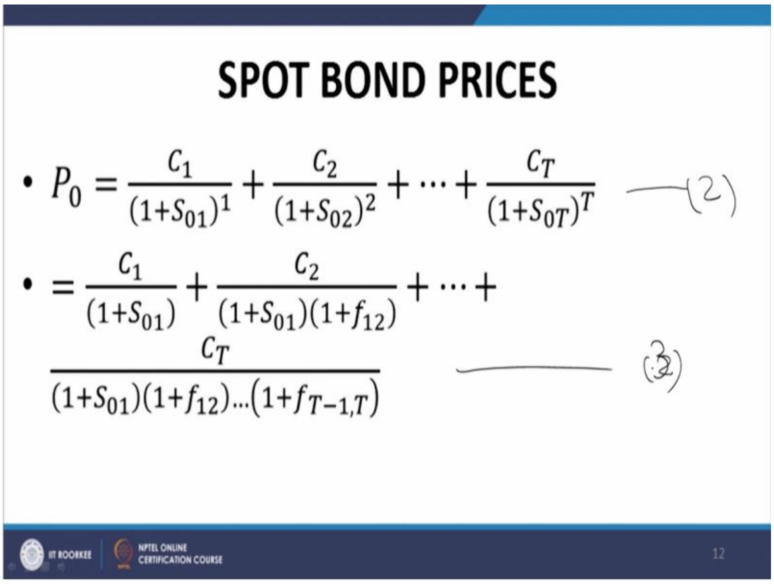

Then forward bond prices, I have discussed this also in detail what would be the price of a bond one year from now, as worked out at t equal to 0. I repeat this is the important part. Obviously, the forward price or the price of the bond at the end of one year from now would be a random variable. But the question is what would be the price as calculated or as fixed at t equal to 0, price of the bond at t equal to 1, when the price is fixed at t equal to two that is called the forward price.

So that works out to, on the considerations of arbitrage free pricing again, if we use the principle of arbitrage free pricing, we arrive at this equation, this is equation number 4 which can be written in the form of equation number 5, using the relationship that we had in the earlier slide that is equation number 2 and 3 here, using these two equations we can write equation number 4 as equation number 5.

And similarly, this is for the price of this is the forward price at t equal to 1 calculated at t equal to 0 for a bond which has the majority of t equal to capital T. Similarly, the price a t equal to 2 calculated at t equal to 0 for a bond of maturity capital T would be given by this equation, let us call it equation number 6.

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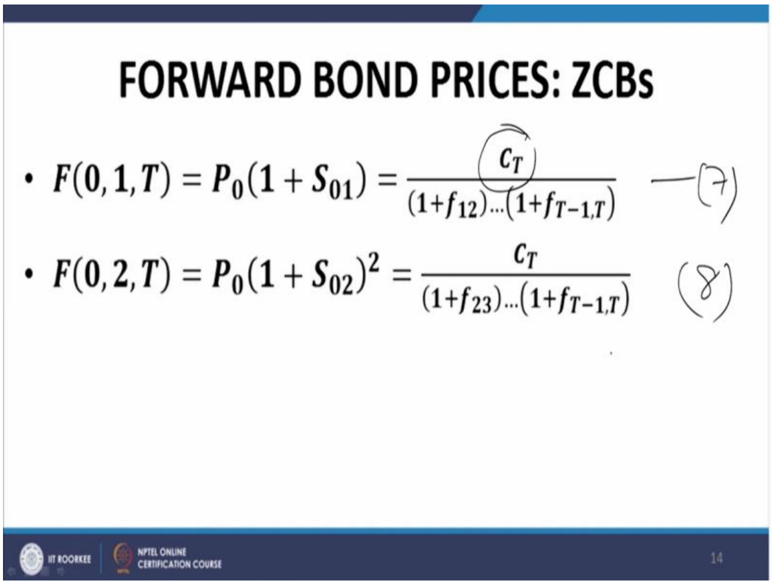

For the zero coupon bonds because there are no intermediate cash flows, what is the zero coupon bond? A zero coupon bond is a bond that has no intermediate cash flows, the entire cash flow that comprises of the maturity value and the interest there on, is paid at the maturity of the bond, which is represented by CT here, and therefore, indicates zero coupon bond the forward prices take the form of equation number 7 here. And similarly, equation number 8 for a bond price at t equal to 2 fixed at t equal to 0 with a maturity of t equal to capital T and having no intermediate coupons.

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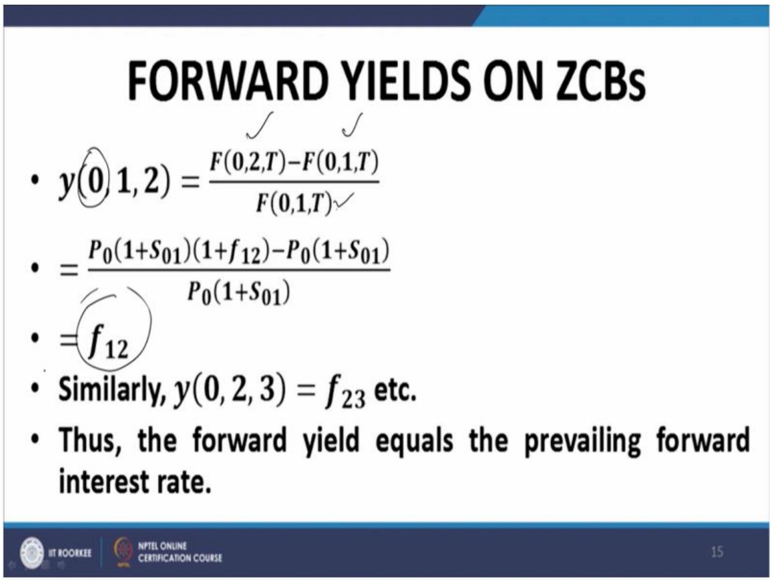

You know, forward yields on zero coupon bonds as you can see, how do you calculate yield? Forward yield means the yield calculated on the basis of the forward prices. Forward yield means the yield calculated on the basis of forward prices. That is the yield that is calculated at t equal to 0 on the basis of prices that will occur at t equal to 1 and obviously t equal to 0. So, the forward yield between for example between t equal to 1 and t equal to 2 will be given by this expression that will be the forward price at t equal to 2 minus the forward price at t equal to 1 divided by the forward price at t equal to 1.

This is the yield remember between t equal to 1 and t equal to 2 as worked out at t equal to 0, this is the forward yield. That is the yield calculated at t equal to 0 corresponding to, corresponding to prices of a bond as determined at t equal to 0 for t equal to 1 and t equal to 2, that is the yield for the period t equal to 1, to t equal to 2, and that when you substitute the respective values that turns out to be the forward rate between at t equal to 1 and t equal to 2.

Forward yields equal the forward rate as indeed it should be. Similarly, you can have the forward yield between t equal to 2 and t equal to 3, you can work it out and you will find that it is equal to f 23. Thus, the forward yield equals the prevailing forward rate.

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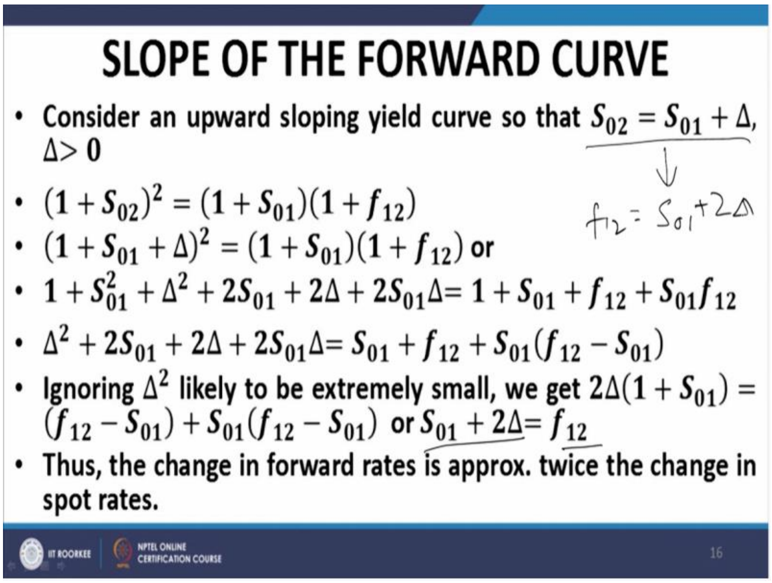

The slope of the forward curve, this is very interesting, if you work out the slope of the forward curve, what we find is using the simple algebra and retaining terms up to first order, what we find is that the change in forward rates is approximately twice the change in spot rates, if you have an upward sloping yield curve for example, that means what, that means S02 is equal to S01 plus delta, where is positive, the interest rates are increasing with maturity that is, we are having an upward sloping interest rate curve.

Then what happens using the formula for arbitrage free relationship between spot rates and forward rates and simplifying a bit, ignoring second order terms, what we end up with is that, S01 plus 2 delta is equal to f12. In other words, the increase in the forward rate is twice the increase in the spot rate, if you compare this expression and this expression f12 is equal to S01 plus 2 delta, you find that the increase in this forward rate is approximately twice the increase in this spot rates. So, I will not go to the algebra. The steps are given here in this slide and they are quite elementary. So, I will leave it as an exercise for the learners.

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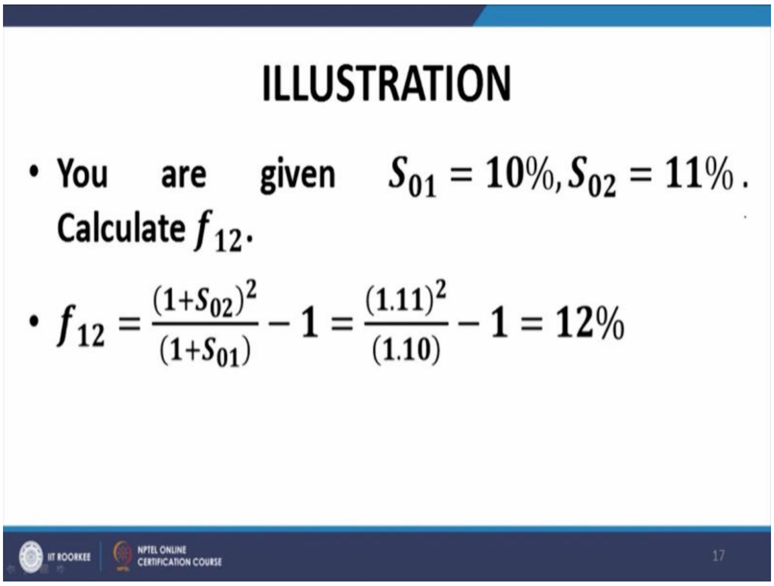

And this is illustrated in this example, if S01 is equal to 10 percent, S02 is equal to 11 percent. If we calculate f12, using what, using the arbitrage free pricing, what we find is that f12 turns out to be 12 percent. Thus, a change in the forward rate which is 2 percent is twice the change in this spot rate, which is 1 percent.

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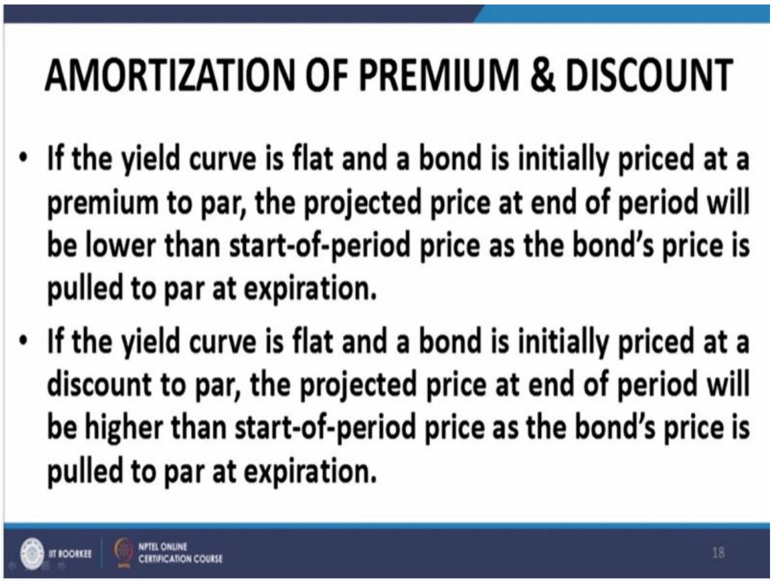

Amortization of premium and discounted, this also I had briefly touched upon earlier. Let us recap this as well. If the yield curve is flat, and a bond is initially priced at a premium to par that is the bond is trading at a premium that is the coupon rates exceed the YTM, the projected price at end of period will be lower than the startup period price, as the bond price is pulled to par at expiration.

You see the important thing one must note is that at the date of maturity of the bond, arbitrage considerations mandate that the bond be traded in the market at the maturity value, it has to be so, because otherwise if the market were price happens to be more than the maturity value, what would you do, you would shot the bond, you have to receive the maturity market price and I guess the shot position in the bond, you get the redemption, you give the redemption value and as a result of it you make an arbitrage profit.

Conversely if the market price is lower, you buy the bond in the market and give it, return it to the company and take the proceeds. Again, you make an arbitrage profit. So, in either case, when the market price exceeds the redemption price or the redemption value, or vice versa, you make an arbitrage profit though, thus, to ensure that such profits are not available in the market and using the philosophy of arbitrage free pricing, as on the date of maturity of the bond, the bond should be quoting at the redemption value, if it is a par value, par redeemed bond it should be quoting at par.

So, if the bond is quoting at any earlier date at a premium, then that amount of premium will necessarily be amortized, would be eliminated, would be reduced as we move towards the maturity of the bond, at maturity the premium will be 0. Now because this amortization is a continuous process, that means that gradually as we approach the maturity of the bond, the amount of the premium declines and same as the situation when we have a discount bond, the amount of discount also declines as the bond is pulled towards par value.

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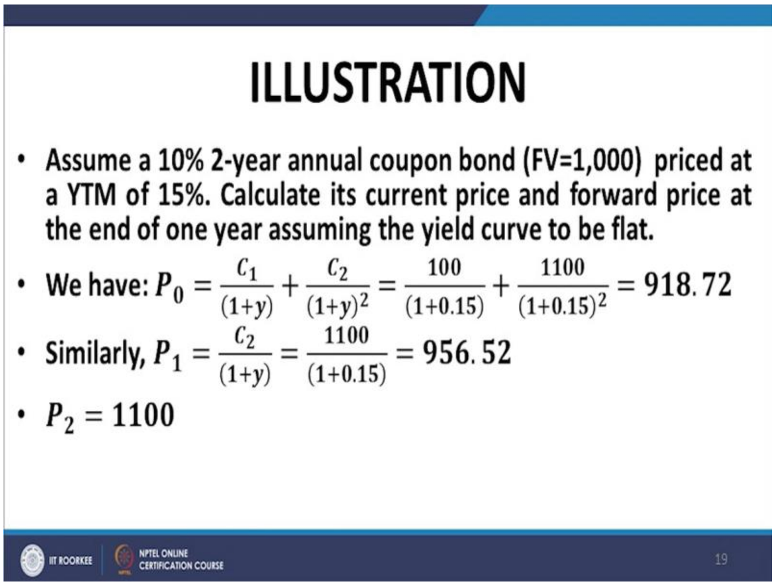

Now, this is an illustration of the concept that I elucidated just a few minutes back. Assume, a 10percent, two-year annual coupon bond face value is 1,000 priced at a YTM of 15percent.

Calculate the current price and the forward price at the end of one year assuming the yield curve to be flat.

Now we have P0 is equal to the discounting of t equal to 1 which is the coupon payment of 10 percent bond so, coupon payment is 100. Assuming the face value to be 1,000 and the yield curve is flat at 15 percent. So, we get a value of 918.72 in the second year, the discounting will be of 1100 it should be 1100 and 1100 discounted for two years, you end up with a price of 918.72. This is a price at t equal to 0 for two-year bond, which has a YTM of 15 percent and which has a coupon rate of 10 percent. Typical level coupon bond.

Let us work out the price at t equal to 1, at t equal to 1, at price will be given by 1100 divided by 1.15. And that turns out to be 956.52. And at t equal to 2, it would be obviously level 1100 that is the amount of the redemption value plus of course the final coupon payment. So, you can clearly see as you move towards the maturity, the price is being pulled back towards its redemption value. And it equals the redemption value at maturity.

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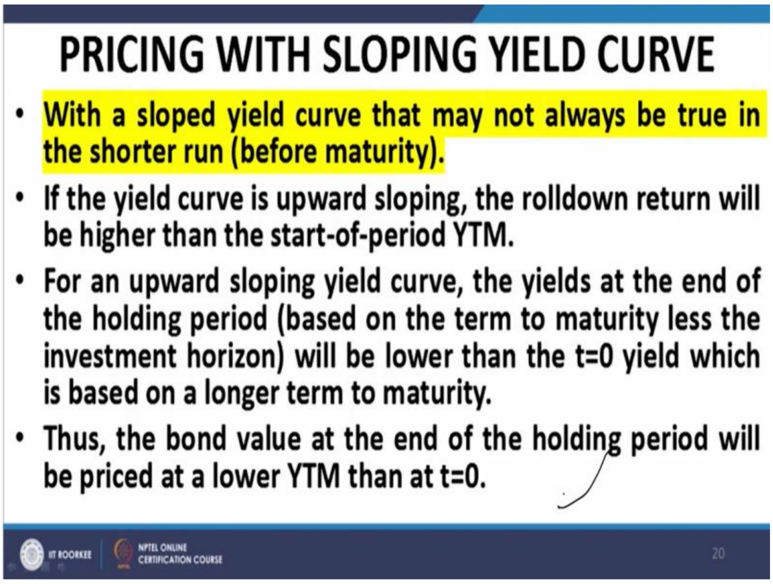

But this situation is slightly tampered with, slightly disturbed when we have a sloping yield curve. The example that I discussed just now was assuming a constant YTM of 15 percent across the maturity. In other words, the term structure was assumed flat at 15 percent. So now, suppose we have a situation where we are encountering a sloping yield curve, that is let us say it is an upward sloping yield curve. So, that short maturity rates are less, and long maturity rates are more, the yield curve is of this form.

So short maturity rates are less and long maturity rates are more. In this case what will happen? This is very interesting. And let us further assume that let us say we have a bond at t equal to 0 and let us say we evaluate the value of the bond, the price of the bond, let us say the maturity is capital T.

Then obviously we will discount all future cash flows at t equal to 1, t equal to 2, 3, 4 up to capital T, at the respect is spot rates S01, S02, S03 and S0t. And please note, due to the upward sloping nature of the curve this spot rate should form an increase in sequence, but that is one part of the story.

Now, at t let us say we want to now work out the price of the 1 at t equal to 1 and let us assume fundamentally, I repeat this point, that the interest rates have not changed over this period. Now, when we work out the price of the bond at t equal to 1, it is not a t year bond, it is not a capital T year bond, one year is already elapsed. So, it is now a t minus one year bond, and t minus one year bond will be the cashflows relating to this t minus 1 year bond let us assume it is a level coupon bond, so, we will have C1, C2, C3 up to Ct minus 1 and that Ct minus 1 is will be the maturity value plus the coupon value.

But the important thing is now, the discount rates will be what, they will be S01, S02, S03 up to S0t minus 1. The impact is that because the interest rates are increasing as maturity increases, that the rate that was S0t will no longer feature in the discounting process, and which was the highest rate, and therefore what will happen is that the value of the bond at t equal to 1, will be higher than the value of the bond when computed at t equal to 0. This is a very interesting feature that we encounter with the upward sloping yield curve, I will take it up with an example in the next lecturer. Thank you.