Quantitative Investment Management

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Lecture 22

Key Rate Duration

Welcome back. So, let us continue with key rate duration, but before I talk about key rate duration, it is necessary to give an introduction about what exactly we want to do using the special measure, or using this peculiar measure of duration.

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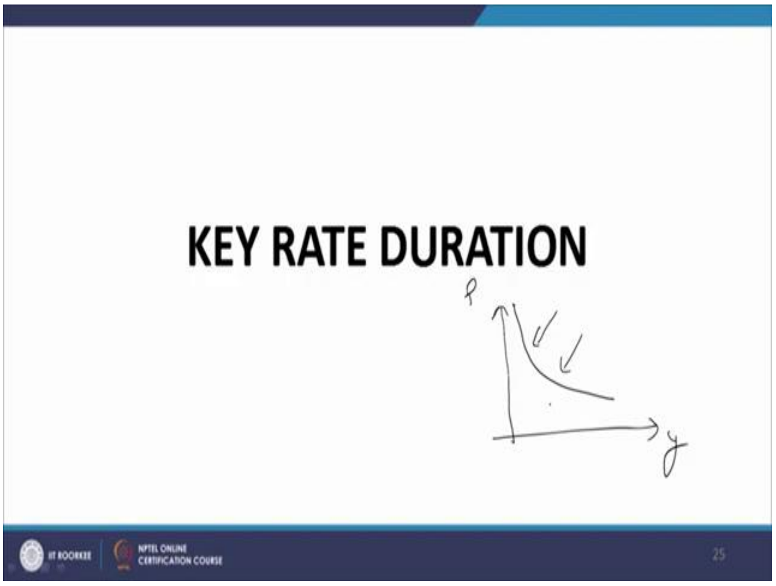

You see, the first thing that I have been emphasizing until now is that the yield price curve is curved, it is not a straight line. Now, that means what? That means that as the yield changes from one point to another, as the yield changes from one value to another, the change in the price does not, is not linear, it is, it follows a convex curve. And because it is not linear, we need to consider when we do a Taylor series expansion, we need to consider terms of higher order than the first order. Had it been a straight line, the second order and the higher order terms would have automatically vanished.

However, it is not a straight line and as a result of which we need to consider higher order terms. So, to account for this curvature, this is the yield and this is price. We have something like this. To account for this curvature that is manifest in the yield price curve, we need to consider second order terms and that gives rise to what we call the convexity. The first order terms gives us the duration or is captured by the duration and the second order is captured by convexity. That is one part of the story.

The second part of the story is even more interesting. The second part says that, you see, when you are evaluating duration or convexity, you are doing so at a particular point on the yield price curve. For example, usually what happens we work out the duration and we work out the convexity at the YTM of the bond. If a bond is quoting a 15 percent YTM we work out the duration and convexity at the YTM of 15 percent, at an yield of 15 percent.

If the YTM is 20 percent, we do the same thing computing duration and convexity at YTM of 20 percent. So, that is important. What does it mean? It means firstly, that we are using YTM as a universal representation of all spot rates. In other words, as I mentioned some time ago, the YTM captures the entire term structure of interest rates, it is some kind of an average, it is a single number that encapsulates the geometry of the term structure of interest rates. So, that is the important thing.

In other words, it is equivalent to saying that we are using a flat term structure curve when we are evaluating the duration. I repeat, when we evaluate the duration, we evaluate it at one single interest rate, which is usually the YTM. That being the case, it implies that the entire term structure is assumed to be flat.

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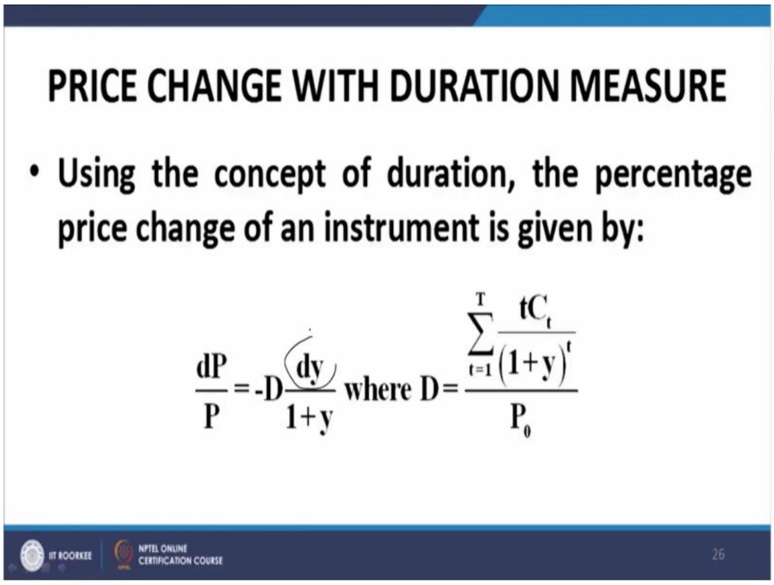

The second thing is the case of parallel shifts. If you look at the definition of duration, you have this quantity dy, this dy is also universal. Universal in the sense it is independent of the various spot rates that are involved in discounting of the bond. So that means what? That means we have two fundamental assumptions that go into the duration convexity model of working out a price, price percentages or price percentage changes, rather and that is number one, we assume a flat yield curve and number two, we assume a parallel shift in the flat yield curve which is captured by this dy.

So, this is very important. This is one issue where we need to be careful where we need to be, which we need to be conversant with. But there is another issue. As I mentioned when we talk about the duration or convexity, we are assuming a flat yield curve, but it is not a flat yield curve. The real life, the empirical yield curve is not a flat yield curve. It is obviously we have a functional relationship between the maturity of the instrument and the corresponding interest rates.

That means what? That means we have to do with some more thinking and we have to introduce a concept which also takes into account the curvature of the term structure of interest rate, not the curvature of the yield price curve. Please note that, we are not talking about the curvature of the yield price curve, which is captured by convexity, which is captured by convexity.

We are talking about the curvature of the term structure of interest rates, how the term structure of interest rates manifests themselves as functions of the maturities, which are relevant, which are relevant for discounting of the cash flows from the bond. So, that is where the issue of key rate duration comes into play. Let us now discuss this. Using the concept of duration, the percentage change in the percentage price change of an instrument is given by this expression. We have talked about it.

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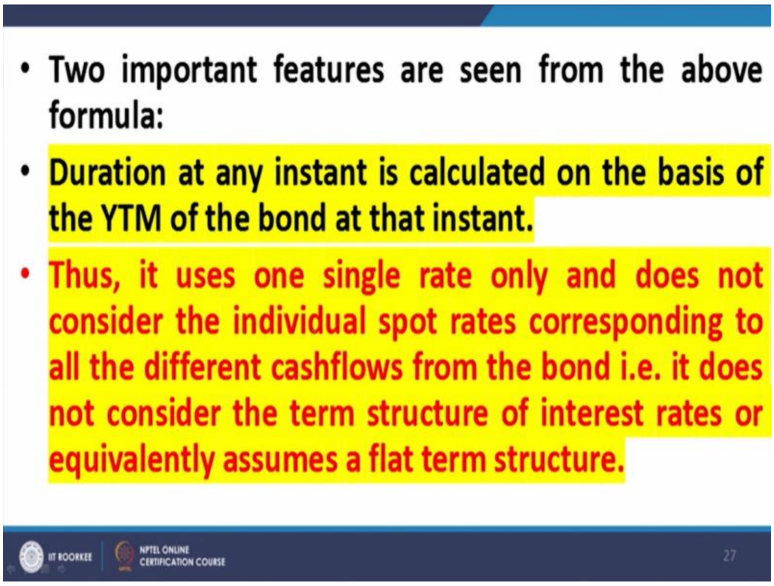

Two important features are seen from this particular formula. Number one, duration at any instant is calculated on the basis of the YTM of the bond at that instant. That is what I mentioned just now. Thus, it uses one single rate only and does not consider the individual spot rates corresponding to all the different cash flows on the bond. It does not consider the term structure of interest rates equivalently; it assumes a flat term structure worth repeating.

Let me repeat it. Thus, it uses one single rate that is the YTM rate and does not consider the individual spot rates which may be different from the YTM, which usually are different from the YTM. As I mentioned, YTM is some kind of an average, some sense of averaging of the various spot rates that are used for discounting of the cash flows on the bond.

Consider, the individual spot rates corresponding to all the different cash flows from the bond, it does not consider the term structure of interest rates or equivalently it assumes a flat term structure.

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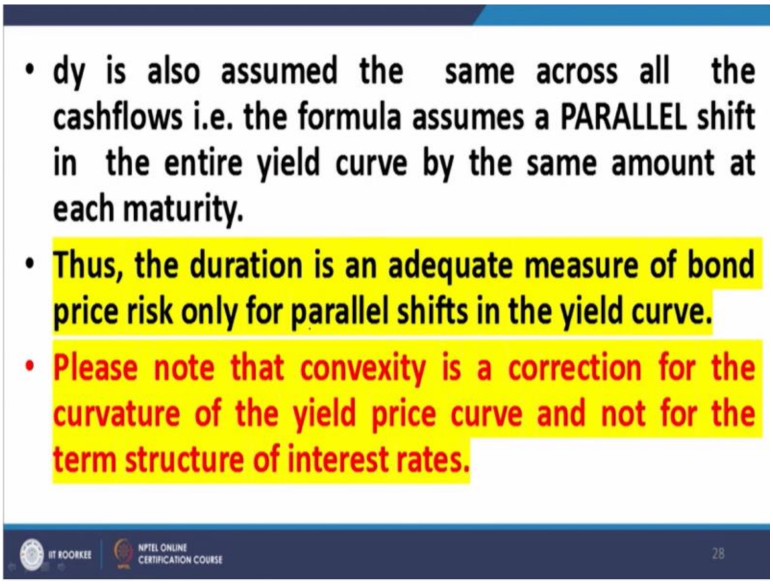

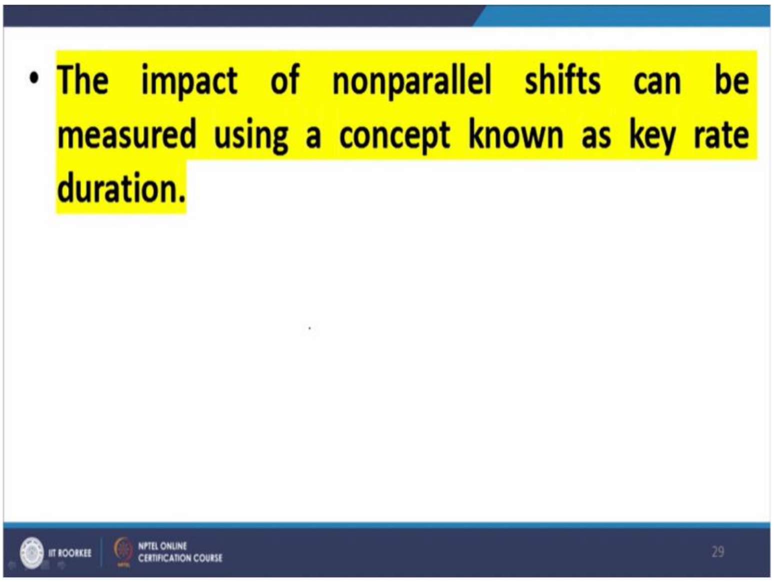

dy is also assumed the same across all cash flows, dy that is the shift in the yields, the shift in the yields is also taken as independent of this, of the changes in spot rates, or different spot rates or the term structure of interest rates. The entire yield curve is assumed to shift parallel to itself, what we discussed at the beginning of the last class. A level shift is what is assumed when we talk about measure the duration and convexity.

So, these two are fundamental assumptions of the duration convexity model. But these are not empirically indicated. dy is also assumed the same across all cash flows that is the formula assumes a parallel shift in the entire yield curve by the same amount at each maturity. Thus, the duration is an adequate measure of bond prices only for parallel shifts in the yield curve where the yield curvature is not significant or the yield curve curvature is not significant. And secondly, the shift is also parallel to itself.

Please note, that convexity is a correction for the curvature of the yield price curve. And not thus spot yield curve or the forward yield curve as the case may be, and not for the term structure of interest rates. The impact of non-parallel shifts can be measured using a concept known as key rate duration. So, that is where the relevance, the importance of key rate duration comes into play.

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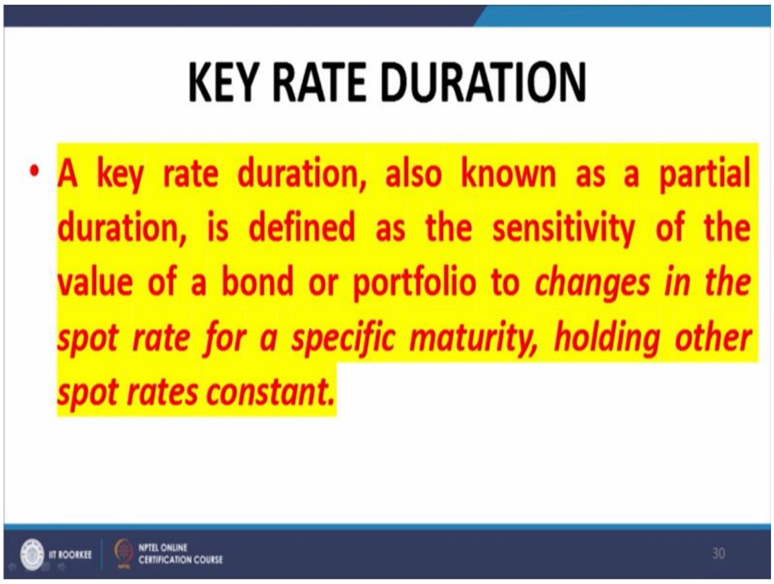

So, what is key rate duration? Key rate duration also known as partial duration is defined as the sensitivity of the value of a bond or portfolio to changes in spot rate, for a given maturity, holding all other rates constant, it is pretty much similar to partial differentiation, where you take, where you consider changes as a function of a number of variables with respect to a particular variable.

Keeping all other variables, keeping all of the independent variables constants, something like a partial derivative, and that is a reason that we call it the partial duration as well. So, the key rate duration is the percentage change in price, a measure of the interest rate sensitivity of the bond with respect to a particular spot rate, keeping all other spot rates constant.

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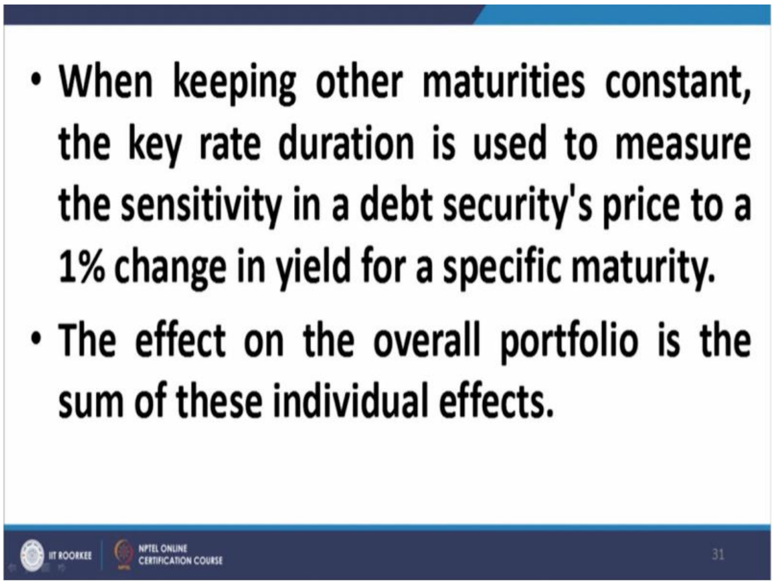

So, when keeping other maturities constant, a key rate duration is used to measure the sensitivity of a debt security's price to 1 percent change in yield for a specific maturity. The definition in effect is pretty much similar to the definition of the modified duration except that we replace the YTM by a particular spot rate.

As you can see, in the next slide, the effect on the overall portfolio is the sum of these individual effects. So let me read it out again. When keeping other maturities constant, the key rate duration is used to measure the sensitivity of a debt security's price to a 1 percent change in the yield for that specific security.

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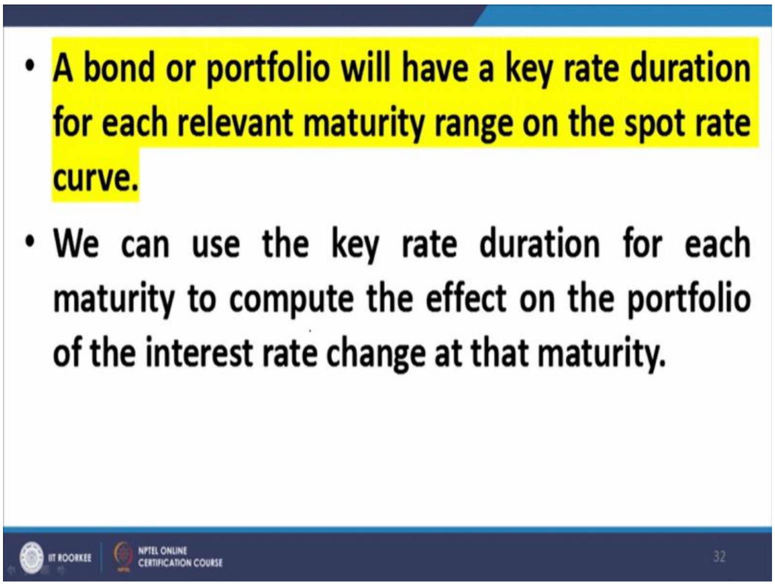

A bond or portfolio will have a key rate duration for each relevant maturity range on the spot rate curve. So, because different spot rates would be used for discounting the cash flows that occur at different points in time, every, the cash flow at t equal to 1 will need to be discounted at S01, the cash flow t equal to 2 will be, need to be discounted at S02 and so on.

So, all this portraits that relate to the points in time at which the cash flows come into the picture or cash flows form the bond emanate would be relevant for discounting and correspondingly we will have a key rate duration with respect to each of these spot rates. We shall, we can use the key rate duration for each maturity to compute the effect on the portfolio of the interest rate changes at that maturity.

So, you have a key rate duration, with respect to each of those spot rates, which are used for discounting of cash flows from the instrument. And correspondingly you have percentage change in the value of the bond with respect to changes in that particular interest rate, spot rate.

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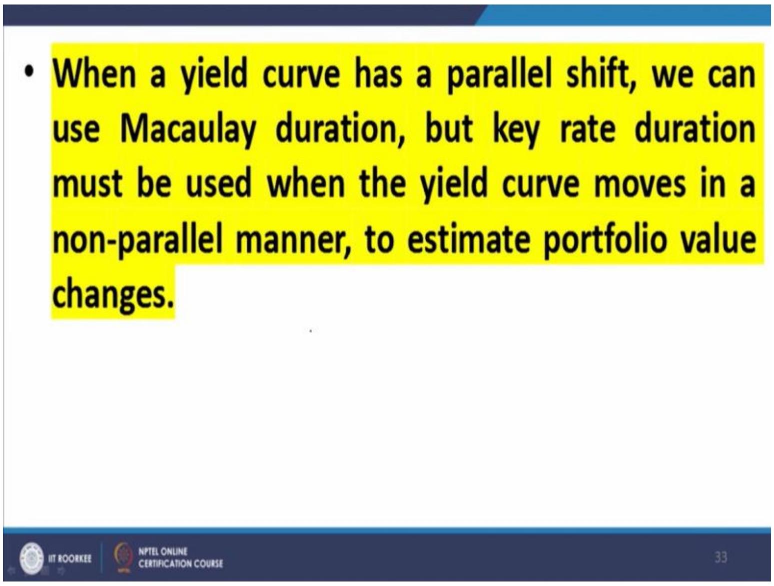

When a yield curve has a parallel shift, we can use Macaulay's duration, but key rate duration must be used when the yield curve moves in a non parallel manner to estimate portfolio value changes. Now that is where the relevance of the key rate duration comes into play. When there is a parallel shift.

Obviously, Macaulay's duration can do a good job, can do an adequate job, but when there is a non-parallel shift in the yield curve, they are different maturity spot rate change by different amounts, then obviously the use of the duration convexity model may not be sufficiently accurate.

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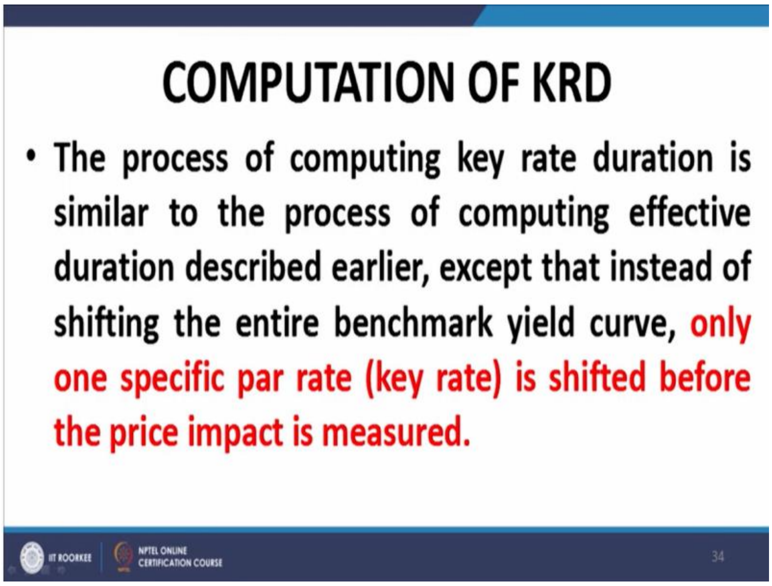

Computation of key rate duration. The process of computing key rate duration is similar to the process of computing the effective duration described earlier, except that instead of shifting the entire benchmark yield curve, only one specific par rate, key rate is shifted before the price impact is measured.

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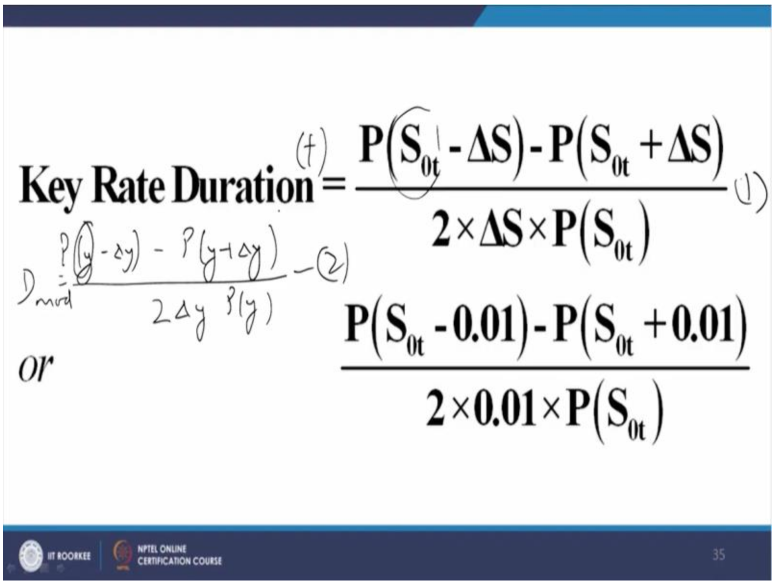

So, this is the formula here. Recall what formula did we have for the effective duration? It was P y minus delta y minus P y plus delta y divided by 2 delta y into P of y. And this is D mod or D effective. You can see a complete one to one parallel between this move formula then let us call this formula 1. And let us call this formula 2.

There is a one-to-one parallelism between one-to-one correspondence rather between the formula 1 and the formula 2, only, the only thing is instead of the YTM, why we are using a specific spot rate S0t with respect to which we are calculating the key rate duration, so you can write it as a function of that particular t. So, the formula for computation of key rate duration is exactly the same.

The formula that is used for computation of effective duration, the only differences in the effective duration, we use a universal rate, the YTM rate, in the case of the key rate duration, we use a specific rate for which we were want to work out the key rate duration for which we want to work out the percentage price change.

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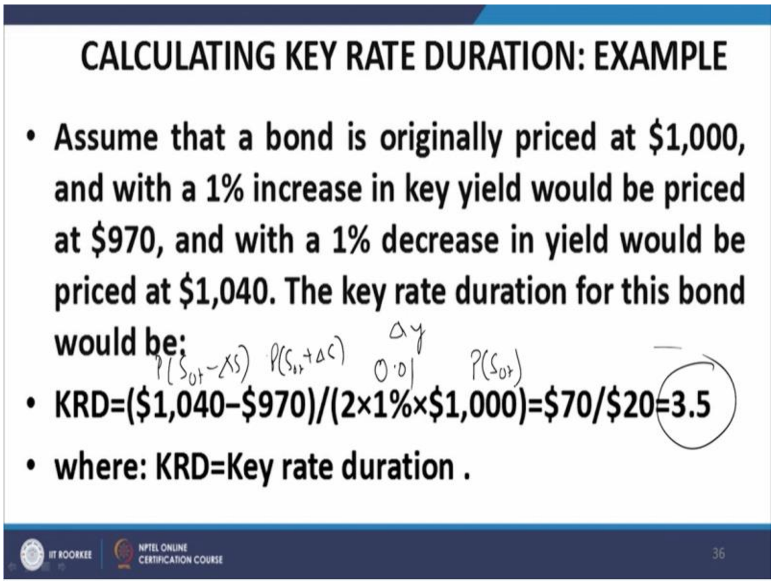

Calculating key rate duration is a trivial example, but nevertheless, let us do it. Assume that a bond is originally priced at dollars 1000. And with a 1 percent increase in key yield, whatever the key yield is, the key rate is would be priced at 970 and with a 1 percent decrease in yield would be priced at 1040.

The key rate duration for this bond would be the percent, the change in price that is P S, whatever that S is, S minus delta S minus P S plus delta S which happens to be 1040 minus 970. That is equal to 70 divided by 2 into 1 percent, 1 percent is the change. So, this will be 0.01, 1 percent increase and this is delta y so delta y is 0.01. The current market price is 1000. This is Py or PS rather this is P S0t minus delta S, this is P S0t plus delta S. Then this is P S0t, this is delta y and that gives you a value of 3.5 years.

In other words, this is the duration with respect to the specific rate whichever is, whichever is used here, not specified in the example, which is used here, we call it S0t. This is the key rate with respect to the interest rate S0t.

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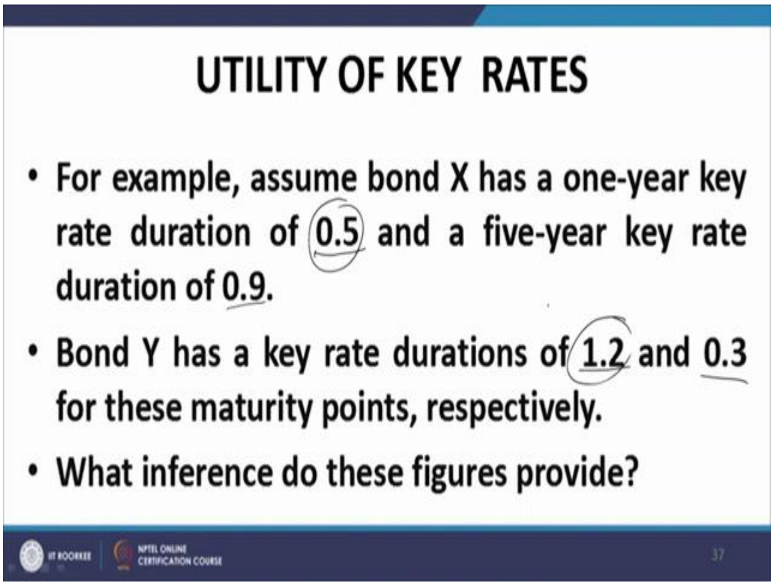

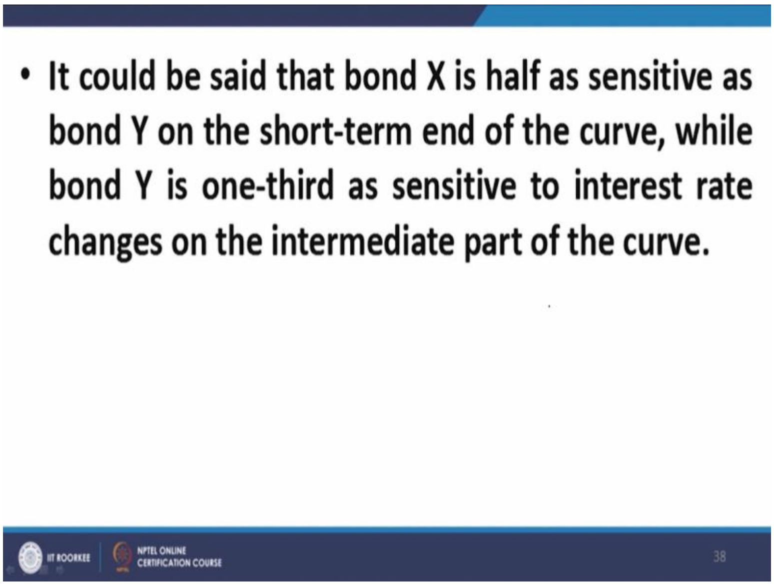

Utility of key rates. For example, assume bond X has a one year key rate, duration of 0.50 and a five year key rate duration of 0.90. Bond Y has a key rate durations of 1.2 and 0.3 for these maturity points, 1.2 for one year maturity, and 0.3 for a five year interest key rate. What inference can we get?

The inference that we can get is that bond X is half as sensitive as Bond Y for the short end of the curve you can see here, as far as the key rate or the short end rate is concerned, the one year rate is concerned bond S's duration is 0.50, bond Y's duration is 1.2. So approximately the sensitivity of bond X to the short end is half of that of the sensitivity of Y. Similarly, we can infer that the sensitivity of X to long end, or the long-term interest rates is almost three times that of Y.

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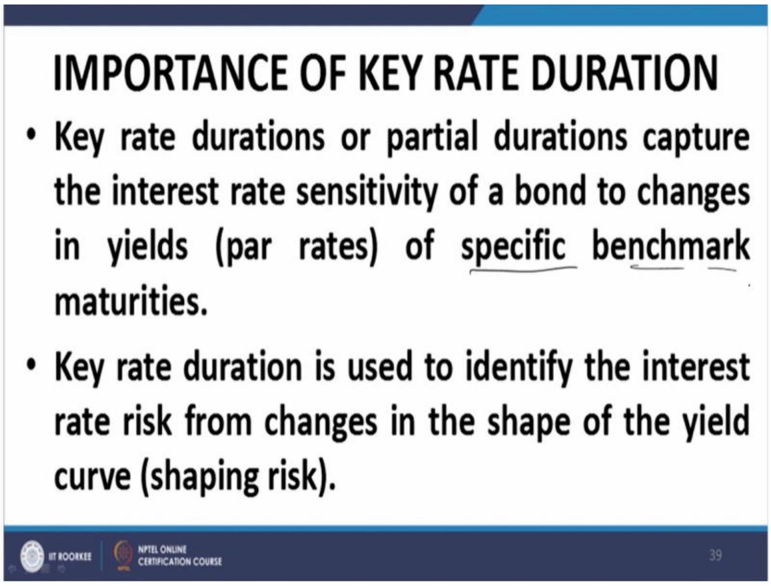

Importance of key rate duration, key rate durations or partial durations capture the interest rate sensitivity of a bond to changes in yields, par rates or specific benchmark maturities, specific interest rates, not a universal or a single figure YTM which captures the entire spectrum of interest rates, no, we do not use the average rates, we use specific rates similar to as I mentioned, I reiterate that point, that it is very similar to the concept of partial differentiation. Key rate duration is used to identify the interest rate risk from changes in the shape of the yield curve.

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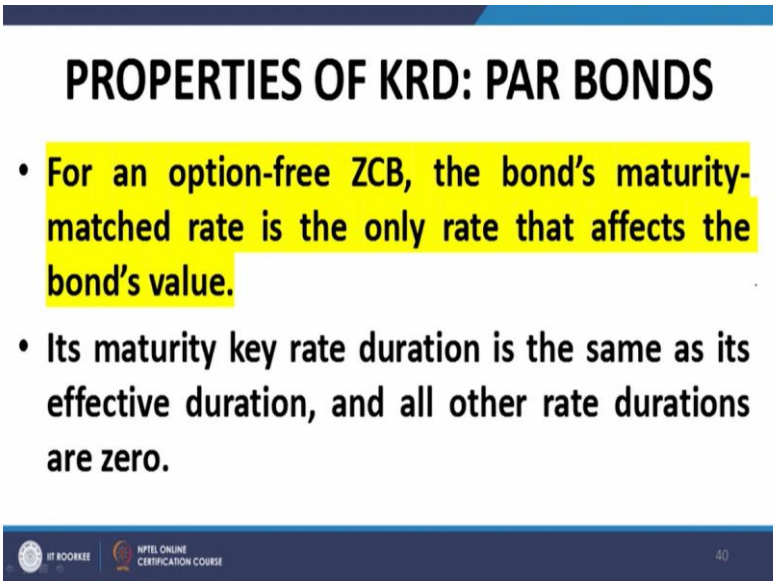

Properties of key rate duration for an option-free zero coupon bond, the bond's maturity matched rate is the only rate that affects the bond's value is quite trivial. It is maturity; therefore, its maturity key rate duration is the same as its effective duration and all other durations are zero, that is for a zero coupon bond. For a zero coupon bond as we know the bonds Macaulay's duration is equal to its maturity. Similarly, the bonds key rate duration with respect to just the one rate which is relevant, which is the maturity rate, maturity match rate which will also be its maturity.

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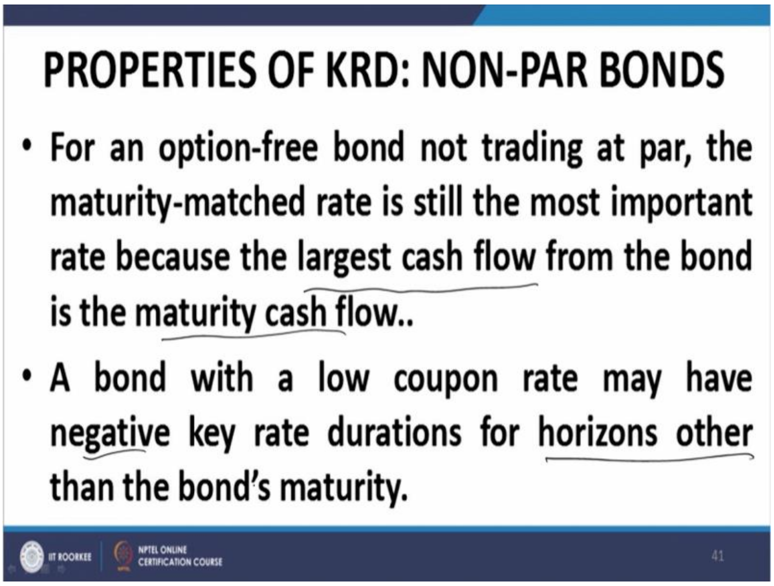

Properties of key rate duration for non-par bonds, non-par bonds, what happens? For an option free bond, not trading at par, the maturity match rate is still the most important rate, why? Because the largest cash flow that is the redemption of principal and the final coupon from the bond is at the maturity, is the maturity cash flow.

Let me repeat, as we assume a conventional bond structure of what is called a level coupon bond or a plain vanilla bond, in that case, what happens is, you get the redemption at maturity together with the last coupon, so, it is quite natural that the largest cash flow that is going to take place is going to take place at maturity and therefore the maturity matched rate would be the most important key rate in so far as these type of bond are concerned.

A bond with a low coupon rate may have negative key rate durations for horizons other than the bond maturity for horizons, other than the bond maturity, the key rates maybe negative in special circumstances, but that can occur only for horizons other than the maturity of the bond.

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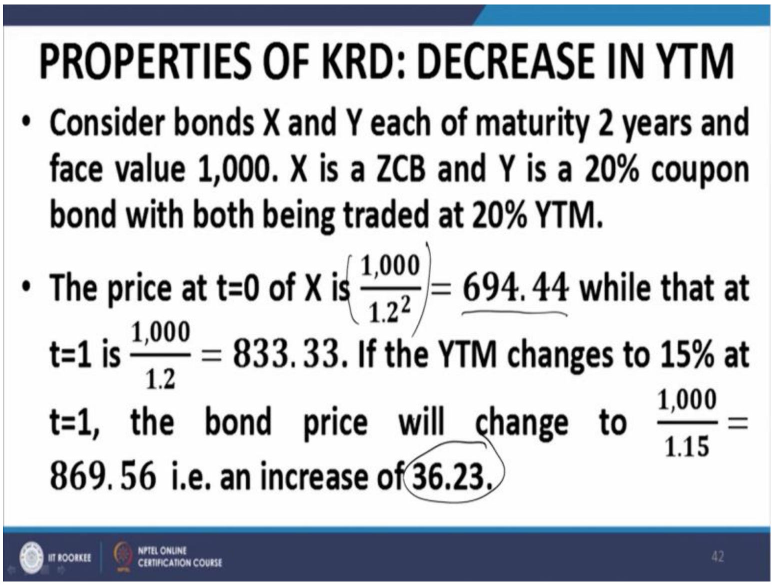

Now, properties of key rate duration decrease in the YTM. Let us look at an example here to illustrate what we are trying to say. We consider two bonds X and Y each of maturity two years and face value 1,000, X is a zero coupon bond. And Y is a 20 percent coupon bond with both being traded at 20 percent YTM.

So, the price of X at t equal to 0 is equal to 1000 divided by the YTM squared because it is a two year bond. That is equal to 694.44. Let me repeat. The zero coupon bond has a redemption value of 1000. It is trading at a YTM of 20 percent. So, it is quite natural that the current market price will be given by this expression. And this expression when solved gives us 694.44.

And the price of the same bond at t equal to one year would be 1000 divided by 1.2. Again, because it is been traded at the YTM of 20 percent that turns out to be 833.33. If the YTM changes to 15 percent, what happens to this bond? At t equal to 1, let us assume that the YTM at t equal to 1 changes from 20 percent to 15 percent. The bond price changes from 1,000 divided by 1.15 and that is equal to 869.56. That is, it is increasing by 36.23. This is as far as bond X is concerned.

What is bond X? It is a zero coupon bond and it is trading at a YTM of 20 percent to start with, it is a two year bond, YTM 20 percent will start with, therefore the market price at t equal to 0 is 694.44 at t equal to 1, the market price at 20 percent YTM is 833.33 and if the YTM changes to 15 percent the price is 869.56 that is there is an increase in price corresponding to a 5 percent decrease in YTM of 36.23.

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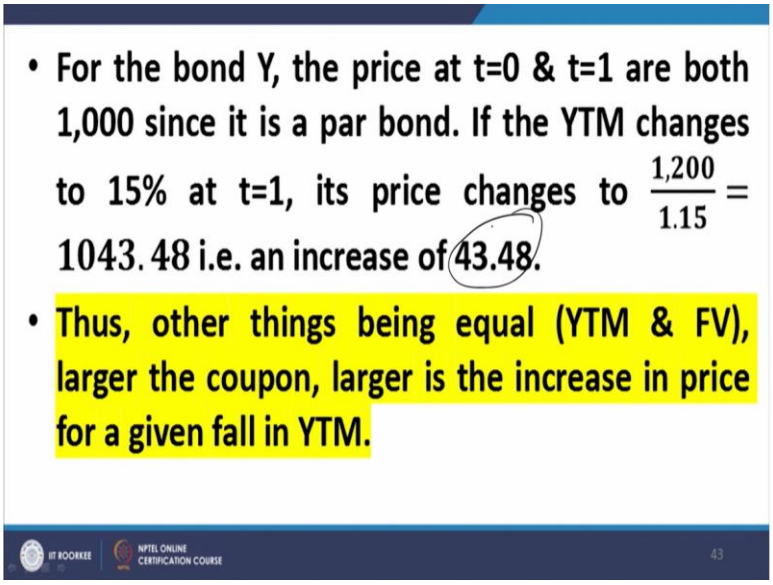

For the bond Y what happens? The price at t equal to 1 and t equal to 0 and t equal to 1 are both 1,000 since it is a par bond and if the YTM changes to 15 percent at t equal to 1 as price will change from 1,200 which is the maturity cash flow comprising a 20 percent coupon and 1,000 of face value. And this will be equal to, this will be discounted at 1.15 because the YTM has changed from 20 percent to 15 percent. We get 1043.48 that means an increase of 43.48.

So, what do we infer from this example? What we infer from this example is that, other things being equal that is YTM and face value being equal, larger the coupon, larger is the increase in price corresponding to a given fall in YTM. Let me repeat, given a YTM to start with, given a face value to start with, if the YTM and face value are identical, larger the coupon, larger is the increase in price corresponding to a given fall in YTM.

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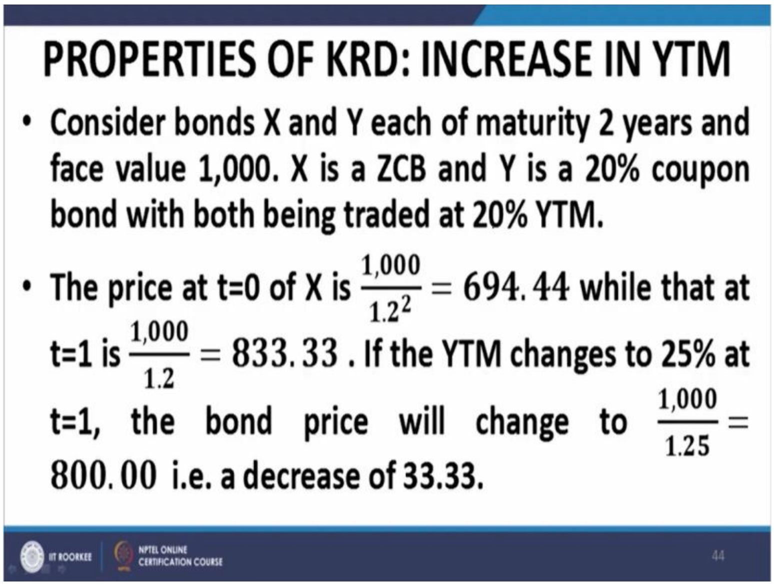

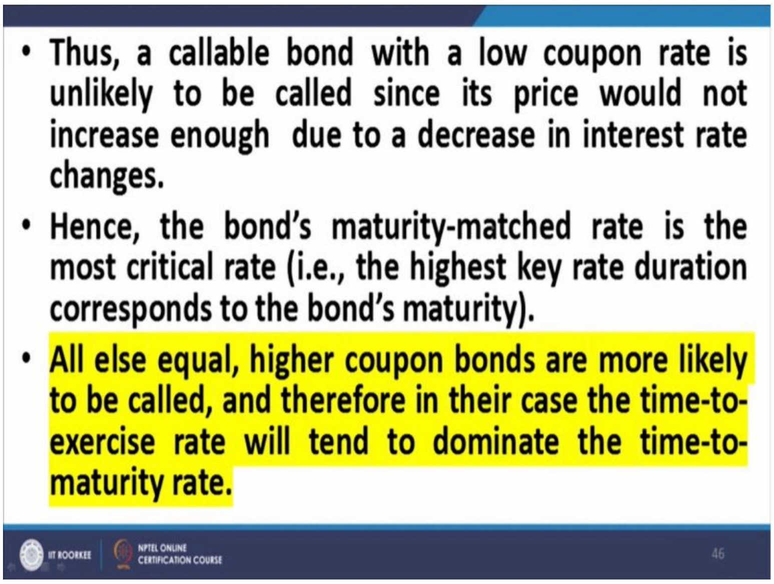

And the same analysis is done for an increase in YTM, I will not go through it, the exercise is absolutely parallel to what we discussed just now. So, no point in repeating the same process. But the inference is very interesting. The inference is that a callable bond with a low coupon rate is unlikely to be called since its price would not increase enough due to a decrease in interest rate changes.

See this is a direct application of the example that we did just now. What was the example? Higher the coupon rate, higher is the change in price, lower the coupon rate, lower is the change in price. So, if there is a lower coupon rate, and there is a change in YTM, the price change will not be very significant and because the price change will not be very significant, the probability of exercise of the callable option or the call option embedded in the bond will not increase significantly.

That being the case the probability of the option related cash flows emanating or manifesting themselves due to a change in yield would not be significant for low coupon bonds. Hence the bonds maturity matched rate is the most important rate, it is the most critical rate. In other words, the exercise matched rate, will not be very significant because the chances of exercise of the call option are not significant and therefore the exercise matched rate will be subservient to the maturity matched rate, which will be the most important rate and therefore the highest key rate duration corresponds to the bond's maturity.

All else equal, higher coupon bonds are more likely to be called because if there is a small change in interest rates, that will manifest as a significant change in price and significant change in price may result in a bond being bond that is out of the money, becoming, in the money as far as the call option embedded in the bond is concerned and therefore result in the call option being exercisable and presumed to be excised. And therefore, in their case, the time to exercise becomes very significant. And the time to maturity may not be very significant.

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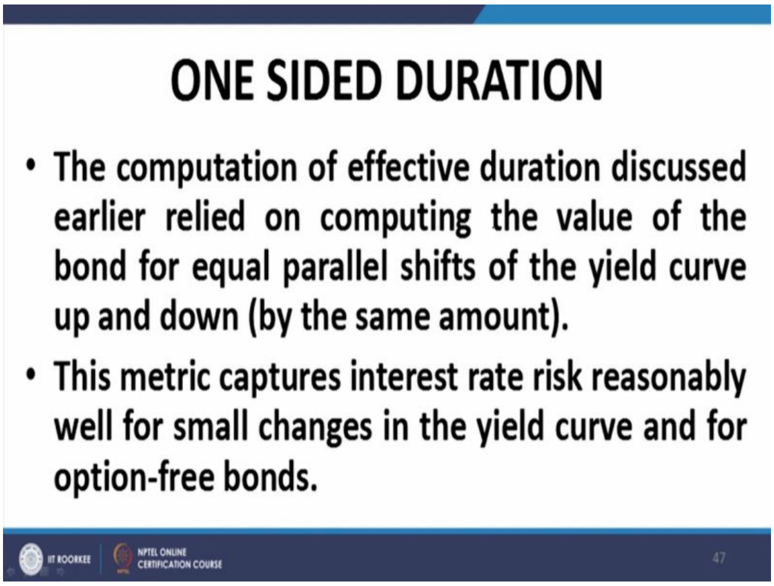

Now we talk about one sided duration. This is another interesting concept. But I need to emphasize that this concept of one-sided duration is relevant, essentially, for bonds that have embedded option features, for bonds that have callable options or portable options embedded in them.

Now, what are one sided duration? You see, in the case of a straight bond that is a bond that does not have any options attached to it, no options embedded in it, the effective duration, as we calculated a few slides back considered the increase in price corresponding to a given decrease in yield and a decrease in price corresponding to an increase in yield, and then average the two, an increase in yield together with a corresponding decrease in price and then a decrease in yield corresponding to an increase in price and then took the average of those two.

So, the duration that we arrive at in this situation is pretty much the average duration which takes into account the changes or whether the changes in YTM whether they be positive or they be negative. Now what happens in the case of bonds with embedded options is that, suppose we had considered a callable bond. Now in the case of a callable bond, if interest rates decrease, then what should happen to the bond price?

The bond price should increase and the bond price does increase. But if there is a callable option embedded in the bond, what happens is that as the price increases, and it goes above the exercise price, the issuer of the bond, that is the person who is long in the call option will exercise the option and truncate the price of the bond, truncate the value of the bond, the value of the bond will decrease.

And as a result of which, in the direction of decreasing interest rates, what happens? The price is truncated, the change in price is truncated and as a result of which the duration in this direction is different from the duration that is there in the other direction, because if the interest rates increase for that matter, the price will decrease. And if the price decreases, the option does not become in the money and it will behave as a straight bond, right?

So let me read it out for you. The competition of efficient, effective duration discussed earlier relied on the computation of the value of the bond for equal parallel ships of the yield curve up and down. This metric captures interest rate risk reasonably well for small changes in the yield curve and for option-free bonds.

So, we, as I mentioned, we use the price which corresponded to a small increase in the yield and we use the price corresponding to a small decrease in yield, took the average and use that average for calculating the effective duration. But so long as it is a straight bond, it does not have any embedded option features, it does not matter it was not a problem.

The problem arises in the case of bonds that have embedded option features. Why? Because in this case, when the option becomes in the money, if there is a for example, in the case of a callable bond, if there is a decrease in interest rates, if there is a decrease in interest rates, what happens? The price increases, and if the price goes above the exercise price, the insurer will call back the bond and if he calls back the bond, the payoff for the price of the bond is truncated is cut down to the exercise price.

And as a result of which the duration that is worked out corresponding to a decrease in price will be different from the duration corresponding to an increase, there will be asymmetry between the two directions of motion of the yield, decrease in price or decrease in rates rather, would give you a different duration compared to an increase in rate. So, for that purpose, what we consider is the one-sided duration.

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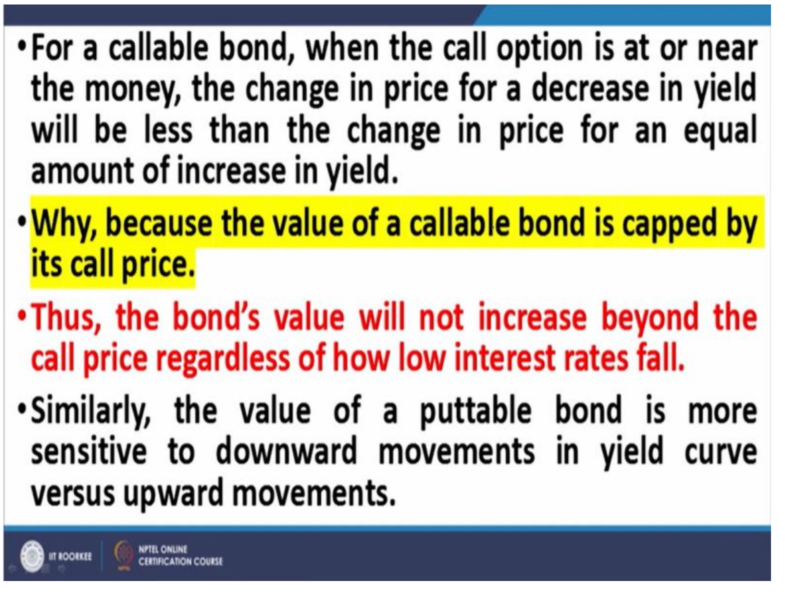

For a callable bond when the call option is in or near the money, the change in price for a decrease in yield will be less than the change in price for an equal amount of increase in yield. Why? Because the price is truncated by the exercise price of the bond. Why? Because the value of a callable bond is kept by the call price, and thus the bond's value will not increase beyond the call price regardless of how interest rates fall. And similarly, the value of a puttable bond is more sensitive to downward movements in yield curve versus upward movements.

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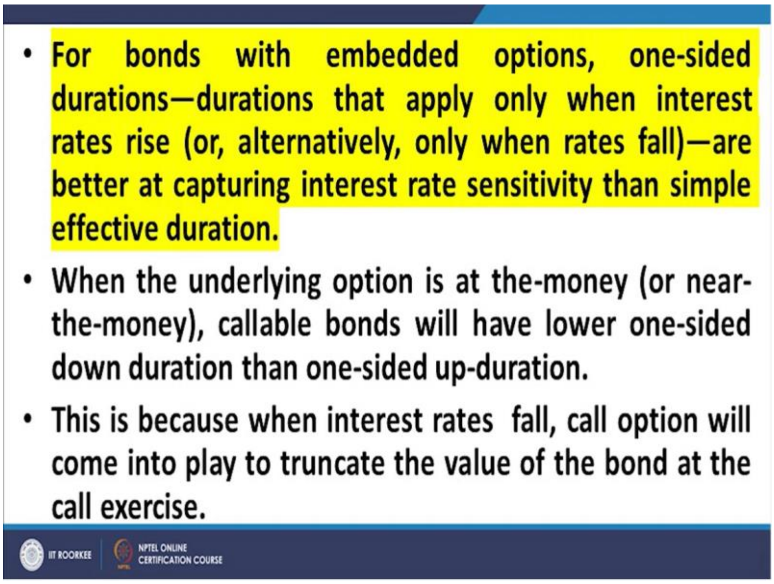

For bonds with embedded options, one sided directions durations, durations that apply only when interest rates rise or alternatively, only when interest rates fall, are better at capturing interest rate sensitivity, then simple effective duration. When the underlying option is at the money or near the money, callable bonds will have lower one-sided duration, then one sided down duration then one-sided up duration, because as I mentioned a fall in interest rates triggers an increase in prices and that causes the chance of exercise of the call option, if the call option may turn out to be in the money, maybe exercised, and that cuts down the price, it truncates the price to the call price.

This is because when interest rates fall, call options will come into play to truncate the value of the bond at the call exercise. So, this is because when interest rates fall, call options will come into play to truncate the value of the bond at the call exercise.

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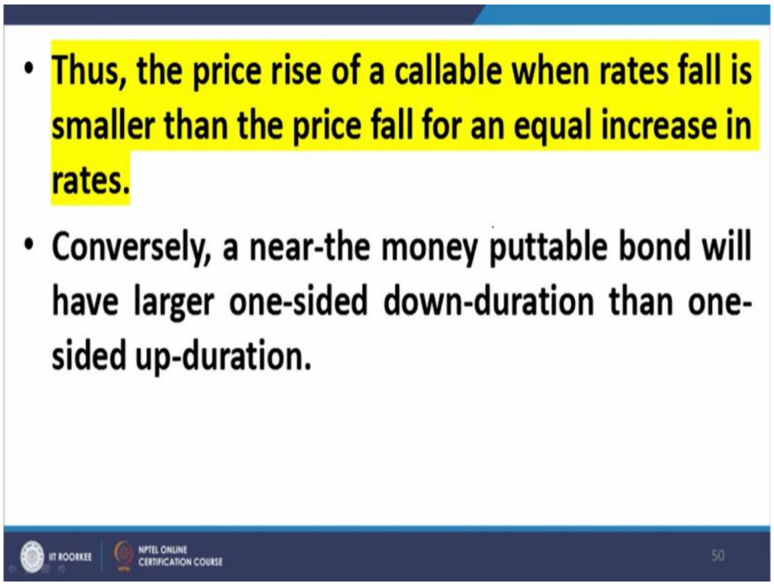

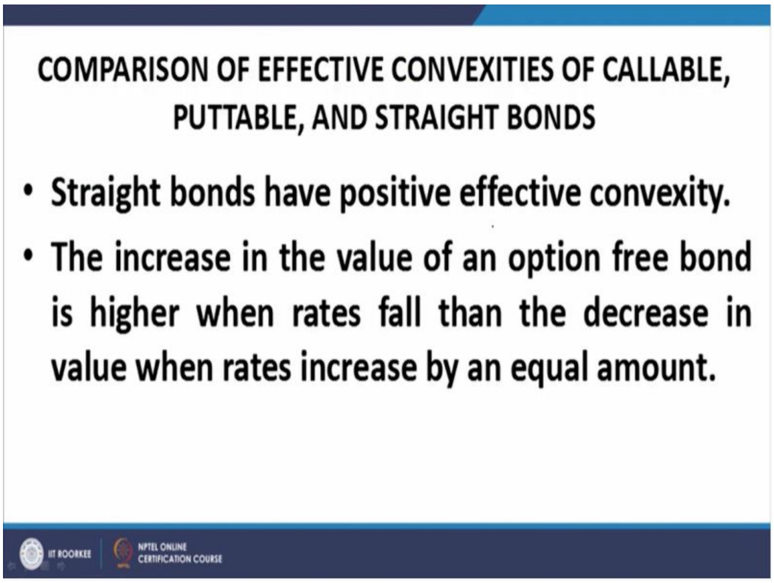

Thus, the price rise of a callable bond when rates fall is smaller than the price fall of an equal increase in rates. Conversely, near the money, puttable bond will have larger one-sided down duration than one sided up duration. We shall continue from here in the next lecture. Thank you.