Quantitative Investment Management
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Lecture No. 19
Immunization & Bond Dynamics

Welcome back. So, in the last lecture, I talked about interest rate risk and I also discussed the various measures of interest rate risk that included DV01 or what we call the dollar value per basis point, then we moved over to Macaulay duration, then modified duration and finally, interest rate elasticity.

These are the four common measures that we use for assessment of interest rate risk of a bond portfolio, of these the cardinal measure is the Macaulay duration. And continuing from where we left off in the last lecture, I will now illustrate this concept of duration and the dynamics of bond investment with an example.

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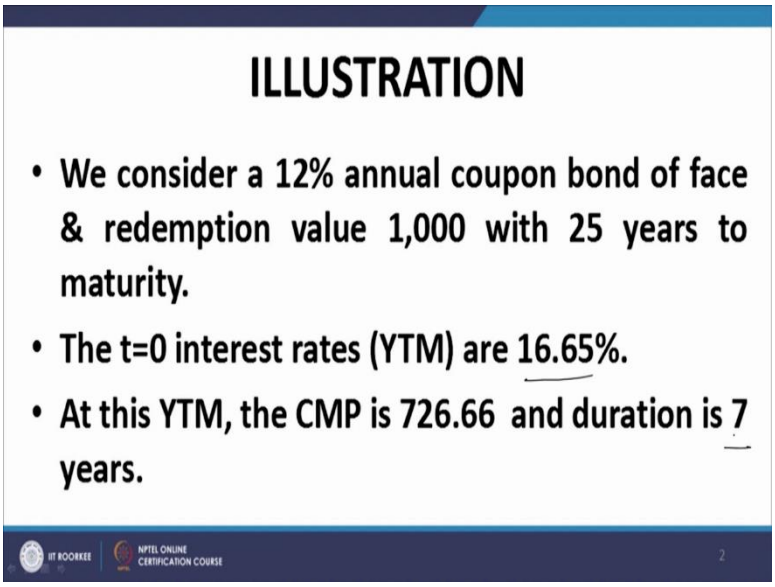

So, for that purpose we consider a 12 percent annual coupon bond of the face value of rupees 1000, which is also its redemption value. It is redeemable at par at the end of 25 years from now. So, let me repeat it is 12 percent annual coupon bond coupon payments are annual and the face value as well as the redemption value is 1000. And the bond is 25 years to maturity, the T equal to 0 interest rates YTM are 16.65 percent.

This figure seems unusual, but the rationale behind it will be apparent in the next sentence, at this YTM the current market price of the bond is 726.66. And the duration is 7 years you get

a whole number for the duration that was the objective behind choosing 16.65 percent as the interest rates. So, this is the basic data on which we build upon in the following slides.

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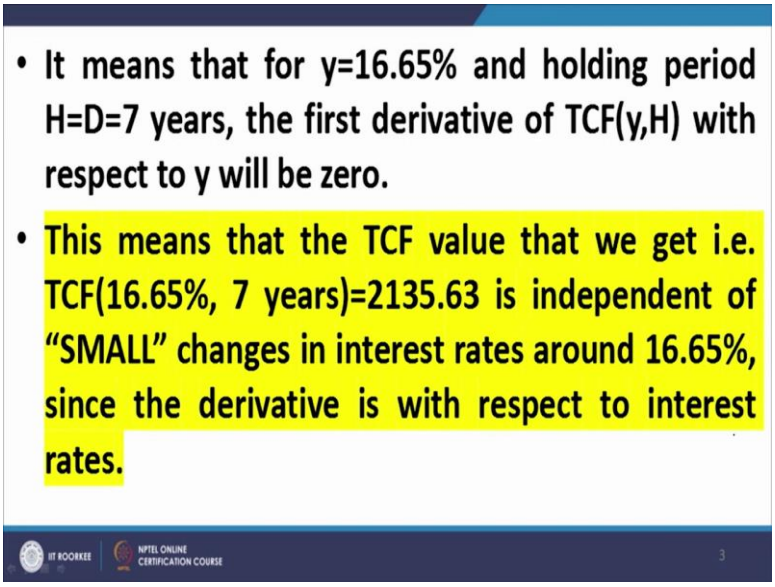

It means that for y equal to 16.65 percent, which is the YTM of the bond and a holding period of 7 years, the first derivative of total cash flows, which is a function of y and the holding period, which is a function of the YTM as well as the holding period with respect to y will be 0 this was the definition of duration this was how we arrived at the fact that if you hold a bond for restoration, you are immunized against small changes in interest rates. We equated the first derivative of the total cash flows to 0 and then on simplification, we arrived at the expression for the duration.

So, if the duration is 7 years, that means the first derivative with respect to total of total cash flows with respect to y will be 0 for this holding period. This means that the total cash flow value that we get that is 2135.63, which will work out the total cash flow value for a period of 7 years for a holding period of 7 years. And at the interest rate of 16.65 percent, you get the total cash flows of 2135.63. And this are 2135.63 independent of small changes in interest rates around 16.65 percent. Since the derivative is with respect to the interest rates.

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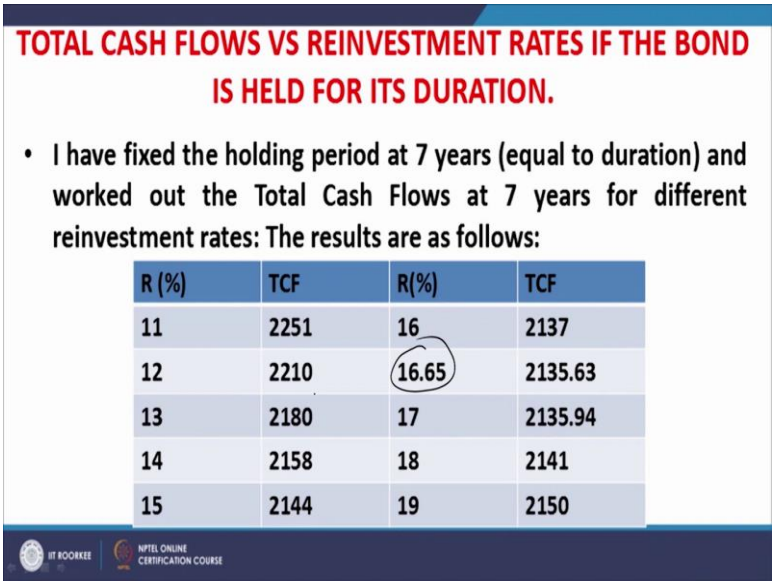

You can see here explicitly also in this table, what I have done is I have worked out the total cash flows corresponding to different interest rates for the same bond for a holding period of 7 years. So, it is… these cash flows correspond to a bond which has annual coupons of at the rate of 12 percent face value of 1000, redemption value of 1000 and maturity of 25 years the holding period is 7 years.

The total cash flows are computed at the end of 7 years corresponding to different YTM different reinvestment rates. As you can see, if the interest rate is 11 percent for on the basis of which we compute the total cash flows, we find that the total cash flows are turn out be 2251. If it is 12 percent, 2210 and so on, the entire table is given in the bottom panel.

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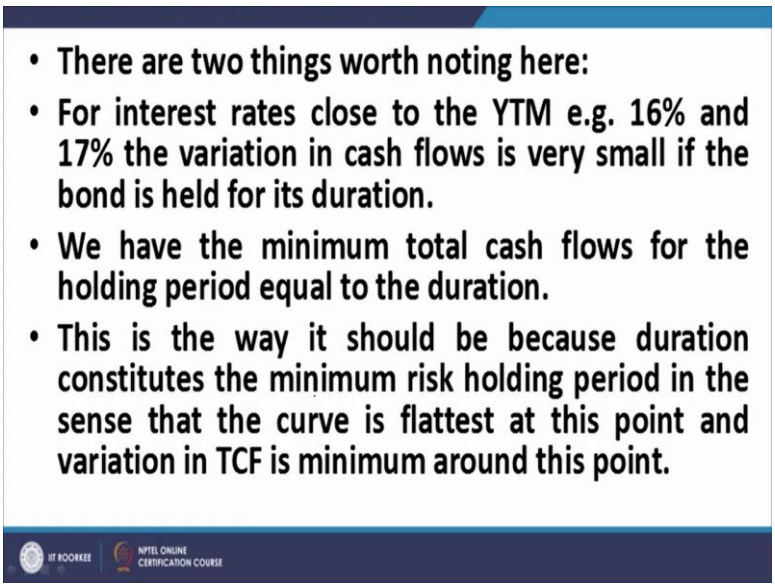

And there are two fundamental things which are which we note immediately from the figures that we have here. The important thing is that around this figure of 16.65 percent, what we find is that if there is small, small change in the interest rates, let us say from 16.65 percent, around 16.65 percent from 16 percent to 17 percent. The total cash flows change by a front change from 2137 22135.94 that is a marginal change of one unit of money.

However, if you observe a similar change of 1 percent from 11 percent to 12 percent, the changes from 2251 to 2210, which is 41 money units change. So, the important point that I want to bring to the learners, that I strongly emphasize here is that around the point at which the derivative turns out to be 0, that is the 16.65 percent point here in this key example, what we find is that if there are marginal changes, either to the left or to the left, the total cash flows remain literally unaffected.

That is one thing. The second thing that we observed from this table is that the total cash flows at this point of 16.65 percent interest rates turned out to be minimum of all total cash flow values that we have in this table. The total cash flows as 16.65 percent is 2135.63. Whereas if you look at any other value in this table, it is above this figure.

So, these are two important observations. The reason behind the second observation or the rationale or the final sense between the secondary or behind the second observation is that notwithstanding the fact that it has the lowest cash flow, it also has the minimum variability. So, this is compatible with our usual assumption that minimal the risk and expected return move in tandem higher there is higher the expected return lower there is lower the expected return.

So, because the variability in cash flows around this figure of 16.65 percent is minimal. Naturally, we expect the expected returns to be lower and that is indicated by the figures that we saw in the previous table, where we saw that around 16.65 percent in the total cash flows turned out to be minimum. So, let me read these points to you once again.

And there are two things worth noting here, the interest rates close to the YTM that is 16 percent for example, 16 percent and 17 percent. The variation in cash flows is very small. If the bond is held for its duration, that is 7 years, we have the minimum total cash flows for the holding period equal to its duration that also explained.

This is the way it should be because duration constitutes the minimum risk holding period in the sense that the curve is flattest at this point, the curve between the interest rates and the YT

and the total cash flows is flattest around this point of 16.65 percent. Therefore, minor variations in the interest rate do not manifest themselves as variations in the cash flows and variation in total cash flows is minimum around this point.

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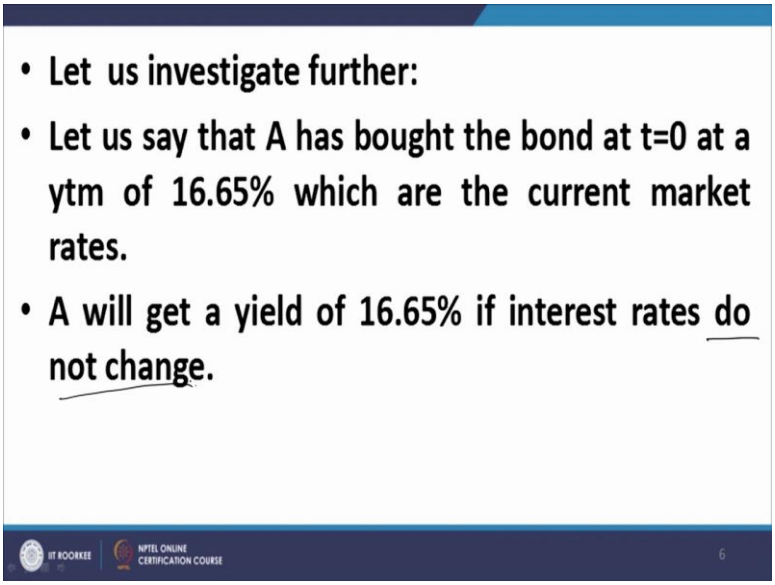

Now, let us investigate these, the dynamics of the bond investment further that is very interesting. Let us say that A has bought the bond at t equal to 0 at a YTM of 16.65 percent, which is the current interest rate, I repeat we buy … A buys the bond at t equal to 0 when the market interest rates are 16.65 percent. The current price of the bond turns out to be 726.6 at which he purchases the bond, redemption is at face value, but we shall be holding the bond for a period of 7 years.

But if no… if A has made the investment at t equal to 0 in this bond at 12 percent annual coupon bond, maturity 25 years at the current market price of 72.66 when the interest rates are 16.65 percent, then if the interest rates do not change over the life of the bond, then A will get a return of 16.65 percent A will get an annual yield of 16.65 percent if interest rates do not change. This is important, if interest rates do not change subsequent with making of the investment A gets a return of 16.65 percent per annum.

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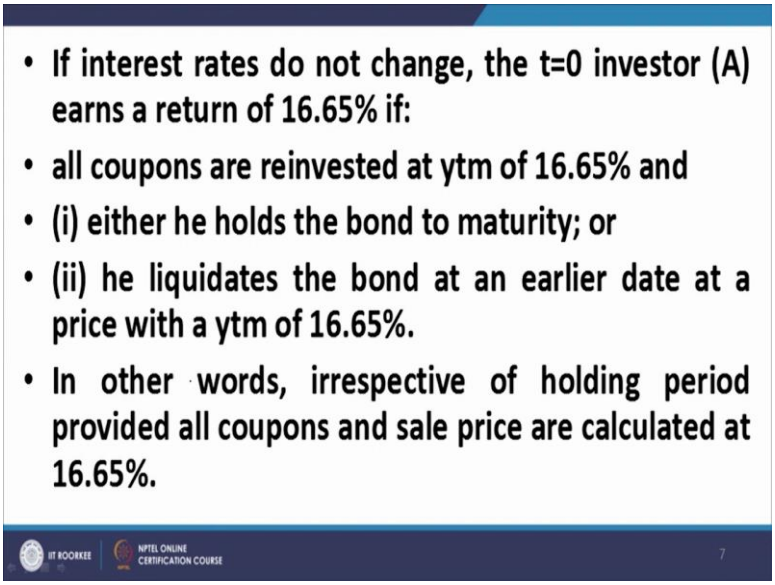

If interest do not change, if interest rates do not change the t equal to 0 investor that is A earns a return of 16.65 percent if all the coupons are reinvested at the YTM of 16.65 percent and either A holds the bond to maturity or he liquidates the bond at an earlier date whichever earlier date it may be, but at a price with which is computed with a YTM of 16.65 percent.

Let me repeat, if interest rates do not change, then t equal to 0 investor A will earn a return of 16.65 percent per annum. If all the coupons are reinvested at the YTM of 16.65 percent, and the price at which A liquidates the bond, if A does not hold it to maturity of course, if you hold it to maturity, it is a different letter, he gets the redemption value, but if he does not hold the bond to maturity liquidates the investment earlier than maturity, then the price at which A liquidates the bond turns out to be 16.65 percent at the rate of 16.65 percent.

If both these conditions are satisfied reinvestment at 16.65 percent, sale in the market liquidation in the market, at YTM of 16.65 percent, if both these conditions are satisfied, then A will get a return of 16.65 percent irrespective of his holding period. In other words, irrespective of the holding period provided all coupons and the sale price are calculated at 16.65 percent A gets a return also of 16.65 percent.

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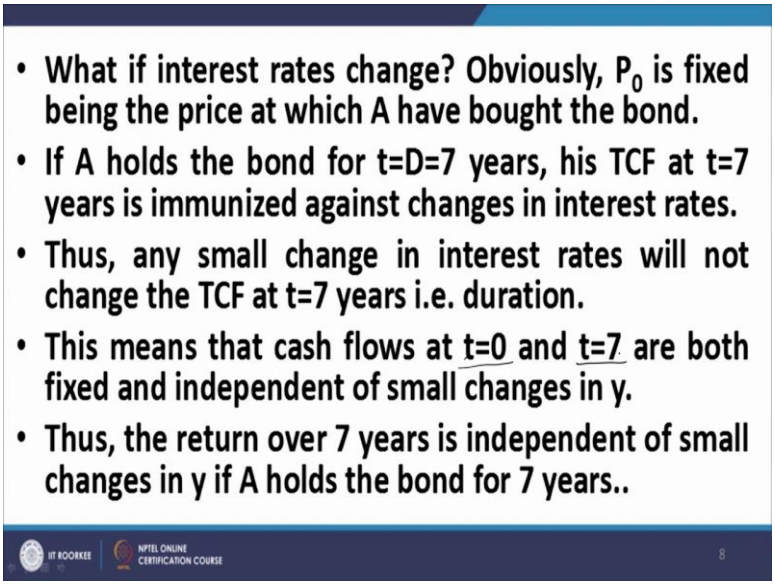

What if interest rates change? What happens if interest rates change? Obviously, the price at which the investment has been made by A that is P0 that is the price at t equal to 0 is fixed being the price at which A has bought the bond which is 726.66 if A holds a bond for t equal to D equal to 7 years, which is the duration of this bond is total cash flows at t equal to 7 years is immunized against changes in interest rates.

Thus, any small change in interest rates will not change the total cash flows at t equal to 7 years that is the duration of the bond. So, if you hold the bond up to its duration, then you will get immunized against any changes in the total cash flow due to changes in interest rates. This means that the cash flows at t equal to 0 and t equals 7 are both fixed t equal to 0 cash flow is obviously fixed because that is the investment that is made for buying the bond that is a known cost that is in a sense that is a cost which has already been incurred.

So, there is no randomness attached to it. And then what we find is that if we hold up onto t equal to 7 years, then their total cash flows become independent of changes in interest rates, that is to some extent. Therefore, the total cash flows at t equal to 0 and t equal to 7 years both become independent of interest rates. And that means what that means that the return over the 7 year period will also become independent of changes in y changes in interest rate if A holds a bond for 7 years.

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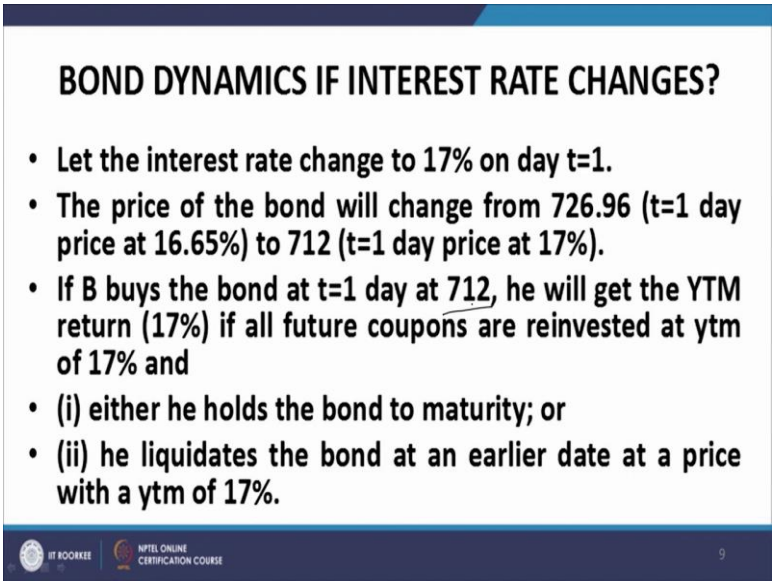

Bond dynamics if interest rate changes, let us assume let us know dwelt further into the bond dynamics, let the interest rate change to 17 percent on day t equal to 1 the investment is made equal t to 0 by a at 16.65 percent at t equal to 1 that is one day after making the investment the interest rates change from 16.65 percent to 17 percent. The price of the bond will change from 726.96. What is this figure?

This figure is the t equal to 1-day price at 16.65 percent to 712, which is the t equal to one day price at 17 percent. I repeat this is very important. Due to the change in interest rates immediately following the change in interest rates, what will happen is that the bond price because there is an increase in interest rates the bond price will fall, it will fall from the price corresponding to 16.65 percent with a maturity of 24 years and 364 days that is at t equal to 1 day and that turns out to be 726.96 from this figure it will drop down to 712 which is the T equal to 1 day price that is up for a bond 12 percent bond with the maturity of 24 years and 364 days at a YTM of 17 percent and that turns out to be 712.

So, those are fall in price, corresponding to an increase in interest rate from 16.65 percent to 17 percent. The price has changed from 726.96 to 712. If B buys the bond at t equal to 1 day at 712 that is after the interest rate change, A will get the YTM of 17 percent. If all future coupons are reinvested at the rate of 17 percent and either he holds a bond to maturity or if we liquidate the bond, he liquidates the bond at a price which is calculated at a YTM of 17 percent. As was the case in is holding.

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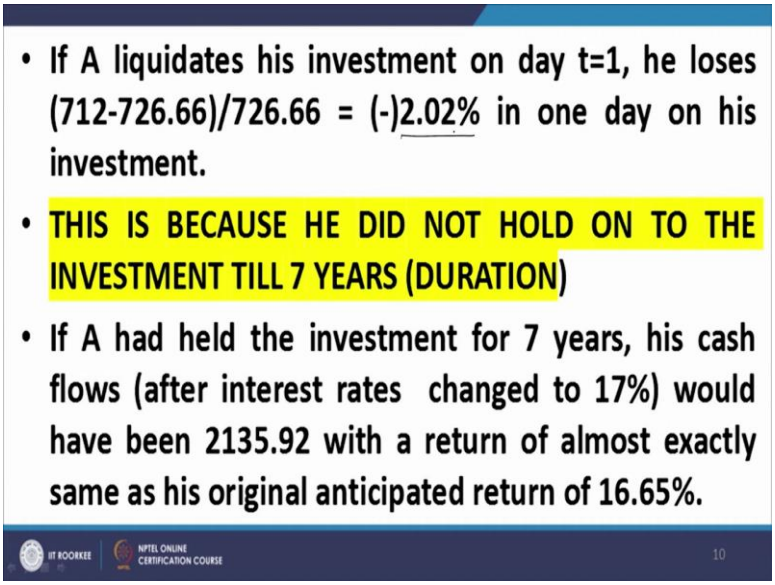

However, if a liquidates his investment and date t equal to 1 what happens to his situation, he loses out significantly the investment that he had made it to 726.66 is now being liquidated at 712, because we will pay only 712 for buying the bond at YTM which is equal to the current market interest rate of 17 percent.

So, A loses out or the loss of A of that occurs because of his one day holding and because of the change in interest rate from 16.65 percent to 17 percent is 2.02 percent. So, in one day he has lost 2.02 percent of the value of his investment due to this change in interest rate, but, important thing is because, why has he lost this much of money, he has lost this much of money, because he has not heard the bond up to 7 years he has not held the bond up to its duration.

Had he held the bond upwards duration, notwithstanding the price change the interest rate change from 16.65 percent to 7.17 percent He would still have made a return or have earned a return of 16.65 percent if he had held the bond for 7 years that is the importance of duration. Let me repeat the sentence again. The important thing is why are they lost this amount of money 2.02 percent because he has held the bond only for one day had he held this bond for 7 years, then notwithstanding the fact that the interest rates have increased from 16.65 percent to 17 percent, he would still have earned a return of 16.65 percent.

Why is that, is even when he liquidates the investment at t equal to 1 day and the interest rates have raisin. The price at which is it is the new price of the price at which the bond is sold is calculated on the basis of discounting all future coupon payments and the terminal

redemption value at the higher rate that is 17 percent. So, this 17 percent operates for a sustained period of time over a 24 coupons and the final price or the final redemption value.

Had he held the bond for 7 years what would have happened? He would have reinvested at least 6 of the coupon at t equal to 1 coupon equal t equal to 2 t equal to 3 up to t equal to 6 at the higher rate of 17 percent, which would have compensated him for some of the laws that were that he would have incurred due to the fall in the market price of the bond corresponding to the change in interest rates.

And secondly, when we work at the work out, the market price also at t equal to 7 years, the fall will not be so gigantic, why? Because the number of coupons and the terminal value that needs to be discounted to work out the value at t equal to 7 years is less compared to what we have at t equal to 1 day. So, if A had held the investment for 7 years, his cash flows after interest rate change to 7 percent would have been 2135.92 that is a return of almost exactly same as originally anticipated return of 16.65 percent as I explained.

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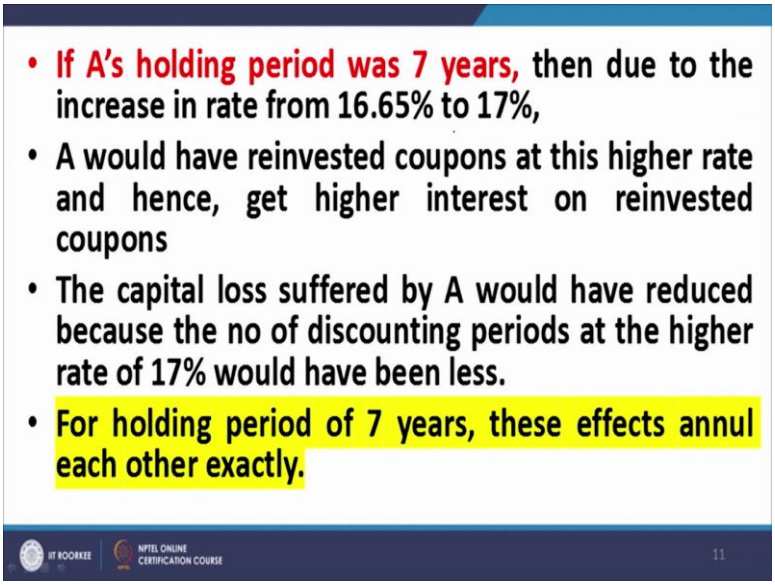

If A is holding period was 7 years then due to the increase in rate from 16.65 percent to 17 percent. A would have reinvested coupons at this higher rate for a longer period t equal to 1, 2, 3, 4, 5, 6, 7. If A has liquidated the investment at t equal to one day, he gets no opportunity to reinvest his coupons and take advantage of the increase in interest rates.

However, the increase in interest rates manufacturers manifests itself by discounting all future cash flows and there are 24 of them, 25 of them rather which need to be discounted or t equal to 1 to t equal to 25 which need to be discounted, had he held the bond for 7 years, then

out of this 25 cash flows that need to be discounted as 7 of them would not be discounted and only 18 of them would be discounted at a higher rate. And at the same time, the coupon payments at 1 to 6 would be reinvested at the rate of 17 percent the higher rate.

So, there would be a significant annulment of the lost here capital loss due to the interest rate increased by the reinvestment income, that is the important thing. A would have reinvested coupons at this higher rate and hence got higher interest on reinvested coupons. The capital loss suffered by year would have been reduced because the number of discounting periods at the higher rate of 17 percent would have been less, the holding period of 7 years. These effects cancel each other these effects annul each other exactly.

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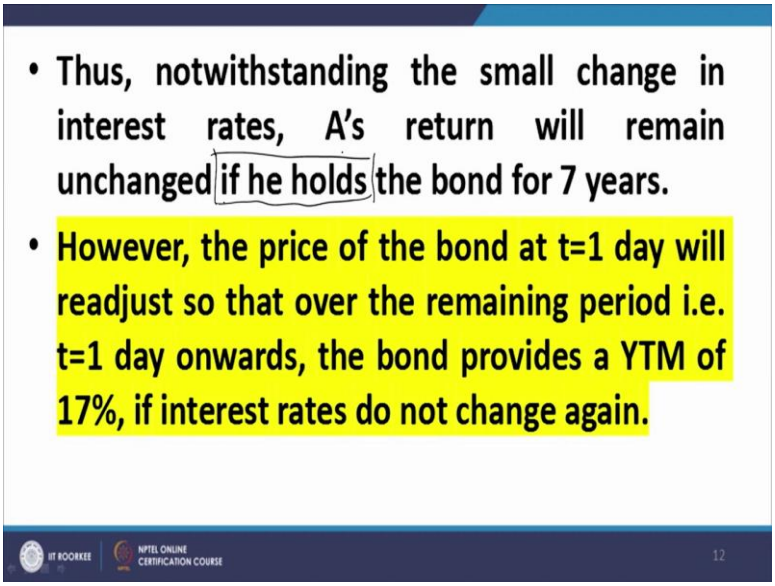

Thus, notwithstanding the small change in interest rates A's return would remain unchanged. If he hold this is the catch, this is the fundamental thing. If he holds a bond for 7 years, but he did not hold the bond for 7 years, he held the bonds for only one day and that is the reason that he lost an amount as significant as to consider 2 percent of his investment value.

However, the price of the bond at t equal to one day will readjust so that remain that over the remaining period that is t equal to one day onwards, the bond provides a YTM of 17 percent because the bond has to give you a return which is commensurate with the market rate of interest, you cannot have a situation where the bond gives you a different rate of interest and the market wants a different rate of interest that will not, that will not be a stable equilibrium you see.

At equilibrium it is necessary that the bond gives you the same rate of return the same YTM or the same yield as the market rate of return corresponding of course, to the same risk class, it is important that the two rates that two dates in order that they be comparable be defined with respect to the same risk class as upon the market risk must be compatible with the risk class of the bond that we are talking about.

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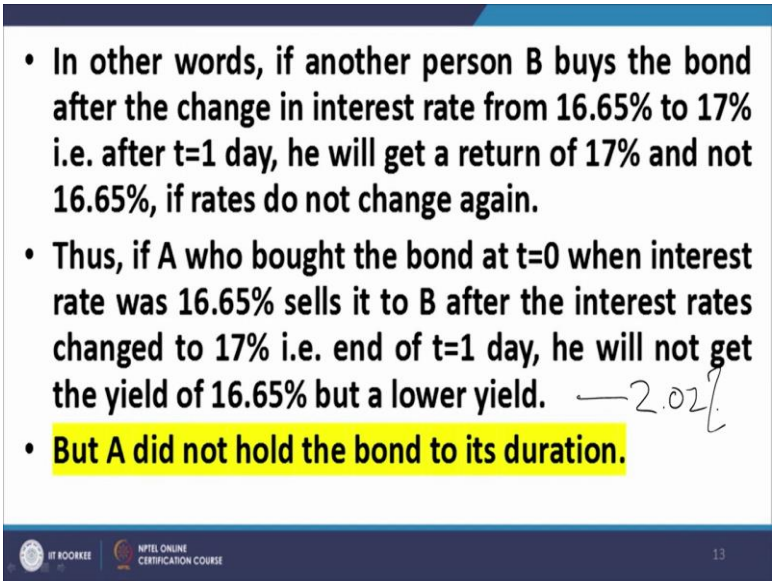

In other words, if another person B buys the bond, after the change in interest rates from 16.65 percent to 17 percent, that is after t equal to one day, you will get a return of 17 percent and not 16.65 percent that is why the price has gone down you see, after the change in the interest rates, whosoever has entered the investment, whosoever has made the investment at 17 percent market rates has to get a YTM of 17 percent.

And because the maturity value is fixed, the only thing that can change is the price of the bond, the price of the bond readjust itself such that if you buy the bond at t equal to one day after the change in interest rate to 17 percent, you get a YTM of 17 percent over the life of the over the remaining life of the bond that is 24 years and 364 days, if the interest rate should not change again of course.

So, let me read it out once again. In other words, if another person B buys the bond after the change in interest rate from 16.65 percent to 17 percent, that is after t equal to one day, he will get a return of 17 percent and not 16.65 percent and you can only get that by the falling of the reduction in the price of the bond at t equal to one day after the change in interest rate, everybody will demand 17 percent return the demand for the bond will decline and therefore, the price will decline.

Thus, if A who bought the bond a t equal to 0 when the interest rate was 16.65 percent sells it to B, after the interest rate changed to 17 percent that is at the end of t equal to one day he will not get the yield of 16.65 percent but a lower yield. But it so happens that the yield turns out to be minus 2.02 percent, the return turns out to be minus 2.02 percent but A did not hold the bond of (())(23:24) duration.

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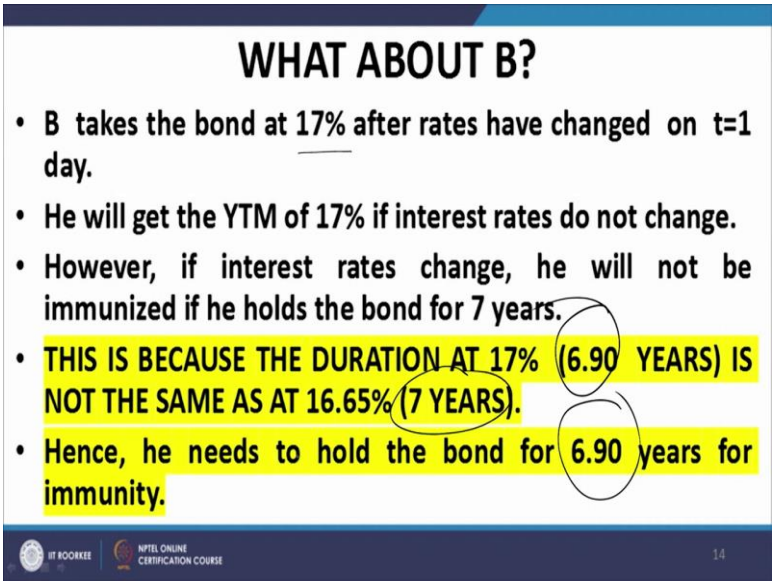

What about B? B takes the bond at 17 percent after the interest rates change and t equal to one day, you will get a YTM of 17 percent if the user conditions are satisfied, what are those conditions? The conditions are that all the coupon payments are invested by B at the rate of 17 percent. And either B hols the bond up to its maturity or if the bond is liquidated earlier, the price at which it is liquidated the price at which it is sold in the market is at the rate of 70 percent YTM.

So, if both these conditions are satisfied, Y will get a return of 17 percent, there is another interesting feature about Y now as far as Y is concerned, if Y hold the bond for 7 years, does he get immunized if he does not get immunized? Why? Because Y has bought the investment Y has taken up the investment at a YTM of 17 percent and if you work out the duration on at the rate of 17 percent it is not 7 years it is not 7 years, it is how much it is about 6.90 years.

So, in order that Y immunize themselves against further changes in interest rate, he should not hold the bond for 7 years. He should hold the bond for 6.90 years. Let me repeat because you see the duration is Y dependent the duration depends on the YTM. Now, the point at which B entered the investment is at a rate of 17 percent. The figure of 7 years that we are calculated for the duration was at 16.65 percent.

So, that is 7-year figure will not hold for B for B, you have to work out the duration again at 17 percent and you find that it is 6.90 years. Therefore, if B is to immunize himself under this philosophy, he has to hold the bond not for 7 years, but for 6.90 years. However, if interest rates change, you will not be immunized if you holds the bond for 7 years, this is because the duration at 17 percent that is 6.90 years is not the same as that at 16.65 percent which is 7 years. Hence, he needs to hold the bond for 6490 years for immunity.

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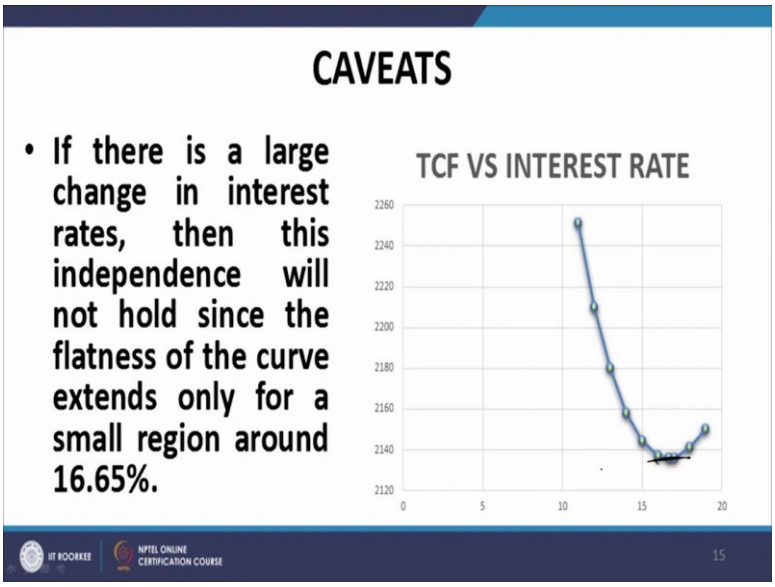

Now, there are certain caveats, which I need to refer to quickly, you see the referred to earlier also in in the last lecture, but the basic thing is that the interest rates, so the changes in interest rates or with immunity operates have to be small in magnitude that is simple, because how did we arrive at this immunization by holding to duration, we worked it out on the basis of taking the first derivative of total cash flows and equating to 0 that is in a sense we worked out the local minimum you may say.

And therefore, this minima will hold only for minor changes for infinitesimal changes in interest rates to the left or to the right the Newtonian differentiation, you have to keep in mind in Newtonian differentiation is the first order correction. As far as the Taylor series are concerned first order derivative we consider we do not consider higher order terms.

And in that sense, we assume that around this point at which we are working out the derivative, the curve is approximated very well approximated adequately well by a straight line as you can see here in this portion. However, if the interest rate changes much more the curvature comes into effect and the curvature introduces certain deviations or distortions

rather and as a result of the immunization will not operate, when we talk about large changes in interest rates.

So, if there is a large change in interest rates, then this independence will not hold, since the flatness of the curve extends only for small regions around 16.65 percent as you can see in the right-hand panel figure as well.

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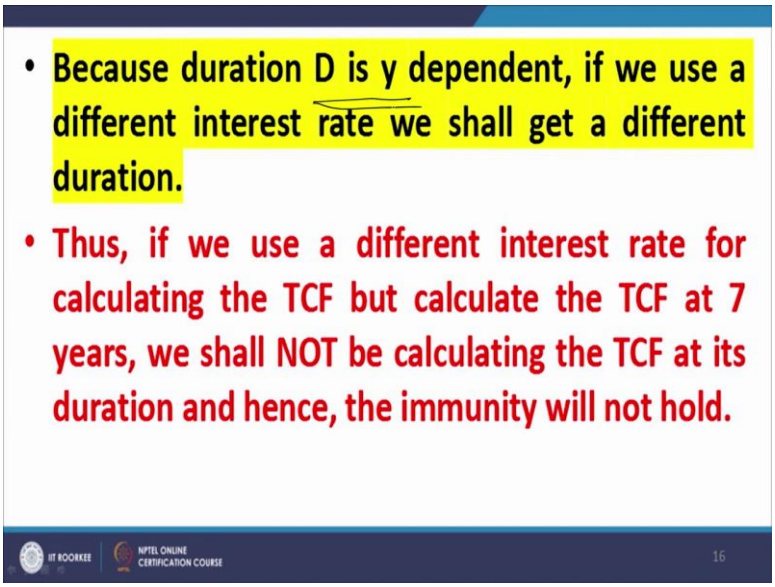

Because generation D is y dependent, if we use a different interest rates, we shall get a different duration, I repeat and duration is y dependent this is very important. Another important issue duration is y dependent, so, if you have a different YTM you naturally get a different duration. And thus, if we use a different interest rate for calculating the total cash flow, but calculate the total cash flow at 7 years, we are not calculating the TCF at its duration and hence, the immunity will not hold.

For example, you will see if you are calculating the total cash flow at a rate of 11 percent or 12 percent or 15 percent, then that duration corresponding to this 11 percent, 12 percent or 15 percent will not be 7 years. So, if you are calculating cash flow for 7 years, using interest rates of 11 percent, 12 percent or 15 percent, you are not calculating those total cash flows at the duration corresponding to the interest rates that you are using for calculating the total cash flows.

You have to try to understand that this has to be compatible, the rate that which you are calculating the total cash flows must be the same as the rate at which the duration is calculated like the example that I gave you that if the rate is 16.65 percent and then the

duration is 7 years. So, if you calculate the total cash flows at 16.65 percent for 7 years, you will get immunized if you calculate the total cash flows at 12 percent. And you calculate for 7 years, there is a incompatibility between the duration the duration at 12 percent is not 7 years.

Therefore, the immunization does not hold, as you saw in the example that the figures in the table that I showed a few minutes earlier. Thus, if we use a different interest rates for calculating the total cash flow, but calculate the total cash flow is t equal to 7 years, we shall not be calculating the total cash flows at the duration and hence the immunity will not hold.

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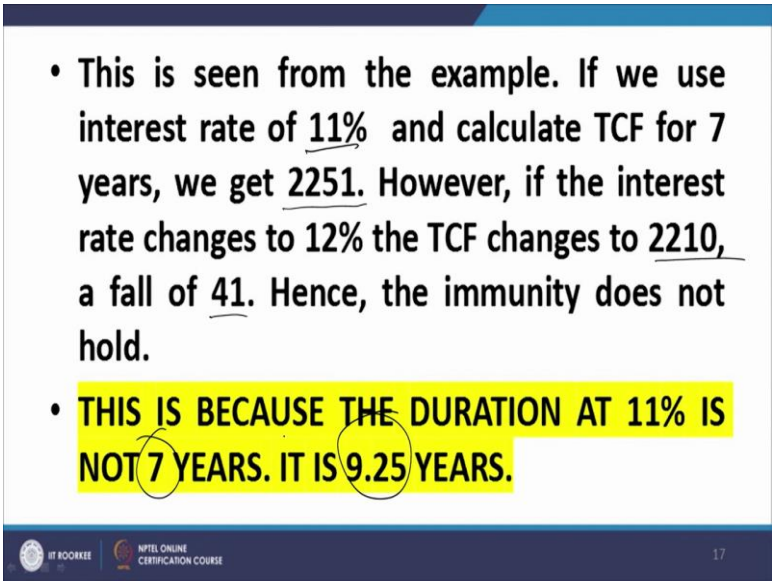

This is seen from the example, if we use the interest rate of 11 percent and we calculate total cash flows for 7 years, we get 2251. However, if the interest rates changes to 12 percent that TCF changes to 2210 that is a change of 41 money units that is quite significant. As I mentioned if there is a change from 16 percent to 17 percent, the total cash flows change by as small as one money unit. This is because the duration at 11 percent is not 7 years, it is 9.25 years.

So, if the interest rates are 11 percent, you will need to work out the duration of at 11 percent which is 9.25 years and thereafter, you need to calculate the total cash flows over 9.25 years and then those cash flows will be immunized against small changes in interest rates around that 11 percent mark.

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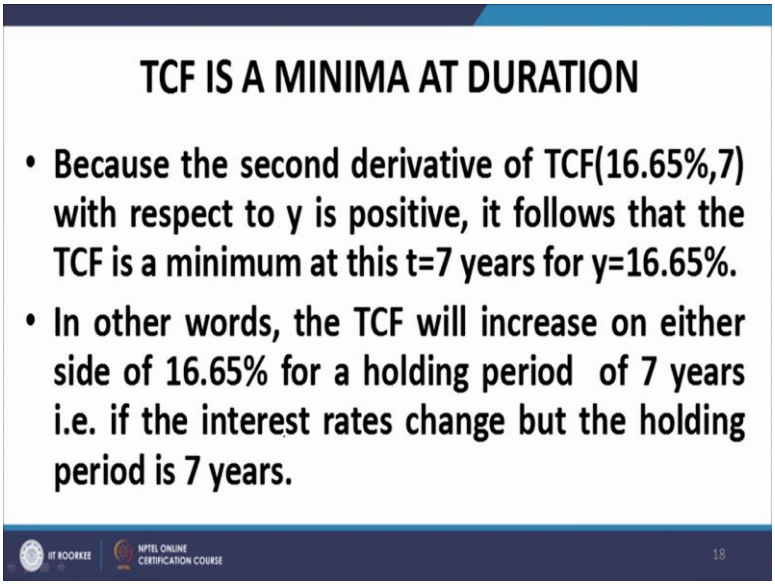

TCF is a minimum duration I have already touched upon this point, let me read it out quickly. Because the second derivative of TCF at the rate of 16.65 percent for 7 years with respect to y is positive it follows that the TCF is a minimum at this t equal to 7 years for y equal to 16.65 percent this also was vindicated by the figures that I showed you in the table. In other words, the TCF will increase on either side of the 16.65 percent mark for a holding period of 7 years. That is if the interest rate change, but the holding remain at 7 years.

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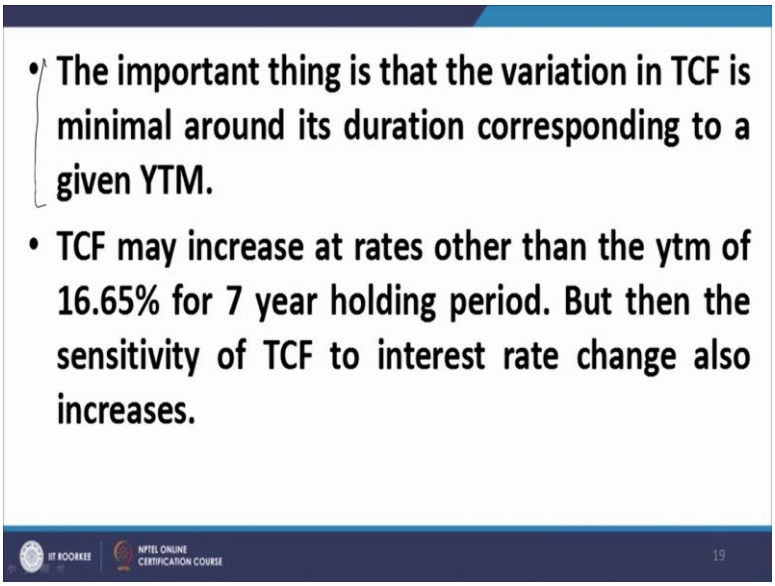

But the important thing is that the variation in TCF is minimal around the duration corresponding to a given YTM. This is important when we are talking about duration,

duration is not a concept which is basically for optimizing return it is a concept for minimization of risk.

And when you are talking of minimization of risk, you are talking about minimization of variability and variability is minimal when you hold the bond around its duration notwithstanding the fact that if you move to either side of it left or right of duration, that total cash flows will increase. So, TCF may increase at rates other than the YTM of 16.65 percent for 7 year holding period, but then the sensitivity of TCF to interest rate changes also increases.

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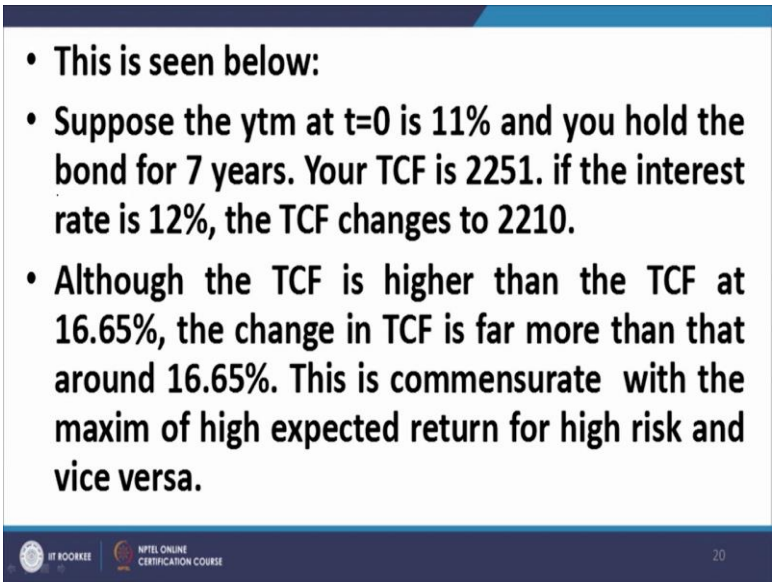

This is seen below, suppose the YTM at equal to 0 is 11 percent and you hold the bond for 7 years. Your TCF turns out to be 2251, if the interest rate changes to 12 percent. The TCF changes to 2210, although the TCF is higher than the TCF at 16.65 percent. The change in TCF of the variability of TCF corresponding to a 1 percent change in interest rate that I mentioned just now is 41 compared to only one for a change from 16 to 17 percent; the change in TCF is far more than around 16.65 percent. This is commensurate with the principle that higher expected return for higher risk, lower expected return for lower risk.

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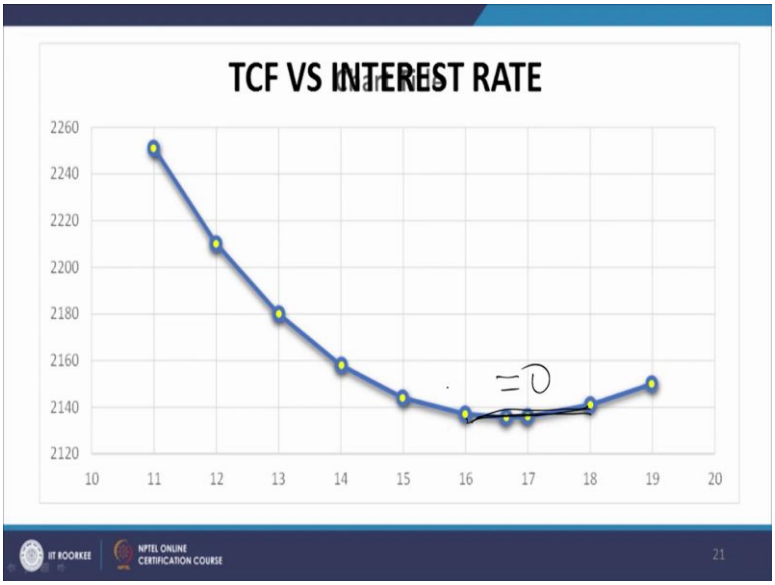

This is the plot of TCF versus interest rate. As you can see, we have a flat region around this point of 16.65 percent. And as you move to the left or to the right, the cash flows tend to increase but the variable… the slope also increases here the slope is approximately 0. However, if you move to the left or to the right, the magnitude of the slope increases. That means what? That means the variability in the cash flows corresponding to a given change in interest rate also increases.

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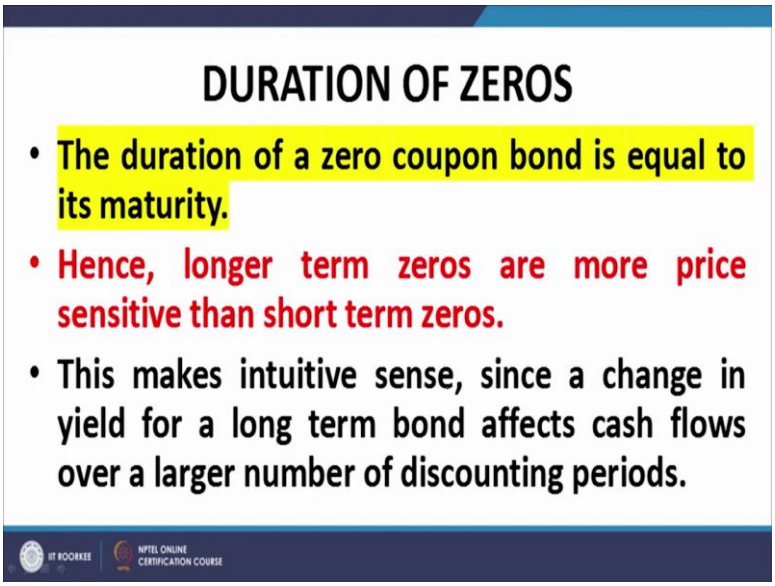

Duration of zeros which will take up in the next lecturer. Thank you.