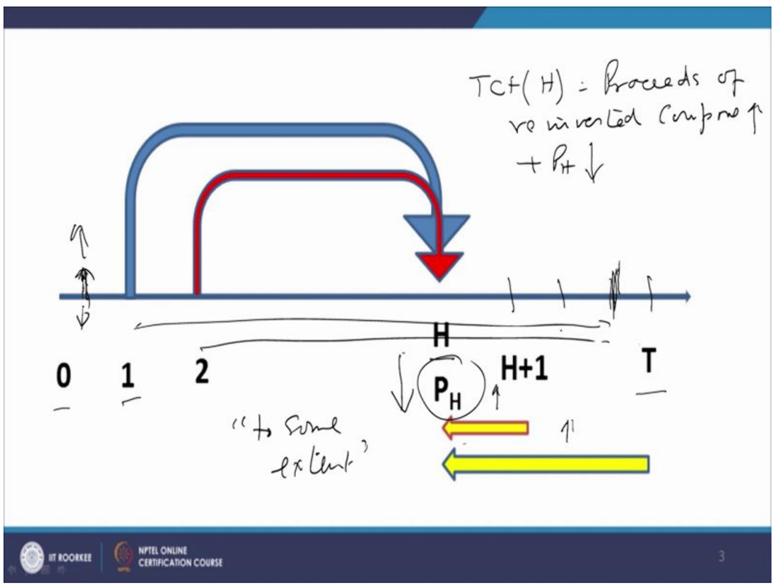
Quantitative Investment Management
Professor. J P Singh
Department of Management Studies
Indian Institute of Technology, Roorkee
Lecture: 17
Interest Rate Risk

Welcome back. So, let us continue from where we left off. In the last lecture, I was explaining the concept of interest rate risk. So, let us start from there.

(Refer Slide Time: 0:37)



Now as I mentioned then, that let us assume that you are investing in a bond at T equal to 0 that is at this point. And then the life of the bond is capital T years this is the life of the bond. So, the tenure of the bond is from 0 to capital T years. But your investment horizon does not coincide with the maturity of the bond. We assume that your investment horizon is up to H. That means you are going to hold the bond for a period, which is less than the tenure of the bond less than the term to maturity of the bond.

Then how would you realize this investment? You would realize this investment by selling it in the market. When you arrive at the end of your investment horizon you go, you will sell the bond in the market and recover the proceeds from the bond. You will not wait until the maturity of the bond because you need the money at an earlier point of time that is at T equal to H. Now let us assume that capital PH is the price at which you sell the bond in the market.

Now two things will happen. Let us assume also that the interest rate changes at some point after you have made the investment in the bond. Let us say at this particular point the interest rate changes from your initial rate to a certain rate, let us assume that the interest rate increases, then what will happen is, that the coupon that you receive on the bond at T equal to 1 at this point, what will you do? You will reinvest the coupon until the end of your investment horizon that is up to the point H.

Similarly, the coupon that you will receive at T equal to 2 you will invest up to the point H, that is for H minus 2 years. The coupon at T equal to 1 you will invest for H minus 1 years. The issue is what is the rate that which you will invest? The rate that you will get is the market rate is the new market rate is the changed rate after the rate has made an upward jump at this point. Because the coupon is being received after the change in the rate takes effect.

Now similarly, so that is one part of the story, that you will get, obviously, because the rate rates have increased. You will get a higher proceeds higher terminal value of the reinvested coupons, when you liquidate the entire investment at T equal to H. Please note this particular point that when you liquidate the entire investment at T equal to capital H not only will you get the proceeds from the selling of the bond that is at the price PH. But you will also get the proceeds of the reinvested coupon that you have invested at T equal to 1, T equal to 2, T equal to 3 and so on.

So, the total proceeds that you are going to get at T equal to capital H will comprise of not only of the market price of the bond, but will also comprise of the proceeds of the reinvested coupons at T equal to 1, 2, 3 upto H minus 1. And also the coupon at T equal to H which will obviously not be reinvested but that nevertheless it constitutes a cash flow and will contribute to the yield of the instrument. So, keeping that in mind the total cash flows that you are going to get Tcf, let me call it Tcf at the point H will be equal to re proceeds of reinvested coupons plus PH.

Obviously higher the reinvestment rate, higher would be the proceeds of the reinvented coupon. And similarly, suppose this rate takes a down swing, at this particular point the reinvestment procedures will also decrease. Because we assume that the reinvestment proceeds are made at the market rates. Now the second thing is, what about PH? As far as PH is concerned, how do we compute the market price of bond?

We compute the market price of a bond by discounting all future cash flows from the bond. In this case what will happen? What are the future cash flows? The cash flow at H plus 1, the cash flow at H plus 2, H plus 3 and so on upto capital T. So, this will be the cash flows which will be discounted not compounded, discounted back to T equal to H as shown by the direction of this arrow.

The cash flow at T equal to H plus 1 will be discounted at T equal to H. And the cash flow at T equal to H plus 2 will be discounted for 2 periods upto T equal to H. And similarly, the cash flow as T equal to capital T will be discounted for T minus H periods to bring it back to T equal to H. And then, we will add all those cash flows that will give us a measure of PH.

Now obviously, now is the important point, now is the catch. If the interest rates share have increased, then the discount rate that we are going to use here, because the discounting here is also going to be at the market rate. So, the discount rate that we are going to use here is also going to increase.

If the interest rate here has increased, let me reiterate, if the interest rate here has increased, then the discount rate that we are going to use for discounting this cash flows will also increase. And if the discount rate increases, what happens to the price? The price declines. So, on the one hand the proceeds from the reinvested coupons will increase, if the interest rate increases the value of PH will decrease, if the interest rate increase.

So, there are two opposing effects. Please note there are two opposing effects. One effect that operates on the reinvested coupons. And the other effect that operates in determining the market price at which the bond will be liquidated. And corresponding to any change in interest rates these effects will operate in opposite directions.

If there is an increase in interest rate, the reinvested coupons will increase the price at which the boundary will be liquidated will decrease. And conversely, if there is a decline in interest rates, the price will increase but the reinvested coupons will give you less income. But there is another point here, the opposing effects that is that of the reinvested interest and that of the capital gains or the price because the ultimately it is the price that determines the capital gains so you can as well use capital gains without any loss of generality.

Now because the these two effects that is the reinvested coupon effect or the reinvestment effect and the capital gains effect are operating in opposite directions they will enter each other to some extent. To some extent I emphasize to some extent is very important to some extent. Why do I use this word to some extent? Because this annulment or this neutralization or this equalization will depend on the period of holding, that is quite obvious in fact.

Suppose you are holding the bond only for 1 year, then obviously you are not reinvesting any coupon. So, if the interest rates increase here that is not going to benefit you in any way the reinvestment income is not going to change because you have not reinvested any income. On the other hand the discounting will be for a number of periods upto T. So, the because the discounting is there for a number of periods and increase an increase in interest rates at this point will affect the market price of the bond very significantly.

So, the capital gains will decline significantly but the reinvestment income will not increase. Because you have not had an opportunity to reinvest at the increased rate of interest. That is what I say, it depends on the holding period. Similarly, if your holding period is very close to maturity, then what happens?

If your holding period is say here, then the reinvested income is going to in corresponding to an increase in interest rates. The reinvest in, the reinvested income is going to increase a lot a significant because you are going to reinvest a significant amount of money at a significant amount for a significant period as well. And T equal to 1 will be invested for this whole period and T equal to 2 will be invested for this period and so on.

So, the point is the basic point is, it depends on your investment horizon to what effect the reinvestment effect and the price effect or the capital gains effect neutralize each other. The closer you are, the closer is your holding period to the point of investment the shorter is your holding period, the shorter would be the reinvestment effect. The smaller would be the reinvestment effect, the larger would be the price effect.

Conversely, if the holding period is very close to the maturity of the bond the reinvestment effect will predominate over the price effect. Because you will have very few coupons to discount. So, if there is an increase in interest rate that will not affect the price very much. Conversely, if there is a decrease in interest that will also not affect the price very much but your reinvestment income will decrease significantly.

So, that is, these are two points, two fundamental points. Number one; reinvestment effect and price effect operate in opposite directions that is number 1. And number two; the amount of the extent of annulment of the reinvestment effect and the price effect depends on your holding period.

(Refer Slide Time: 10:53)

So, now given in this introduction let me read out what exactly we mean by interest rate risk. So, these effects obviously, the if interest rate is a random variable and in the interest rate cannot be predicted, then we are susceptible to this risk, to this change in return that manifests itself when a change in interest rate takes place.

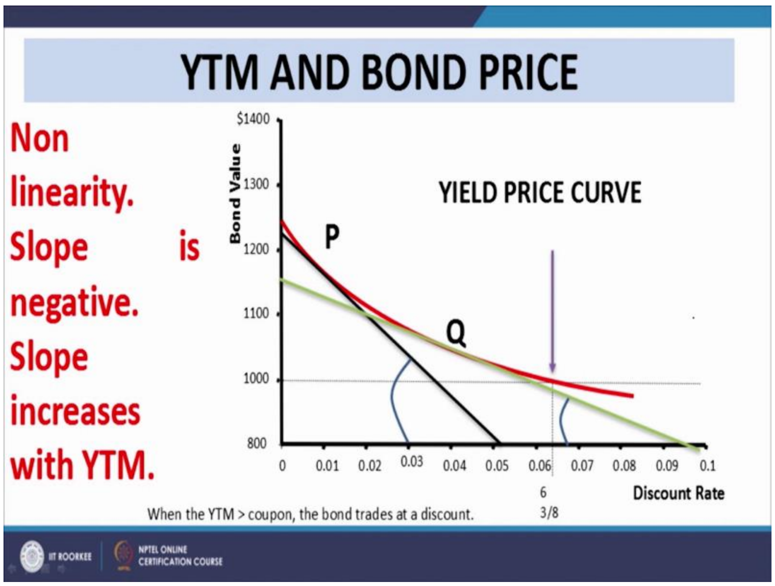
So, since the price of a bond fluctuates with the market interest rates the risk that an investor faces, when investing in a bond portfolio is that number one. The price of the bond at the end of the holding period that is PH will decline, if the market interest rates rise unanticipatedly and this risk is referred to as the price risk.

So, if the market rates rise, obviously the price is going to fall. Then and this rate risk is called price risk. Please note this word unanticipatedly. Why do we use this word unanticipatedly? We use this word unanticipatedly because if the change is anticipated then it is already built into the price and if that change then materializes it will not influence the price.

So, that is the important thing that changes that are unanticipated affect the price or the reinvestment income as I mentioned just now. If a change is anticipated it is expected then obviously it forms a part of the price of the security. The income from reinvestment of coupons may not need the desired return if interest rates fall.

Reinvestment income is moves in tandem with the direction of movement of the interest rates as is quite elementary and the price moves in the inverse direction to the direction of movement of the interest rates. And this reinvestment change or the fall and reinvestment income of the change in reinvestment income is called reinvestment rate risk.

(Refer Slide Time: 12:58)



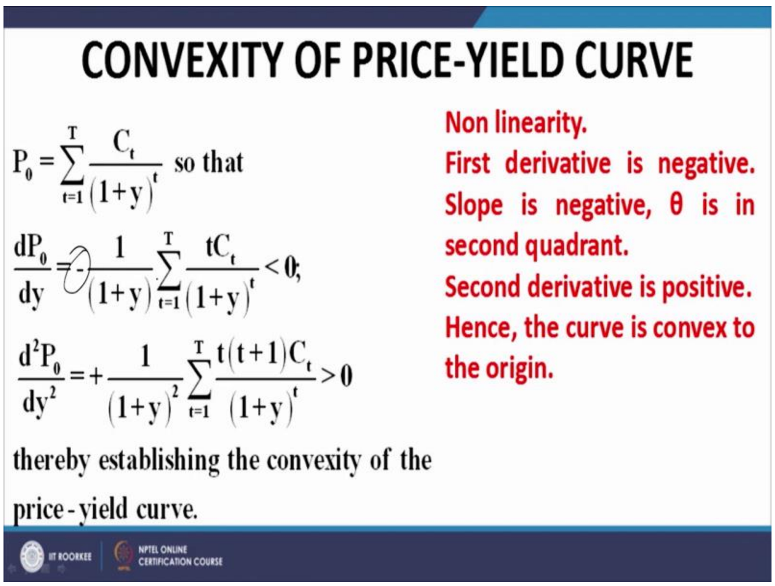
Now we talk about some more technicalities. You please look at this figure. If you look at this figure, what is this figure? This figure is the plot of the bonds price against the YTM of the bond the YTM is along the x axis and the price of the bond is along the y axis. And the red curve is the curve that represents the plot of a given bond along the given bond against the YTM of the bond.

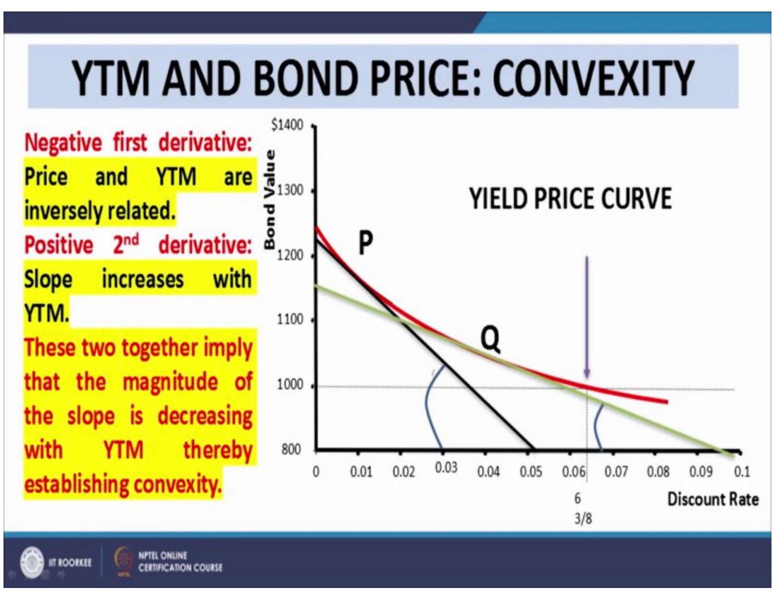
What the most important part that I want to emphasize here is, two things. 1: the plot is downward sloping, 2: the plot is not a straight line it is not a straight line. In fact as you can see here, it is convex towards the origin, it has a certain amount of convexity in it. Certain amount of curvature and the direction of that curvature is that it is convex towards the origin and therefore it has a positive convexity.

So, this is what I have highlighted on the left hand panel non-linearity. The slope is negative, it is downward sloping towards the right. And the slope increases with YTM. Please note the slope is negative. So, as you can see here as YTM increases the slope approaches 0 and therefore from a negative value to 0 the slope is increasing.

Although, the absolute value of the slope is decreasing. I repeat the slope is increasing but the absolute value of the slope is decreasing. Why is that? Because the slope is negative.

(Refer Slide Time: 14:32)



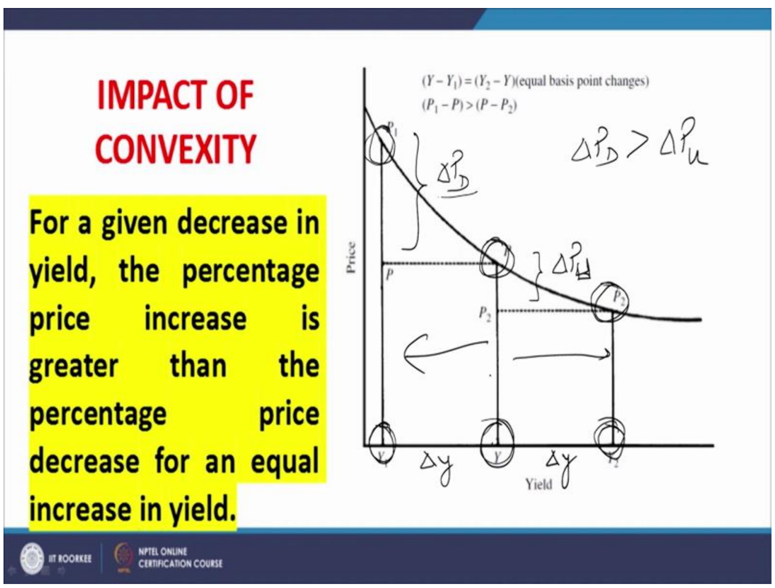


So, you can see here this slide analytically shows you both the points that I have mentioned here, in the earlier version, earlier slide that firstly that the curve is downward sloping, the slope is negative. You can see here that the first derivative of price with respect to the YTM is less than 0, you can see here this negative sign. And the rest of the expression is clearly positive. So, the entire expression will invariably return a negative sign, the slope will be negative.

And if you work out the second derivative of price with respect to the YTM you find that it is invariably positive. The two things combined together tell us that this loop is downward sloping and the slope has a certain amount of curvature. It is not linear had it been linear the second derivative would have been 0.

So, YTM and bond price convexity negative first derivative that means price and YTM are inversely related the slope is negative the curve is downward sloping. Then positive second derivative, the slope increases with YTM. But being negative, the magnitude of the slope or the absolute value of the slope decreases as the YTM increases. The flatness of the slope in becomes more and more pronounced as we increase the YTM.

(Refer Slide Time: 16:00)



Now what is the impact of convexity that is very interesting. Let us look at this diagram carefully. Let us assume that we are at this point corresponding to which our price is equal to given by this, this particular value. Now let us assume that there is a decline in YTM then we move from this point to this point. And corresponding to this point the price now changes to this. So, clearly the change in price is given by this expression. The expression that let us call it delta P because we have assumed a downward shift. So, let us call it delta PD.

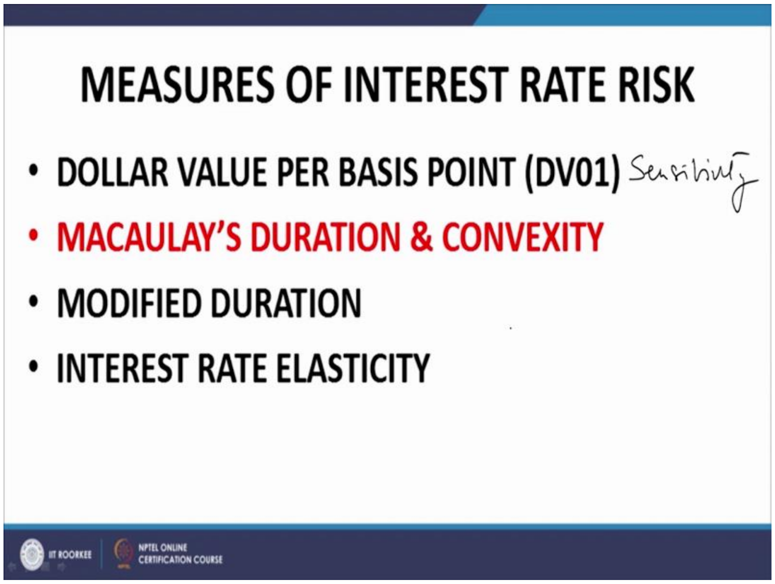
Now we look at the other side there is an increase in YTMs. So, we assume that the YTM shift is in the opposite direction the YTM increases, so the we are at this point and now the YTM has gone up to this point. And corresponding to this increase in YTM the price has gone down obviously they are inversely related, so the price has gone down to this point. And therefore this is the change in price corresponding to an, corresponding to an equivalent increase in YTM.

Let us call this delta PL. So, let me repeat the diagram. What is this diagram shows obviously the plot of the price against YTM is a yield price curve for a given bond. We are we start at this point, we start at this point and we assume that there is a decline in YTM by an amount, let us say delta y. As a result of which we are now at this point. And the price obviously has change has moved from this point to this point.

In other words, there is an increase in price because the YTM has decreased. So, there is an increase in price, let us say we call it delta PD. And now we reverse the direction of movement of the YTM from a decrease delta y from a given YTM, we assume that there is an increase delta y. Same magnitude but there is an increase in this case and as a result of which we now arrive at this point for the YTM and the corresponding price is given by this point.

Let us call it P delta u then because of the convexity of the curve. Because it is not a linear curve, because its convex towards the origin, what happens? We find that delta PD is greater than delta PU. So, what we find is that, the impact of convexity or the outcome of convexity is that for a given decrease in yield, given decrease in yield that is this direction. The percentage price increase is greater than the percentage price decrease for an equal increase in yield that is what the impact of convexity gives us.

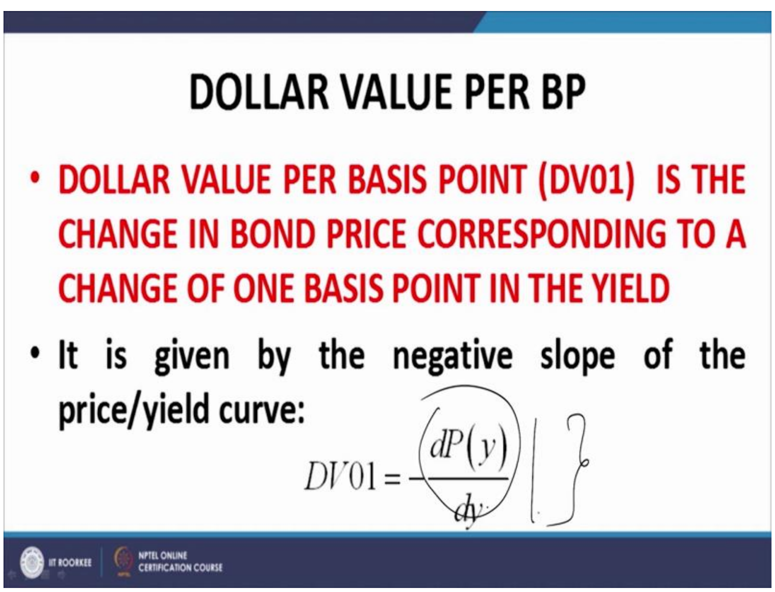
(Refer Slide Time: 19:11)



Now we talk of the measures of interest rate risk we talk about the measures of interest rate risk. There are 4 common measures of interest rate risk. The dollar value per basis point that is also called the sensitivity. Macaulay's duration and convexity this is the most common and the most rich in terms of literature, the most rich measure of interest rate risk.

We have the modified duration which is also called the volatility. And then we have the interest rate elasticity. DV01 is also called sensitivity. And modified duration is also called volatility. We shall come back to these terms later on.

(Refer Slide Time: 20:01)

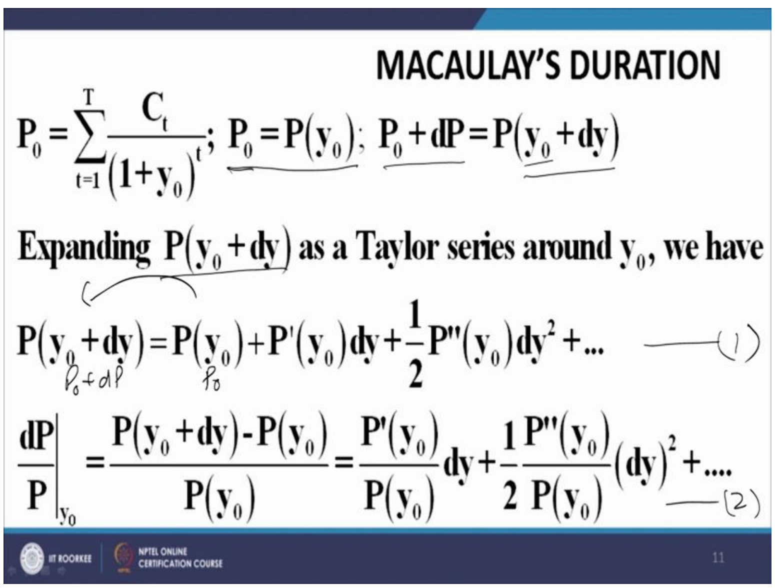


The dollar value per basis point is quite simple. It is given by this expression.It is the change in price the absolute change is price corresponding to one point change in the YTM of the bond. It is given by the slope of the yield price curve at any given point. So, of course it is a, it carries a negative sign.

Why does it carry a negative sign? Because this expression is invariably negative price and YTM move in opposite directions. Therefore, whenever there is an increase in price, the YTM will, I am sorry whenever there is an increase in YTM the price will decrease and vice versa and we it is generally convenient to work with a positive figure.

So, we add a minus sign, so that the DV01 figure that we talk about returns a positive number. So, this is the definition of DV01. Dollar value per basis point DV01 is the change in bond price corresponding to change of one basis point in the yield or one unit in the yield. It is given by the negative slope of the price yield curve.

(Refer Slide Time: 21:11)



Now we talk about the Macaulay's duration. This is the most important part. Let us assume that we are at the YTM as of today, as of now, right now is y0 corresponding to which the current market price is P0 of a given bond. We have a given bond the current interest rates are market interest rates relevant to that bond or y0, the YTM of the bond is y0 and as a result of which the price of the bond is P0. So, P can be written as a function of I0 in the form of this expression.

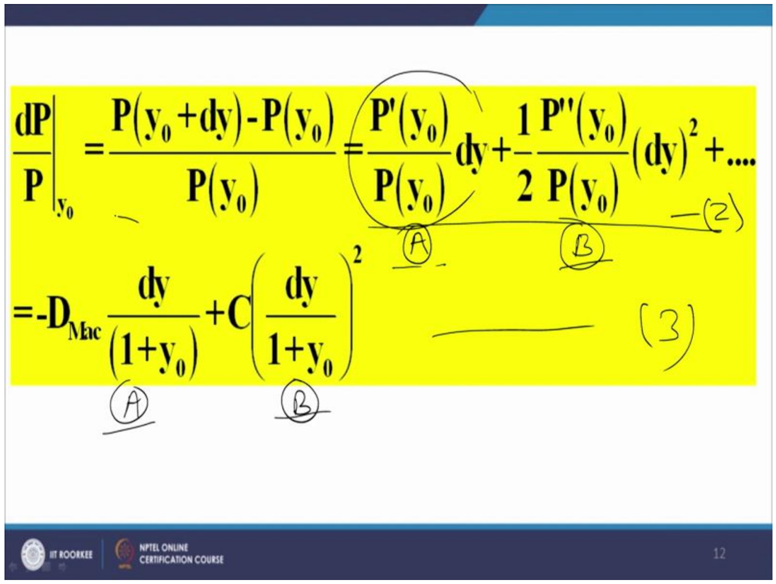
Let us assume that there is a minor shift in the interest rates as a result of which the YTM changes from y0 to y0 plus dy the YTM changes from y0 to y0 plus dy there is a change dy in the market interest rates and the y time of the bond. And corresponding to that the change in price is given by dP, so that the new price turns out to be P0 plus dP. So, P0 is the price at y equal to y0 and P0 plus dP is the price at y0 plus dy.

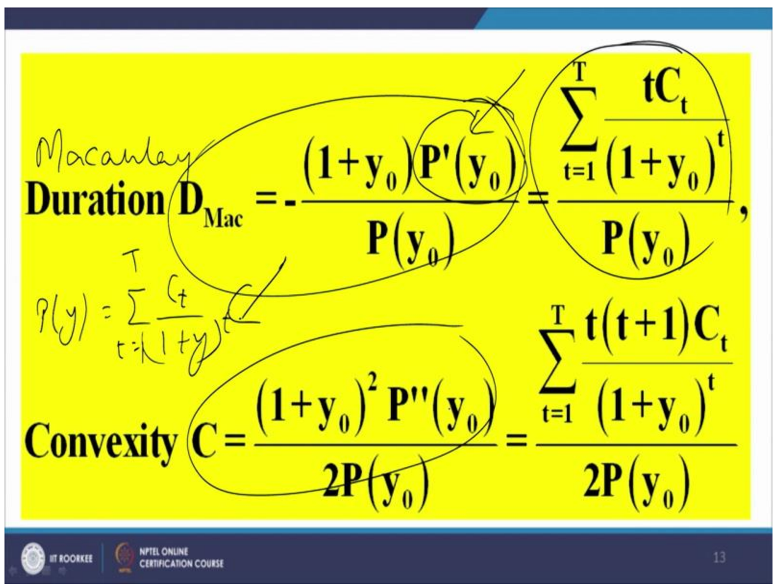
We expand P y0 plus dy we expand P y0 plus dy of P0 plus dP around y0 as a Taylor series. Around y0 we expand P of y0 plus dy, dy is assumed small and we expand it as a Taylor series. The result that up to two terms up to two orders of magnitude and what we have is equation number 1. P of y0 plus dy is equal to P of y0 plus P dash y0 into dy plus 1 by 2 P double dash y0 into dy square plus further derivatives terms of y.

Now if we simplify equation number 1 a little bit and write it in terms of the percentage price that is we divide throughout by P and take this expression P y 0 to the left, to the left what we get is, dP upon P. Please note P of y0 plus dy is nothing but P0 plus dP P0 plus dP and this is P0. So, if you take this to the left and divide throughout by P P0 or what we end up with is, dP upon P at the point y equal to y0 and this is given by equation number 2. I repeat what we have done is, we have taken this expression to the left hand side P of y0 to the left hand side. And then we have divided throughout by P0.

So, using that P of y0 plus dy is equal to P0 plus dP and P of y0 is equal to P0 the P0 and the P0 cancel out. We get dP upon P0 which is the factor that we are dividing throughout. And we get dP upon P that is the percentage change in price due to a change dy in the YTM that is given by the right hand side, which is this expression. Now we make a substitution. What is that substitution?

(Refer Slide Time: 24:36)





We write this expression which I have picked up from the previous slide. This is equation number two of the previous slide. Absolutely as it is, no change. We write these two expression, this is expression number A, this is expression number B in the form of this expression. This is the corresponding expression A and this is the corresponding expression B.

We write them in terms of two quantities Dmac and C, two parameters we have introduced Dmac and C. What is the magnitude? And what is the definition of these two parameters will come back to in a 2 sec, in the next slide. But the important thing is that here what we have simply done is, we have substituted certain terms such that we write equation number 2 in the form of equation number 3.

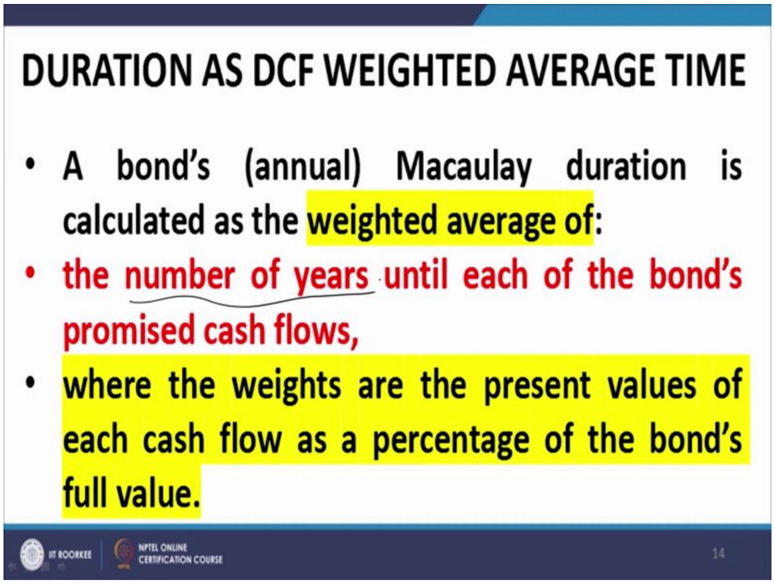
We introduced two terms, two quantities, two parameters Dmac and C. How they are defined? We can obtain by comparing equations 2 and 3. I repeat we have written equation 2 in an equivalent form introducing two parameters Dmac and C and the definitions emerge from comparison of equations 2 and 3 put term by term.

This is this term A goes to this term A, this term B goes to this term B, which enables us on comparison to arrive at definitions of Dmac and C. What are those terms? We get here. So, Dmac if you compare this, if you compare this expression with this expression we get Dmac is equal to this expression.

Similarly, if you compare this expression with this expression, we get this expression for c. And what do we call this Dmac and C? Dmac is called the Macaulay’s duration. Macaulay’s and C is called the convexity of the bond. Dmac is called Macaulay’s duration and C is called the convexity.

And when you substitute the value of P dash of y using P y is equal to summation Ct upon 1 plus y to the power t. And when you differentiate this expression that I have written just now. And put it here, put it in this expression P dash y0 what you get is, the expression on the right hand side. And if you take the second derivative of this expression here, this one and put it in P double dash and y0 you get the expression for the convexity. So, that is how these terms have been arrived at.

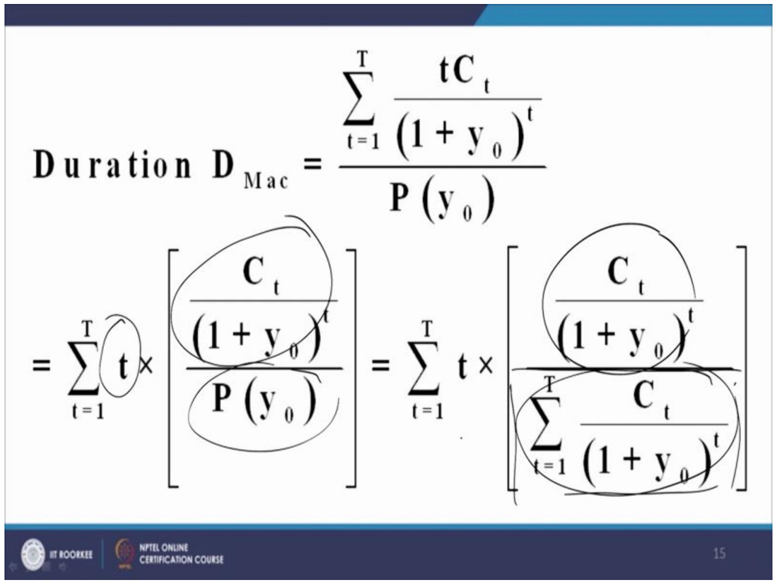
(Refer Slide Time: 27:38)



Duration as discounted cash flow weighted average time. So, this is another equivalent definition of duration I have introduced duration in an analytical manner. Here is a possible interpretation of the duration, so let us see what we are trying to say, a bonds annual Macaulay duration is calculated as the weighted average of, as the weighted average of what, the number of years until each of these bonds promised cash flow.

It is a time weighted, it is a the measure of unit is time please note this, the number of years that is time that is the variable that is being weighted with something, what is that something? That something is where the weights are the present values of each cash flow as a percentage of the bonds full value or the bonds price. So, you can how we have arrived at this definition is clear in the next slide.

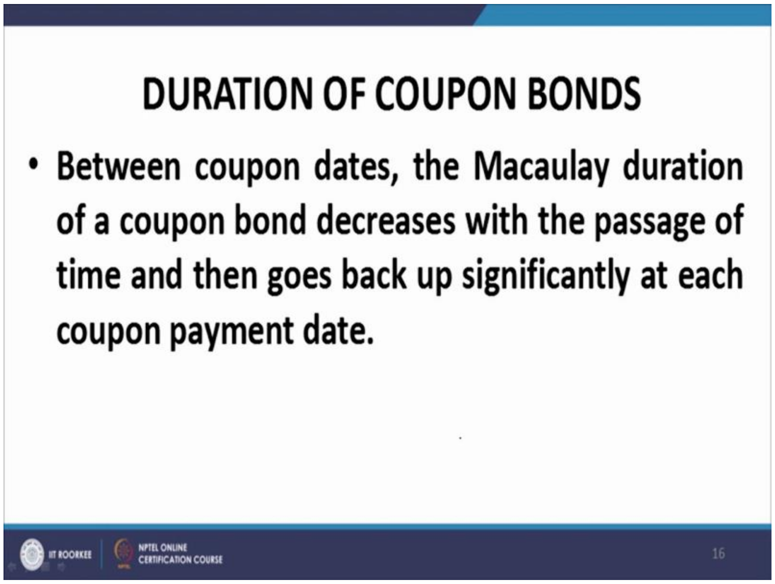
(Refer Slide Time: 28:36)



This is the basis, so this is the analytical analytics corresponding to the interpretation of the duration as a weighted time period. So, you can see here this time period t is being weighted by this expression divided by this expression. And this please note this is the aggregate of all these cash flows. So, the total of the weights obviously turn out to be 1.

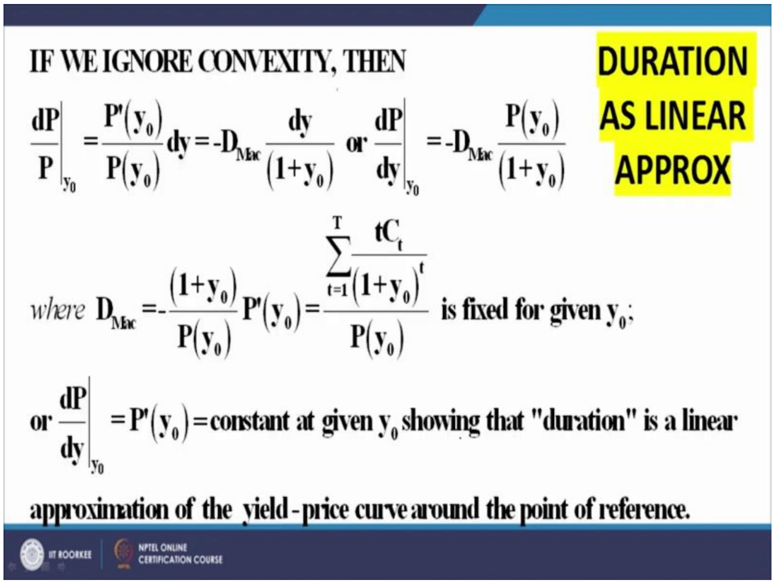
Now what I am trying to say is that we can write P0 in this form and if you look at this form carefully, this is a weight and this is the sum of the weights, so obviously this expression the weight sum of the weights will turn out to be 1. If you sum all these weights you will add a sigma here and the terms will become exactly the same. And therefore, we will get the weighted average or the weighted or the sum of the weights will be equal to 1.

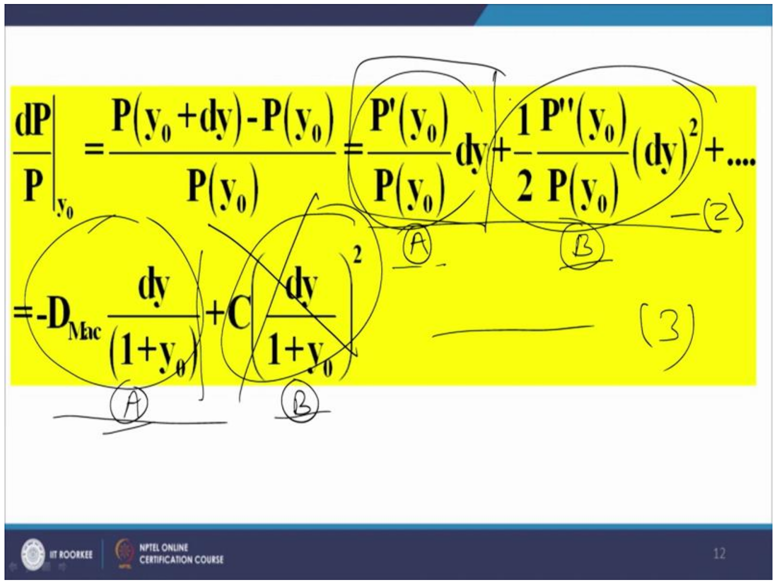
(Refer Slide Time: 29:37)



Duration of coupon bonds. Between coupon dates, the Macaulay duration of a coupon bond decreases with the passage of time and then goes back up significantly at each coupon payment date. So, this is the behavior of duration of a coupon bond. As the previous coupon date goes far behind or it goes more and more distant the duration decreases. But immediately after the coupon when the next coupon is made the bond duration again increases.

(Refer Slide Time: 30:15)





Now if you look at this expression. Let me go back you, look at this expression carefully. If we ignore this term then clearly what we make out of this that make out is that, if we restrict ourselves to this term what we are doing is we are confining ourselves to this term only and we are, when we confine ourselves to this term only we are assuming that duration is the linear portion of the curve or duration assumes a linear relationship between the yield and the price of a bond.

Around in proximity in the neighborhood of the point at which the duration is being calculated. That is explicitly shown, in this expression. You can see here that if we ignore the convexity, if we ignore the convexity, we have this expression from the Taylor series. Taylor series expansion and this expression, if I write in terms of the Macaulay duration gives me this expression or we can write this as Dmac and dP upon dy by transposing P and this dy cross multiplying or shifting the two to other, to the other side of the equation.

We get the slope of the yield curve at the given point, at which the duration is being calculated is given of this expression. And now this expression as it, so happens is constant at a given point P of y0 is constant 1 plus y0 is obviously constant at a given y0. And if I work out the duration at y0 that will also give me a number a constant number and that is dependent upon y0 but as far as given y0 is concerned this is also constant.

So, the slope happens to be constant at a given point. If we exclude the convexity part, that is if we confine ourselves to the first order expansion of the Taylor series. We find that the expansion gives us the inference that at the point, at which the duration is calculated, we assume that the curve is linear. We shall continue from here in the next lecture. Thank you.