Quantitative Investment Management
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Lecture 14
Bond Yields & Yield Spreads

So, let us continue from where we left in last lecturer. Before that I mentioned that as far as YTM goes, we make two fundamental assumptions one implicit, one explicit.

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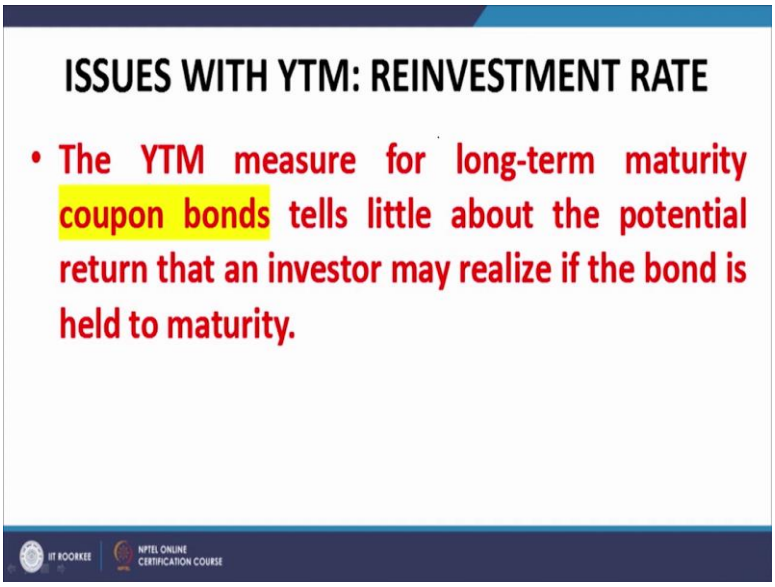

The implicit one is that all intermediate cash flows are reinvested at the YTM rate. And explicitly we assume that the bond is held by the investors, to be held by the investor up to the maturity of the bond. In other words, the investment horizon coincides with the bonds maturity. Now, as a consequence of these two assumptions, what we find is that in the event that intermediate cash flows are not reinvested at the reinvestment rate or the investors or either does not coincide with the bonds maturity.

The YTM of the bond may not give you the correct picture insofar as the yield on your investment is concerned. Let me illustrate this with an example.

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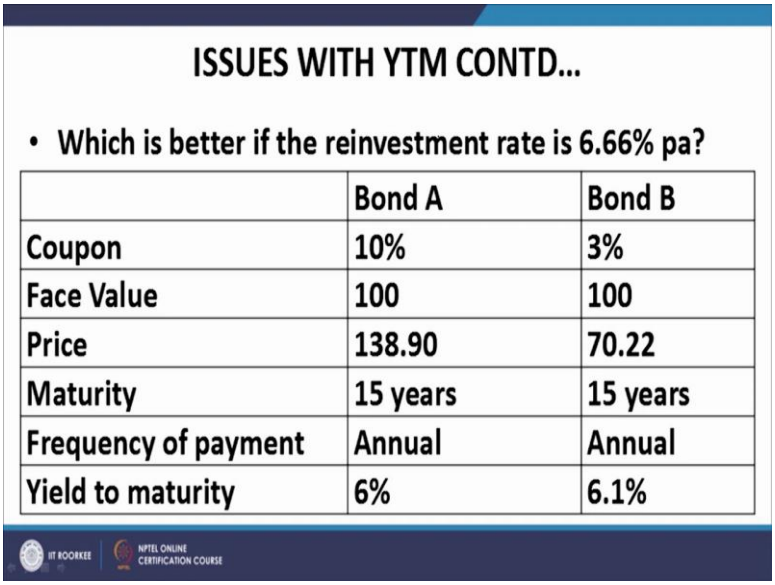

We have two bonds A and B, the coupon rate of A is 10 percent, the coupon rate of B is 3 percent. Face value of both of them is 100. And the price is 138.90 and 70.22 respectively, at the end at t equal to 0 that is today. And the maturity of both the bonds is 15 years and you are going to hold the bonds up to their maturity, the frequency of coupon payments is annual in both cases and what the YTM if you work out on these parameters, we find that the YTM of bond A turns out to be 6 percent and the YTM of bond B turns out to be 6.1 percent out.

Therefore, it is clear that if YTM was to be the deciding criterion, then bond B would be this superior investment compared to bond A, but we now add one more data to our problem and we say we assume that the reinvestment of coupons during the life of bond A as well as that a bond B is at 6.66 percent, then we with this additional assumption, let us examine whether the which is the better investment.

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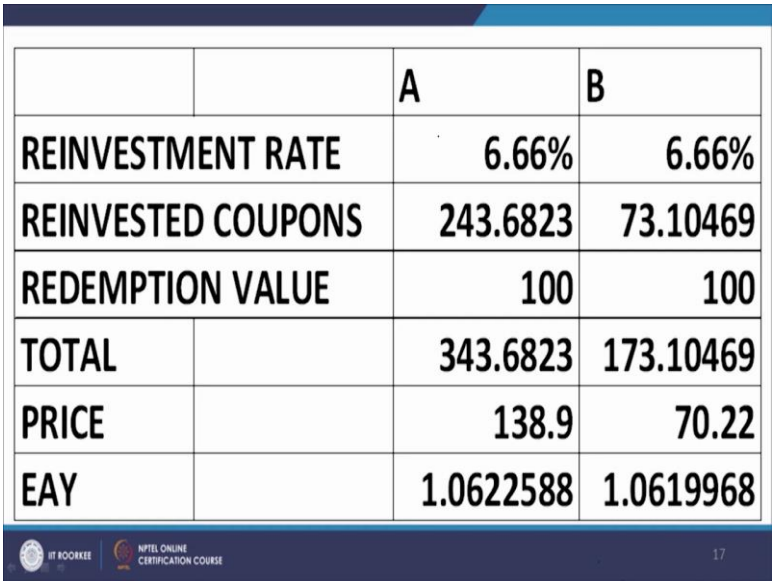

If you do so, the reinvestment rate is 6.66 percent coupon rate remember for A is 10 percent for B is 3 percent. And the final value or the terminal value of all the reinvested coupons if we work it out at the reinvestment rate of 6.66 percent. We find that for bond A it turns out to be 243.6823 and for B turns out to be 173.10469.

They are invested at the rate, which is given to us, which is 6.66 percent I reiterate the investment is of the coupons is at 6.6 percent and not the YTM rates.

So, add to that the redemption value of 100 and we get the total cash flows for bond A at t equal to 15 years that is the end of the that is on the date of maturity of the bonds for A it is 343.6823 for B it is 173.10467. The current market price, let us recall is 138.94 for A and 7.22 for B. When we work out the effective annual yield we find that the effective annual yield for A is 6.22 percent for B it is 6.199 percent.

So clearly, notwithstanding the fact that the YTM of B was higher than the YTM of A the effective annual yield of A tends to be higher than B. Why is that? Because the reinvestment rate, actual reinvestment rate or projected reinvestment rate happens to be different from the YTM. The projected reinvestment rate in our case is 6.66 percent it is given to us. However, if we assume the YTM rate, then A's coupons would have been invested at 6 percent, B's coupon would have been invested at 6.1 percent.

And now, the reinvestment rate of both the coupons is 6.66 percent. And because A's coupons are more, the coupon rate of A is 10 percent. That means every year you are getting

a coupon of 10. Whereas in the case of B, you are getting a coupon of 3. Therefore, the amount that is reinvested in the case of your turns out to be more, the rate is more and therefore, the effective annual yield turns out to be more for A compared to B then compared to the YTMs of the two bonds.

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PRICE	A	138.9	B	70.22		
REINV RATE	TOTAL CASH FLOW		TCF/PRICE		RETURN	
	A	B	A	B	A	B
0.06	332.88	170.15	2.39654	2.4231	0.06	0.0608
0.061	334.66	170.68	2.40936	2.43065	0.0604	0.061
0.065	341.94	172.87	2.46177	2.46183	0.0619	0.0619
0.066	343.682	173.1047	2.47431	2.46518	0.0622	0.06212
0.07	351.41	175.71	2.52995	2.50228	0.0638	0.0631

This is the table which shows you the effective annual return or the effective annual yield corresponding to various reinvestment rates. At 6 percent you find that the return, the effective annual yield is 6 percent for A, 6.08 percent for B, and in fact, up to 0.066 percent reinvestment rate we find that bond B continues to be superior than bond A.

But once we add 6.66 percent and thereafter, we find that bond A turns out to be superior than B, I repeat, this is because of the higher coupon rate of A which means higher investment at this higher reinvestment rate and therefore, a higher effective annual higher terminal value of coupons and higher effective annual yield.

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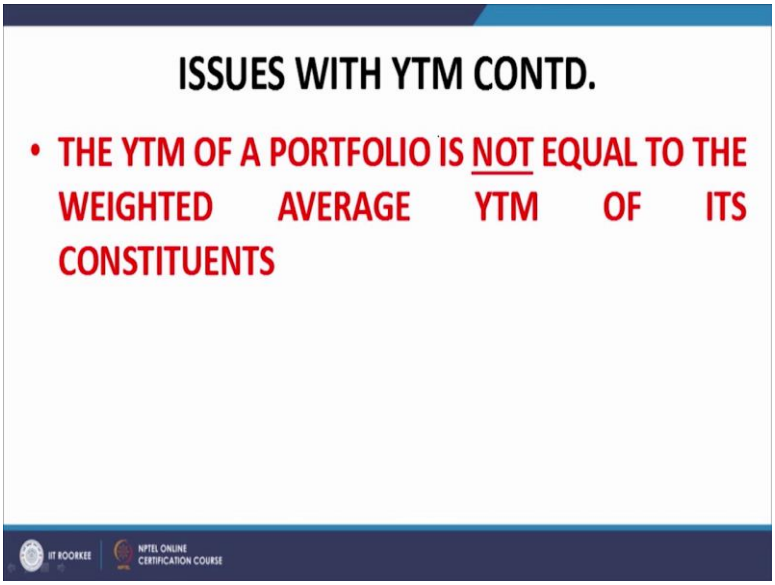

The second problem is YTM is rather more technical, it means it says that the YTM of a portfolio, the YTM of a portfolio is not equal to the weighted average of the YTM of the constituent's securities, let me repeat the YTM of a portfolio is not equal to the weighted average YTM of the constituent securities.

This makes it YTM an incompatible measure of return in so far as the portfolios are concerned, because as far as the expected returns are concerned, this expected return on a portfolio is equal to the weighted average of the expected return of its constituents. However, the YTM of a portfolio is not equal to the weighted average YTM of the constituent securities.

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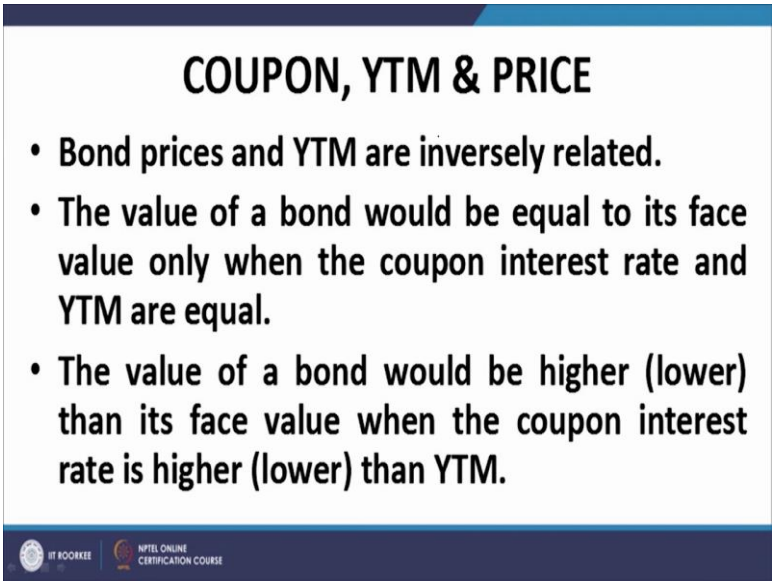

So, a quick recall about the relationship between coupon YTM and price. Bond prices and YTM are inversely related this is quite obvious from the formula. The value of a bond would be equal to the face value, only this applies to the conventional bond, which is redeemed at par and others level coupon.

The value of a bond would be equal to its face value only when the coupon interest rate and YTM are equal. The value of a bond would be higher than the face value when the coupon interest rate is more than the YTM and vice versa.

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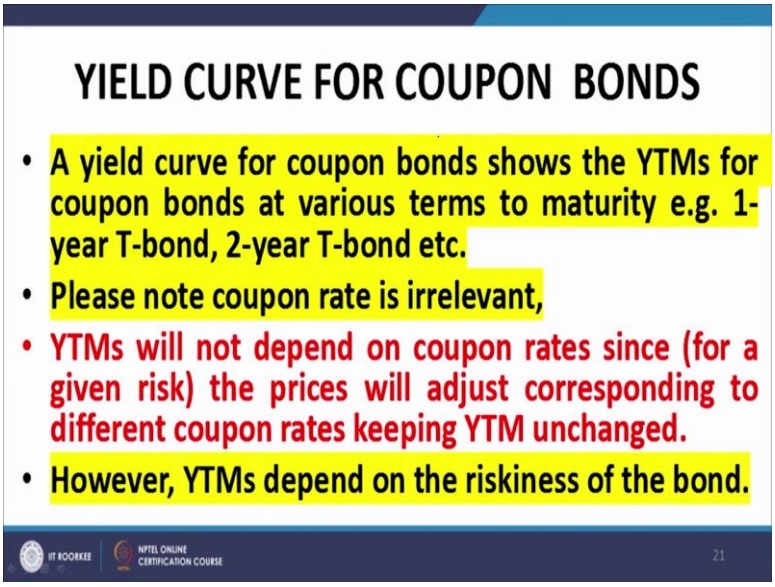

Now, we talk about yield curve for coupon bonds, or yield curve for coupon bonds shows the YTM's for coupon bonds at various terms to maturity, e.g. the 1 year T-bond, the 2 year T-bond and so on, please note the coupon rate is irrelevant. I have been saying that again and again, the YTM is a risk adjusted rate. So, if you are talking about T bonds, it would be the risk-free rate.

But the important thing is that it is a rate which depends on risk, it does not depend on the coupon payment. The price adjust itself in tandem with the pattern of coupon payments are corresponding to a given risk profile. So, coupon rate is irrelevant. YTM's will not depend on coupon rates. Since for a given risk the prices will adjust corresponding to different coupon rates keeping the YTM unchanged, this is very fundamental. However, YTM depends on the riskiness of the instrument.

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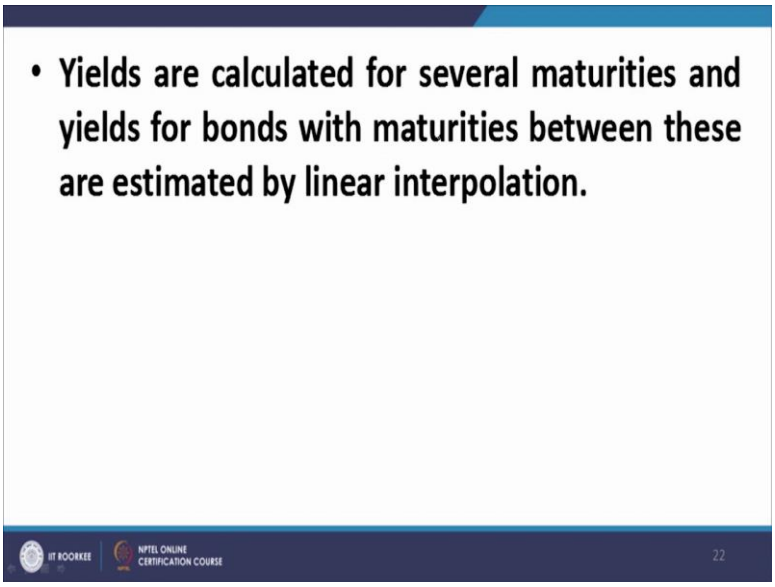

Yields are calculated for several maturities and yield for bonds with maturities between them are estimated by linear interpolation, more of a procedural issue.

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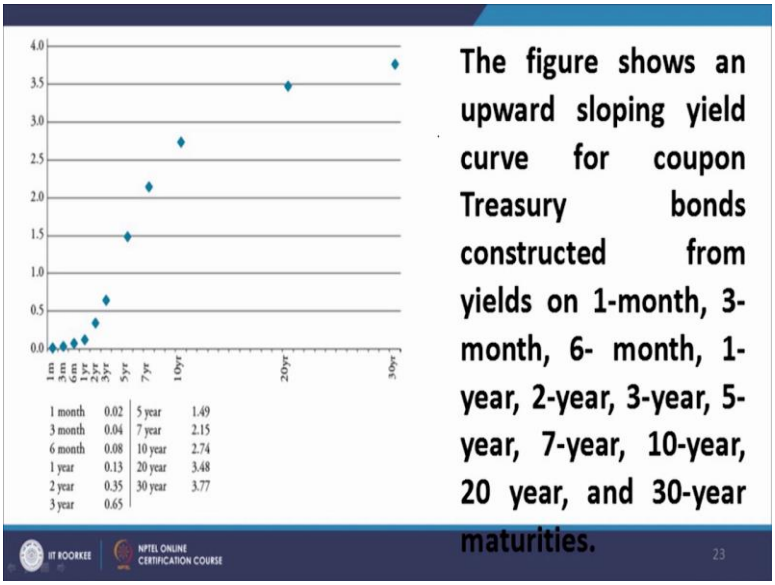

This is an example of a spot curve. That is this is an example of the curve that is based on; for coupon bonds that as you can see here in the right-hand panel, and this is an upward sloping yield curve. And the figure shows that upward sloping yield curve for coupon treasury bonds constructed from yield zone, yields, please note yield, YTM's, 1 month, 3 months, 6 months, 1 year, 2-year, 3-year, 5-year, 7-year, 10-year, 20-year and 30-year maturities. So, there is no mention about coupon rates here. Please note this fact because the YTM does not depend on coupon rates.

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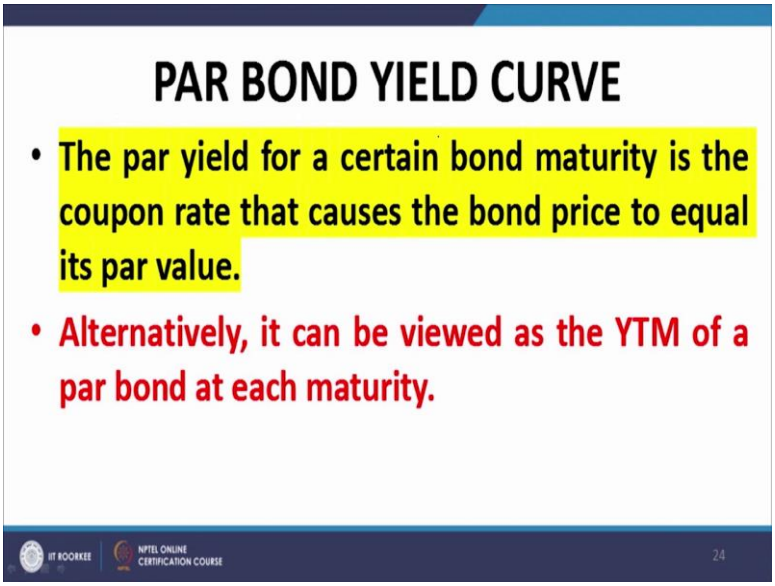

Par bond yield curve - What is par rate? I mentioned a little bit about par rates in an earlier lecture. Let us recall the par yield on a certain bond maturity is the coupon rate that causes the bond price to equal its par value. It is the coupon rate at which; the hypothetical coupon rate, not the actual coupon rate; it is a hypothetical coupon rate, such that, if that were the coupon rate on the given instrument, the instrument would be trading at par value.

So, let me read it out again, the par yield for a certain bond maturity is the coupon rate that causes the bond price to be equal to its par value. Alternatively, please note for a par bond, for a par level coupon bond the coupon rate is equal to the YTM. So, this explain, this definition that I mentioned just now can also be related to the YTM.

And we can say that par rate can be viewed as the YTM of a bond at each maturity. I repeat for a par bond, the coupon rate and YTM must be equal. And therefore, we can also describe or we can also define the par rate in terms of the YTM as it can be viewed as the YTM of a bond at each maturity. This is how we calculate the par rate.

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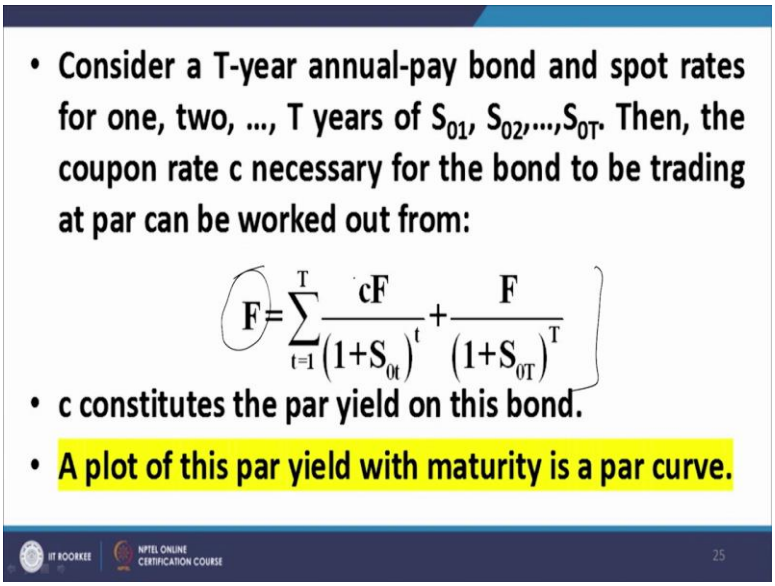

The C is the coupon rate which will incidentally turn out to be the par rate and we solve the equation that we have here, this is a familiar equation simply represents the intrinsic value of the instrument. And but, when the important thing is the left-hand side, the left-hand side is the C will be the coupon rate, the C will be the coupon rate, if the value of the bond that is the right-hand side equals the face value of the bond or the par value of the bond.

In other words, if the bond is quoting at par, it is that coupon rate it is that value of C such that the, the valuation on the right-hand side turns out to be equal to the face value of the bond, in other words, is such that the bond is quoting at par. So, C constitutes the par yield on the bond. Plot of the par yield with maturity is a par curve, a plot of the par yield along the y axis, maturity along the x axis gives you the par curve.

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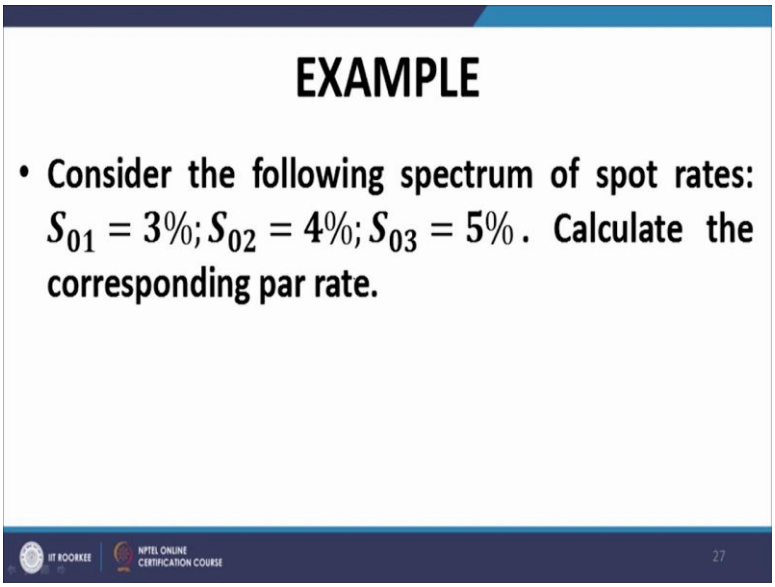

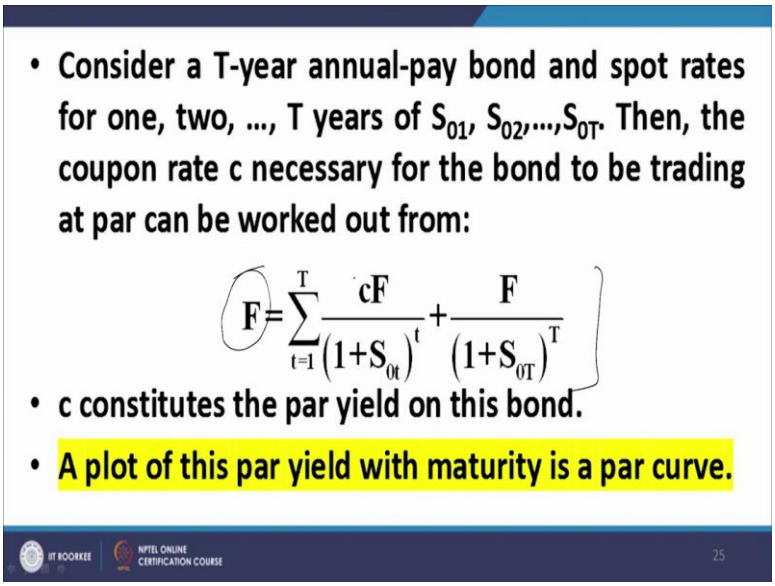

Par bond yield curve, or par curve is not calculated from yields on actual bonds, but is constructed from this spot curve. As you can see here, from this formula, all we need is the spot rate and the spectrum of spot rates corresponding to the cash flow pattern of the bond and we can work out the value of c at which the bond is coating at par.

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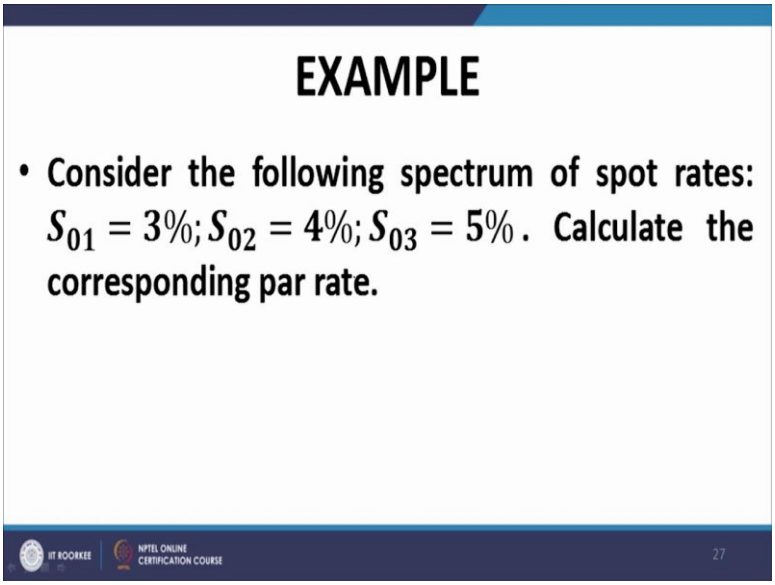

This is an example which illustrates what I have just talked about. Consider the following spectrum of spot rates: S01 is equal to 3 percent; S02 is equal to 4 percent; S03 is equal to 5 percent. Calculate the 3 year par rate.

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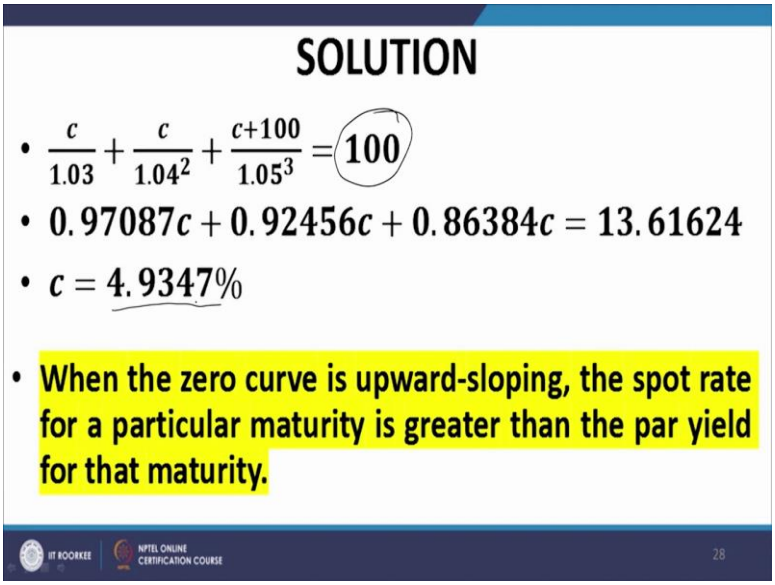

So, we assume that the par rate is c, then the value of the bond by discounting the future cash flows, what are those cash flows? c for the first year, c for the second year and c plus 100 assuming the face value of 100, it is c plus 100 for the third year, redemption at face value. The first c that is c corresponding the first, to the first year, is discounted at the first-year spot rate, which is 3 percent.

The second c that is the coupon payment at the end of the second year is discounted at the 2-year spot rate that is 4 percent and the third payment of c plus 100 that includes the redemption value of 100 will be discounted at 5 percent that is a 3-year spot rate. And what is this, has to be equal to, what has this got to be equal to? This has got to be equal to the face value of the bond or the par value of the bond.

On solving this equation for c, what we find is the c is equal to 4.9347 percent. So, this is quite simple, but this is how the par can be calculated from spot rates or vice versa as well. When the 0 curve was upward-sloping, as it is in this case, you can see here as 01 is 3 percent, as 02 is 4 percent, as 05 is 5 percent.

So, clearly the spot curve is upward sloping. The spot rate for a given maturity, for example, we have a maturity of 3 years, the 3-year spot rate is 5 percent and this is greater than the par yield for that maturity, which is 4.9347 percent. So, given an upward sloping yield curve, the spot rate for a given maturity exceeds the par yield corresponding to that maturity.

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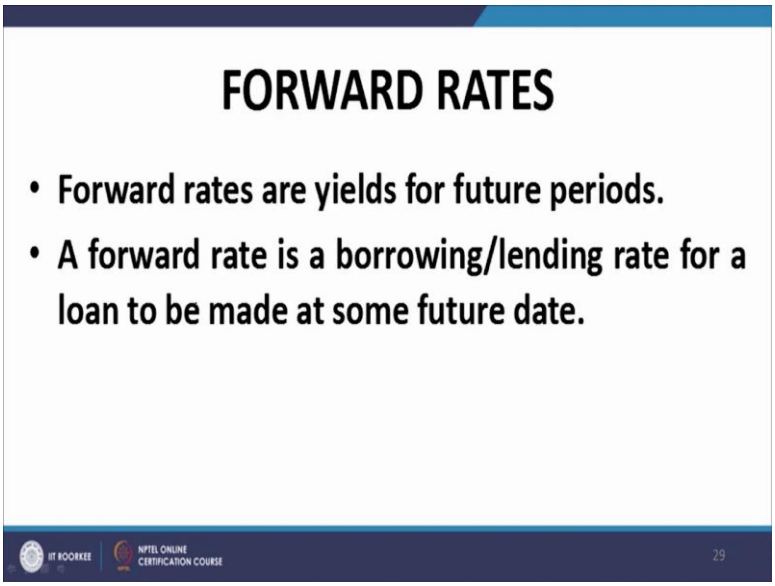

Forward rate I have discussed in a lot of detail earlier so I will not spend time on it.

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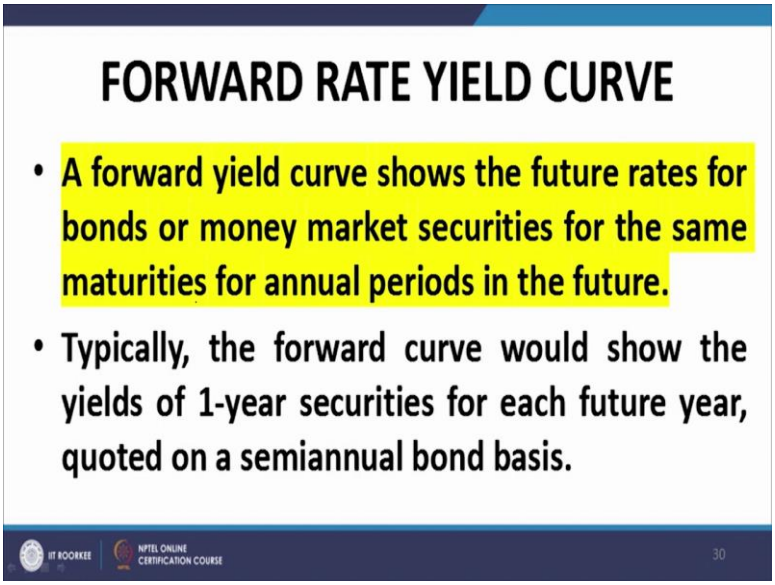

Forward Rate yield curve: The forward rate yield curve shows the future rates for bonds or money market securities are the same maturities for annual periods in the future. So, for example, typically we have the forward curve that shows the yields on 1-year securities for each future year quoted on a semi-annual basis.

A typical illustration is a curve which shows 1-year maturity rates for an investment, which begins at T at equal to 1 year, investment begins at T equal to 2 years that is from 2 to 3 years. Firstly, from 1 to 2 years and 2 to 3 years and 3 to 4 years, 4 to 5 years and so on. So, these are 1 year forward rates, which commenced at different points in the future 1 year apart.

I repeat, we have the, we work out the forward for F12 that is the investment initiating at T equal to 1 and terminating a T equal to 2, then F23 that is investment initiating at T equal to 2 terminating at T equal to 3 and similarly, F34, F45 and so on. And we plot these rates F12, F23, F34 against the corresponding mature, corresponding points in time that is the forward rate curve.

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Now, we talk about yield spreads, what are yield spreads?

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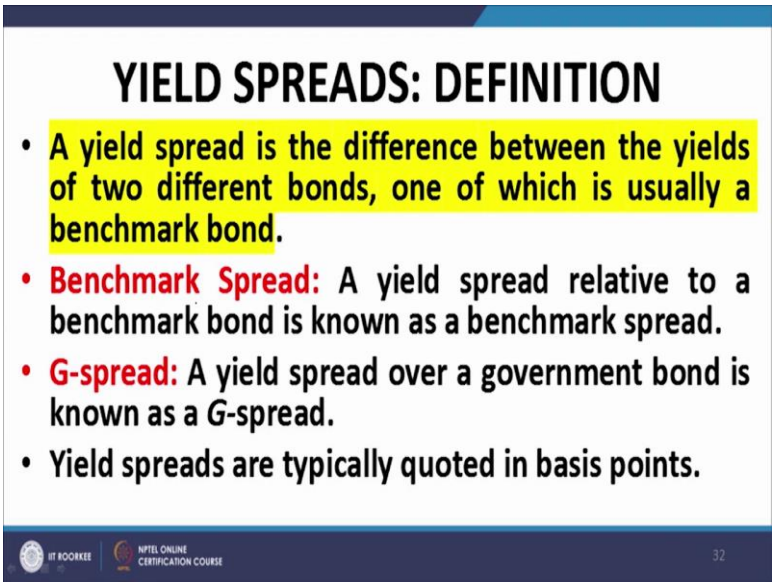

A yield spread is the difference between the yields of two different bonds, one of which is usually a benchmark bond. So, there is little to explain here. Let me repeat, a yield spread is the difference between the yields of two different bonds, one of which is a benchmark bond. The benchmark spread, a yield spread of a given bond, given risky bond with reference to a benchmark bond is known as a benchmark spread.

Similarly, a G spread, a yield spread over a government bond is called a G spread. So, if you have a risky bond, and you have a benchmark bond, the YTMs, I repeat the YTMs or the

difference in YTMs between these two bombs gives you the yield spread or the benchmark yield spread. If the benchmark bond is a government bond, and you have another risky bond, the difference in YTM between these two bonds is called the G spread.

Let me repeat, you have a risky bond with you, you have a benchmark bond which you may choose as the Treasury bond or LIBOR… whatever. But the basic thing is, the YTM between these two bonds gives you the benchmark spread. If that benchmark bond is a government bond, is a T bond, is a government treasury, then the difference between the risk, the YTM of the risky bond and the YTM of the government bond gives you the G spread.

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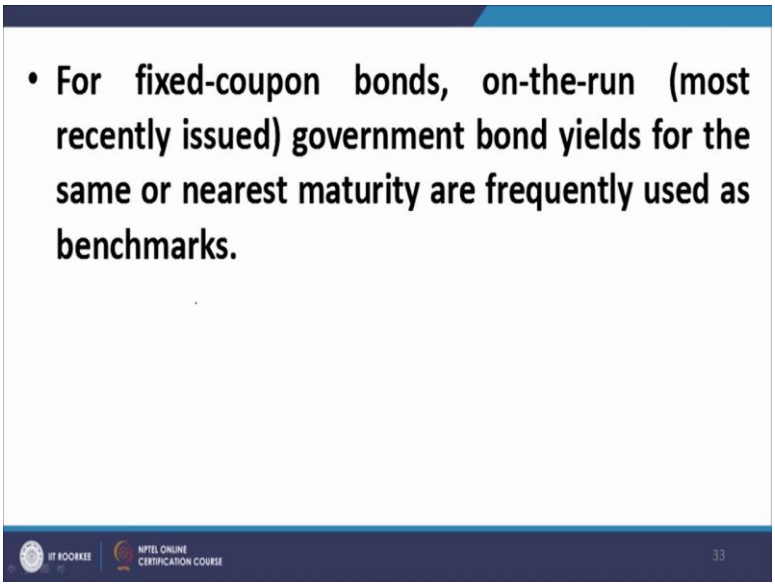

For fixed coupon bonds, on-the-run, on-the-run means the most recently issued common securities that government bond yields. On-the-run securities means the most recent securities that are available in the market, the yields on those securities are usually used as the benchmarks. So, for fixed income bonds, on-the-run government bond yields for the same or nearest maturity are frequently used as benchmarks.

But you see, the basic thing is benchmark is a broader term. It may include some other benchmarks that may be relevant to your investments. For example, in the case of the floating rate bond, we use the LIBOR as the benchmark. However, for fixed income bonds, we use the most recent traded government securities of equivalent maturity as the benchmark as a common practice.

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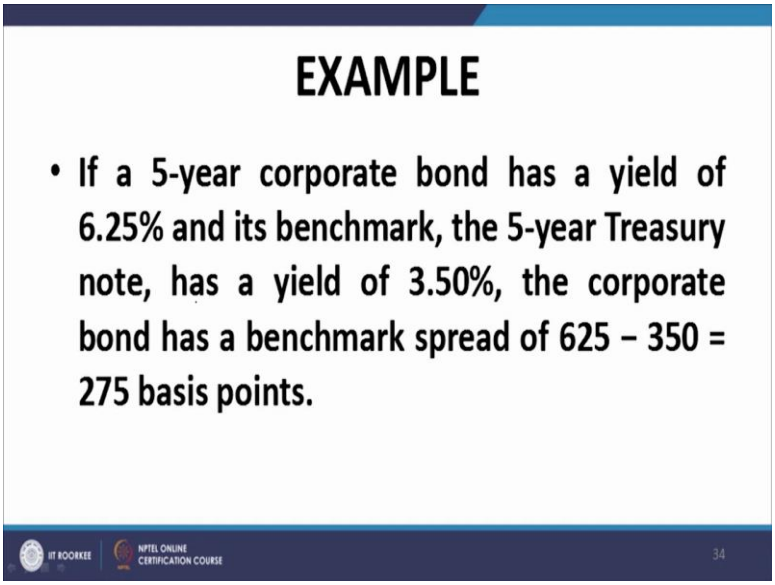

If example, if a 5-year corporate bond has a yield of 6.25 percent and its benchmark, the 5-year Treasury note, has a yield of 3.50 percent, then the corporate bond has a benchmark spread of, even you can also call it a G spread in this particular case of 275 basis points. The spreads that is the benchmark or the G spreads are usually represented in basis points.

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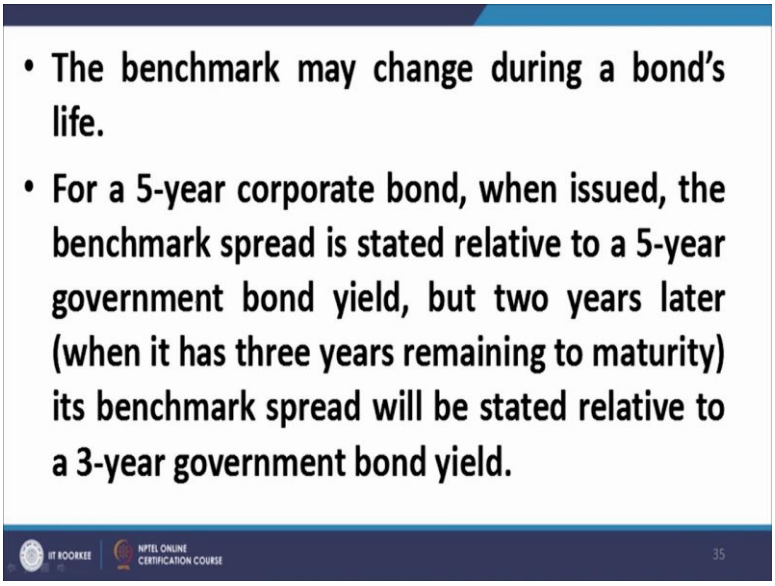

The benchmark spread may change during a bonds life. For a 5-year corporate bond, when issues, the benchmark spread is stated relative to the 5-year government bond yield. This is quite logical also, the basic thing is the remaining maturity of our bond, the bond whose spread is going to be worked out that has to be compared or that is the relevant maturity

insofar as the benchmark bond is concerned; not the total life of the bond, not the life of the bond at the point of issue.

It is the remaining life of the bond, the remaining tenure of the bond, the term to maturity, as of today, as on the date of comparison, as on the date of calculation of the spread, which is relevant. So, that is what this statement says that if the bond has a remaining maturity of say 3 years, it is a 5-year bond, but it has a remaining maturity of 3 years, then the benchmark should be a T bond of maturity of 3 years and not 5 years.

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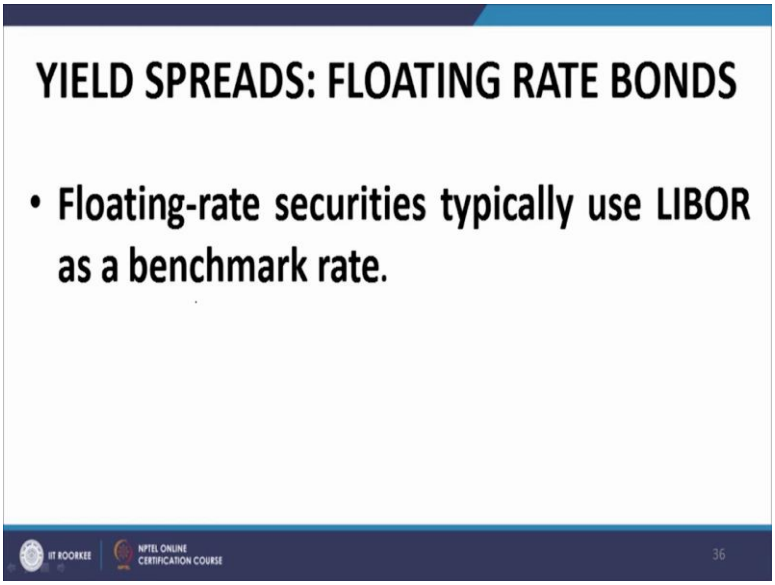

Yields spread for floating rate bonds. As I mentioned just know floating rate securities typically use LIBOR as a benchmark rate, London Interbank Offer Rate.

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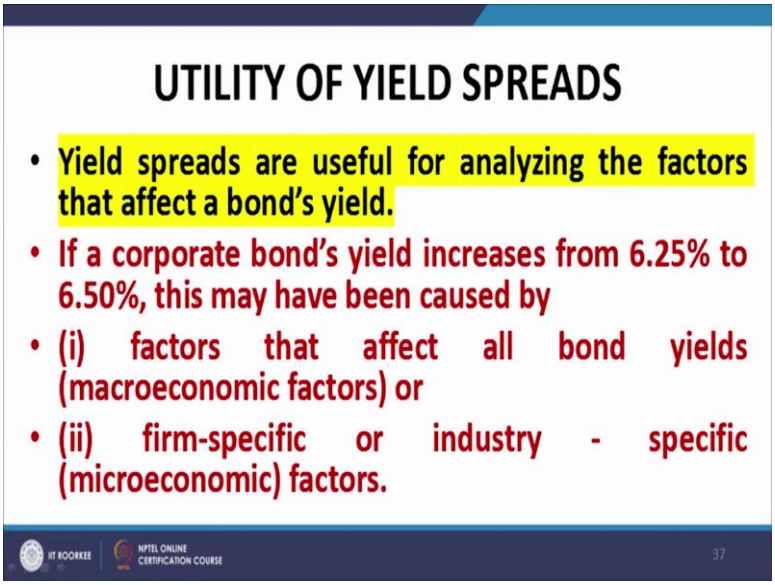

Utility of benchmark. So, what is the interpretation of benchmark spreads? What is the interpretation of yield spreads rather? That is interesting, but quite simple. Yield spreads are useful for analysing the factors that affect a bond's yield. If a corporate bond's yield, let us, this is an example which illustrates the interpretation of yield spreads.

If a corporate bond yield increases from 6 to, 6.25 percent to 6.50 percent, this could have been caused by factors that affect all bond's yield, that is macroeconomic factors for specific or industry specific factors that is microeconomic factors. So, if a corporate bond's yield increases, for example, the increase could be due to macroeconomic factors or it could be due to microeconomic factors.

For example, if this increase from 6.25 percent to 6.50 percent, is accompanied by an increase in the yields, it is accompanied by no change in the yield spread, that means what, that means the benchmark yield has also increased by the same amount. Let me repeat, if this increase from 6.25 percent to 6.50 percent in the corporate bonds yield is accompanied by no change in the yield spread. That means what?

That means, the benchmark yield has also increased by the same amount. That means what? That means the increase is due to macroeconomic factors. On the other hand, if the benchmark yield remains the same, and the yield on our instrument, that is this corporate bond has increased from 6.25 to 6.5 percent. It means that the bonds riskiness has increased.

It is due to factors which are specific to this instrument. Why? Because the benchmark yield has not changed. Therefore, because the benchmark yield is indicative of the macroeconomic factors, the macroeconomic impact is not prevalent, is not there, insofar as the change in yield on our corporate bond is concerned. In other words, the change in yield on the corporate bond is due to factors that are intrinsic, that are internal, that are relevant solely to our corporate bond.

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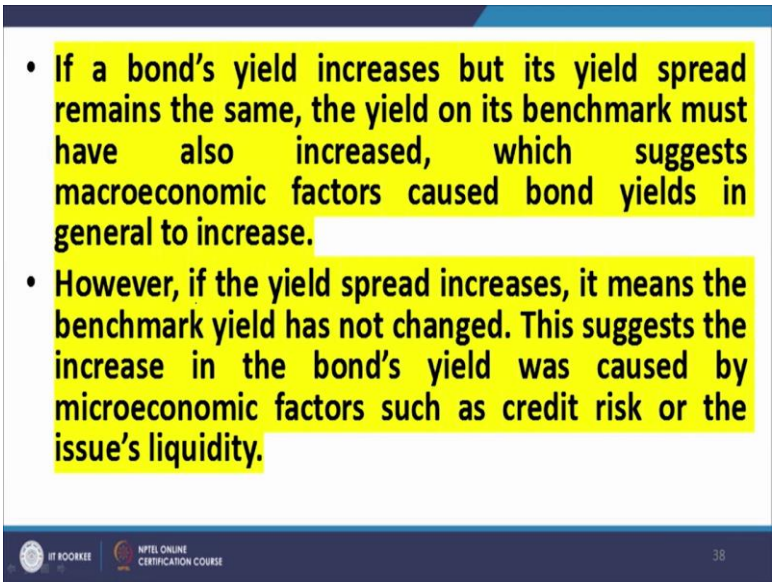

So, let me read it out for you. If a bond's yield increases, but the yield spread remains the same, but the yield spread remains the same that means what? That means, the yield on its benchmark must have also increased, which hint, suggests that macroeconomic factors caused the bond yields to change in general, how, to increase in general. However, if the yield spread increases, that means, the benchmark yield is not changed.

And if that benchmark yield does not change, that means what? That means the measures, the change or the change in the yield in our corporate bond is due to macroeconomic factor. Let me repeat once more, this is very important. If the, if there is an increase in the yield in our instrument in our corporate bond, let us say from 6.24 percent to 6.50 percent, there could be two situations number 1, the yield spread remains the same.

If the yield spread remains the same, that means, what that means, the benchmark yield has also increased. If the benchmark yield has increased it is due to macroeconomic factors. On the other hand, if the benchmark yield remains the same, that is the yield spread has increased

benchmark yield does not change that means, the change is due to change in factors which are intrinsic or internal to our corporate bond.

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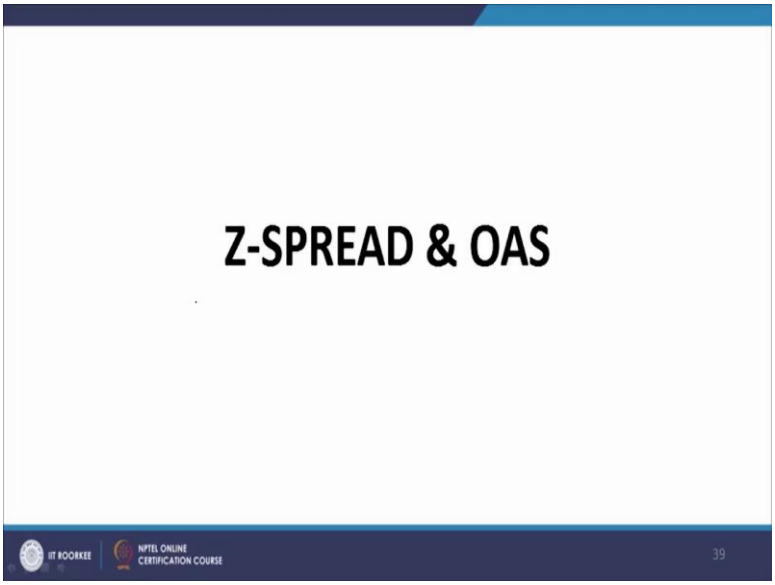

Z spread and Option Adjusted Spread.

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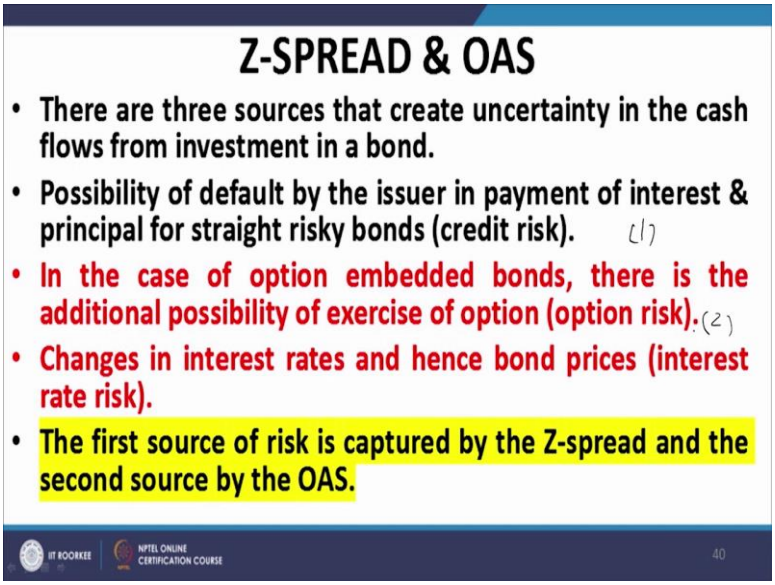

Now, this Z spread is very interesting. So, when I talked about the G spread and the benchmark spread, I mentioned the, I explicitly mentioned the word ‘YTMs’. Yield, wherever the word ‘yield’ is use in the context of the G spread and the benchmark spread, I refer to them as YTMs. And what is the property of YTM? YTM is one single rate that

encapsulates the entire spectrum or that encapsulates the impact of the entire spectrum of spot rates.

In other words, it assumes in some sense, it assumes that the yield curve is flat, it does not account for the change in the shape of the yield curve explicitly. So, that is a important drawback of the benchmarks spread and the G spread. In the Z spread, that is the 0 volatility spread. Z means 0 volatility. So, 0 volatility spread, what we try to do is we try to eliminate this fallacy or flaw in the G spread. How do we do it? We will come to it. Let me read out the slide first.

There are three sources that create uncertainty in the cash flows from investment in a bond. Number 1, possibility of default, that is the credit risk in the bond, possibility of default by the issuer insofar as payment of interest and redemption of principal is concerned. Number 2, in the case of option embedded bonds, bonds that are callable or portable or have some other complex option attached to them, there is the optionality risk or the risk of possibility of exercise of the option.

Changes in interest rates also result in or the possibility or the prospect of changes in interest rate during the life of the investment in a bond in a fixed income security also give rise to a certain amount of risk that is called the interest rate risk. The issue of interest rate risk, I will talk about in a lot of detail in subsequent lectures.

For the moment let us focus on credit risk and optionality risk that is this point, point 1 and point 2. The first source of risk is captured by the 0-volatility spread or the Z spread. And the second source of risk is captured by the option adjusted spread.

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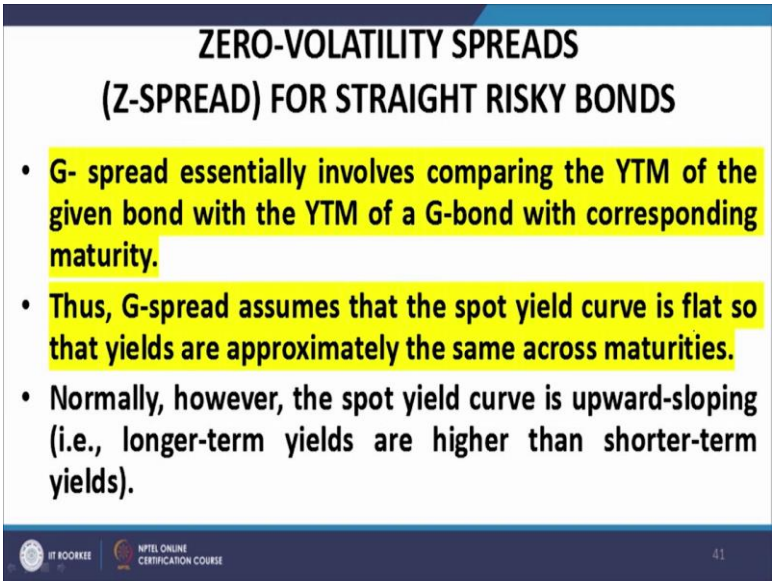

Now how do we work out the 0-volatility spread? The important thing is I reiterate, when I talked about say benchmark spread and G spread, I talked about YTMs. When we talk about YTMs, we talk about a single interest rate, we do not talk about the entire spectrum of interest rate, that single interest rate is deemed to encapsulate the entire information contained in this spot rate.

But obviously, there is a loss of information, when we use one number to represent the entire spectrum of spot rates. To obviate this problem, to remedy this flaw, we have these 0 volatility spread. What do we do? To each, we first of all, we identify the spectrum of benchmarks spot rates, we identify the spectrum of benchmarks spot rates.

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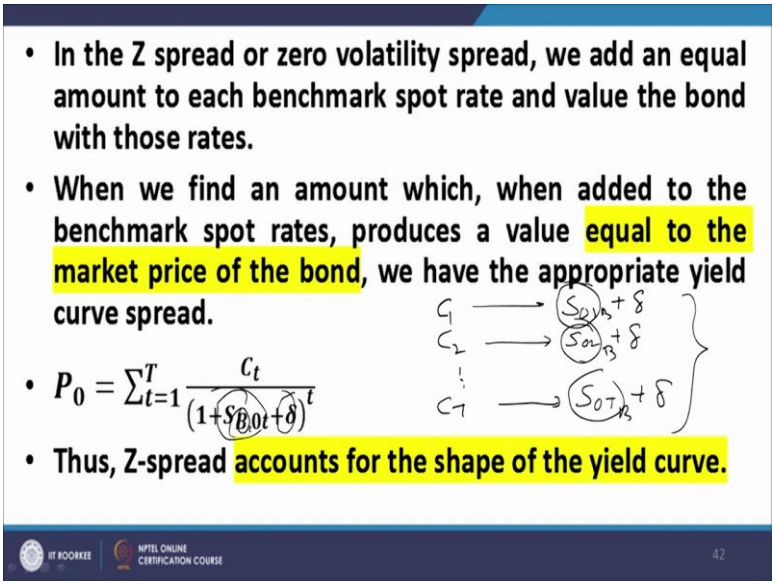

These are the benchmarks spot rates. Then what we do is, we add to each of these benchmarks spot rates, the same quantity a single constant, in this case it is delta, in this expression. So, to each, no, delta is an unknown quantity, please note this point. D1elta is something which will work out as part of this equation, and then we will say that this delta is what we call this Z spread.

Let me repeat, we have the entire spectrum of benchmarks spot rates, that is for example, the T bond rates or the risk-free rates, we add to each of them a quantity delta, and we discount all the futures cash flows arising from the bond at the respective spot rates plus this delta. For example, C1 will be discounted at S01. Let me write it down, C1 will be discounted at S01 plus delta, C2 will be discounted at S02 plus delta.

And similarly, CT will be discounted at S0T plus delta. But we do not know what delta is. How do we find out delta? We find out delta by equating the aggregate of these discounted values to the current market price of the bond. So, I repeat these are benchmark rates, these are not the risk adjusted rates, these are risk free rates or benchmark rates. Let me add a subscript B to the them, B, B, B, all these are benchmark rates as you can see here, this B is here.

So, this B represents that these are benchmark rates. You add to each of these benchmark rates a quantity delta and then you discount all future cash flows at S0T plus delta T ranges from 1 to capital T and then you equate this right-hand side to the current market price. This

is an equation in one unknown. What is that unknown? That unknown is delta and you can solve this one equation in one unknown and look at the value of delta and this delta is called the Z spread.

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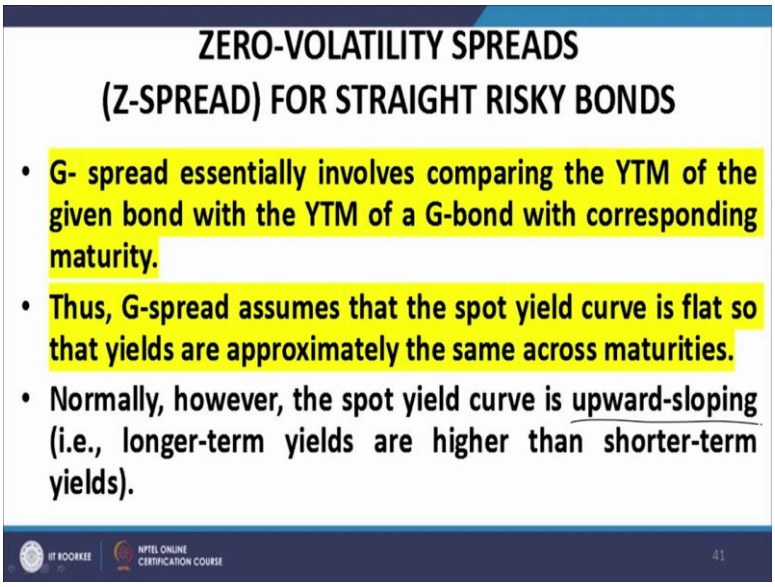

So, let me read it out, G spread essentially involves comparing the YTM of the given bond with the YTM of… with the YTM of a G bond, government bond with corresponding maturity. The G spread assumes that this spot yield curve is flat because it says only one figure here, that is the YTM figure. So that yields are approximately same across all maturities.

Normally, however, the yield, the spot yield curve is upward sloping. That is, as you go into the future the spot rate increase. So, spot yield curve is upward sloping that is long term yields are higher than short term yields.

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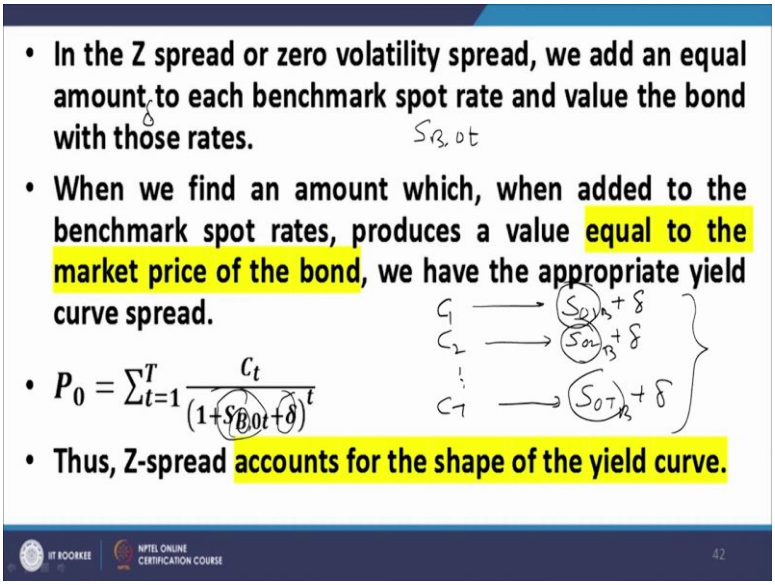

In the Z spread or the 0 volatility spread or the Z spread, we add an equal amount this delta to each benchmarks spot rate what SB0t. So, add an equal amount delta 2 each benchmark spot rate SB comma 0t and value the bond with these rates, work out the valuation by discounting the future cash flows on the bond and the respective spot rates plus delta.

When we find that delta such that these discounted values on the right-hand side equal the current market price of the bond we get what is called the Z spread. So, when we find an amount which, when added to the benchmarks spot rates, produces a value equal to the market price of the bond, we have the appropriate yield curve spread. The Z spread accounts for the shape of the yield curve. In other words, what we have done here is.

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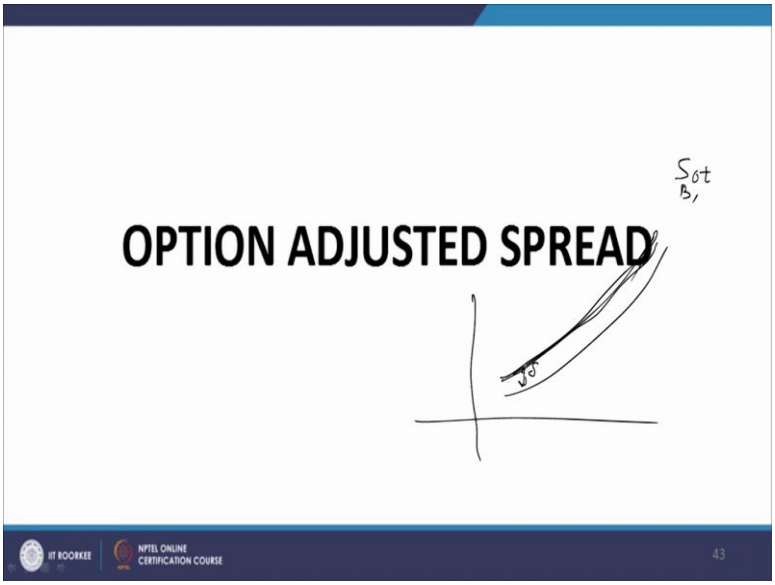

Let us try to understand, let us look at upward sloping yield curve. This has certain, this is a government or the riskless, upward sloping yield curve of T bonds or riskless bond. So, this will give you the various values of S0t, or rather with the benchmark SB0t. You will get the all the values of SP0t from this government spot curve, spot curve relating to government rates.

Now, what I do is, I lift it up with parallel to itself. I lift up this curve parallel to itself by a certain amount delta, such that when I lift up this yield curve parallel to itself by delta, and I use these new rates corresponding to this curve for discounting my cash flows. What I get on the left-hand side is the current market price of the bond. That will give me the value of delta and that is what is called as Z spreads. I will continue from here in the next lecture. Thank you.