Quantitative Investment Management
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Lecture: 13
Yield to Maturity

Welcome back, So, let us continue from where we left off a quick recap towards the end of the last lecture we were talking about yield to maturity. How do we define yield to maturity that is given on the slide.

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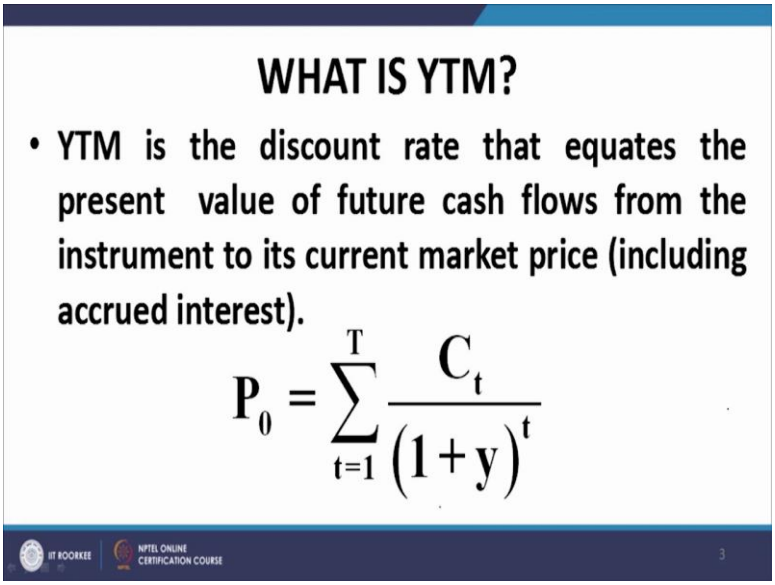

Yield to maturity is the discount rate that equates the present value of future cash flows attributable to that instrument to the current market price. Let me read it out again it is very important.

So, the formula for YTM is here on your, at the bottom of your slide P0 is equal to summation of Ct divided by 1 plus y to the power t. Now clearly it is an interaction between 4 variables, number 1, C that is the magnitude of the cash flows. Number 2, t the timing of the cash flows. I shall collectively refer to this as the pattern of cash flows or the spectrum of cash flows. Then we have the current market price that is P0 and we have the YTM that is y.

So, this equation represents an interaction of three quantities. In essence what I am trying to say is that if I fix the price of the instrument or if I have two instruments or two bonds with the same market price then the YTM will be determined by the pattern of cash flows that is the magnitude of the cash flows and the timing of the cash flows.

On the other hand, if I leave P0 as free and have Ct fixed. In other words, I have two bonds with the same pattern of cash flows then the YTM will be determined by the price and there will be a one to one correspondence between price and YTM. I would like to emphasize at this point that YTM or y in this equation is essentially the risk adjusted discount rate.

In other words, y encapsulates the riskiness in the realization of the cash flows from the financial instrument from the bond. And you see when we talk about investment, as I have emphasized again and again that we talk about a two dimensional framework along the x axis we talk about risk and along the y axis we talk about expected returns.

So, depending on the riskiness of the cash flows or the realizability of the cash flows from the instrument, the which is the, which is one of the very important variables one of the two variables that the market uses for appraising the investment. So, the riskiness is an information which is very much available or very much assessed by the market. And as a result of which, the price would incorporate the riskiness of the instrument.

And that would be reflected in the disc risk adjusted discount rate, why? That we call the YTM. In other words, let me put it simply, that if we have two bonds with identical cash flow patterns then the riskiness in the realizability of this cash flow patterns, if this is different then that riskiness will be absorbed into the price and will therefore be reflected in the YTM as well. So, that is what I have in store for you in the following slides. So, let us read it out.

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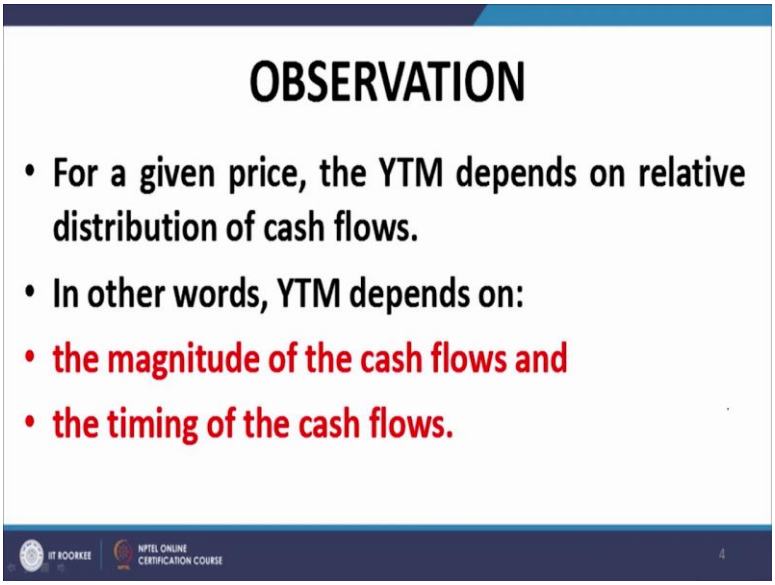

For a given price the YTM depends on the relative distribution of cash flows that is the pattern of cash flows. In other words YTM depends on the magnitude of the cash flows and the timing of the cash flows.

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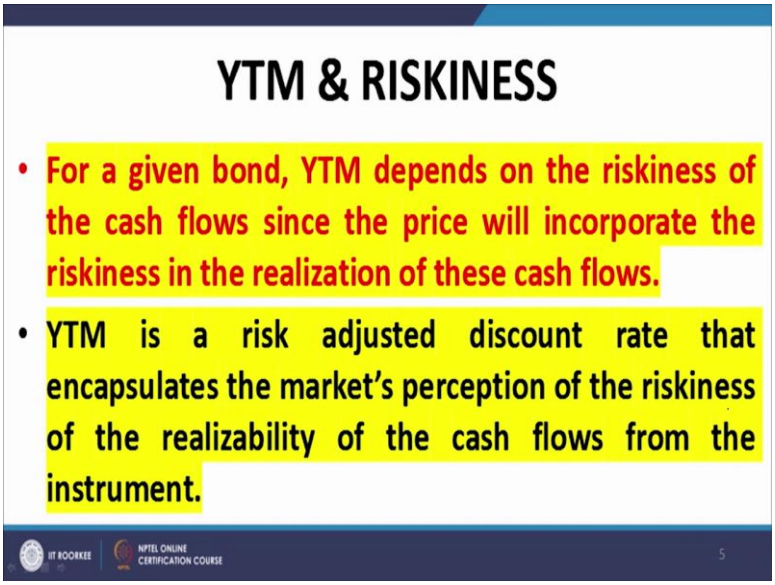

Then as I mentioned YTM is the risk adjusted discount rate. So, for a given bond YTM depends on the riskiness of the cash flows. Given bond means, given cash flow pattern. YTM depends on the riskiness of the cash flows, since the price will incorporate the riskiness in the realizability of these cash flows. A bond in which the realizability of the cash flows is suspect or there is a significant possibility of default in the realizability of the cash flows will be quoting at a lower price and therefore the YTM would be higher.

And a bond which is a little which is risk free would be quoting at a higher price and the YTM would accordingly be lower. I reiterate YTM is the risk adjusted discount rate. YTM is the risk adjusted discount rate that encapsulates the markets perception of the riskiness of the realizability of the cash flows from the instrument. So, this is important this needs to be understood very carefully.

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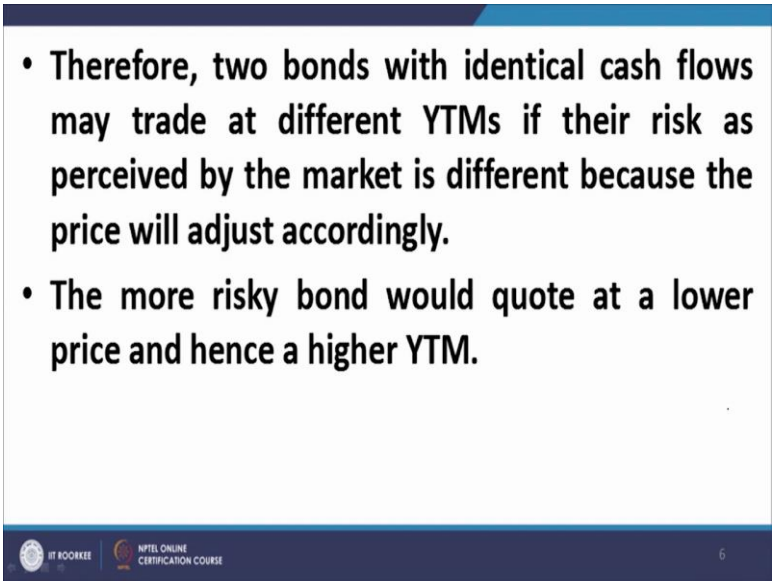

Therefore, two bonds with identical cash flow patterns may trade at different YTMs, if their risk as perceived by the market is different, because the price will adjust accordingly. The higher risk bond will quote at a lower price and the lower response will quote at a higher price with inverse YTMs.

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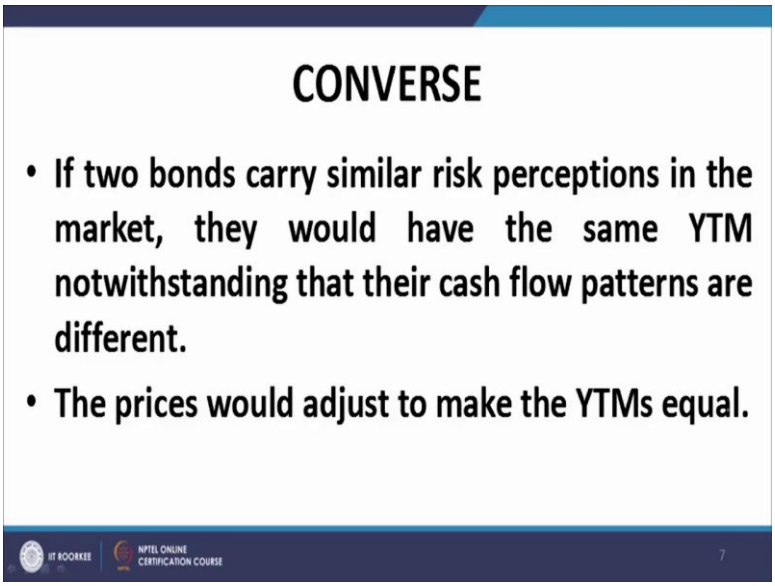

If two points carry similar risk perceptions in the market, they would have the same YTM. Now this is another interesting feature, if two bonds carry similar risk perceptions in the market they would trade it at the same YTM notwithstanding the fact that their cash flow patterns are different.

The cash flow patterns would adjust or would manifest themselves in a change in the price the YTM will remain unchanged. YTM is a measure of the riskiness, as I mentioned again YTM is the risk adjusted discount rate, it is a measure of the riskiness of the bond. And therefore if there are two bonds which have a different cash flow patterns but they carry say riskiness as far as the realizability of those cash flows is concerned then the YTM would remain the same for both the instruments. The prices would adjust to make the YTM equal.

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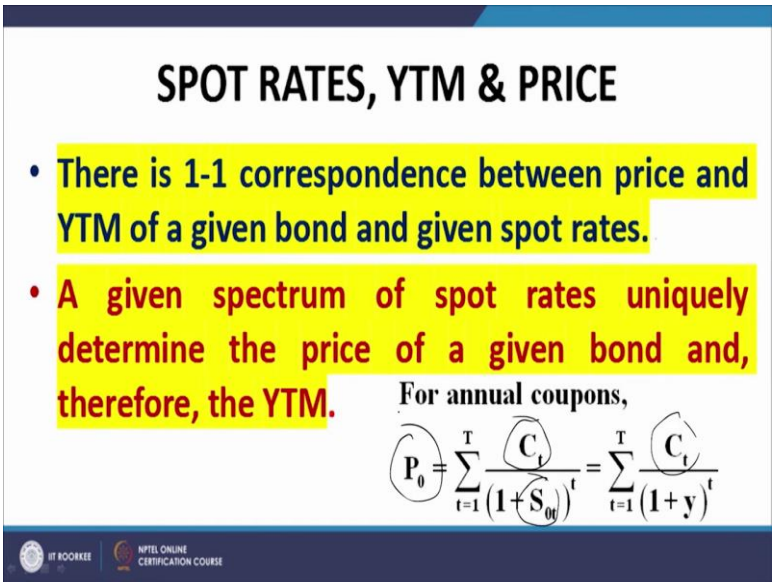

Now we talk about an interpretation of YTM. For that I would like the learners to look at the equation that we have here. Now if you look at the left hand side equation that is P0 is equal to summation t upon 1 plus S0t to the power t. This is what? This is the discounting of the various cash flows emanating from the instrument at the respective spot rates.

Ct corresponding to a given t is being discounted at the spot rate S0t. However, if you look at the right hand side of this equation this particular part what you find is that there is just a single figure y.

We do not have spot rates corresponding to different maturities, we simply are discounting all the cash flows from the instrument irrespective of the point of time at which they are occurring at this same rate. C11 is being discounted at y, C2 is being discounted y, C3 is being discounted y and up to the majority c capital T is also being discounted at y. There is only one rate that is being used throughout the spectrum of cash flows that arise from the instrument.

However, and the result of these two discountings that is discounting at their respective spot rates and discounting at a common rate at a single universal rate results in the market price of the instrument. So, what is the inference? The inference is that this y that we have here this expression y that we have here is the is some kind of an averaging of the various spot rates that are relevant for the discounting of cash flows at various points in time at which those cash flows arise.

It is some kind of an averaging. It is not a conventional arithmetic average or a geometric average. But nevertheless it is some kind of averaging over of the entire spectrum of spot rates that are relevant, that are used for discounting the cash flows from the instrument. So, let me read it out now, the YTM may be interpreted as some kind of average for the term structure of interest market interest rates.

It is just it is one single rate which is equivalent in some sense of the entire spectrum of market rates that go into the discounting of the cash flows that results in the current market price of the bond. So, spot rates YTM and price there is a as I mentioned there is this 1 to 1 correspondence between the price and YTM of a given bond and given spot rates.

So, you can see it from this equation again which we had it on the previous slide as well. You see if this spot rate spectrum is given that means this quantity is given, if the bond is given that means this quantity is also given to us. So, both Ct or all the values of C that is C1 C2 C3 are given to us because we are given a particular bond, then the corresponding spot rates we assume that they are also given to us, that means P0 is given to us and if P0 is given to us and for a given Ct which we already have taken for granted.

We find that P0 and y have an inverse have a 1 to 1 correspondence. So, given a bond given a spectrum of spot rates the or given the relevant spectrum of spot rates then we have a 1 to 1 correspondence between the price and the YTM of the bond.

Which in in essence represents the riskiness or which manifest, which is a manifestation of the riskiness in the realizability of the cash flows from the bond. So, a given spectrum of spot rates uniquely determines the price of the given bond as you can see from the left hand side equality and therefore the YTM that arises from the right hand side equality.

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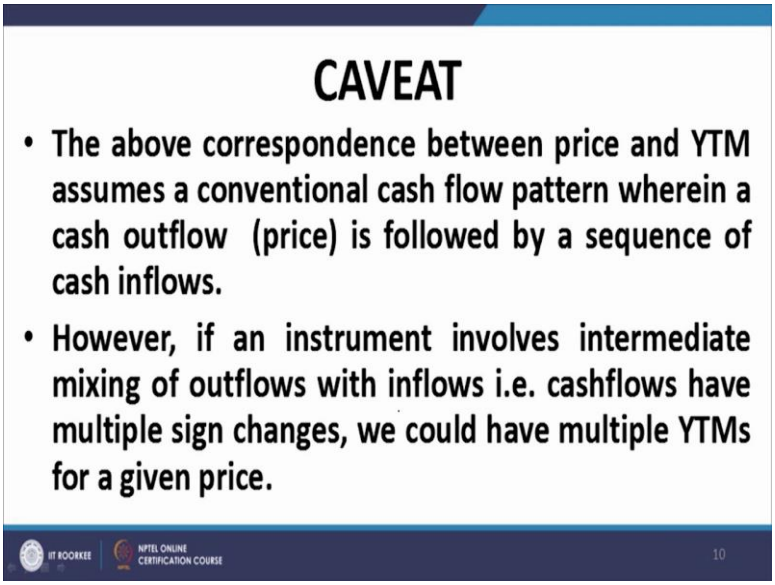

However, there is a caveat here. The caveat is that the above correspondence between price and YTM assumes a conventional cash flow pattern. What is the conventional cash flow pattern? A conventional cash flow pattern is a pattern, in which a cash outflow and investment paying the price of the instrument that is the cash outflow at t equal to 0 is followed by a sequence of cash inflows or of course you may have 0 or you may have a sequence of cash inflow.

In other words, we have we have just one sign change during the life of the instrument. The first flow is negative that is when you make the investment. And then we have positive cash flows or non-negative cash flows rather which represent the cash inflows from the life of the instrument from holding the instrument as an investment.

However, if an instrument this is now more of a hypothetical conjecture, that if you have an instrument that entails multiple sign changes for example maybe we can look at an investment in equity where the or in a preference share which is called up at different points in time. You may have a call at t equal to one year and then you may have a call at t equal to five years in between the instrument may be paying dividends.

So, that could be a situation possible situation where we encounter this kind of cash flow pattern. So, however if an instrument involves intermediate mixing of outflows with inflows you have the preference dividend at t equal to 1 t equal to 2 3 4 and then at 5 you have to pay the call money. So, that is a cash outflow then 6 you have a cash inflows and so on.

If you have this kind of multiple sign changes during the life of the instrument you could end up with multiple YTMs. So, let me read it out however if an instrument involves intermediate mixing of cash flows mixing of outflows with inflows that is cash flows and multiple sign changes we could have multiple YTMs for a given price.

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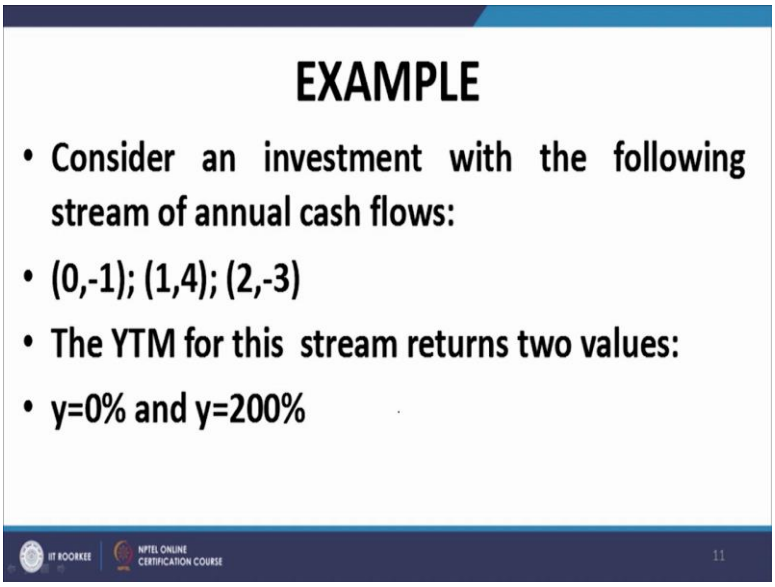

This is an example here where we have a cash outflow at t equal to 0 of 1 that is t equal to 0 out flow is my minus 1 at t equal to 1 there is a cash inflow of 4 and then at t equal to 2 there is a cash outflow of 3. So, there are multiple signs there are two sign changes one from t equal to 0 to t equal to 1 and the other from t equal to 1 to t equal to 2.

What we find is, if you solve this the corresponding equation for the YTM what we find is YTM turns out to be 0 as you can see here minus 1 minus 1 that is 4 and 4 in the second in the first year. There is another YTM which is t equal, which is y equal to 200 percent. So, you have two YTMs y equal to 0 percent that is y equal to 0.

Because you can see here minus 1 and minus 3 equal minus 4 and we have plus 4 in the middle corresponding to t equal to 1 here. So, if you have 0 discounting if you have no discounting then clearly minus 1 plus 4 minus 3 is 0. So, that gives you the YTM of 0 percent. Similarly, if solved if you solve the quadratic you also end up with a YTM of 200 percent. So, here our contention breaks down.

So, let me reiterate whatever I have said earlier about a 1 to 1 correspondence between the price and YTM given bonds cash flows, given a bonds pattern of cash flows and given the corresponding spectrum of spot rates holds for conventional cash flow patterns from a bond.

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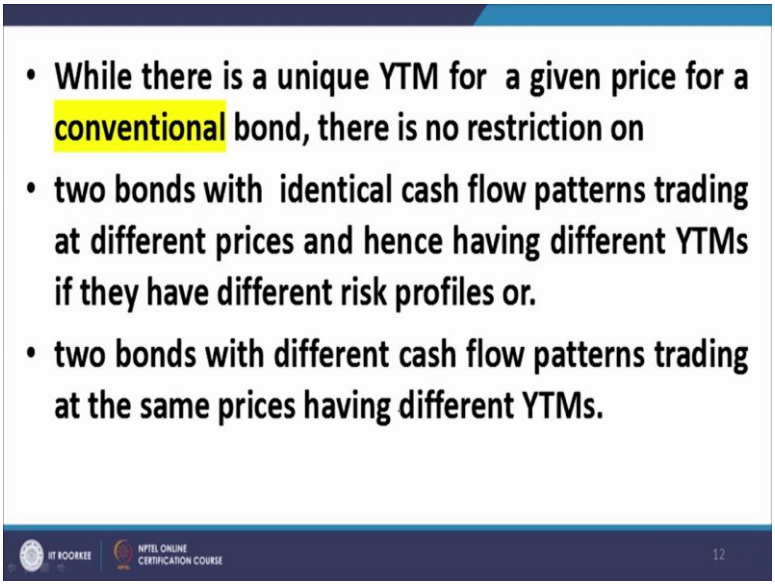

While there is a unique YTM for a given price for a conventional bond there is no restriction on two bonds with identical cash flow patterns with identical cash flow patterns trading at different prices. Why would they trade at different prices? They would trade at different prices when the riskiness corresponding to the cash flow patterns is different.

And when the market perceives that one of the cash flow patterns the realizability of one of the cash flow patterns is lower compared to the other the one with the greater possibility of default would be trading at a lower price and the one with the higher the lesser possibility of default would be trading at the higher price.

Similarly, we can have two bonds with different cash flow patterns trading at the same price and they may yield different YTMs as well. Let me read it out again two bonds with different cash flow patterns can be trading at the same price of course there is no restriction on that and they will of course yield different YTMs.

And therefore they will have different risk profiles. Why I emphasize here again? Why is that, because information on the riskiness of a bond is part of market information part of information that market players or traders make use of investment investors make use of, when they enter into an investment decision.

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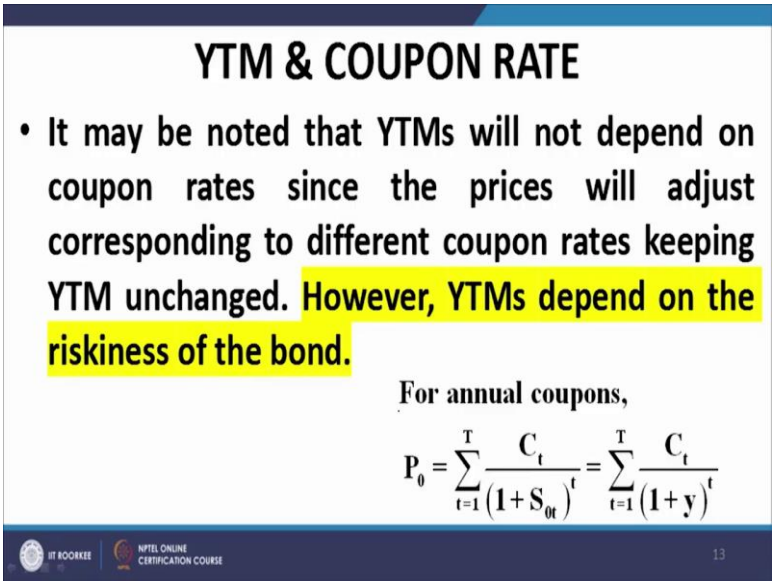

YTM and coupon rate it may be noted that YTMs will not depend on the coupon rates since the prices will adjust corresponding to different coupon rates keeping the YTM unchanged. Let me illustrate this with an example. Suppose you have two bonds A and B and the cash flows that arise from the bond A are exactly twice the cash flow that arise from the bond B. And the market perceives that the riskiness in the realizability of the cash flows of bonds A and bB is identical. Then what will happen? What will happen is that the price of A would turn out to be twice the price of B.

And as a result of which, as a consequence of which, the YTM of both A and B would remain the same. YTM as I mentioned is in a sense at this discount rate but a risk adjusted discount rate. So, if there are two bonds which have similar riskiness they would return the same YTMs. However, YTM depends on the riskiness of the bond that is what I have emphasized again and again YTM is the risk just to discount rate.

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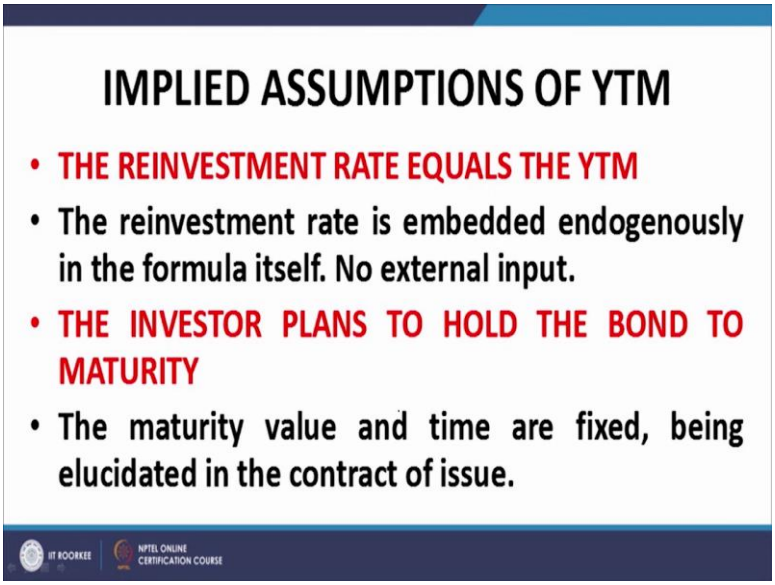

Implied assumptions that go into the YTM. Now there are two fundamental assumptions that go into the YTM. One of them is explicitly made the other is implicit in the formula that we have for the YTM. Let us look at these. The first assumption which is implicit which is embedded in the very formula of YTM is that all cash all intermediate cash flows that arise during the life of the instrument are reinvested at the YTM rate.

You can see this quite easily what is the YTM formula? P0 is equal to summation Ct upon 1 plus y to the power t. If and the summation goes from, let us say t equal to 1 to t equal to capital T. Let us multiply both sides of this equation plus 1 plus y to the power capital T. What do I get? I get P0 into 1 plus y to the power capital T is equal to summation from t equal to 1 to capital T Ct into 1 plus y to the power capital T minus small t.

Now this clearly shows what this clearly shows that all intermediate cash flows are being invested at the YTM rate. So, this is what the assumption tells us that all intermediate cash flows are reinvested at the YTM rate. Then we talk about the second assumption, and once one thing more at this in this assumption is a part of the formula as you have seen in the derivation that I just showed you.

The second assumption is that we make explicitly is that the investor holds the bond up to the maturity of the bond or holds the investment up to the maturity of the bond. The holding period of the investor coincides with the maturity of the instrument that means, what is the implication of this you see the redemption value of the bond at maturity is a part of the offer document.

It is contained in the offer document. Whether the redemption will be at par or premium or whatever the case may be or a discount whatever the case may be that is contained in the offer document. So, that is known upfront that is known at the time of making the investment. If we assume that the investor is going to hold the bond to maturity that means the uncertainty in so far as the market price of the bond at maturity is removed.

In other words, what I am trying to say is that if the bond was held for a period less than its maturity then obviously the investor would have had to sell the bond in the market price to realize the proceeds. And that in that market price is a random variable that has a certain amount of that has obviously a probability distribution and a certain amount of uncertainty in it.

That means what? That means at t equal to 0 we cannot be sure about what the price at which I am able to sell the bond at the end of my investment horizon is going to materialize. However, if my in investment horizon coincides with the maturity of the bond then I am certain what the investment is going to give me when I liquidate the investment.

Because that would be nothing but the redemption value and the redemption value is known at t equal to 0. And under this assumption when we make this both these assumptions in fact what we find is that the return that we get or the yield that we get on the bond equals the holding period or the effective annual yield.

I repeat under these two assumptions if we make both these assumptions then the yield that we get on the bond is equal to the effective annual yield on the instrument. We will continue in the next lecturer. Thank you.