Quantitative Investment Management
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Lecture: 12
Features of Option Embedded Bonds

We now talk about the effect of changes in the level of the yield curve on the value of a Callable or a Puttable bonds.

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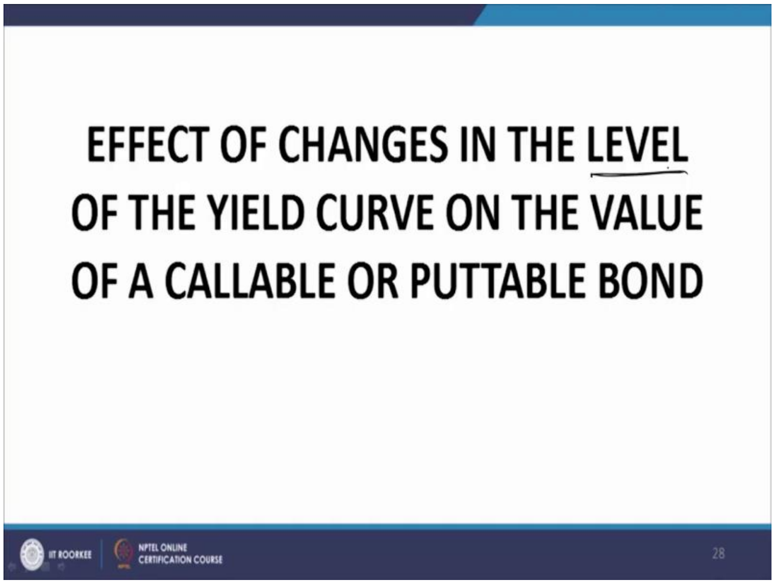

Speaking simply we want to know the impact of a change in the interest rates on the value of a Callable or a Puttable bond. This is very interesting as we shall see very soon.

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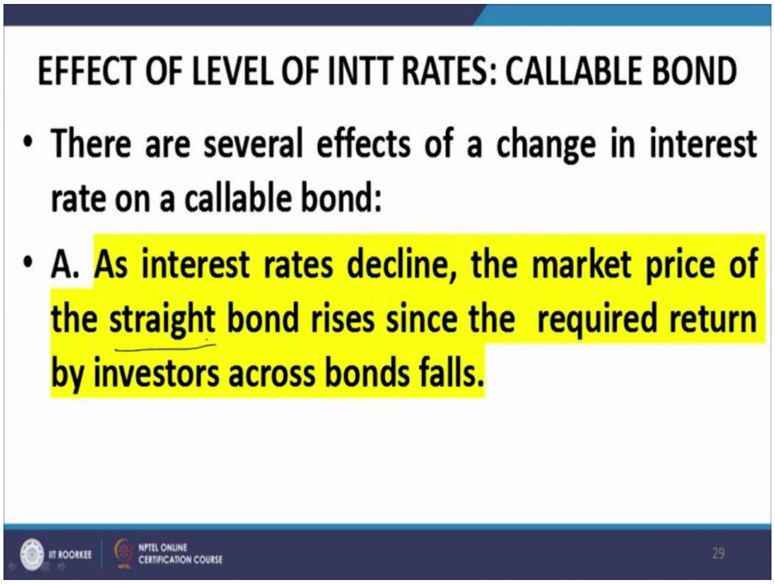

Now there are different effects of a change in interest rate on a Callable bond. We shall examine each of this in detail. Number 1 or A, as you must see, as interest rates decline the market value of the straight bond rises market value of the straight bond. Please note this the market value of the state bond rises since the required return demanded by investors across the bonds falls.

It is quite simple as the level of market interest rates fall, declines, it falls the people in the market who are in who are willing to invest for traders or investors in bonds would demand a lower return and as a result of which the state bonds price will increase.

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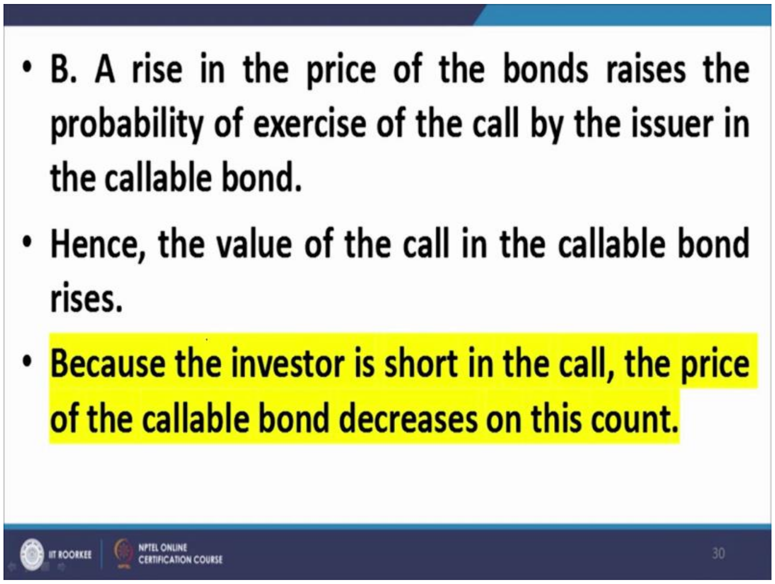

As a result, now the second thing that second impact that is very interesting, a rise in the price of the bonds raises the probability of exercise of the call by the issuer in the Callable bond. Now what is the Callable bond? It gives the right to the issuer to take back the bond to call back the bond if the option turns out to be in the money whenever if it decides to exercise the option with the discretion is with the issuer of the bond.

Now obviously because it is a call option it would be exercised, if the market price of the bond is higher than the exercise price of the option. Now therefore now, suppose the interest rates are decreasing, that means what that means the prices of the bonds across bonds would tend to increase for the reason that I have just elicited that is the market return goes down.

Market the return demanded by investors in the market goes down and as a result of the with the price increases. Price increases means there could be more situations, there could be more scenarios in which the price of the bond would be higher than the exercise price of the option and as a result of which the option would be in the money and as a result of which, it would be profitable for the issuer to call back the bond.

Therefore, what we are trying to say simply that if the interest rates fall the increase in prices across bonds would mean that would that there is a greater chance that the at any particular node for that matter, that the exercise price would be lower and the market price would be higher and as a result of which the call holder that is the issuer of the bond would prefer exercising the option.

In other words, because these probabilities ultimately determine the price at which the embedded option is going to be valued, greater the probability of exercise of the option greater would be the value attached to the option because greater the chance of making a profit out of that option and therefore greater would be the value of the call option in the bond.

And as a result of which on this count because the investor is short in the call option on this particular count the value of the Callable bond would decline. Why? Because the probability of its exercise increases why because the prices across bonds increases. And that means what?

Because the investor is short in the call option therefore and the call option has become darer, therefore the investor the investor would be paying to pay even lesser for that particular bond. In other words, the price of the Callable bond on this particular count on the count on the reason that the call option becomes more costly will decrease.

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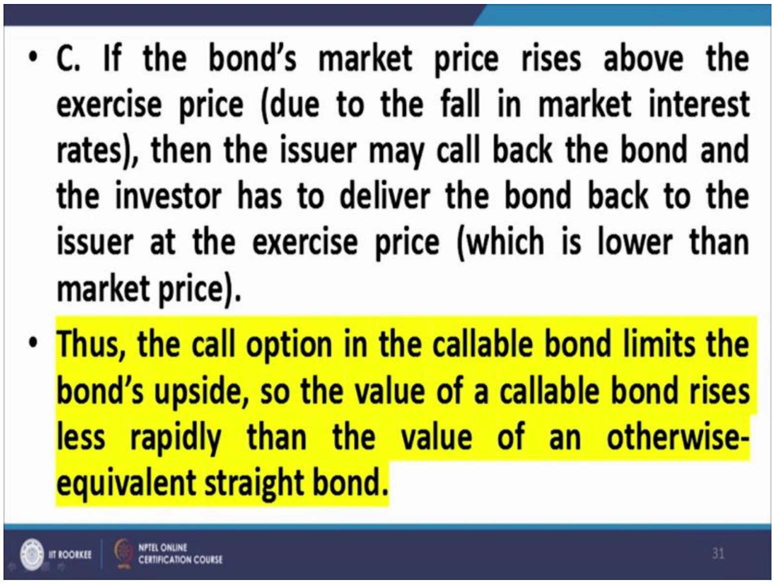

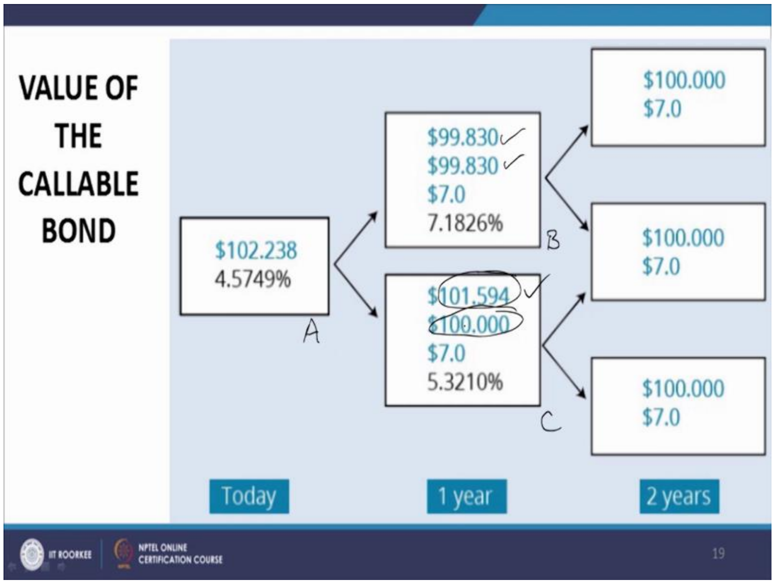

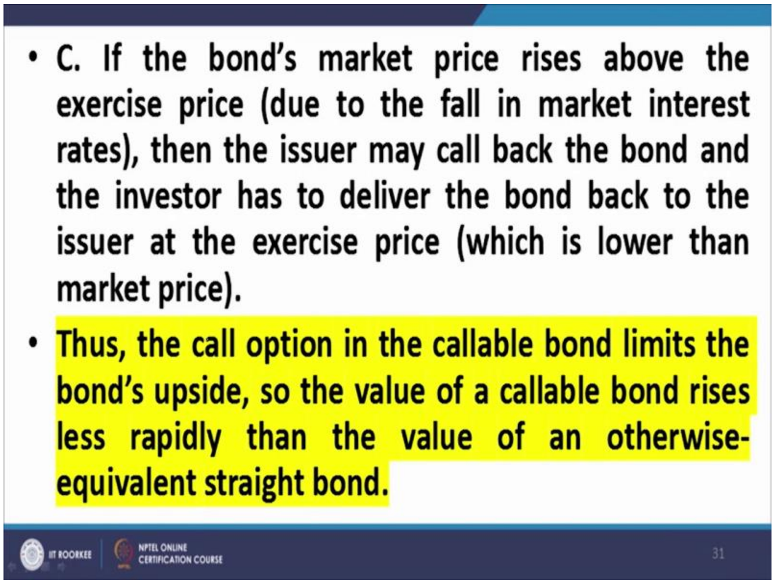

See if the bonds market price increases now this is another interesting feature, which we can look at by going back to the example. Look at this example. Now you what you observe here is, just a minute, this particular slide, look at this particular slide. What is happening here is that when there is an embedded call option as soon as the price at any particular node goes above the exercise price the computed price is replaced by the exercise price.

What does that mean? It means that as far as the investor is concerned the upside that is generated or created on the bond decreases. Let me repeat had this bond not had a call option the payoff at this particular node at the node C would have been 101.594 but because of the embedded call option we assume that the because the computed price is higher than the exercise price, the issuer will access that option and as a result of which, the price which should have been 101.594 now goes down to 100.

Therefore, the upside of the prices at various nodes is curtailed, is segregated, it is reduced by the exercise price by a ceiling of exercise price whenever at any particular node the option is exercisable and the computed price exceeds the exercise price you have to replace it by the exercise price. In other words, in totality the bonds upside is curtailed by the existence of the option.

If the bonds market price rises above the exercise price due to fall in market interest rates then the issuer may call back the bond and the investor has to deliver the bond back to the issuer at the exercise price. Please note the market price is higher but because there is the embedded option and the issuer has access the option the investor has to return back the bond at the exercise price not the market price.

So, the value of a Callable bond rises less rapidly than the value of an otherwise equivalent straight bond. Why? Because of the call option being there as the interest rates decline the value of the bond increases, the value of the call option also increases, but because the investor is short in the call option and the overall the price of the bond or the Callable bond increases in price but at a slower rate.

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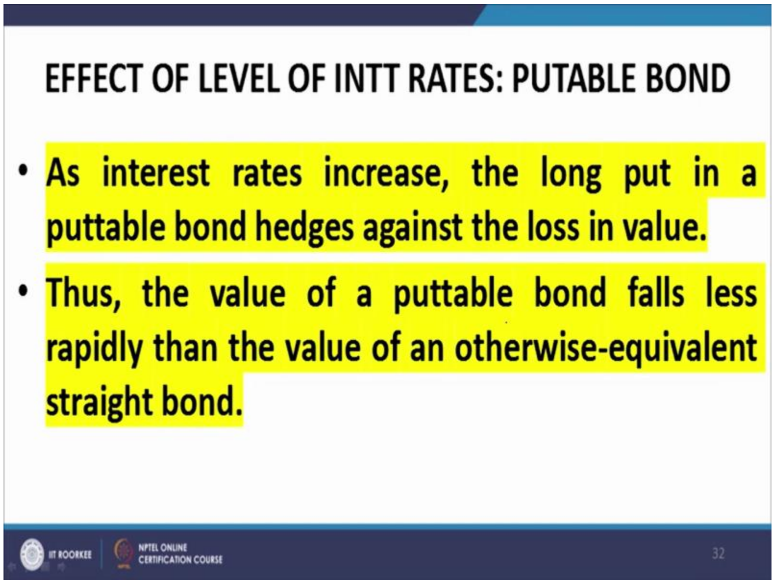

Similarly, we can analyze the put option as interest rates increase the long put in a Puttable bond hedges against the loss and value. Therefore, as the interest rates increase because the put is long now and interest rate increase means the prices are declining therefore the put option acts as a protection act to the hedge. So, where if the prices go down below the exercise price of the put option then the investor can exercise the put option and can get back the exercise price instead of the market price.

Thus the value of a Puttable bond falls less rapidly than the value of an otherwise equivalent straight bond. Because the value of the put option as the interest rates increase, what happens price declines, price declines of what, of the state bond but the put options probability it gets the probability of exercise increases and as a result of it the value of the put option increases.

Now the investor is long in the put option and because the investor is long in the put option and the value of the put option has also increased. Therefore, whereas therefore there is a cumulative effect and the therefore there is an cancelling effect rather the price of the bond has decreased, the value of the put option has increased, and therefore the price of the bond decreases slower than that of a straight bond.

So, that is what is stated here. The value of a Puttable bond falls less rapidly than the value of an otherwise equivalent straight bond. Why? Because the value of the put option increases whereas the value of the state bond decreases if there is an increase in interest rates.

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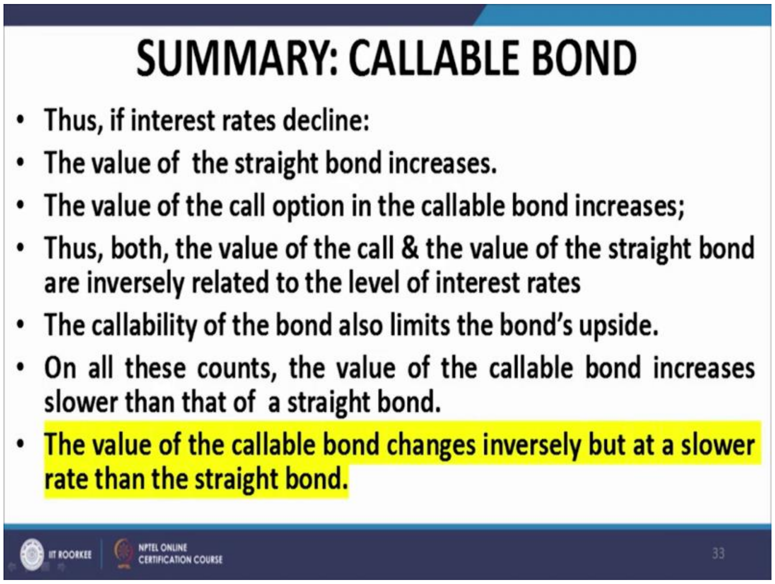

So, this summary in this slide summarizes whatever I have been discussing so far and let us read it out slowly and carefully. Thus if the interest rates decline the value of the straight bond increases, this is as far as the Callable bond is concerned. If the interest rates decline we all know there is an inverse relationship between the price of a bond and the interest rates and therefore if interest rates fall the price increases.

The price increases means what, the value of the call option in the call Callable bond increases why because there is a greater probability of the exercise of the call option. However, thus both the value of the call and the value of the state bond are inversely related to the level of interest rates lower as interest rates decrease, both the value of the straight bond increases and the value of the call also increases because of the greater probability of exercise of the counts.

The callability of the bond also limits the bonds upside at no node where the call become exercisable you can exceed the exercise price you will have to assume that the call is exercise and therefore the payoff at that particular point will be restricted to will be truncated to the exercise price on the call.

And all these counts the value of the Callable bond increases slower than that of a straight bond. On all these counts the value of the Callable bond increases slower than that of a straight bond, if the interest rates decline, if the interest rates decline let me repeat call the value of the Callable bond, the value of the straight bond increases the value of the call increases but because the investor is short in the call the rate of increase of the Callable bond price is less than the rate of increase of the straight bond price.

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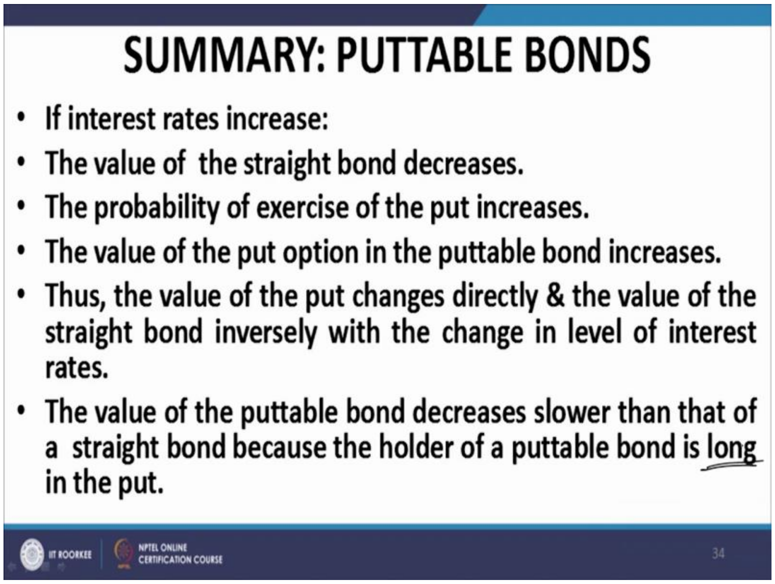

Now for the Puttable bond if interest rates increase the value of the state bond decreases the value of the straight bond decreases inverse relationship. The probability of exercise of the put increases. The probability of exercise of the put increases why? Because the value of the straight bond value of the bond has decreased due to increase in interest rate.

Therefore, there could be greater therefore there could be more situations so there could be a possibility of the profit, possibility of a situation being more probable where the price of the bond is less and the exercise price of the put option is more and as a result of which the investor will find it profitable to exercise the put option and sell it back to the to the issuer.

Therefore, the probability of exercise of the put option increases the value of the put option in the Puttable bond increases. So, thus the value of the put changes directly and the value of the bond changes inversely with respect to changes in the interest rates. Overall the value of the Puttable bond decreases slower than that of a straight bond because the holder of the Puttable bond is long in the put option.

So, that is as far as the level of interest the impact of the level of interest rates on bonds with embedded options is multi-fold. First of all the impact on the straight bond the impact on the number one the impact of the state bond is inverse in all cases the impact on the option as you can see here because the probability of exercise increases whether it is a call or a put therefore in both cases the value of the put value of the option increases.

Number three the bonds upside is limited in the case of a call option embedded instrument or a Callable bond and the downside is limited in the case of an case of a bond with a put option embedded in it. Overall what happens is that the value of the bond in both cases moves inversely in relation to the interest rates but at a slower rate compared to the corresponding straight bonds.

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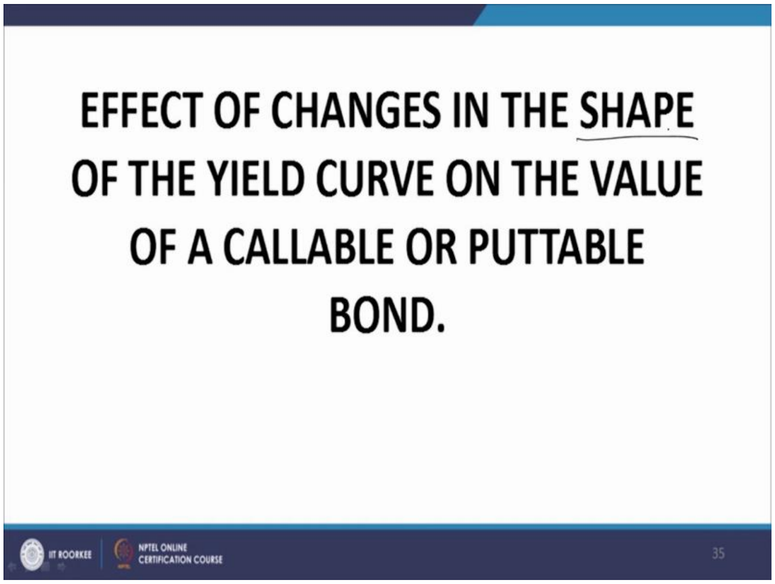

Effect of changes in the shape of the yield curve on the value of Callable or Puttable bonds. Effect of changes in the shape, please note we have talked about the volatility, we have talked about the level of interest rates and number now we talk about the shape of the yield curve on the value of the Callable or Puttable bonds.

Now it is generally the case that the yield curves which I shall explain in detail very soon but the yield curves are basically plots of spot rates or interest rates corresponding to various maturities we work out the interest rates corresponding to maturity of one year, six months, one year, one and a half year, two years and so on and plot them as a graph and that is what constitutes a spot yield curve with the standard yield curve.

I repeat standard yield curve is the plot of spot interest rates of different maturities interest rates against corresponding maturities or rather putting it the other way around maturities with the corresponding interest rates along the y-axis.

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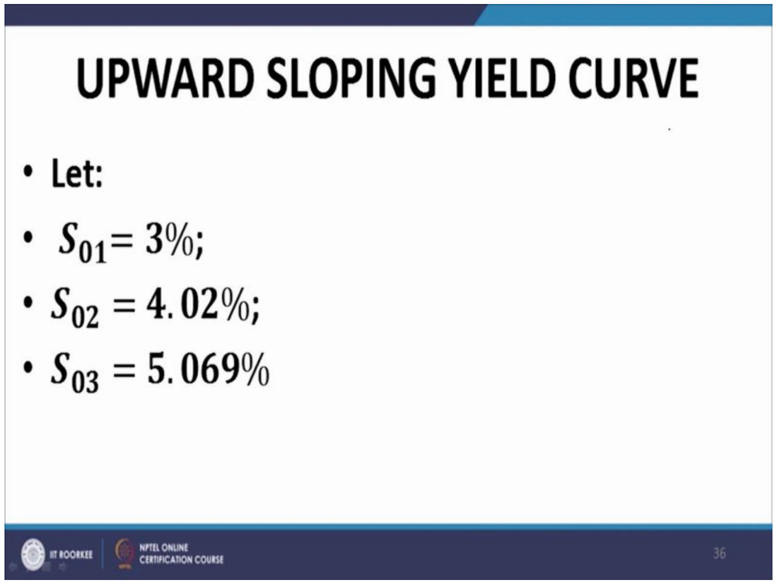

Now let us assume the following let us assume S01 that is a spot rate for a one year deposit to be 3 percent spot rate for a two year deposit let us assume it to be 4.02 percent and the spot rate for a three year deposit let us assume it to be 5.069 percent, these are given figures.

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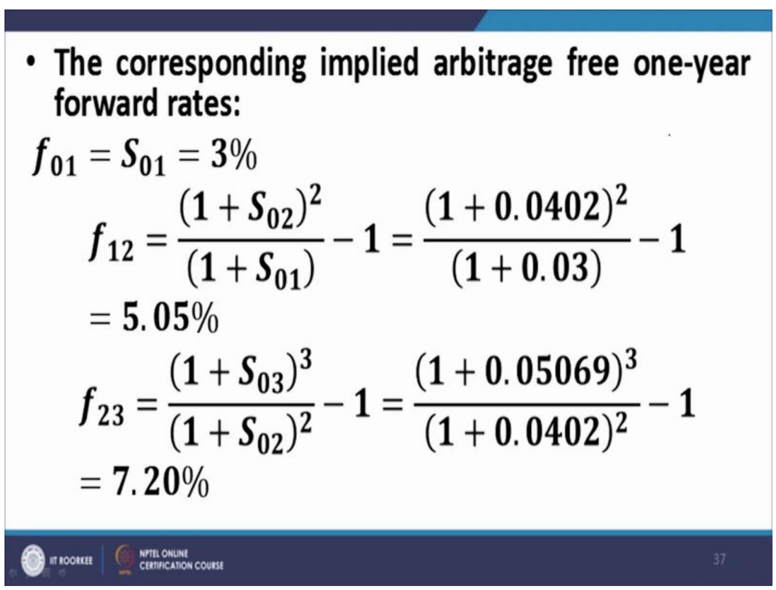

Then on the basis of these, if we work out the arbitrage free forward rates, what do we get? What we get is if s f01 is obviously coinciding with the spot rate which is 3 percent f12 that is a forward rate that operates between t equal to 1 to t equal to that is from for a deposit that is initiated at t equal to 1 and up to t equal to 2 that is 5.05 percent and what we find for f23 that is the forward rate that operates between t equal to 2 and t equal to 3 on the basis of arbitrage free pricing what we find is that turns out to be 7.20 percent.

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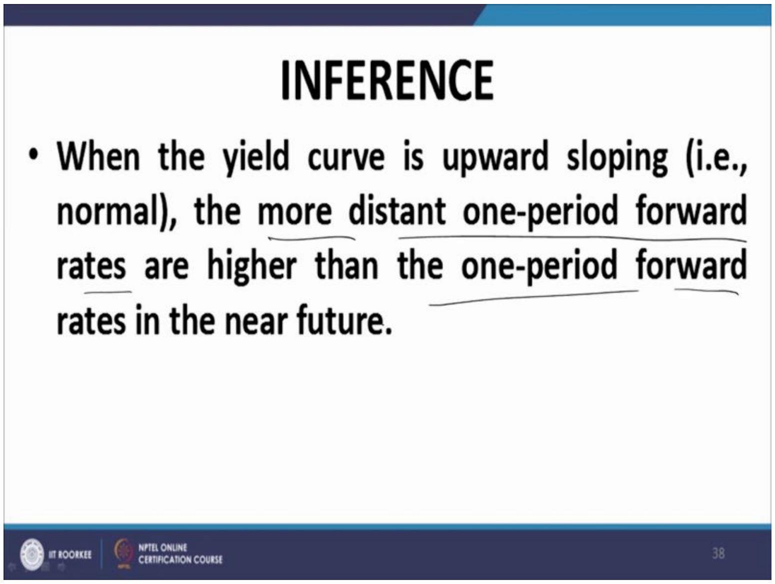

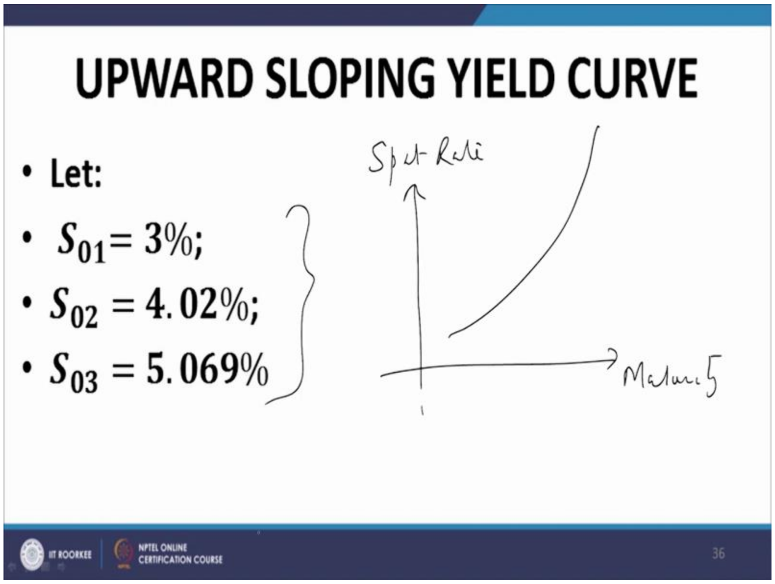

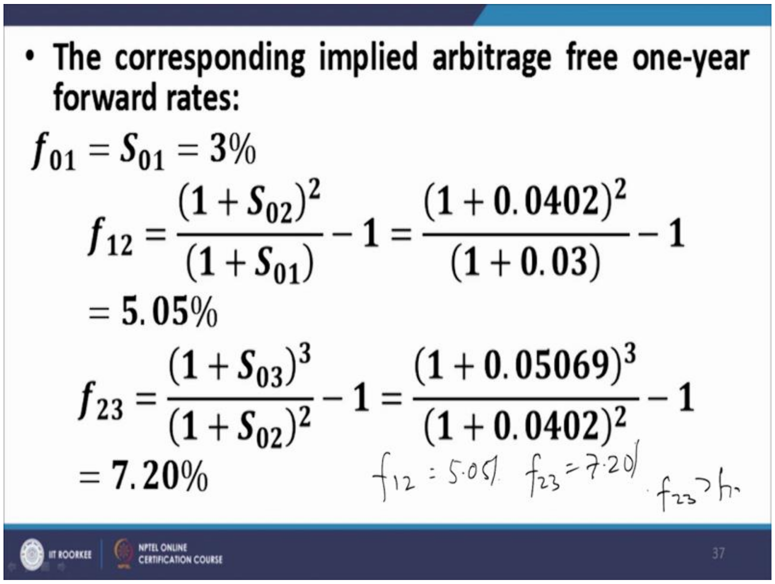

There is one thing what we observe is that as when the yield curve is upward sloping. Please note this is a upward sloping yield curve. If you look at this spots rates the spot rates are increasing with maturity spot rates increasing with maturity this is called an upward sloping yield curve with something like this.

This is maturity this is spot rate therefore it will be something like this. So, as this maturity increases as the maturity increases the rate of interest increases which is depicted in this example this illustration okay. So, when the yield curve is upward sloping as we saw just now the more distant the more distant one period forward rates are higher than the one period forward rates of the near future. As you can see here in this example, what do we have?

We have f12 is equal to 5.05 percent f23 is equal to 5 7.20 percent and clearly f23 is greater than f12. So, the more distance the rate relates to higher it is going to be compared to the rate which is at a closer to today provided the underlying premise is that it is an upward sloping yield curve it is an upward sloping yield curve this phenomenon relates to the upward sloping yield curve, the more distance you are in the future the higher would be the forward rate compared to the closer you are to today.

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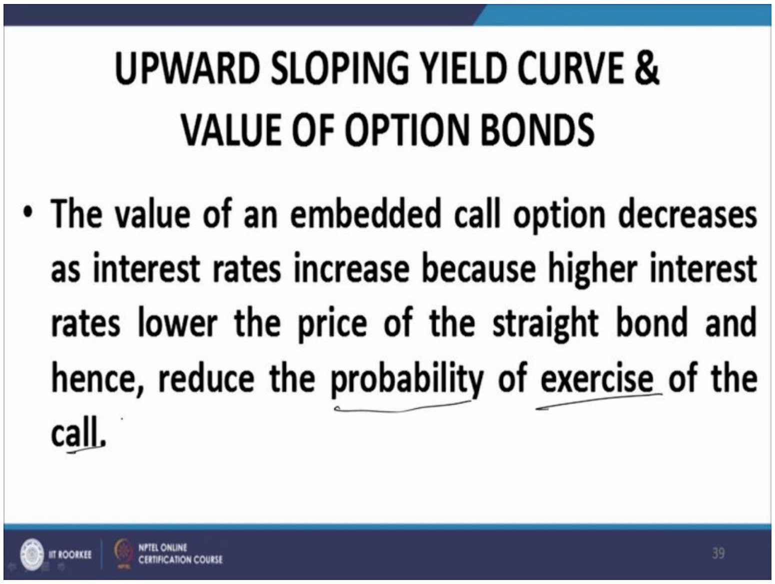

Now upward sloping yield curve and the value of option bonds. We shall discuss the value of option words or the impact of option impact on the price or the value of option bonds in the case of upward sloping yield curve. The value of an embedded call option decreases as interest rates increase. Because higher interest rates lower the price of the straight bond and hence reduce the probability of exercise of the call.

I have discussed this in a lot of detail already, as interest rates increase the prices of the bonds across the market decreases and as a result of which, if given a certain exercise price the chance of a bond x one price exceeding the exercise price reduces and that translates to a fall in the value of the call option.

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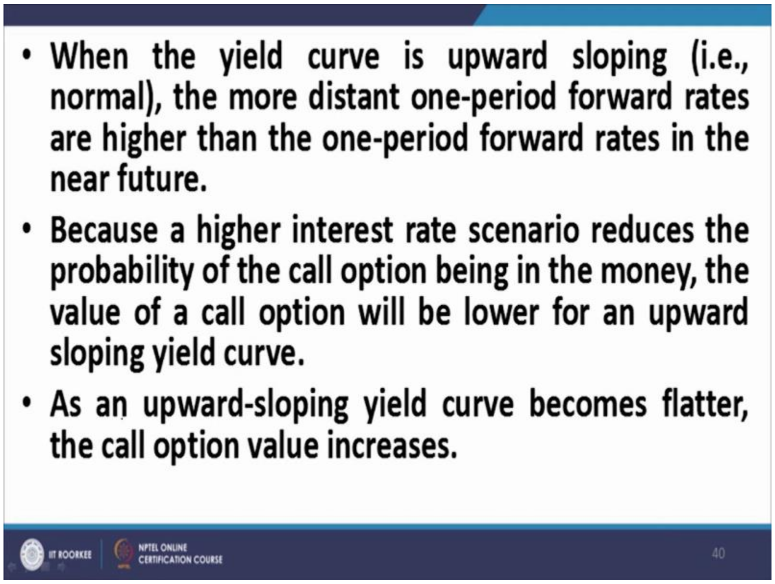

When the yield curve is upward sloping the more distant, one period forward rates are higher the more I have just explained this with the with by virtue of that example, the more distance one period forward rates are higher than the near forward rates or less distant forward rates. We saw that f23 was 7.20 percent f13 was 5 point some 02 percent I think and it was clearly seen that the f12 is much less than f23 or f23 is much greater than f12.

Because the higher interest rate scenario reduces the probability of the call option being in the money the value of a call option will be lower for an upward sloping yield curve. Because a higher interest rate scenario reduces the probability of the call option being in the money the value of the call option will be lower for an upward sloping yield curve.

As an upward sloping yield curve becomes flatter the call option value increases. So, what do we have on the one hand higher interest rates means lower prices of bonds, lower probability of exercise of the call option that is one impact. And secondly higher interest rates also mean that the call option will also decrease in value the bonds will also decrease in value.

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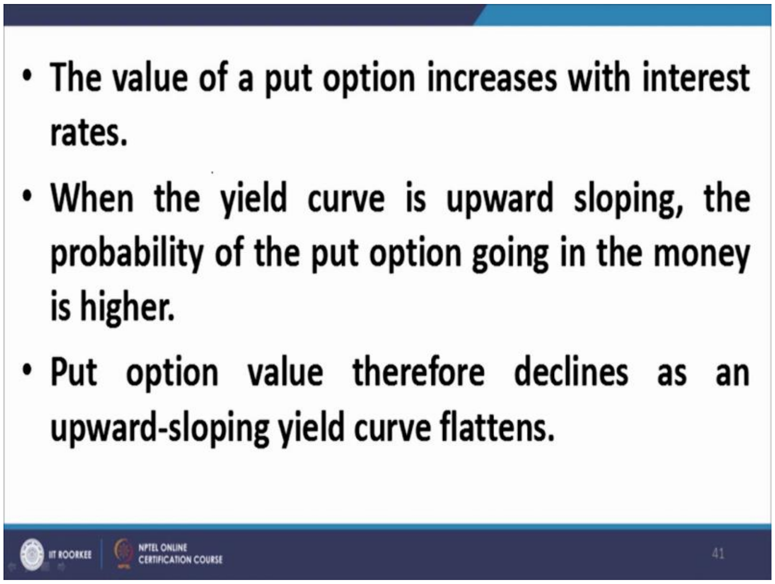

Thus the value of on the other hand, what we have for the put option, the value of a put option increases with interest rates. When the yield curve is upward sloping, the probability of the put option going in the money is more, why, because the prices of the underlying assets are going down they are suppressed.

And they are depressed because the interest rates are going up. So, if the interest rates increase the probability of exercise of the put option increases. And therefore the put value also increases put option value therefore increases when the curve flattens, what happens? The put option value decreases.

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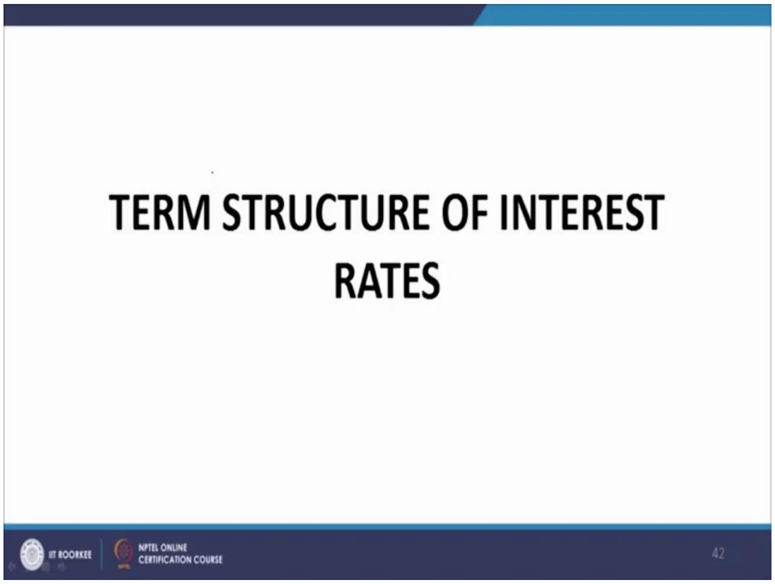

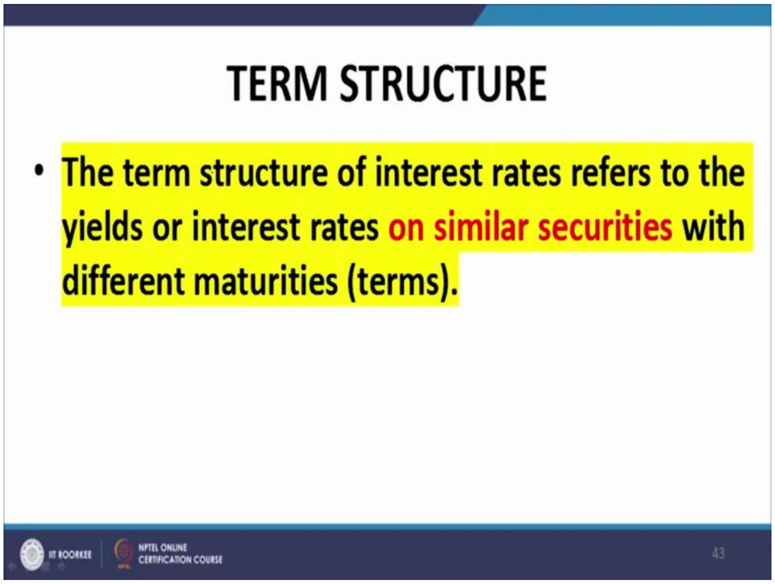

Now we talk about term structure of interest rates. What exactly is term structure? I have touched upon it in an earlier lecture. But basically when we talk about the term structure of interest rates what we are trying to say is that the interest rates are a function of the maturity of the instrument. The when you go to a bank and you make a deposit what happens depending on the tenure of their deposit depending on the tenure that you are planning to deposit or leave the money with the bank the bank offers you a spectrum of rates different rates corresponding to different maturities.

This is what is called a term structure of interest rates. So, basically interest rates are a function of maturity of the underlying deposit and that is what we call term structure. The term structure of interest rate refers to the yields or interest rates on similar securities with different maturities. So, this is important similar securities, similar in terms of the risk profile.

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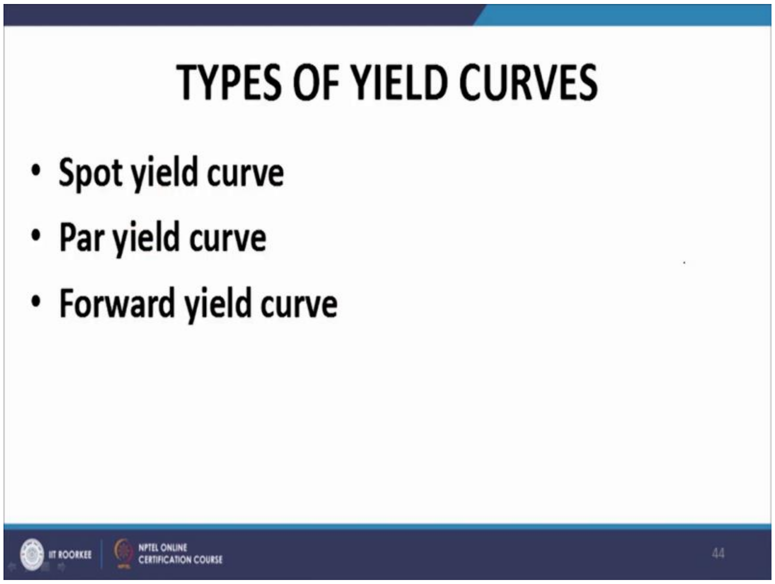

Now there are different types of yield curves I have touched upon the spot yield curve a few minutes back. We also have the power yield curve. And we also have the forward yield curve. Spot yield curve, this yield curve shows spot rates by maturity as I mentioned on the x axis we have maturity and on the y axis we have the corresponding spot rates. So, what are spot rates?

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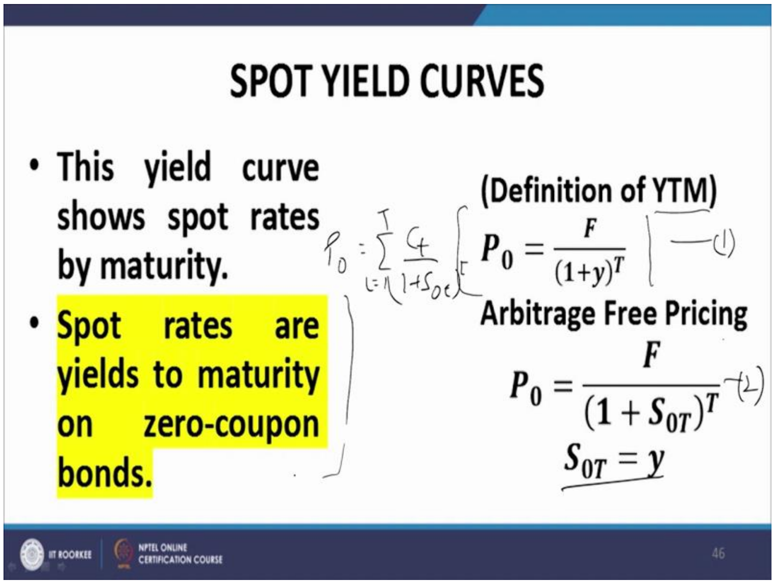

Spot rates are yield to maturity on 0 coupon bonds. And what is the yield to maturity? I will come back to it in a minute, but spot rates are basically yield to maturities on 0 coupon bonds. As you can see here from the definition of field to maturity which I will elucidate in a minute but which I have covered in a lot of detail in my course on SAPM security analysis and portfolio management and the interested learner I would advise could refer to those lectures.

But basically talking about YTM, YTM is that discount rate which such that when we discount all future cash flows attributable to a given instrument at that discount rate, we arrive at the current market price of the instrument. Let me repeat for the convenience of learners, it is that discount rate such that when we discount all future cash flows emanating from a security at that discount rate we arrive at the current market price of the bond.

So, for a 0 coupon bond which has no intermittent cash flows only the maturity cash flows we arrive at this formula for YTM. Because all the intermediate cash flows are 0, we just have the initial price the cash outflow that when you take a position in the security and the cash inflow when you get the maturity proceeds from the check security at the end of its tenure.

And therefore we have this equation all the other terms will be 0. Now also we have using the formula for arbitrage free pricing and adapting that formula to the context of 0 coupon bonds. Let me write down the formula using arbitrage free pricing we have p0 is equal to summation Ct upon 1 plus S0t to the power t t equal to 1 to capital T.

If you use this formula, so put in all C1 C2 C3 equal to 0 except for the maturity cash flow that is at capital T and that is equal to the face value, we are assuming that, and then we get this expression. Let us call this equation number 1, let us call this equation number 2. Then clearly on comparing equation number 1 and 2 we get S0t is equal to y and that is the rational for this definition.

Spot rates are the yield to maturity on 0 coupon bonds. And what are spot yield curves spot yield curves gives you the, gives you the plot of various spot rates corresponding to different maturities. The maturities are along the x axis and the corresponding spot rates are along the y axis.

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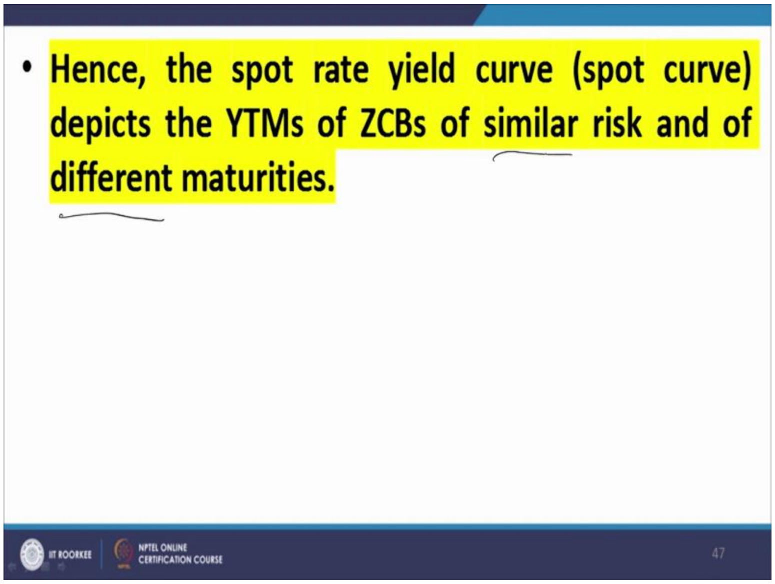

Hence the spot rate yield curve spot curve depicts the YTMS of 0 coupon bonds of similar risk and of different maturity different maturities, similar risk but different maturities.

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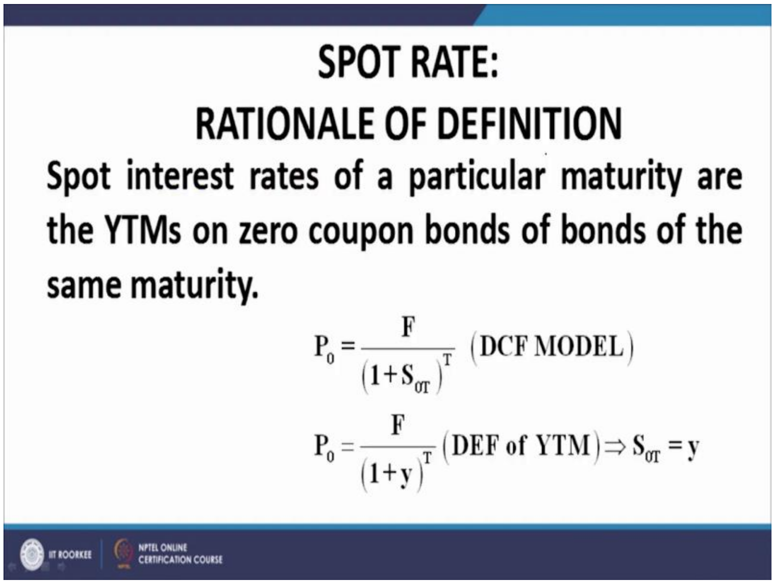

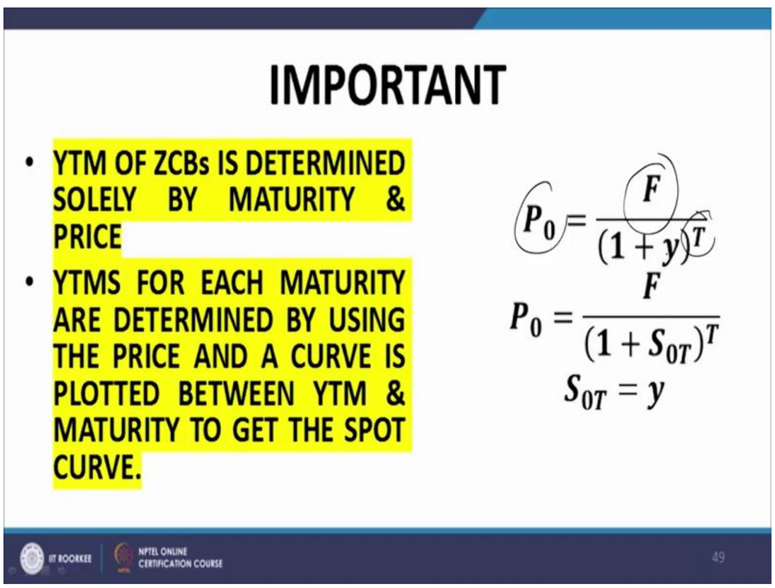

The rational I have already explained here. YTMS of 0 coupon bonds are determined solely by maturity and price. You can see here there are only two variables there assuming that this is fixed. We have only p0 and capital T y is a function of p0 and capital T, the current price and the term to maturity. YTMS for each maturity are determined by using the price by using the price we work out the YTM for a one year bond for a two year bond usually it is at six months of six month bond one year bond 1 point five year bond two year one and so on.

We work it out on the basis of the price and 0 coupon bonds of appropriate maturities, this gives you this spot rates corresponding to those maturities and then we plot those spot rates with respect to the maturities to get the spot curve spot yield curve, we call it also the zero curve or the strip curve.

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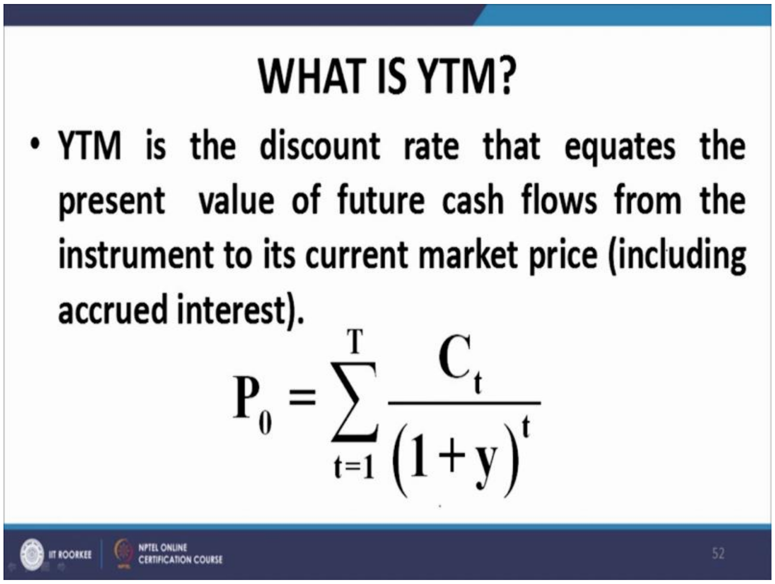

As far as YTM is concerned YTM I have already defined for you this is the explicit definition that I mentioned a few minutes back. YTM is the discount rate that equates the present value of future cash flows from the instrument to its current market price.

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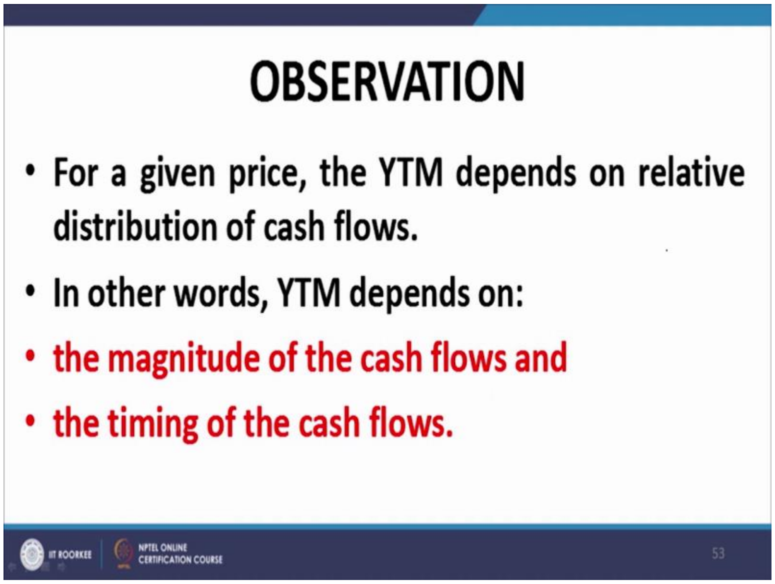

Therefore, for a given price, the YTM depends on what? YTM depends on the magnitude of the cash flows and the timing of the cash flows.

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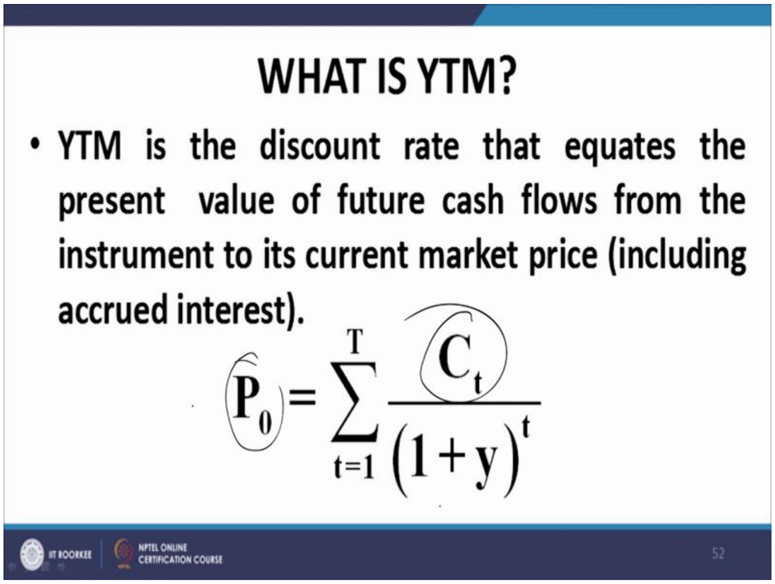

Obvious this is quite obvious from this it depends on C and t and given p0 and given the spectrum, given a particular bond it depends on the magnitude of cash flows from that bond as well as the timing of those cash flows.

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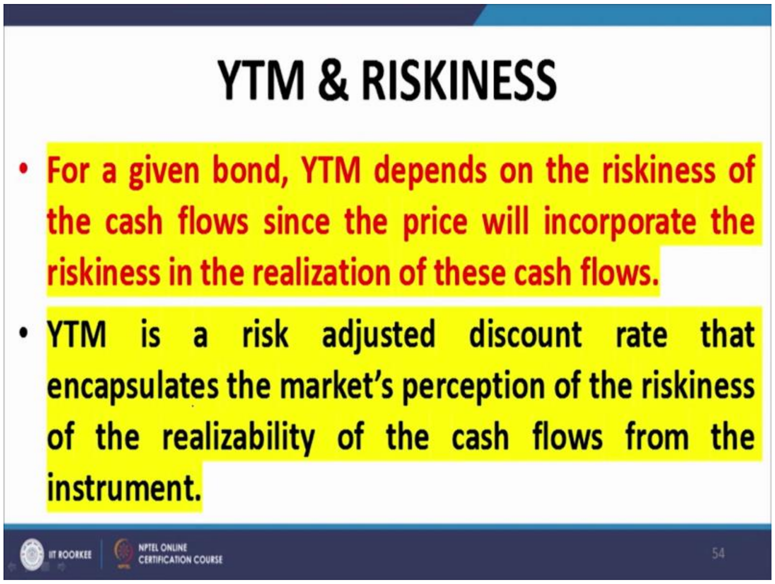

YTM and riskiness now this is important and this is interesting. For a given bond YTM depends on the riskiness of the cash flows since the price will incorporate the riskiness in the realization of these cash flows. You see the basic thing is that price is that riskiness is a fundamental variable when we do any kind of investment analysis as I mentioned we do it investment analysis on a two dimensional framework. The number 1 the expected return and number 2 the riskiness of the in the realization of that expected return.

So, the market has information about the riskiness of various securities that are traded in the market and as a result of which, the market incorporates that riskiness into the price of those securities. And therefore the price adapts itself to the riskiness and as a result of which the YTM depends on the riskiness of the cash flows, that are going to be generated from a particular security.

In other words, the YTM reflects the riskiness of the cash flows that are generated or that are expected to emanate from investment in a given security, when because the price will adjust itself accordingly. YTM is a risk adjusted discount rate that encapsulates the markets perception of the riskiness of the realizability of the cash flows from the instrument, this is what I have explained just now.

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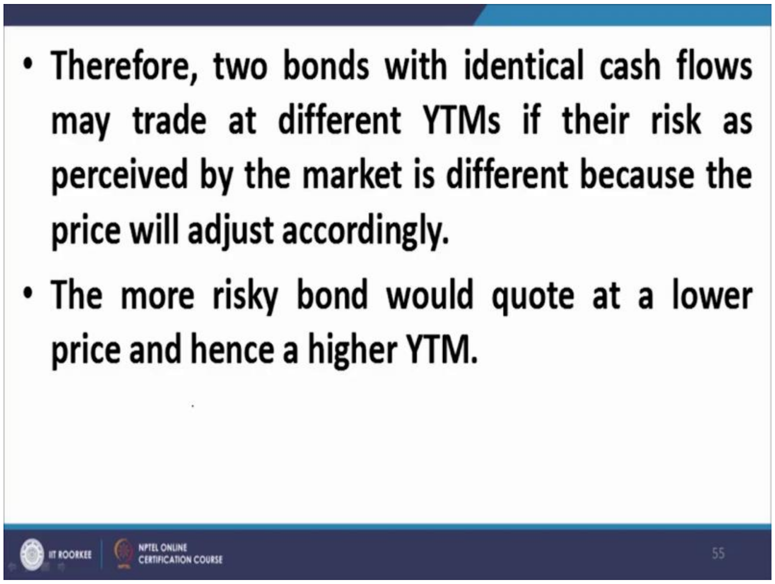

Therefore, two bonds with identical cash flows may trade at different YTMS if their risk as perceived by the market is different because the price will adjust accordingly. So, if you have two identical bonds which absolutely identical cash flows in terms of timing and magnitude you may still find that those bonds are being traded at different prices because the prices will encapsulate riskiness.

And therefore because they are being traded at different prices although they are identical in terms of the timing and magnitude of cash flows they will return different YTMS. In other words, the inferences that two cash flows which are identical but have different riskiness in the realizability of those cash flows will yield different YTMS. The more risky bond would quote at a lower price and hence the higher YTM.

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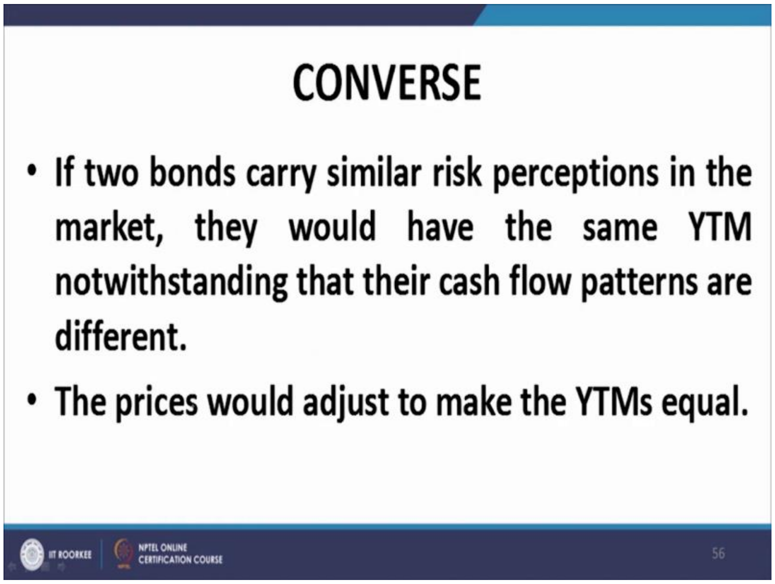

Similarly, if two bonds carry similar risk perception. This is more interesting, if two bonds carry similar risk perceptions in the market they would have the same YTM notwithstanding that their cash flow patterns are different. This part is very interesting, if you have two bonds with different cash flows but with identical risk in the realizability of cash flows the prices will align themselves in such a way that the YTM or the discount rate that is the that we call the YTM happens to be the same.

Prices will organize themselves in such a way, you see the basic thing is that this discount rate or the YTM whatever we call it is representative of the return that an investor gets from a security. So, and this return is obviously a function of the riskiness in the realizability of the return. So, the bottom line is that if you, if two bonds they have carry similar risk perceptions they will give you the same YTM notwithstanding the fact that they have different cash flows.

Why? Because the prices will adjust accordingly. Suppose you have two points A and B, A A has a cash flow of 1 unit over the three years 1 and 1 it is an annuity of three years 1 unit the B has an annuity of 2 2 and 2 at the end of year, 1 2 and 3 is an annuity of 2 2 units of money then and they are identical in terms of their realizability risk.

Then what we will find is that the price of B bond is towards twice the price of the A bond. The prices will adjust themselves in tandem with the cash flows if they have the same riskiness the YTM will remain the same.

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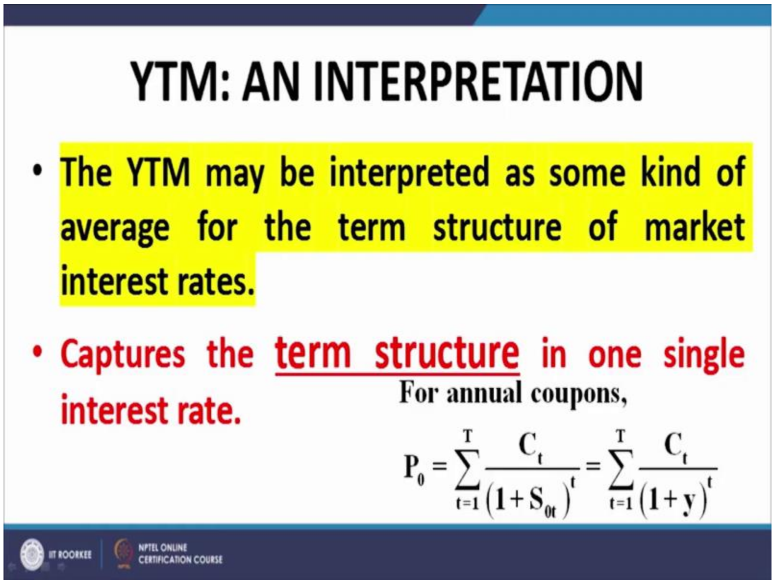

An interpretation of YTM this is where I will take up in the next lecture. Thank you.