Quantitative Investment Management
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Lecture: 11
Valuation of Bonds with Embedded Options

Valuation of bonds embedded with options:

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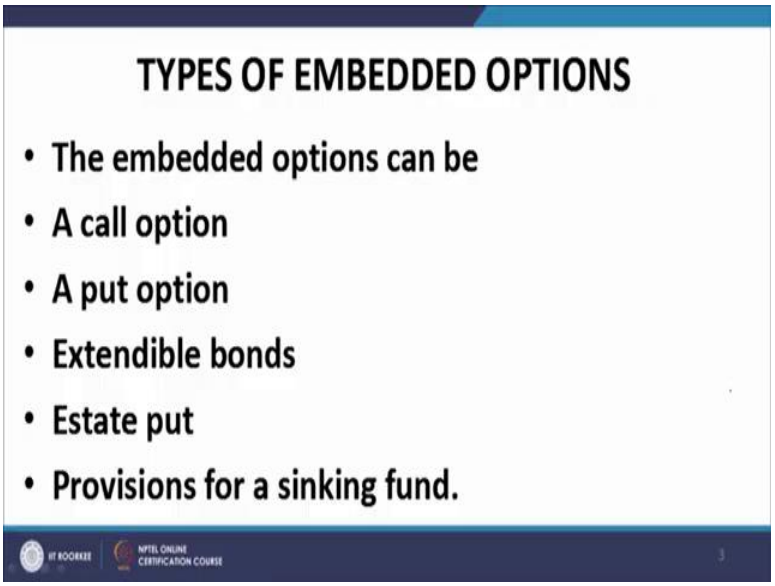

A quick recap before we continue so types of embedded options which can be embedded in bonds. Typically, we have callable bonds that have a callable option attached to the bonds which can be excised by the issuer of the bond if the interest rates in the market go fall substantially and as a result of it the issuer can consider the possibility of issuing a fresh debt by retiring the earlier debt which is at a higher rate of interest, higher contracted rate of interest that can be done by a invoking the call option if it is embedded in the debt instrument as per the document of issue.

Then we have a put option a put option entitles the bond holder the right to return back the bonds and ask for the repayment as per the terms contained in the issue document. We also have extendable bonds where the tenure of the bonds can be extended at the discretion or the choice at the option of the bond holder. Then there is the state put where on the demise of the original bond holder the heirs get a right to put back the bond that is to exercise the put option and return the bond and retire the debt from the issuer.

Then there is a possibility or there is an option which enables or which requires that the bond issuer maintain a sinking fund maintain set out or segregate out a certain amount from the profits of the company each year and in invest them in approved securities to be used for the retirement of the of the bond that have been issued by the issuer. So, these are typical options that are contained or that may be attached to bonds.

But the important thing that I would like to mention here is that all these options including the terms of exercise thereof, the price at which they can be exercise, the manner and the timing of the exercise will all be contained in the offer document. Everything has to be pre-specified at the time of issue of the bond, it is not that one fine morning the issuer of the bond gets up and involve and says that I have a call option and I will exercise the call option.

No, that is not the case. The case is that whatever rights or specialties are attached to these instruments attached to this bonds must be specified in the contract of issues so that the investor can take an informed decision.

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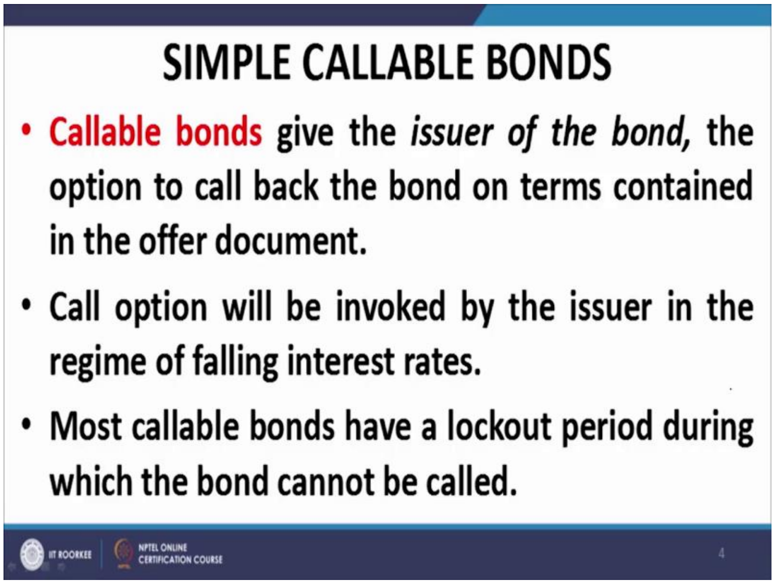

Simple callable bonds: Give the issuer of the bond, the option to call back the bond on terms contained in the offer document, call option will be invoked by the issuer in the regime of falling interested because as I mentioned the issuer would be interested in replacing the existing debt which is at a higher rate which was contracted at a higher rate by a fresh debt, which is now could be contracted at a lower rate because of the following interest rates.

Most callable bonds have a lockout period during which the bond cannot be called. This is the standard practice.

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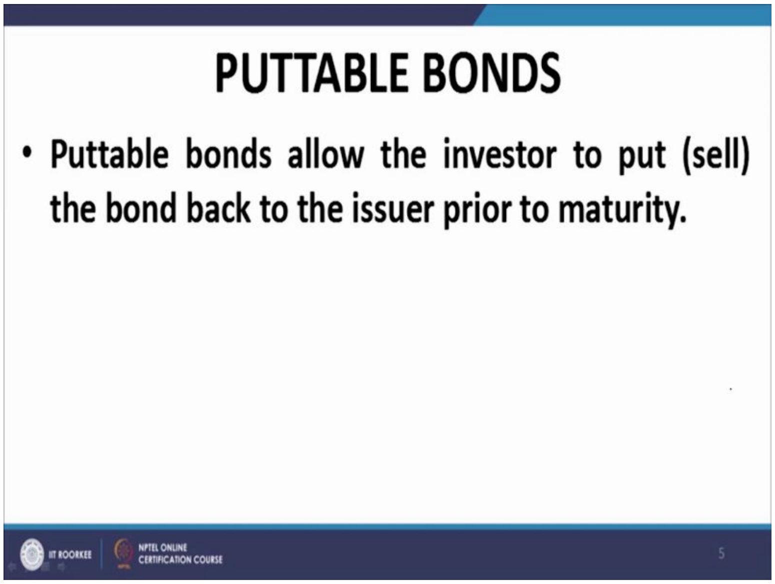

Puttable bonds: Puttable bonds allow the investor to put or sell back the bond to the issuer and ask for a repayment of the amount due on the bonds as per the terms contained in the offer documents which give the issuer the put option right. Now obviously the put option would be exercise when the interest rates are increasing. Why is that?

Because increase in interest rate; firstly, it would operate to reduce the market price of the bonds, interest rates and market price of the bonds are inversely related, that is one thing. And the second thing is that the holder of the bond could retire or could ask for the repayment of the bond as per the exercise provisions of the put option and invest the proceeds or invest the repayment proceeds in another avenue, which may provide a relatively higher return.

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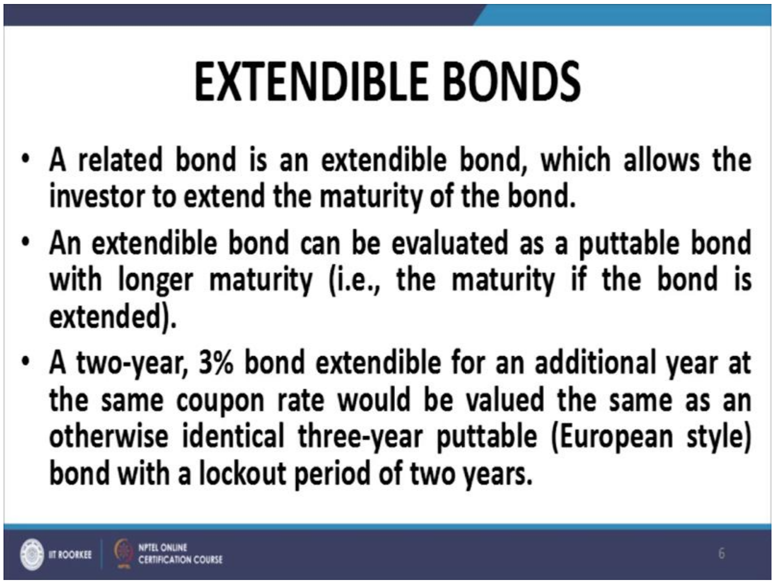

Then extendible bonds, I explained, bonds which are extended or which are extendable at the discretion of the bond holder as per the terms of issue are called extendible bonds. Straight put also I have explained a few minutes back in sinking fund also I have just now explained.

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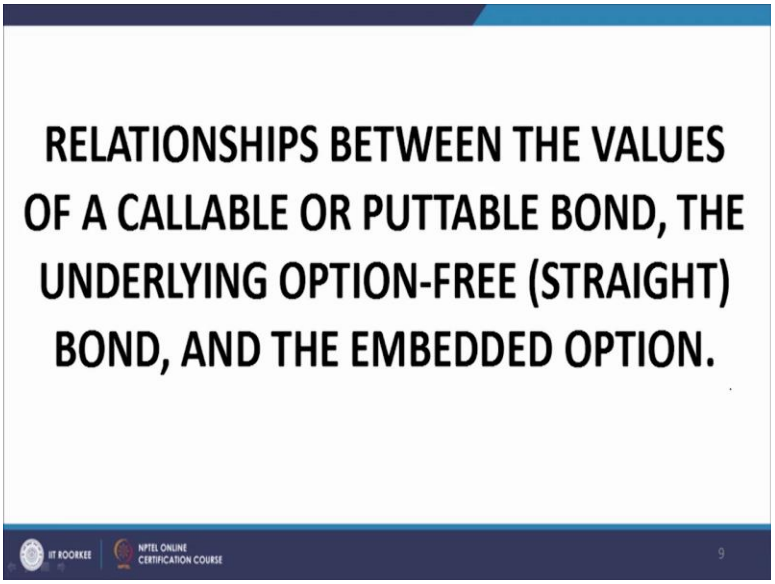

Now relationship between the values of a callable or puttable bond, the underlying option-free that is called the straight bond and the embedded option.

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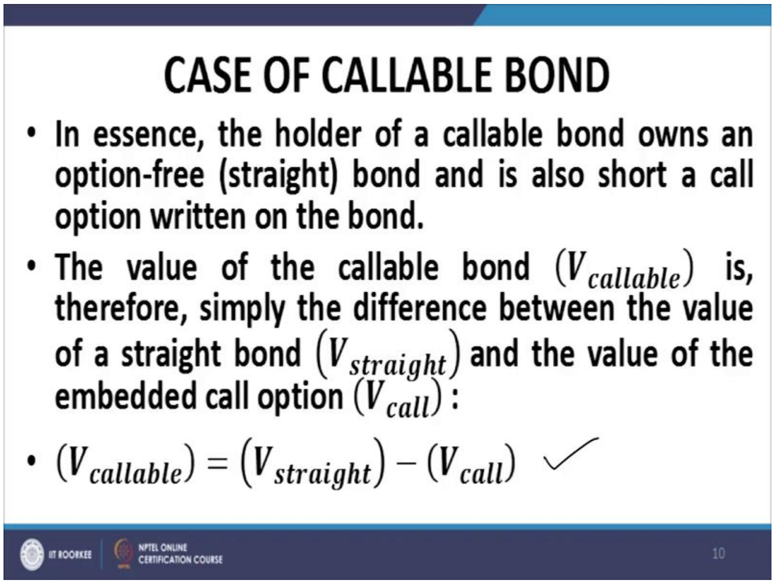

Well, in so far as the callable bond is concerned, obviously the call option confers a right on the bond issuer. Therefore, we can say that the bond holder is short in the call option, as a result of which what will happen is that the value of the callable bond would be less than the value of the straight bond by the amount of the value of the call option embedded in that particular bond and hence we have this particular relationship.

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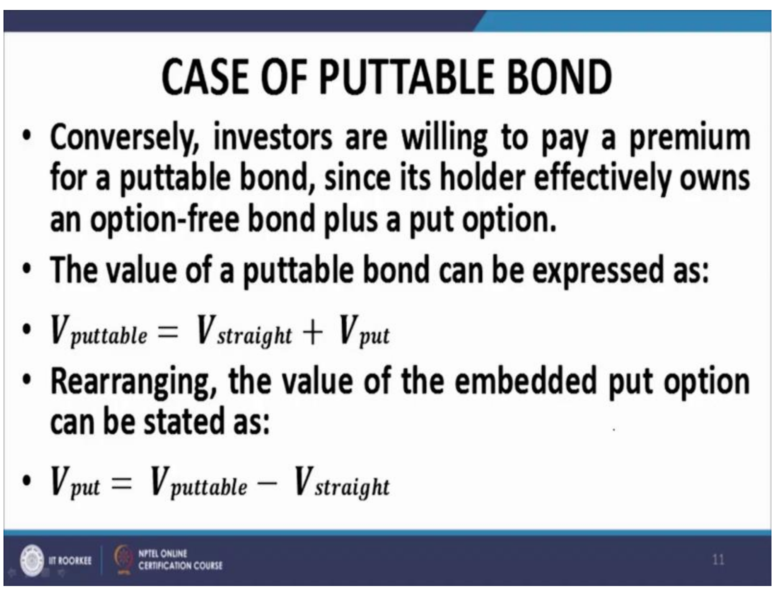

In the case of a puttable bond it is the other way around. What happens is that the investor or the bond holder has a discretion, has the power, has the right, has the choice to return back the bond as per the terms in the issue document by exercising the put option. And therefore, by virtue of this right he would be willing to pay a slightly higher price than the price of the straight bond is long in the put option, in other words and as a result of which the relationship that would operate in this case is given by this expression.

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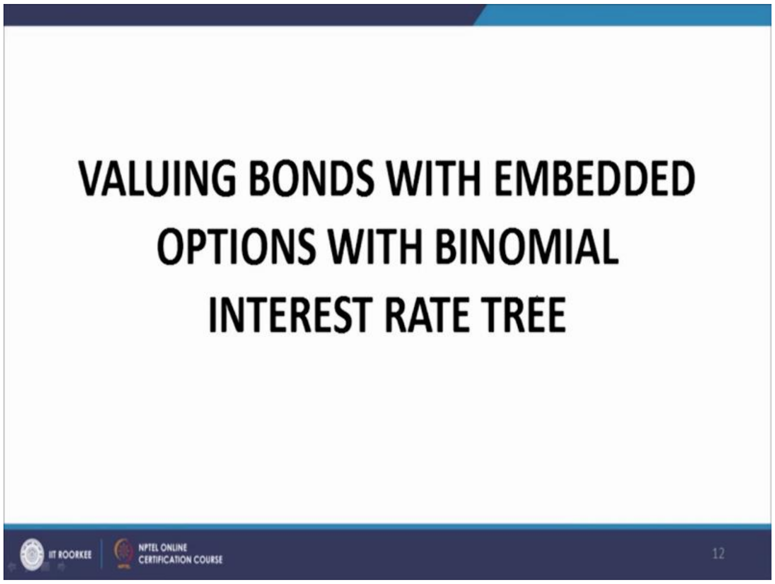

Now we start talking about the valuation of bonds with embedded options using the binomial interest rate tree. I have discussed in detail the valuation of straight bonds that is the valuation of bonds which do not have any options, any embedded options in them using the binomial interest rate tree.

Today what we will do is we will talk about bonds which have embedded options, the valuation of bonds which have embedded options in them using the binomial interest rate tree.

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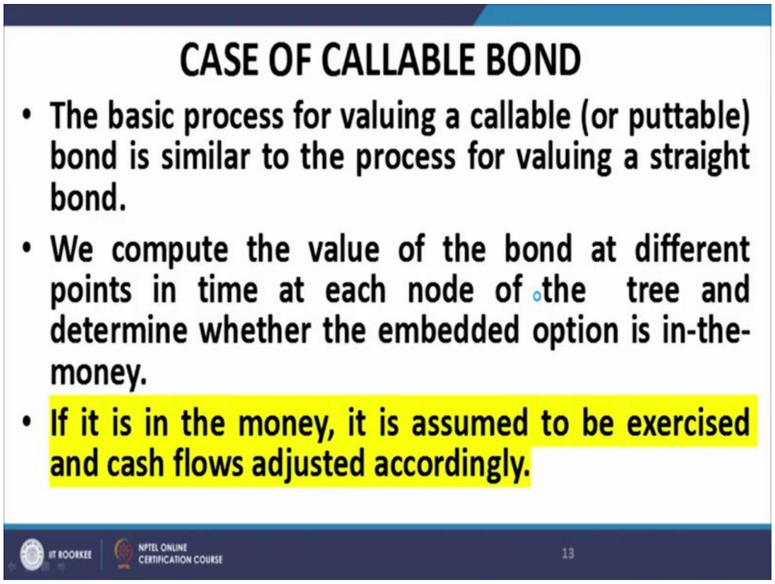

The basic process for valuing a callable or puttable bond is similar to the process for valuing a straight bond which we have discussed in the preceding lecture. We compute the value of the bond at different points in time at each node of the tree and determine whether the embedded option is in the money or out of the money.

Now the important thing is in the case of the straight bond we need not consider any options because there are no options attached to the bond, there are no options embedded in the bond. However, when we talk about bonds with embedded options, what happens is that we have to consider the value of the state bond at each node of the interest rate tree. And then compare it with the strike prices of the various options embedded in the bond.

And thereby, if we find that at any point of time the option becomes exercisable, we shall assume that the option is exercised and we shall adjust the cash flows accordingly. So, if the option is in the money, at any node the value of this straight bond turns out to be such that compared to the exercise price of the bond.

It is such that the party, either party who is holding them right, who is holding, who is long in the option in the case of the call option the and the issuer and in the case of the put option the investor, whatever the case may be if the option becomes exercisable, if the option is in the money, then what happens is that we assume that the option is exercised and what it is, the cash flows are adjusted accordingly. It will be more clear, when we take up an example on this particular issue.

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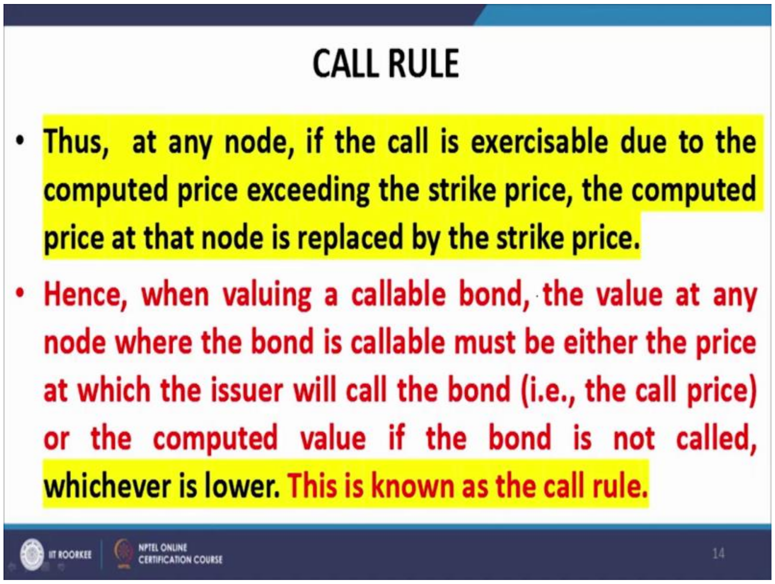

Now as far as the call is, callable bonds are concerned, as far as bonds which have a callable option embedded in them is concerned. Let me recall these bonds confer a right, confer a discretion on the issuer of the instrument to call back the debt, to retire the debt and repay the investor as per the terms of exercise of the call option at a point in time which is earlier to the normal order to the specified tenure of the bonds.

So, in this case what is happen is if at any node, if at any node if the call becomes exercisable, that is the computed price of the bond becomes more, the price at which the call option becomes exercisable is less, then the call would be in the money and while valuing the bond at that particular node will assume that the issuer will exercise the call option. And as a result of it that particular valuation at that node will be replaced or the computed value at that node will be replaced by the exercise price of the call option.

So, let me read it out for you - Thus, at any node, if the call is exercisable due to the computed price exceeding the strike price, the computed price at that node is replaced by this strike price. Because we assume that the issuer of the bond who is long in the option, who has the right, who has the discretion, will exercise that discretion, will exercise the call option and as a result of which he will call back the bond at the exercise price notwithstanding the fact that the computed price of the bond is higher.

Hence, when valuing a callable bond, the value at any node where the bond is callable must be either the price at which the issuer will call back the bond, that is the exercise price of the option or the computed price if the bond is not called, whichever is lower. I repeat, at every node we will compare the computed value with the exercise price of the call option at that relevant to that particular node.

And we will see whether the exercise price is lower than the computed price both at a given node and if it so happens that the exercise price is lower, we shall replace the computed price with the exercise price on the premise that the issuer of the instrument will exercise the call option and call back the bond at that particular node or if that particular node materializes. This rule is called the call rule.

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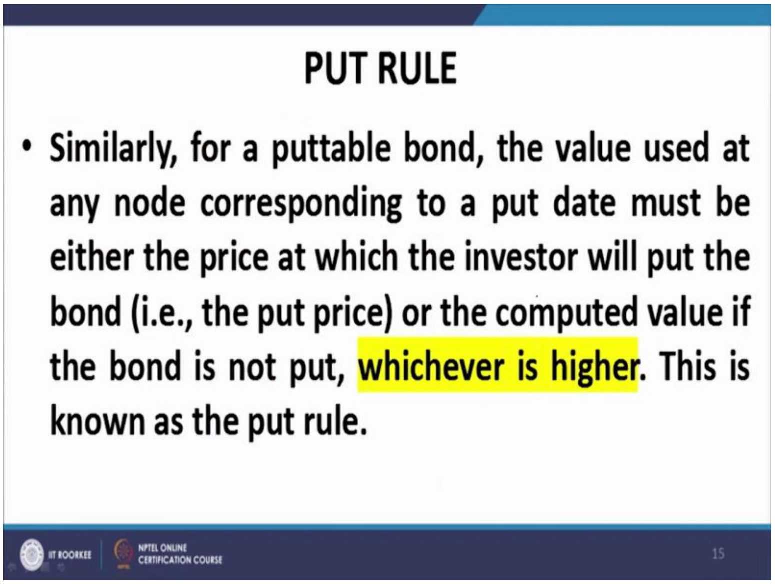

Similarly, we have the put rule. Let me read it out for you again. Similarly, in the case of a puttable bond, the value used at any node corresponding to a put date must be either the price at which the investor will put the bond or the computed price if the bond is not put, whichever is higher. Because what is the put option? A put option confers the right on the investor to return the bond to the issuer and demand back the repayment of the relevant amount of debt.

Now obviously the put holder or the investor who will be the same party, the investor who happens to be long in the put and is also a put holder, therefore, the put holder or the investor will exercise the put option only if the price of the bond in the market that is or the computed price that is lower and the put option price or the exercise price of the put option is higher, so that he can get a higher value for the bond corresponding to that node.

He will exercise the option and he will get a higher value. The value that is the exercise price of the bond, exercise price of the put option and therefore, when we value such a bond at every node we compare the exercise price of the put option with the computed price and if the exercise price turns out to be higher than the computed price, we shall replace the computed price with the exercise price which as I mentioned is higher.

If of course, it turns out that at any particular node the computed price is higher, the exercise price is lower, then obviously the option will not be exercised, will not be deemed to be exercised and as a result of it no adjustment, no correction needs to be made. So this is called the put rule.

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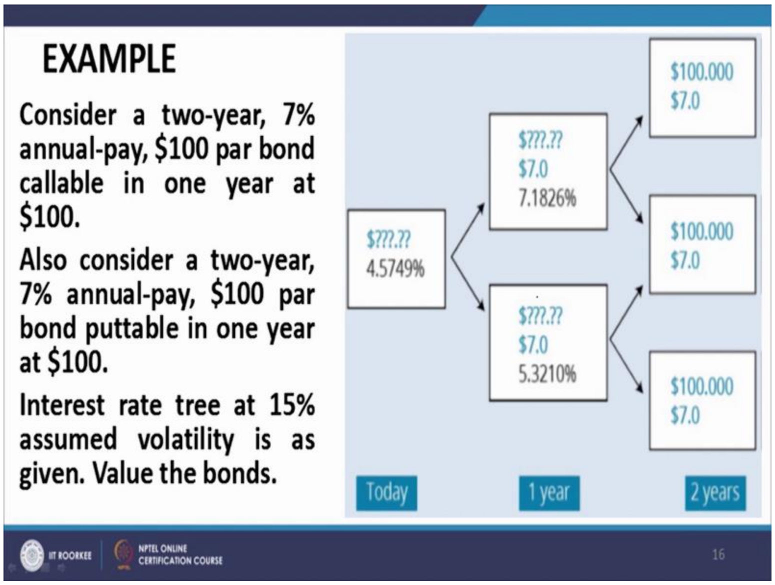

We now illustrate both the call rule and the put rule by an example. Let us consider this example; part of which we have done earlier. Consider a two-year 7 percent annual pay dollar 100 par bond callable in one year at 100. Also consider a two-year 7 percent annual pay in dollar 100 par bond; that is the same bond, but in this case instead of the call option it has a put option embedded in it and which is also exercisable at the strike price of 100.

Interest rate tree at 15 percent assumed volatility is given in the right hand side panel. You can see it here on the right hand side panel. We need to ascribe a value to the bonds. Let us see the solution now.

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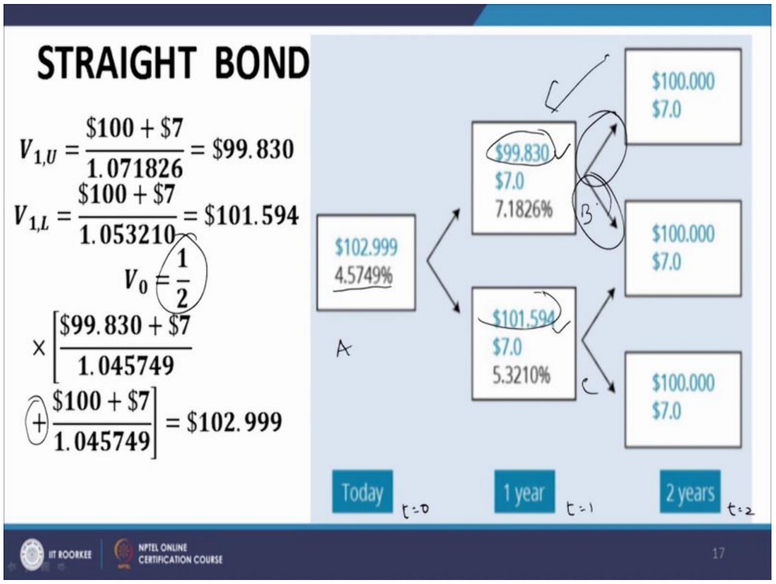

We start by valuing the straight bond. We ignore the options and as we had done earlier in an example, this quite simple, the cash flows that occurred, this is t equal to 0. Let me write it down. This is t equal to 0. This is t equal to 1, and this is t equal to 2, t equal to 0 that is today, t equal to 1 that is 1 year from now and t equal to 2 that is 2 years from now.

Now at t equal to 2 the cash flow that is going to arise from; the again we use the method of backward induction, at t equal to 2 the cash flow that is going to arise on the bond is going to be equal to the redemption value that is 100 assuming that the bond is redeemed at par value and the coupon payment at t equal to 2 that is 7. So the total cash flow here at t equal to is 107 as you can see in the box itself.

In all the three scenarios, whatever be the interest rate scenario, the cash flow at t equal to 2 will be 107. Now the interest rates for the period t equal to 1 to t equal to 2 that is f 1 2, the forward rate operating in the region, in the range of t equal to 1 to t equal 2 that is for the second year, you are starting at t equal to 1 and terminating at t equal to 2, the forward rates can take any of 2 values. The two values are 7.1826 percent and 5.3210 percent.

The forward rate at, let me repeat, the forward rate at t equal to 1 that is the rate that is relevant for the period from t equal to 1 to t equal to 2 can take any of these two values that I mentioned just now. So, we get two values corresponding to the these two interest rates corresponding to these two discount rates; by discounting 107 at 7.1826 we get a valuation of dollars 99.830.

And if we discount 107 at the interest rate of 5.3210 percent we get a valuation of 101.594. Let us call this node A, let us call this node B and let us call this node C, for the purpose of identification. Now, so this is the value of the bond. Please note at the moment we are valuing the straight bond so there is no issue relating to options, we are ignoring the embedded options for the moment.

Now we have got the valuation of the bond at the node B. Now, but at the node B, in addition to this value, see this value that we have now is simply the value that has been calculated on the basis of the pay of the payment of 107 at t equal to 2. It does not include the interest or the coupon payment of 7 that is going to occur at t equal to 1.

I repeat, this very important point in solving this problem. The valuation that we have got here, both of them 99.830 and 101.594, both these valuations have been obtained by discounting 107, which is the cash flow at t equal to 2 at the relevant interest rates, relevant forward rates. So therefore, these values do not include the coupon payment that we are going to get at t equal to 1.

Therefore, when we are going to discount these values back to t equal to 0 for arriving at today's value of the bond, we will have to add the coupon payments at t equal to 1, which do not form part of this value. So the discounting that what we will have to do for going from t equal to 1 to t equal to 0 will be cash flows of 99.830 plus 7 that is 106.830 and the rate that would operate would obviously be this rate, 4.5749 percent.

And similarly, when we discount the proceeds at node 6, at node C, I am sorry, at node C back to node A, we will consider the value of 101.594 plus dollar 7, that is 108.594 and we will discount it again at the rate of 4.5749 percent. Please note this rate 4.5749 percent operates with respect to both the nodes B and the node C.

It is relevant to the discounting of the computed values of the proceeds at node B as well as at node C. So these, the evaluation process or the calculations are given in the left hand panel and what we arrive at is v0 is equal to 102.999. Of course, in this case when we go from node B and node C to node A, will have to take the average of the two valuations, the valuation that is arrived at by discounting the value at node B.

Including the coupon payment at node B and to t equal to 0 and the value that we have from the node C back to node A, we need to take the average of both these values, which is shown here. This is the averaging process, we divide the sum of the two evaluation, the sum of the two evaluations here by 1 by 2. That was not required when we moved over from node 2 to node 1, because the cash flows were the same and the discounting rates were the same.

If you look at these 2 legs of the tree, these 2 branches of the tree, the figures are identical. 100 and 7 is discounted at 7.18226 percent when we go from here to here and the same is done when we go from here to here. So, we need not take the average in that case. Both the values are identical, so if you take the average you still get the same value.

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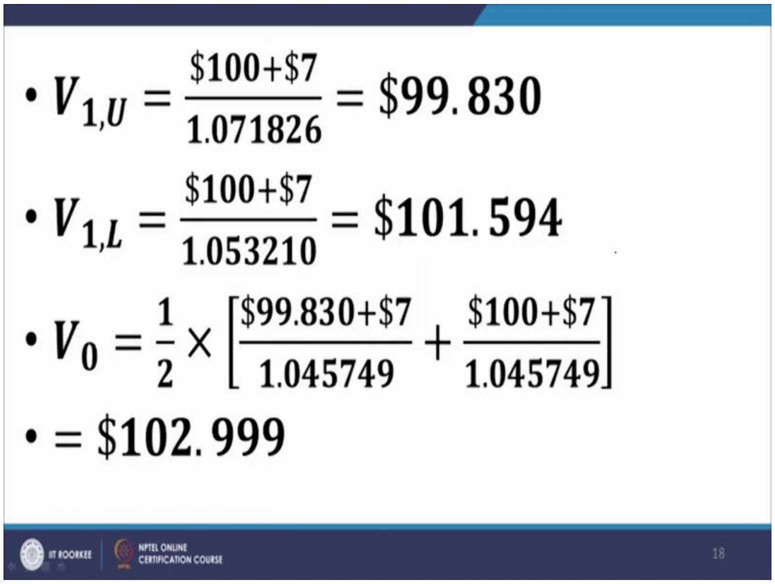

Now we do the, this is the calculations of the whatever I have explained just now.

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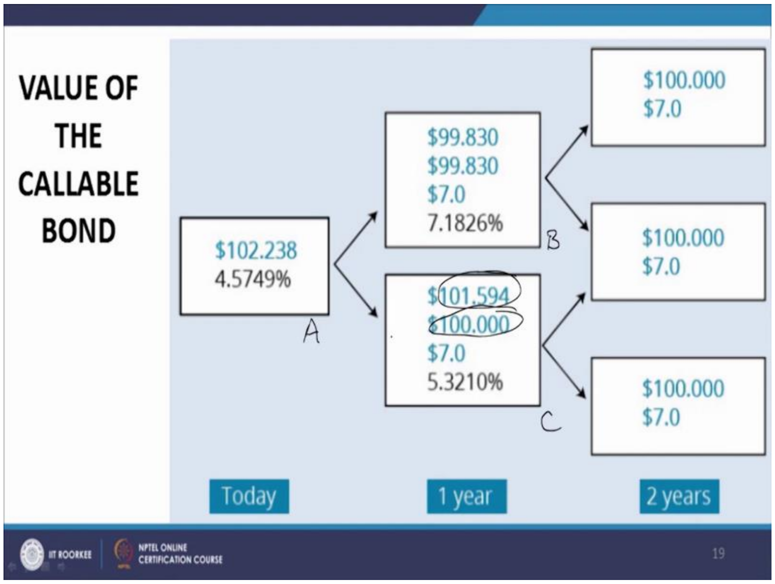

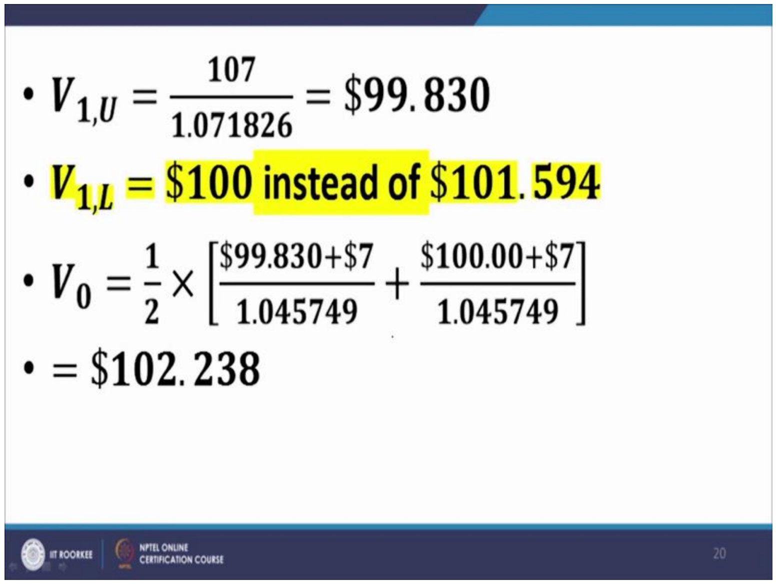

Now we talk about the callable bond. What is the callable bond? Let me recap. A callable bond gives the right to the issuer to call back the bond. We are given that the bond can be called back by the issuer at t equal to 1 at 100. That means what? That means if the price of the bond, if the value of the bond exceeds 100, then the issuer would prefer calling back the bond at 100. In other words he would prefer that he would retire the debt at 100 instead of the computed price which is 100, which is more than 100.

Now if you look at this carefully, if you look at this tree carefully, you find that it is at node C. Let me again rename, name the nodes; this is node A, this is node B, and this is node C. Now if you look at node B what you find is that the computed price that is 99.830 is less than 100. So obviously if the path followed by the evolution process of the bond price turns out to be such that it passes through node B, then the price of the bond would be 99.30.

And therefore, the call option will not be exercised. I repeat, it will not be exercised. Call option is exercised when the computed price exceeds the exercise price; here the exercise price is given as 100, the computed price is 99.83 and therefore, the call option will not be exercised. However, when we look at node C, we find that the computed price is 101.594, which is more than the exercise price of the call option embedded in the bond.

And as a result of which it would be in the interest of the issuer that he exercises the call option. And when he exercises the call option he has to pay an amount of only 100 and as a result of which this 101.594 will be replaced by 100 in our calculations of the value of the bond at t equal to 0. I repeat, we assume that because this call option is in the money at node C, because its computed value is more than the exercised price, we will assume that the issuer exercises the option, retires the bond at the exercised price which is 100.

And as a result of which the computed value becomes redundant at this point and we go backwards from here when we move from t equal to 1 to t equal to 0 with the call option exercise value of 100. And when we do that what we find is that the value of the bond at t equal to 0 turns out to be 102.238. Thus, here is the valuation process, what we find is that this 101.594 is replaced by 100 as you can see here this corresponds to node C. This is node C, this is node B, and this is node A. So, that is how a bond with an embedded call option is to be valued.

We compute the value as per the binomial tree assuming that it is a straight bond and then moving backwards from right to left we examine at each node, whether at that node the value of the bond that is the computed value of the bond exceeds the exercise price and if it does exceed the exercise price, we substitute that computed value by the exercise price on the premise, on the assumption that the issuer of the bond was long in the call option, was the right under the call option will exercise the call option.

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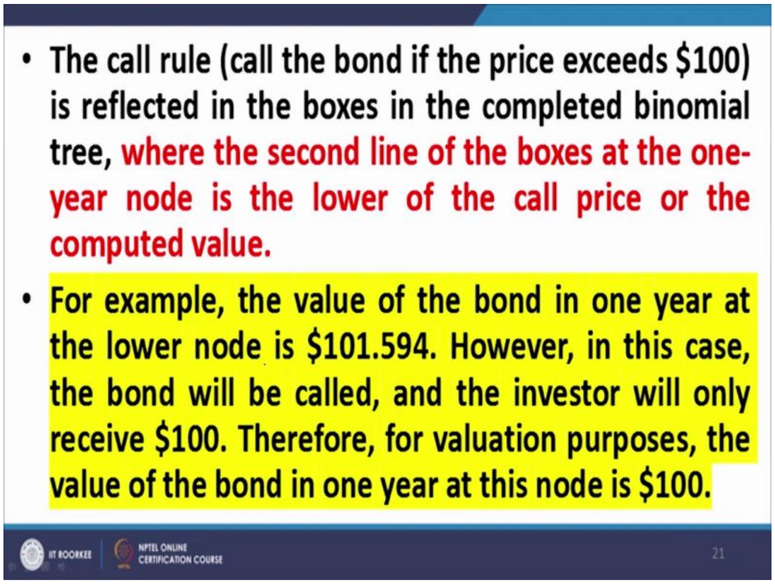

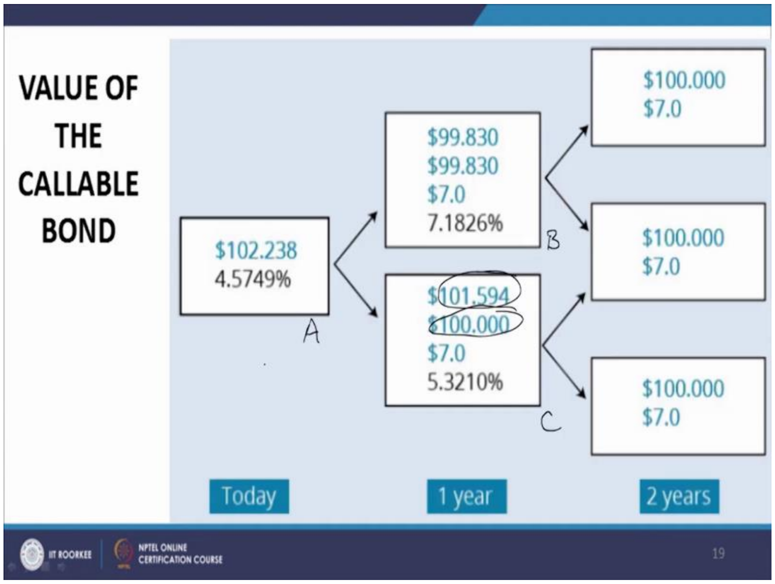

So, let me read it out what I have explained just now. The call rule call the bond if the price exceeds dollar 100 is reflected in the boxes in the computed binomial tree where the second line of the boxes at the one year node is the lower of the call price or the computed value. As you can see here in this this particular example.

Here you can see that the computed value is 99.83 and here we have the lower of the computed value and the exercise price which happens to be 99.83 because the exercise price is 100. Here the computed value is 101.594 and the exercise price is 100, therefore we replace the value here by the lower of the two which is the exercise price of 100. For example the value of the bond in one year at the lower node is 101.594 however in this case the bond will be called and the investor will only receive dollars 100.

Therefore, for valuation purposes the value of the bond in one year at this node is 100. Please note this point, this valuation is node by node basis moving from right to left in line, in tandem with the backward induction rule.

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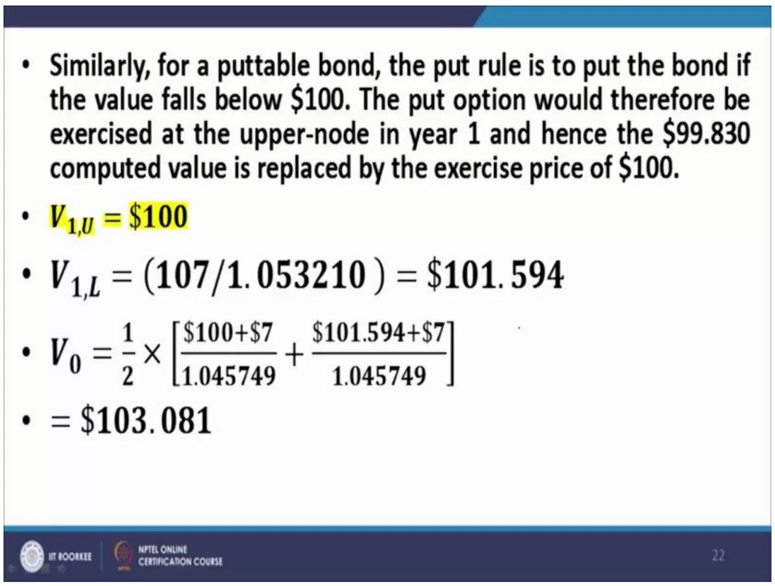

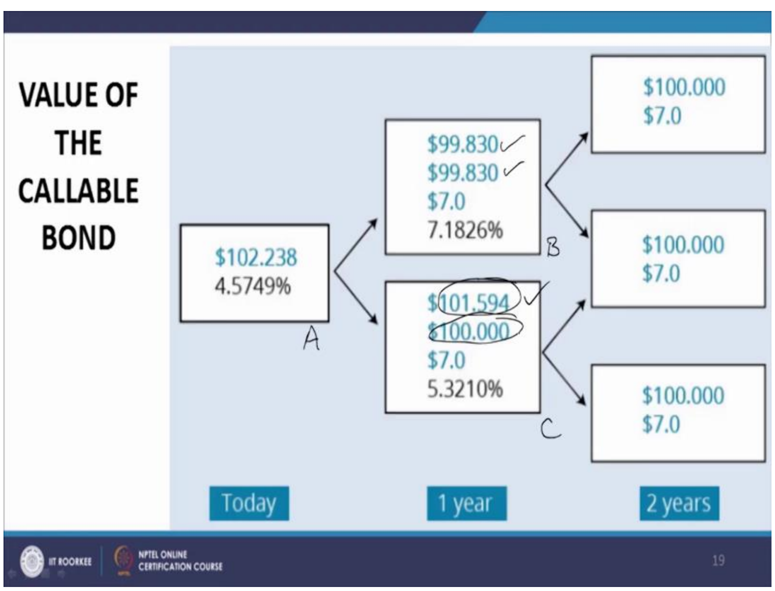

Similarly, for the puttable bond I leave it as an exercise for the student, here what we will do is we will replace the exercise price at the node where the exercise price turns out to be to be higher and the computed price turns out to be lower. As you can see here now look at this example let us go back to it. In this example if you look at the upper node that is the node B.

If you look at the node B, what you find is that the exercise price is 100 and the computed price is 99.83 and therefore, we presume that the investor who is long in the put option will exercise the put option, deliver the bond back to the issuer and receive the exercise price rather than the computed value of 100. And as a result of which by the valuation of the put option this 99.83 is replaced by 100.

However, when we look at the node C, what we find is that the exercise price is 100, the price of the bond or the computed value of the bond is 101.594. Therefore, the issuer of the bond, I am sorry, the investor in the bond will not exercise the bond and I will not exercise the put option and as a result of which here the figures will remain unchanged.

And that is what is depicted here. The value at the node B, this is node B, the value of the, at the node B is replaced by the exercise price, the computed price is 99.83, the exercise price is 100, therefore we replace the computed price by the exercise price. However, at the node C we do not do any such thing why because the computed price exceeds the exercise price. And then we proceed from t equal to 1 to t equal to 0 as we did in the earlier case.

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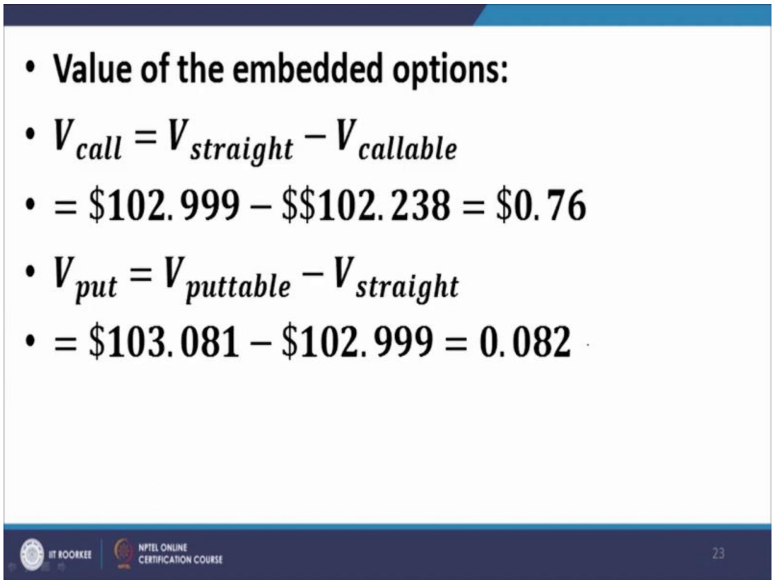

We can easily work out the value of the embedded options as far as the value of the call option is concerned, it is equal to v value of the straight bond minus value of the callable bond that turns out to be 0.76 and in the case of a put option value of the puttable bond minus value of the straight bond gives you the value of the put option which turns out to be 0.082.

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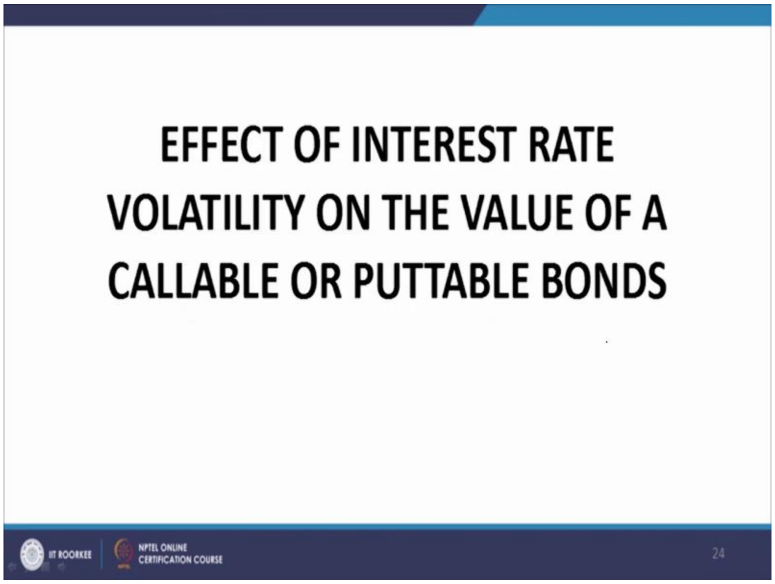

Effect of interest rate volatility on the value of a callable or puttable bonds. I repeat we now study the effect of interest rate volatility, interest rate volatility. What is volatility? Volatility is a major measure of the rapidity of fluctuations in a given variable. Primarily it is used to measure the fluctuations of the changes in stock prices, although its use in interest rates is also quite common. Basically it is the standard deviation per unit time that is what is volatility of a given stochastic given random variable.

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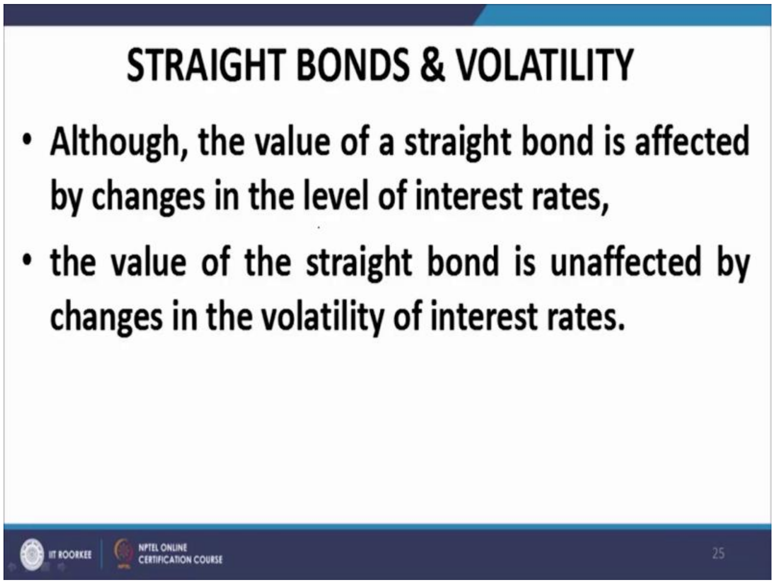

So as far as the straight bond is concerned the value of the straight bond is affected by changes in the level of interest rate. Please note this point these are things which are the nuances of all this and we need to take care of that. Obviously if the interest rates increase the value of a bond decreases, if the interest rate decreases the value of the bond increases that is quite natural.

Why does it happen? It happens because the, with the increase in market interest rates the required return of the investor increases and therefore, given a certain face value, given a certain redemption value he is not willing to pay less on the bond and as a result of which the market price of the bond declines.

However, if the interest rates fall the demand of the investor in terms of return on an investment decreases and as a result of which is willing to pay a higher price on the investment in the bond. Therefore, prices and market interest rates of bonds are inversely related, please note this fundamental thing.

However, the important thing for our perspective at the moment is that the value of a straight bond is unaffected by changes in the volatility of interest rates. The value of the straight bond is unaffected as far as the volatility is concerned. The value does change when the interest rate increases or decreases by the level corresponding to the increase or decrease.

However, the fluctuation themselves do not add value, do not result in any change in the value of the bond. However, as when we consider the value of options one of the, if you look at the Black-Scholes formula or even the basic philosophy of options, what we find is that greater is the fluctuations of the variables, of the underlying variable of an option greater is the uncertainty with respect to a certain price in the future of the underlying.

And as a result of which greater would be the amount that a particular investor or a trader would be willing to pay on the option position. Thus, what I am trying to simply say is that if the volatility of the underlying increases, the option value also increases, irrespective of whether it is a call option or a put option. Option values across the board increase when the volatility of the underlying asset increases.

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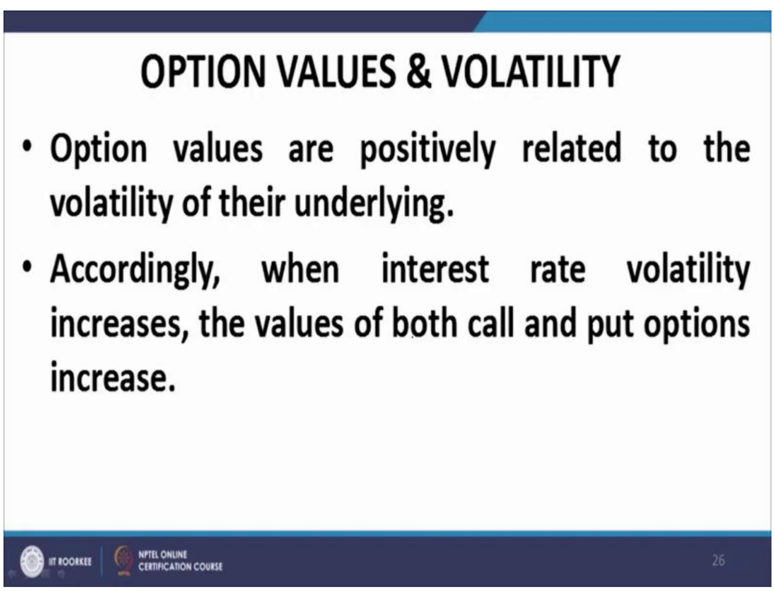

Therefore, when interest rate volatility increases the value of both the call and put option increases. Because the uncertainty as to the value or the price of the underlying asset on the date of maturity of the option becomes more, higher the fluctuations greater the frequency, greater the amplitude greater with the uncertainty associated with the asset or the underlying asset taking a particular value. And therefore, the options which are coverages for the risk resulting from such fluctuations also increase in value.

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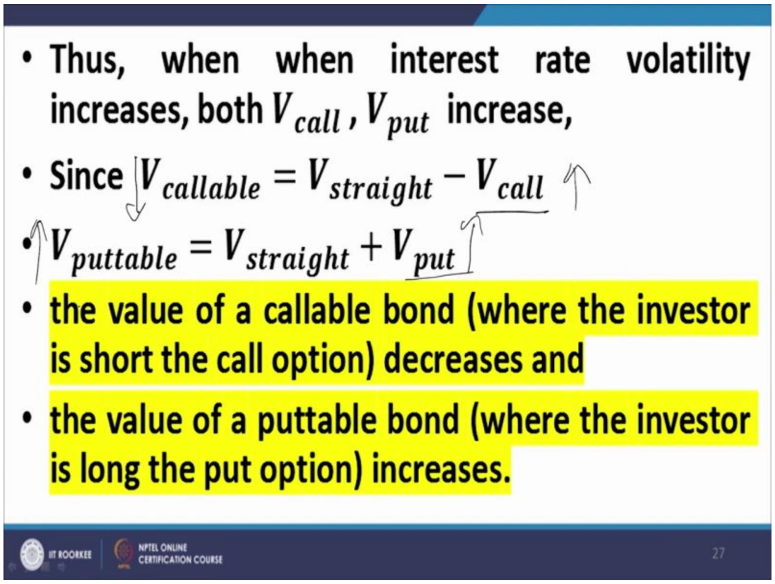

Therefore, when the interest rate volatility increases both, V call and V put increase, this increases as a result of it the value of a callable bond where the investor is short in the call option decreases because if this goes up, then obviously this would go down. And if this goes up the value of a callable bond where the investor is short in the call option decreases and the value of a puttable bond where the investor is long in the put option increases.

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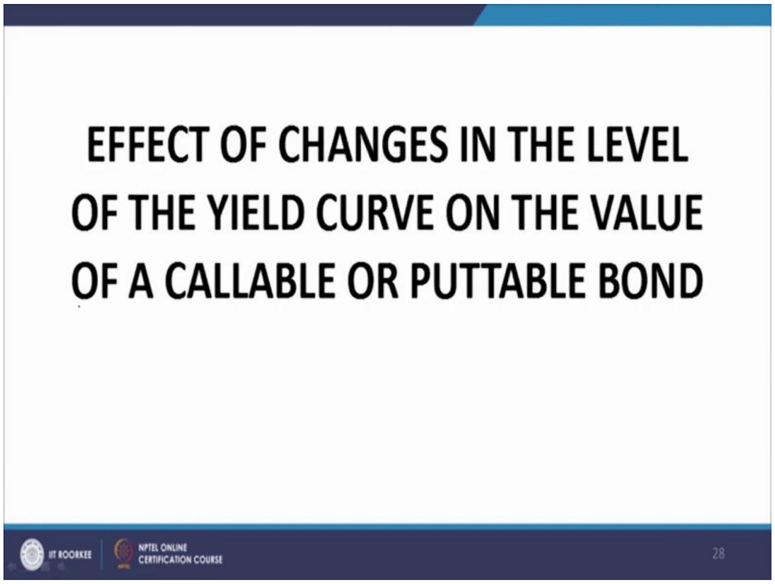

Then we talk about changes in the level we shall come back to in next lecturer. Thank you.