Quantitative Investment Management
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Lecture 10
Bond Pricing Contd….

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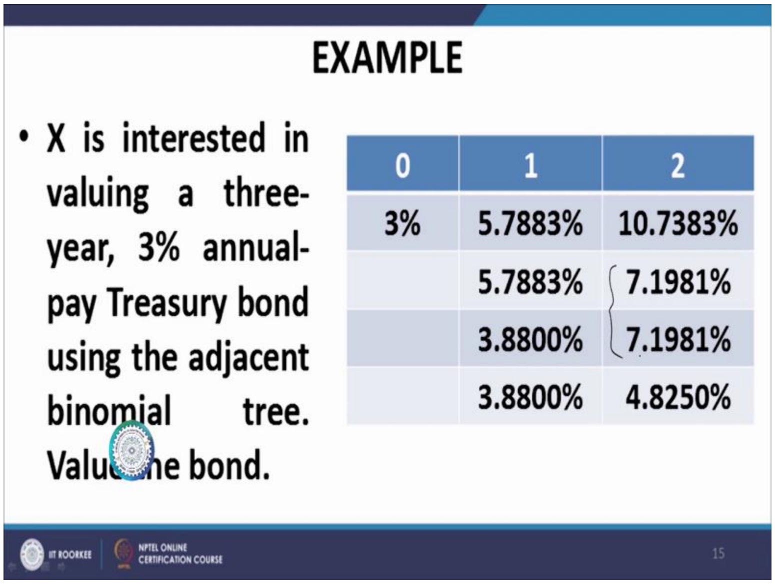

So, before the break, we were talking about this particular example, we have a given tree, and we have a 3 year 3 percent annual coupon treasury bond, and we need to ascribe a value to that bond.

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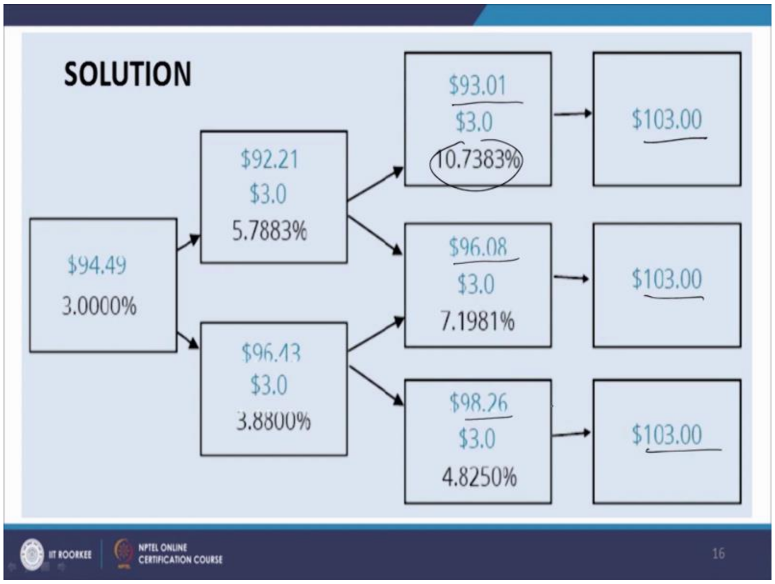

This is the graphical depiction of the tree at t equal to 0, the interest rate is 3 percent. This rate covers the period from 0 to 1, for the period from 1 to 2, the interest rates could take any of 2 values 5.788 percent or 3.88 percent. Both are equally likely and both are forward rates. Then at t equal to 2 for the period from t equal to 2 to t equal to 3, the interest rates could move either from 5.788 to 10.7383 percent, this would be the rate for forward rate for t equal to 2 to t equal to 3 deposit or it could go from 5.788 percent to 7.1981 percent for the period from t equal to 2 to t equal to 3.

Similarly, from the lower node at t equal to 1, the interest rate could go up to 7.1981 or it could go up to 4.825 percent. So, let us not talk about the valuation of the bond, we have discussed the interest rate tree that we have with us, let us talk about the valuation of the bond. Again we move we work by backward induction, we start working out the value of the bond from the last or the final cash flow ascribed to the bond or arising from the bond that is the maturity cash flows. Now, this is a 3 year bond.

So, we start with the cash flow at t equal to 3 years. At t equal to 3 years what cash flow are we going to get? We are going to get back a principal that is 100 assumed of course, and we are going to get a coupon payment of 3 percent on that face value of 100 that is the total cash flow that will occur at t equal to 3 103.

And, this cash flow is going to occur irrespective of what the interest rates. So, in each case, we have a cash flow of 103. Now, what let us work out the value of this cash flow at t equal to 2 corresponding to each path that it could take or the interest rates could take for the period from t equal to 2 to t equal to 3.

That means, if this particular value materializes, then we have to discount 103 at this rate, and we arrive at the value at t equal to 2 of 103 divided by 1.10783, which is this value. So, this is the value at t equal to 2. Let me repeat, this is the value at t equal to 2, if the interest rate from t equal to 2 to t equal to 3 takes the value 10.7383 percent.

If the interest rates do not take this value, if the interest rate is the value 7.1981 percent at t equal to 2 for the period from t equal to 2 to t equal to 3, then this 103 will be discounted at 7.1981 percent. And the value that will that we will arrive at a t equal to 2 corresponding to this interest rate is 96.08. And finally, if the interest rate at t equal to 2 for the period t equal to 2 to t equal to

3, take the values 4.8250 percent, then this 103 will be discounted at this rate, and we shall arrive at a value of 98.26.

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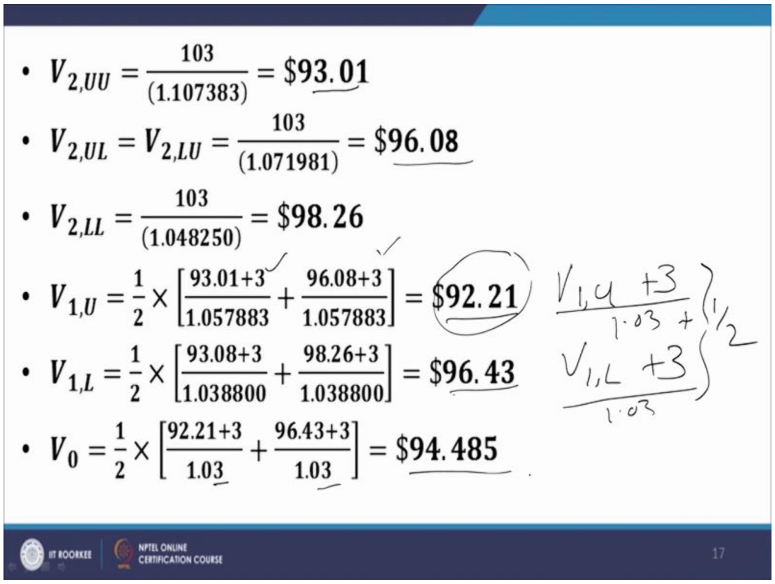

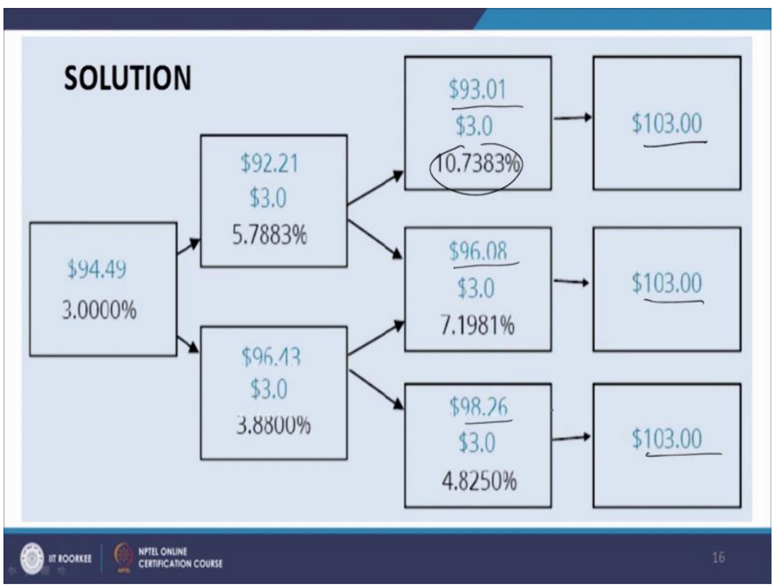

This is shown in this particular diagram 93.01 corresponding to the interest rate of 10.7383 percent, 96.08 corresponding to the interest rate of 7.1981 percent and 98.26 corresponding to the interest rate of 4.8250 percent. Now, we move one step back backwards, we have got the values of the bond at this at t equal to 2, but at t equal to 2 there will be a coupon payment as well of 3 rupees or 3 units of money. So, that has to be added to the valuations that we have received from there discounting the value at t equal to 3.

In other words, the value at t equal to 2, which we need to carry backwards for discount through discounting or for discounting will be equal to the value that we have obtained from t equal to 3 through discounting plus the coupon value that is there at t equal to 2, because this additional cash flow is also going to occur, this is going to add value to whatever is the value of the bond at t equal to 2 by discounting the cash flow that is going to occur at t equal to 3 using the interest rate f23.

So, in each of these cases, we need to add the coupon rate of 3 percent and then discount the total cash flow. For example, 93.01 will become 96.01 and then it will be discounted at 5.7883 percent. Similarly, 96.08 plus 3 it will become 99.08 and then it will be discounted at 5.7833 percent. So, we get as a 2 valuations corresponding to V 1U and we take the average of these two values to arrive at the value of the bond at t equal to 1 upper node.

Similarly, we do the valuation for t equal to 1 lower node by discounting 96.08 plus 3 that is 99.08 it at the rate of 7.1981 percent and 98.26 plus 3 that is 101.26 at the rate of 4.825 percent and we arrive at the values at the lower node by taking the average of these two values at the lower node at t equal to 1. This is given here, V1U is equal to 93.01 this value that is plus this is the coupon payment and the discount rate is 57883 percent 5.7833 percent I am sorry, and the second valuation is taken away 96.08 and this again this coupon is added of 3 units of money and again that is counting it is at 5.7883 percent and we get 90.

And then we take the average of these two, and we get the value at t equal to 1 upper node and that is this valuation. Similarly, we get the value at t equal to 1 lower node that is equal to 96.43. And finally, we bring it back from t equal to 1 to t equal to 0 using the spot rate which is 3 percent.

Here is the spot rate of 3 percent here is the spot rate of 3 percent. And again, because we are going to have a coupon payment at t equal to 1 of 3 units of money, whatever the value is here, for example, this at the upper node, this is V1,U this is V1,L and we need 3 units to add 3 units on account of the coupon payment that occurs at t equal to 1.

And then discount this whole thing by the rate spot rate that is 3 percent and then take the average. So, this is how this figure of 94.485 which is the value of the bond at t equal to 0, this is how we arrive at the value of this bond using backward induction. So, this example, comprehensively illustrates the method of backward induction.

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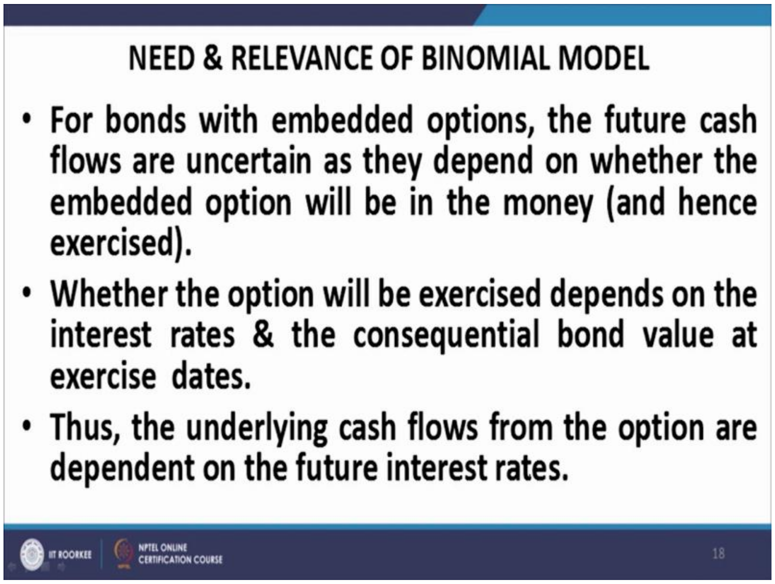

Now need and relevance of the binomial model. I had briefly touched upon this for bonds with embedded you see this model as primary applications the main purpose of developing this model is to ascribe a value to bonds which are embedded with some kind of options. So, for bonds with embedded options, the future cash flows are uncertain as they depend on whether the embedded option will be in the money and hence exercised.

So, if they if the bond is embedded with a call option or a put option, then depending on what the price, estimated price or expected price at a particular node is we need to determine whether the option that is option feature that is attached to the bond is exercisable or not and if it is exercisable, then obviously, we need to reorganize or modify the cash flows that are going to arise from the bond because the exercise of the option will naturally change the cash flows that are incorporated or that are embedded in the straight bond.

So, whether the option will be exercised depends on the interest rate because interest rates have a direct impact on the prices of the bonds, higher the interest rate, lower the price and vice versa. So, depending on what the interest rates are, at that point in time at with the option become exercisable, it needs to be decided, or it would determine whether the option ends up in the money or out of the money and if the option is in the money, naturally, the presumption is that it would be exercised. So, whether the option will be exercised depends on the interest rates and the consequential bond value at the exercised rates. Thus the underlying cash flows on the option are dependent on the future interest rates.

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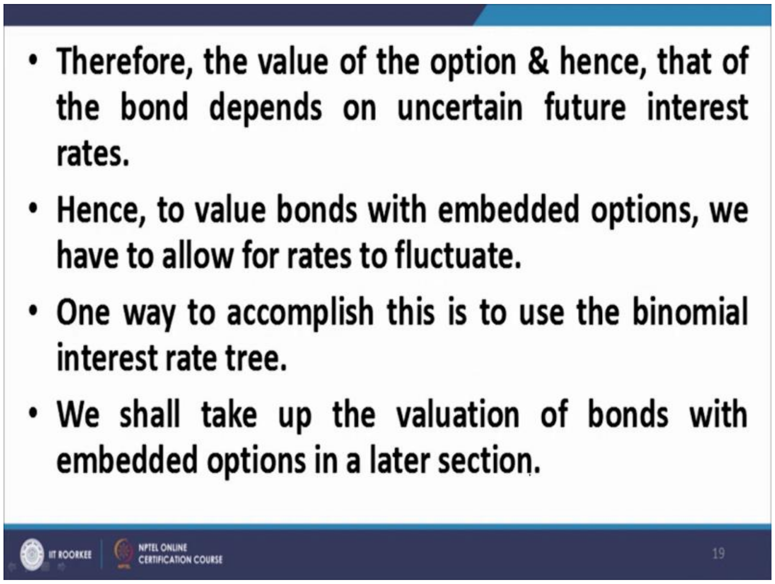

Therefore, the value of the option and hence that of the bond depends on uncertain future interest rates. Hence, to value bonds with embedded options, we have to allow for rates to fluctuate this facility, this flexibility is not available in the arbitrage pricing model and therefore, the development, the devising of the binomial pricing model. One way to accomplish this is to use the binomial interest rate tree which will take up the valuation of bonds with embedded options in a later section.

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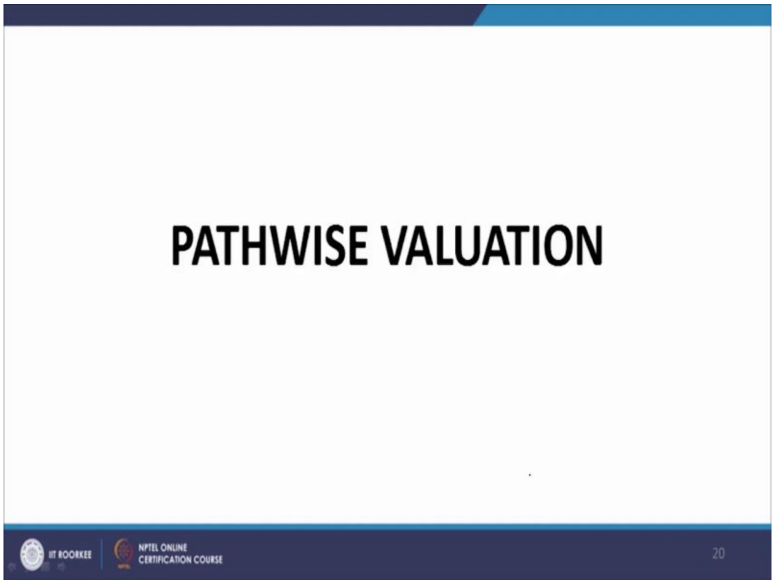

Path-wise valuation, now this method is equivalent to backward induction. Let me tell you at the outset that this method, the Path-wise valuation is equivalent to backward induction.

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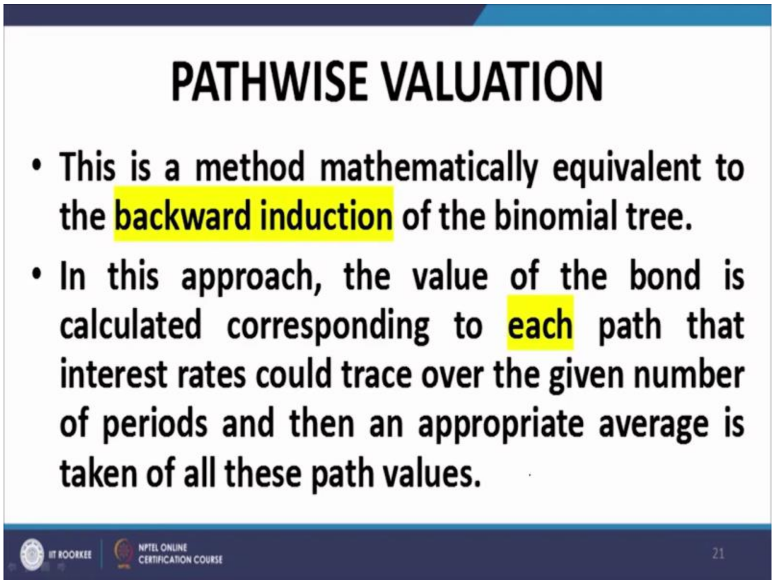

What we do in Path-wise valuation, the value of the bond is calculated correspond to corresponding to each path in the backward induction approach, what did we do? We had the cash flows corresponding to the bond or arising from the bond at the maturity of the bond. And then we from there, we move backwards, one step in time and then we calculated the value at the various nodes at time say t minus 1 if t is the maturity.

And then using those values and the coupon payments arising at t minus 1 we worked backwards again and arrived at t equal to capital T minus 2 point and there we get the valuation and this process, we continued backwards until we arrived at T equal to 0. However, this is a slight variant. This is a variant where we consider each path in isolation, each path that the interest rate tree has in it, embedded in it. And corresponding to each path, we arrived at the value of the bond at t equal to 0, and then we take the average over all the paths.

So, let me read it out. In this approach, the value of the bond is calculated corresponding to each path that interest rates could trace over the given number of periods and then an appropriate average is taken over all those path values.

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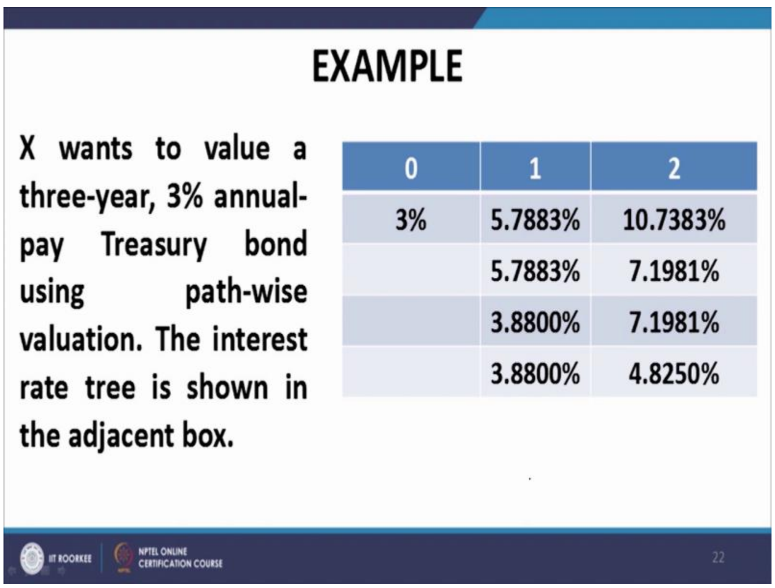

As I explained just now, this will be more clear from this particular example. The interest rate spectrum or the interest rate tree is the same that we encountered at the beginning of this class in the example that we discussed at the beginning of this class. And the problem is also the same X wants to value a 3 year 3 percent annual pay treasury bond using the pathways valuation. The interest rate tree is here.

Now obviously, there are four possible paths, we can go from 3 percent at t equal to 0 to 5.7833 percent, at t equal to 1 and then to 10.7383 percent at t equal to 2. Let us, say this is this is path number 1, the second path could be 3 percent at t equal to 0, 5.783 percent 883 percent at t equal to 1 and 7.1981 percent at t equal to 2, the third path could be 3 percent to t equal to 0, 3.8800 percent at t equal to 1 and 7.1981 percent at t equal to 2.

And the final part that is the fourth part could be 3 percent these are possible paths. Please note, these are not actual path. We do not know the actual path because we are doing working out this problem at t equal to 0, not at the maturity of the instrument. So, 3 percent a t equal to 0 there is the fourth part 3 percent at t equal to 0, 3.88 percent at t equal to 1 for the period from t equal to 1 to t equal to 2 and then 4.825 percent from t equal to 2 to t equal to 3 and this is a 3 year bond that needs to be valued.

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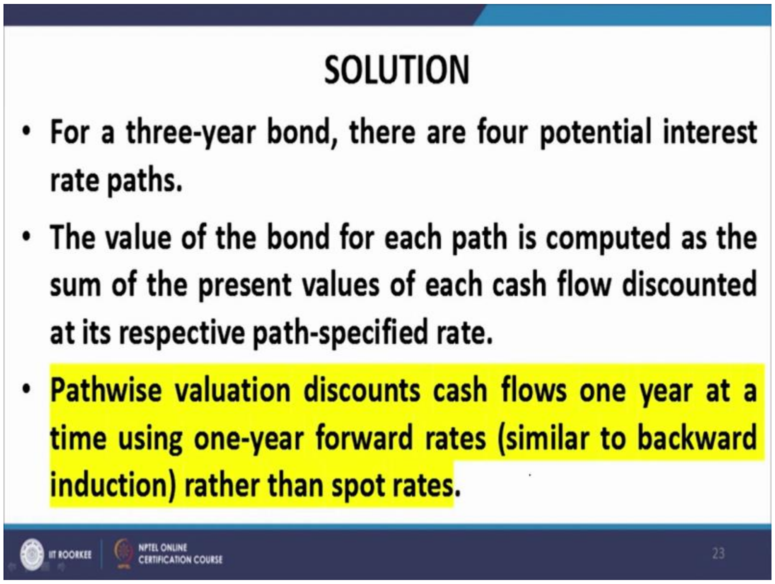

So, for a 3 year bond there are four potential interest rate paths that is what I mentioned just now. The value of the bond for each path is computed as the sum of the present values of each cash flow discounted at its respective path specified rate, path-wise valuation discount cash flows, 1 year at a time using 1 year forward rates similar to the backward induction rather than spot rates.

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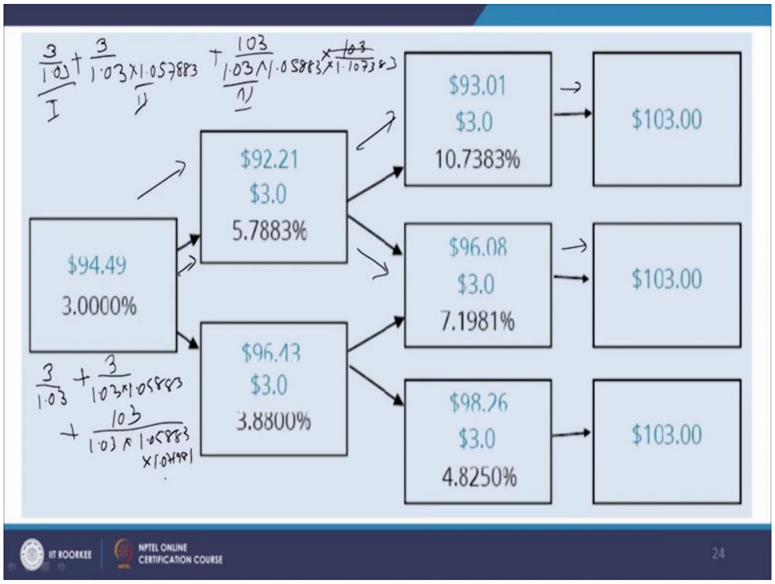

So, let us see how this this is the tree that you recall. So, let me just preview on how we are going to do the evaluation. Now, a in this case, we should proceed from left to right. Let us start with the first path that I mentioned 3 percent operates from t equal to 0 to t equal to 1. Therefore, the cash flow at t equal to 1 that is, how much is the cash flow at t equal to 1? It is simply and only the coupon payment of 3 units of money.

So, the 3 units of money will be discounted at 3 percent and we get, what do we get? We get 3 divided by 1.03 plus at t equal to 2 we get another cash flow of 3 units of money, which is the coupon payment that occurs at t equal to 2 and we discount this at the appropriate forward rate what is the appropriate forward 57883 percent. So, into 1.057883 plus. Then, what is the cash flow at t equal to 3? The cash flow t equal to 3 now please it is 3 of coupon and redemption of the principle of 100.

So, the total cash flow at t equal to 3 is 103 and it will be discounted corresponding to the first path, please note, this is the first path corresponding to the first path, what will it be? It will be 1.03 into 1.05883, 1.107383. So, this is the value that is the path valuation for, first path that is the first part.

So, we discount this at the interest rate 3 percent and that is my term number 1, then we move from t equal to 1 to t equal to 2 there again I get a cash flow of 3 that is the coupon payment. So, a discount it further at the relevant rate, which in this case in this path is 5.7883 percent. And then finally, at t equal to 3 I got a cash flow of 103 which will be discounted in this path at 10.7383 percent in this gives me this is term number 2, this gives me term number 3. So, this is the total value of the bond. Similarly, if I work out the value of the bond corresponding the second part, what will it be? Let us work it out.

For clarification, we start with t equal to 1 the cash flow is 3, so 3 divided by 1.03 plus at t equal to 2 again, there I s a cash flow of 3. So, 3 divided by 1.03. And what is the second path? The second path is this and then this and then this. So, this is equal to 1.03 into 1.05883. And finally, 103 divided by 1.03 into 1.05883 into 1.071981. So, this is a this is the valuation as for path 2. Similarly, we will do the valuation for path 3 and then path 4 and finally, we will take an average of all these four values that will give us the value of the bond at t equal to 0 that is V0.

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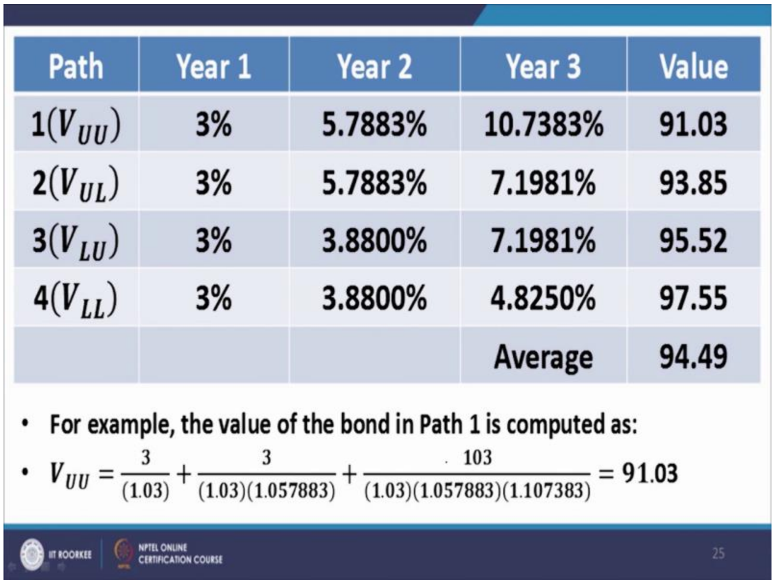

Path	Year 1	Year 2	Year 3	Value
$1(V_{UU})$	3%	5.7883%	10.7383%	91.03
$2(V_{UL})$	3%	5.7883%	7.1981%	93.85
$3(V_{LU})$	3%	3.8800%	7.1981%	95.52
$4(V_{LL})$	3%	3.8800%	4.8250%	97.55
			Average	94.49

- For example, the value of the bond in Path 1 is computed as:
- $V_{UU} = \frac{3}{(1.03)} + \frac{3}{(1.03)(1.057883)} + \frac{103}{(1.03)(1.057883)(1.107383)} = 91.03$

This is the working of this problem, which I have explained in the earlier few minutes. This is how we arrived at the value corresponding to the first path, this is the first path valuation, similarly, we have the second path valuation, and the third path pollution and the fourth path valuation as well and then we take the average of all these four path.

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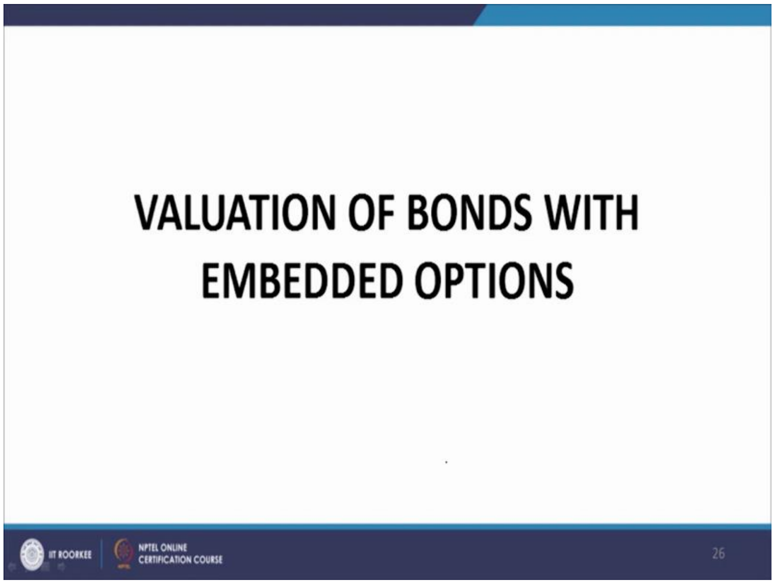

Now we start a new topic we talk about valuation of bonds with embedded options, this is where the actual beauty of the binomial tree model manifests itself. It becomes apparent the utility of the binomial tree interest rate model becomes more obvious when we talk about valuation of bonds with embedded options.

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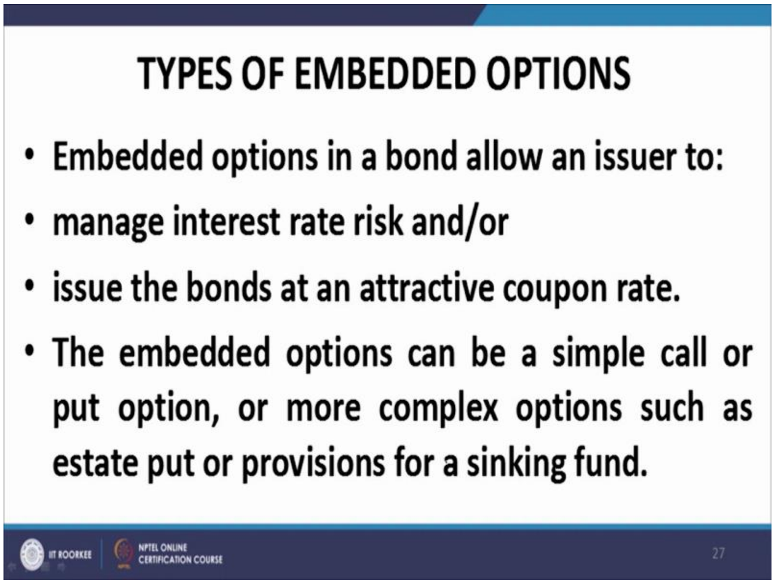

So, embedded options in a bond allow the issuer to manage the interest rate risk and issue the bond at an attractive coupon rate. In fact, venture to say that not only the issuer, but the depositor also has the opportunity to measure manage his portfolio to introduce flexibility into his portfolio. If the bonds have portable options attached to them, then the embedded options can be a simple call, or a put option, or more complex options such as estate put or provisions for a sinking fund. I will come back to them one by one.

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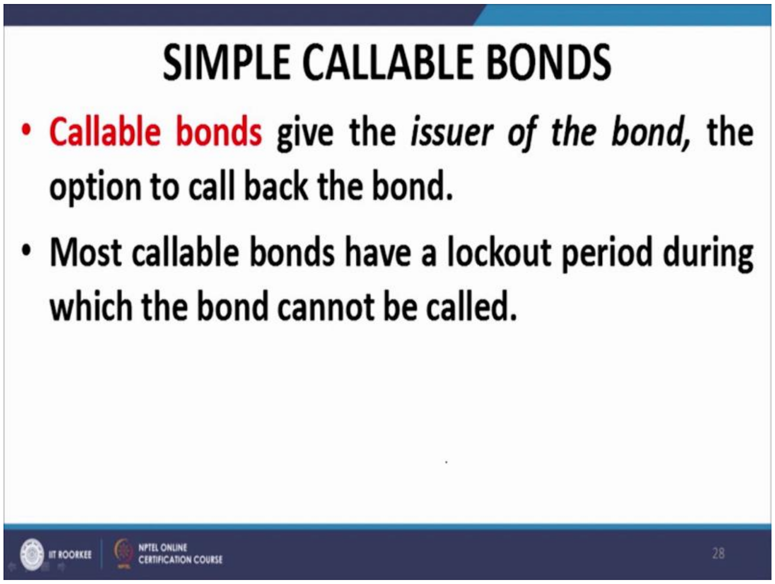

Callable bonds gives the issuer of the bond the option to call back the bond. So, when a bond has an option attached to it has a choice given to the issuer of the bond, that he can call back the depth, he can call back the borrowing that is he can return the money and he can call back the bond then it is called a callable bond.

Obviously, an issuer would consider exercising the call when the interest rates are falling, when there is a decline in interest rates, as a result of it the price of the bond has increased. And also, if the issuer has plans or looks, finds an opportunity to replace his existing debt, which is at a higher rate by a lower cost debt, which is the low prevailing interest rate.

In other words, if the interest rates go down at a certain point in time during the life of the bond, the bond issuer with the bonds which have a callable option, can exercise the call option and call back the bonds or repay them money and borrow the money from somewhere else at a lower rate.

That is a possibility that can be considered provided the bonds carry a callable option. So, what is the callable option or what is the callable bond rather? A callable bond is a bond that has attached to it or has embedded in it, an option which gives the right to the issuer of the bond to call back the depth that is to pay the money and withdraw the bond. As per terms contained in the issue document. The most callable bonds have a lockout period during which the bond cannot be called.

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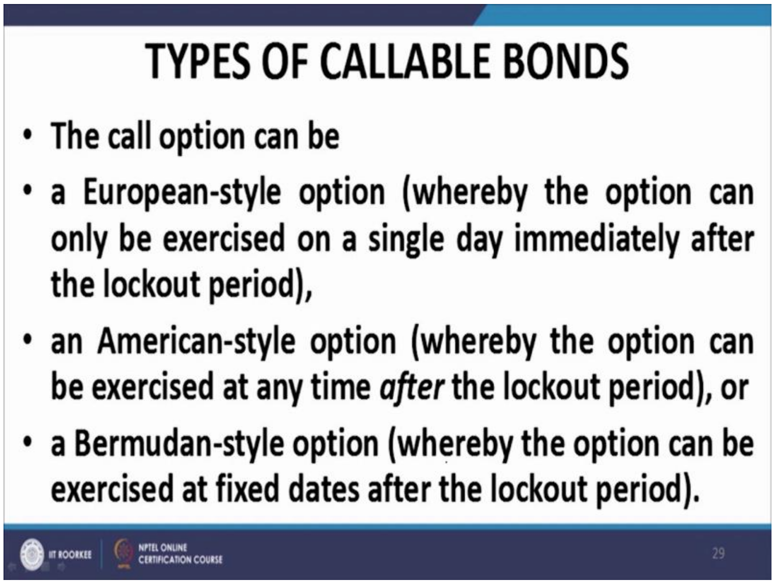

Obviously, as I mentioned, some time back call options are basically of 2 types, the European option and the American option. European options are accessible only at a particular date, and American options are accessible at any time during the life of the option. So, the call options that are embedded in the embedded in the bond can also be of 2 types, a European style option, whereby the option can only be exercised on a single day immediately after the lockout period.

Or an American style option whereby the option can be exercised at any time after the lockout period. Of course, we can also have a Bermudan style option where the option can be exercised at fixed dates after the lockout period. So we can have a European style option call option embedded in a bond, American style option embedded in callable bond and Bermudan style option embedded in the callable bond.

But the important thing is whatever is the nature of the option whatever is the maturity of the option or at what point the option can be exercised on what terms the option can be exercised, at what price the bonds can be bought back, all these things must be structured in the offer document. It is not at one fine morning, the issuer of the bond wakes up and says that I wanted to call back the debt and return the money and call back the bond. No, that is not allowed everything relating to the terms of exercised of the option, the exercise price, the exercise date or the method of exercise and all other models are need to be specified in the offer document. So that is what callable bonds are about.

Callable bonds are bonds which give flexibility give added flexibility to the issuer of the bonds whereby you can call back the bond before its actual maturity whenever, as per the terms of issue, now the option with the fact that it is called an option. Please note means that the issuer has the discretion the issuer has the choice, whether to exercise the option or whether or not to exercise the option.

It is not that it becomes a necessary component of the issue that the issuer must prepay the money or on the option date option exercised and call back the bond. No, that is not the issuer has the discretion, either he may exercise the option, he may repay the money and call back the boring or call back the bonds or he may let the option lapse and let the debt continue up to its normal maturity. That is why we use the word option.

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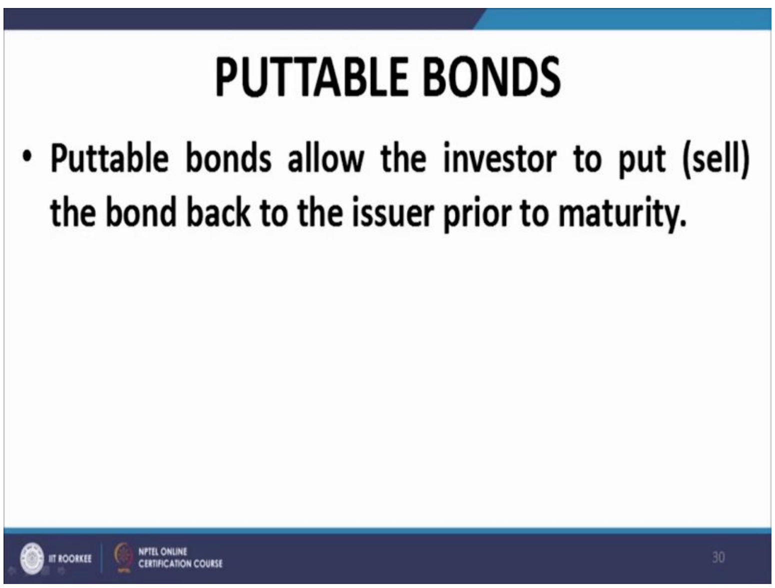

Puttable bond gives the investor give the lender the right the discretion to give back the bonds and recover the money from the issuer as per the terms contained in the issue document. So, that is the difference between the callable bond and puttable bond. A callable bond, the issuer has the discretion the issuer has a choice, whether he wants to pay back the depth and call back the bonds. In the case of puttable bond, the investor has the discretion the investor has the choice, if he so wants as per the terms of the issue document. If he wants, he can return the bonds to the issuer and take back the money that he has lent to the issuer of the bonds.

Just to reiterate, callable bonds means the issuer has the discretion the issuer has the choice to give back the money and call back the bonds. In the case of puttable bond the lender has the choice to give back the bonds and take back the money.

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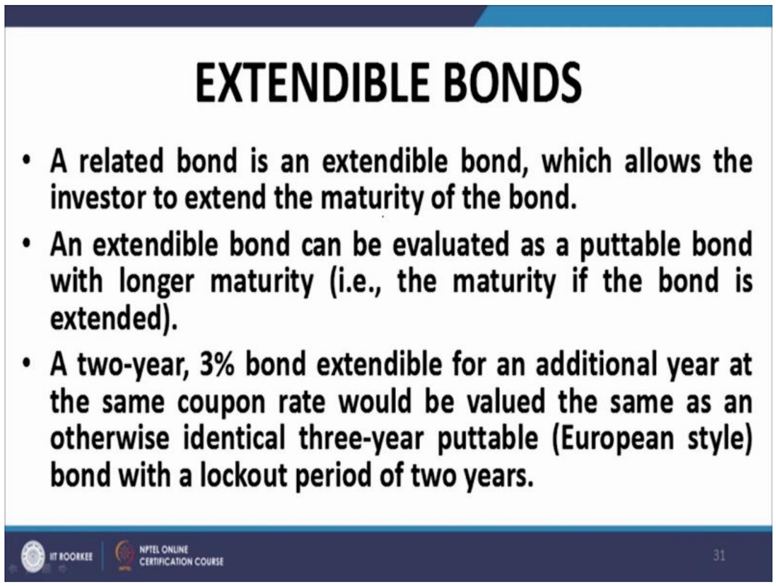

Then we have extendable bonds, which are bonds whose maturity can be extended at the discretion of the investor rather, not the issuer. So, but the valuation of an extendable document is bond is pretty much on the same lines as the puttable bond as I will explain in a minute.

Let me read it read this out first, a related bond is an extendible bond, which allows the investor to extend the maturity of the bond, that is to extend the maturity of the debt, it does not want the money back here for at the normal date of maturity or the actual date of maturity of the bond and he gets the right, he has the option, he has the discretion to ask the issuer to continue with the borrowing until a future date.

Now, an extendible bond can be evaluated as a puttable bond with longer maturity. If we let us take an example. Let us say we have a 2 year, a 2 year extendable bond in which the life of the bond can be extended at the choice at the discretion at the instance of the investor for a further period of 1 year to a total of 3 years. Then we can value this bond as a puttable bond with a maturity of 3 years with the early exercise of the put at t equal to 2 years. So, the framework that we have for the valuation of puttable bonds is pretty much implementable in the case of valuation of extendable bonds.

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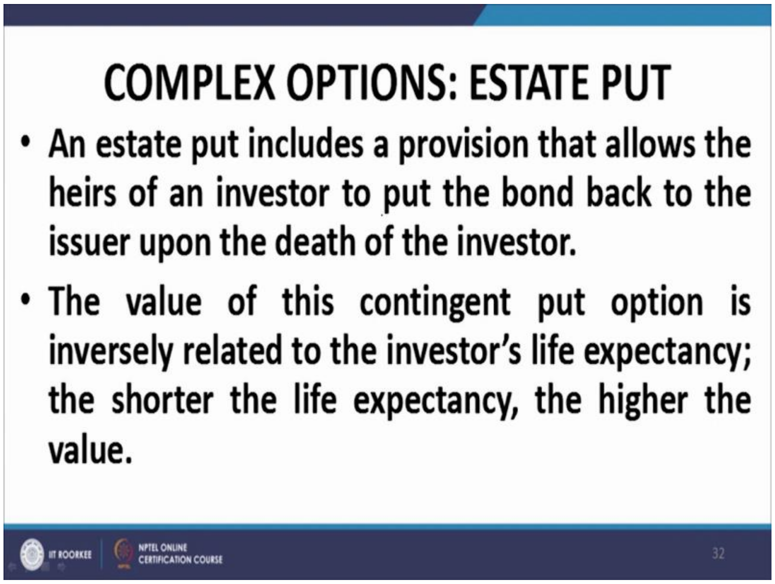

Complex options, where the estate put an estate put includes a provision that allows the heirs of an investor to put that bond back that is to give back the bond to the issuer on the death of the investor. So, this is on the demise of the investor original investor the heirs of the investor, get the right, get the discretion, get the choice to deliver back the bonds to the issuer and take back the debt to close out the debt. The value of this contingent put option is inversely related to the investor's life expectancy, the shorter the life expectancy, the higher the value.

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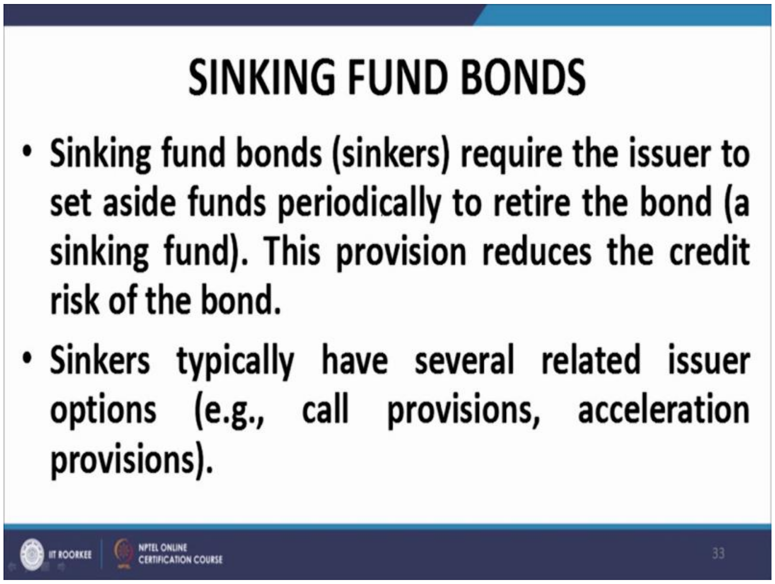

A sinking fund bonds, sinking fund bonds these are normally called sinkers require the issuer to set aside funds periodically normally out of his profit and loss accounts of out of the operations, profits arising from operations, set aside funds periodically to retire the bond and create a sinking fund. And this provision reduces the credit risk of the bond.

So, if such a provision exists in the issued document, the issuer of the bond has the mandate is required to at the end of each year to set aside a certain amount of out of his profits out of its operations for the redemption of the board, of course is the funds that are kept aside that are separated that are segregated out from the profits need may be reinvested in appropriate securities short term or long term instruments depending on the maturity of the bond.

So, that on the date of maturity of the bonds, these instruments can be liquidated and the proceeds utilized for the payment of the bond. So, this gives additional protection to the persons who have lent money to the issuer against the bonds which are sinking fund bonds.

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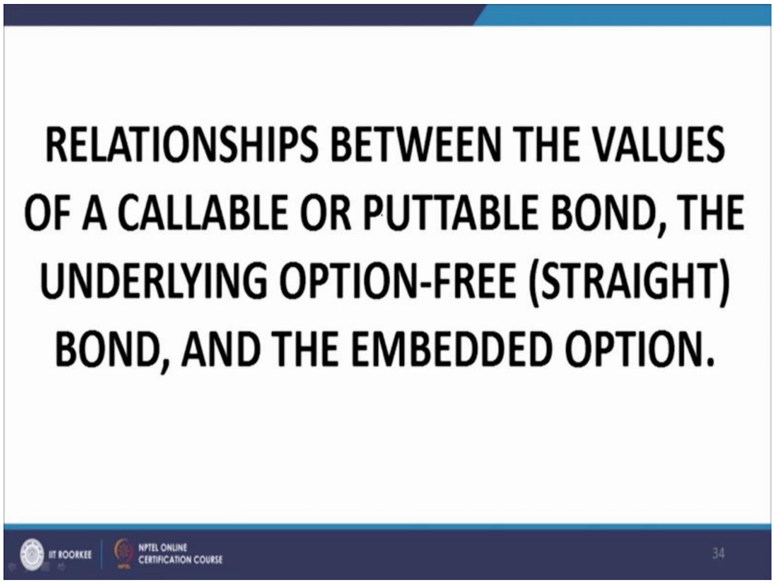

Now, we talk about the relationship between the values of a callable or a puttable bond comma, the underlying option free that is the straight bond and the embedded option. This is quite simple.

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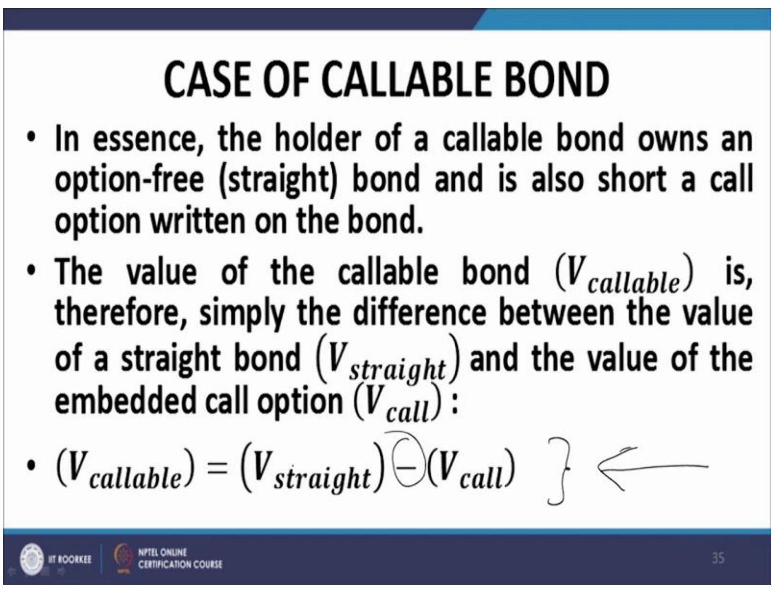

In fact, let us recall what is a callable bond? A callable bond is a bond where the issuer of the bond as the discretion as the choice has the prerogative to decide to call back a debt and call back bond and give back the money to the person who is holding the bond. So, this is the property of the call option. Of course, it has to be exercise, I reiterate, it has to be exercise in terms of the provisions that are contained in the issue on document, we need to take care of that.

But the important thing is that the prerogative or the right lies with the issuer, not with the investor, not with the lender. Therefore, from the lenders perspective, in short in the call option, he has written the call option, and the issuer has bought the call option, the issuer has the prerogative.

So, his long in the call option, the lender has the obligation that if the issuer calls back the bonds, he has to give back the bond and take the money that the lender is going to give him as per the terms of the issue document. Then therefore, the lender is short in the call and the issuer is long in the call.

Therefore, from the perspective of the lender, what happened, the value of the callable bond is less than the value of an equivalent bond, which does not have the call option attached to it.

So, from the perspective of the lender, his rights are curtailed and as a result of which his he pays less for the callable bond compared to a bond which does not have that option, which is called a straight bond. So, in other words, we can write this in the form of this equation here, this equation, value of the callable bond is equal to the value of the straight bond minus value of the call this minus sign is very significant.

Why minus? Because the investor is short in the call option therefore we have the minus sign here. And in any case a call option is an obligation from the perspective of the investor and therefore it demands the price for that, it demands cuts for that and as a result of which the value of the bond with the call option is lesser than the value of straight bond by the amount of the call

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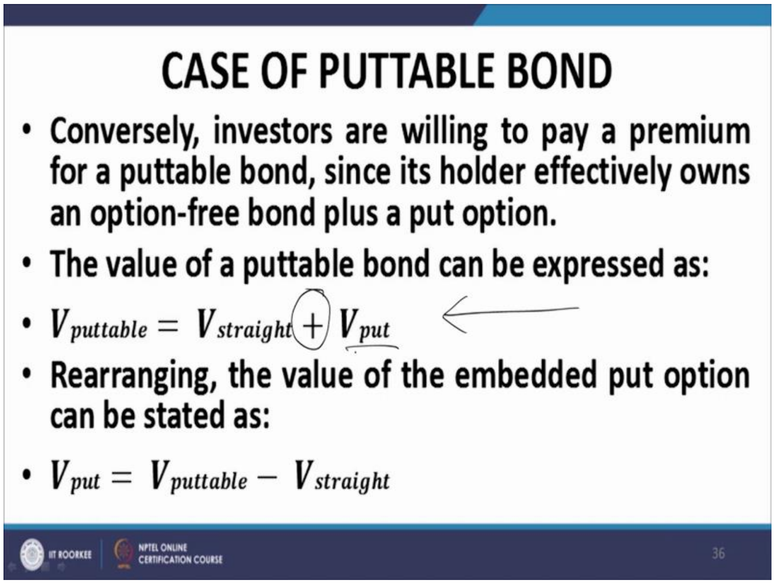

However, the converses the case when you talk about puttable bond, why? Because, what is the puttable bond? The puttable bond gives the right to the investor to the investor has that right he does not have the obligation, the investor has right the to give back bond and demands his money so that prerogative to the investor therefore he is willing to pay a price for that and as a result of which the value of the puttable bond becomes more than the value of the straight bond with the amount of the put. Putting it in another way the investors is long in the put option and the seller or the issuer of the bond is shot in the put option.

Because investor is long in the put option because he has a additional right. Because he enjoys an additional flexibility under the put option which he may or may not exercise. It is not mandatory that the invester exercises the put option he can as well let the put option lapse.

And therefore because of the discretion because that choice, what happened is, that the investor has a pays a prize for the put option and as a result of which the value of a puttable bond is equal to the value of the straight bond that is without the put option plus, this plus is significant value of put option.

So, this is where we conclude. For today's lecture we will continue from here in the next lecture. Thank you.