Decision Making With Spreadsheet
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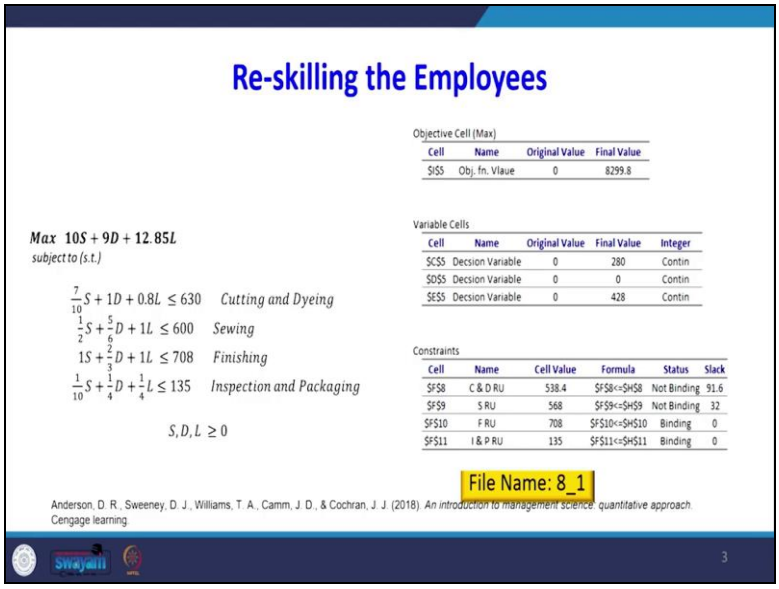
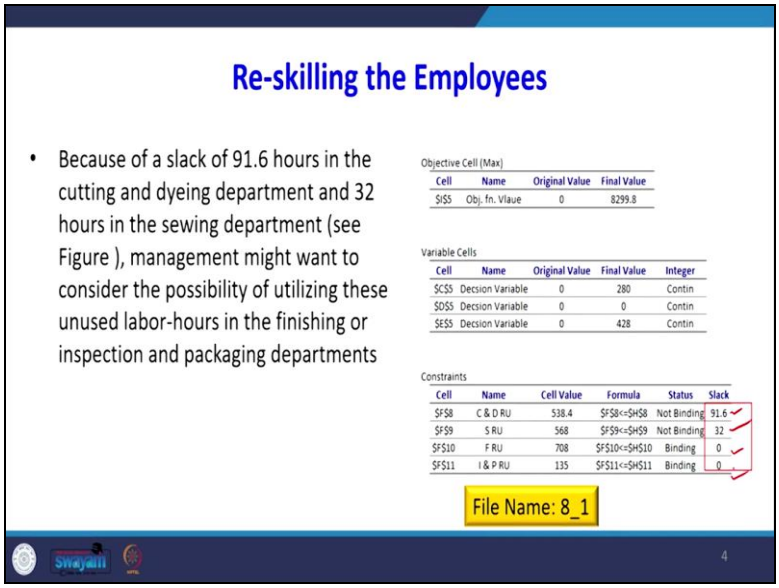
Lecture-08
Sensitivity Analysis - 3

Dear students, the previous class we have done some sensitivity analysis will continue with the sensitivity analysis in this class also. In this class we will discuss about what to do with the unused resources. Then the major portion of this lecture will be on explaining what the limitations are; of our classical sensitivity analysis that is what we are going to see in this lecture.



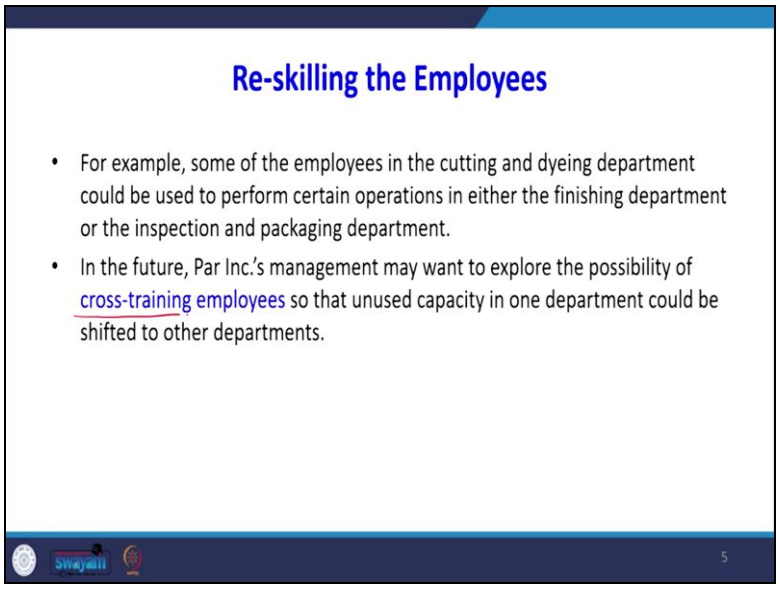
Look at the problem that we can re-skill the employees this problem we have discussed in the previous lectures. There are three bags the company produces: standard bag, deluxe bag, and lightweight bag. There are some resources utilized. So, when I run this one using solver, the objective function value is this much, and there you are getting some slack variable. What is the slack variable? Because all the constraints are less than or equal to type. See this less than or equal to type.

This is our slack variable, the slack variable the slack value represents the unutilized resources. How we can make use of these unutilized resources.



So, because of the slack of 91.6 hours in the cutting and drain department and 32 hours in the sewing department, look at this figure. This way, the management might want to consider the possibility of utilizing these unused labor hours in the finishing inspection, and packaging departments. So, there are some unused resources there, this resources can be reused here wherever there is a binding constraint is there.

So, that is how we can reuse that. We have to re-skill this employee who are working on cutting and dying and the sewing department so that they can able to work in the finishing department and inspection and packaging department.



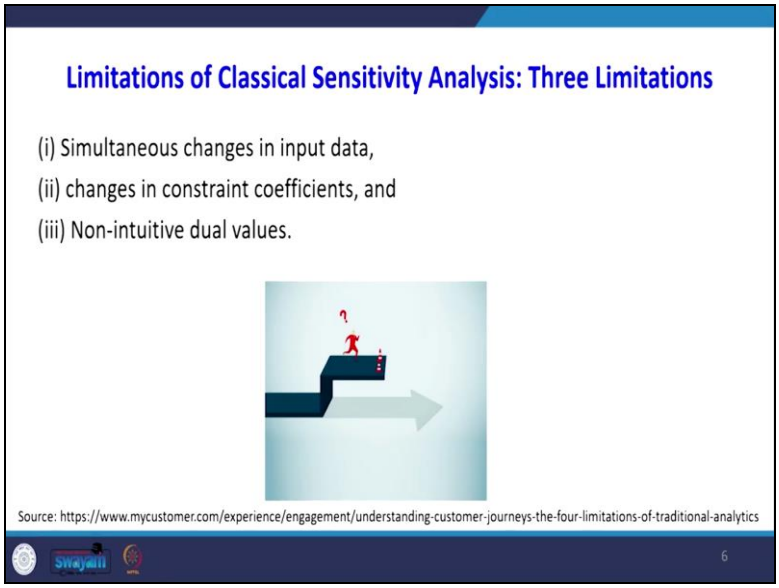
For example, some of the employees in the cutting and dying department could be used to perform certain operations in either the finishing department or the inspection and packaging department. In the future also, the management may want to explore the possibility of cross-training the employees. So, that unused capacity in one department could be shifted to another department. This is the meaning of re-skilling employees.

What is re-skilling? In certain departments, there are some unutilized resources there. So, that many resources those employees can be re-skilled so that they can work in the other department.

Now, I am going to explain what is these unutilized resources. This is the problem which I have taken I am maximizing this; this is a problem taken. So, decision variables initially will put to 0 0 0 coefficient of objective function I have written because I have taken these values from our problem where we are doing already this one. Then, I entered the constraint using my resources, and I wrote the objective function value.

When I solve this, go to data solver. When you solve this, yeah, I need this answer: sensitivity analysis limits. So, when I run this. So, here is the answer I am getting this yes here; I am getting slack values that are 91.6 and 32. So, in these 2 departments, this many hours are not utilized. That is, the employees from this department are kept idle. So, these employees can be retrained or re-skilled so that they can work in the finishing department and inspection package department where we need extra resources.

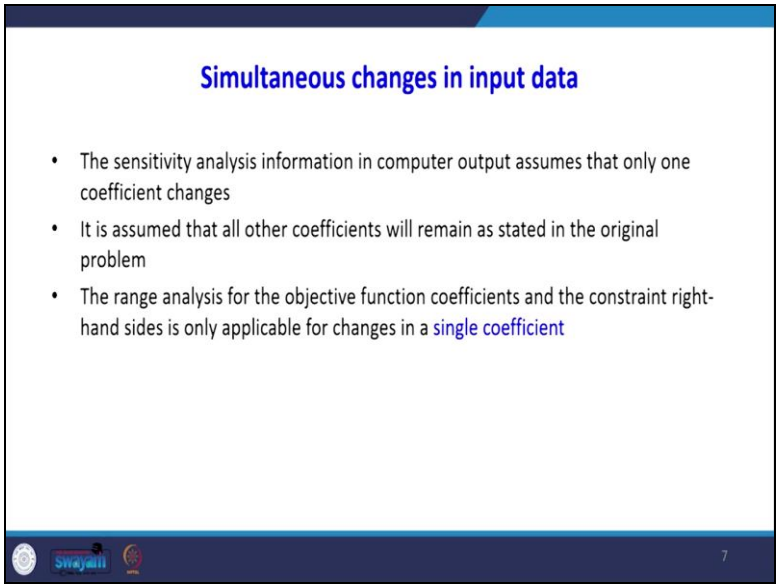
We will be studying how to re-skill from one stage to another stage in detail in the coming lectures, but at present, you can understand that these slack resources can be reutilized by proper training.



Students, we are going to see three limitations of our sensitivity analysis. One is simultaneous changes in the input data and changes in constrained coefficients and nonintuitive dual values. What is the simultaneous changes? Suppose z is equal to (10S + 9D). So, we have seen the effect of the coefficient of S, then we have seen what is on our z value and our our final value of S and D. Then similarly, we have seen the effect of D coefficient of D on our z value, but we did not consider simultaneous changes.

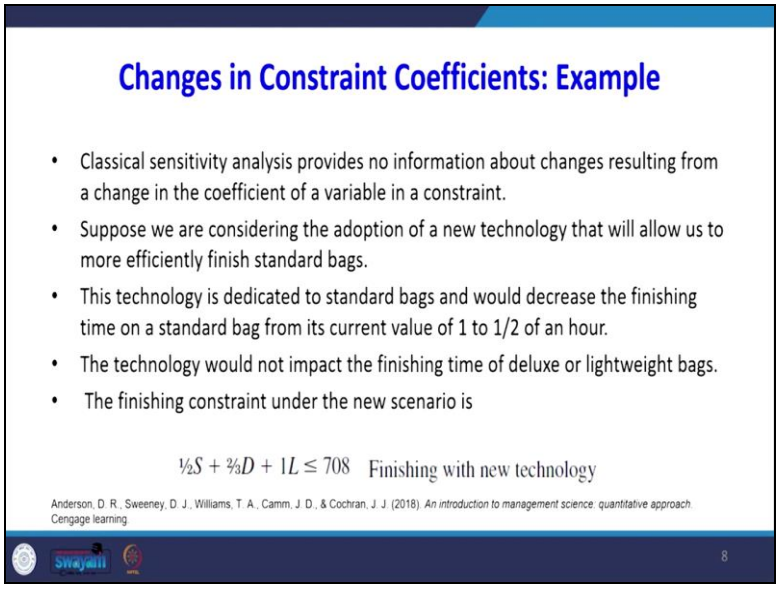
So, what will happen if there is a ceramic change that we can see? The second one changes in constrained coefficients. So, we have seen there are some values S, D something say, for example, 5S says 2D is a thing. So, we did not consider if there were any changes in the constraint of the equation. We did not see the effect of that. Then we will see what will happen and how to handle this. So, then non, intuitive dual values, what is the dual value?

If the right-hand side is increased by one unit, what are the changes on our z value? So, these things there will be a problem in interpreting these limitations that we will see in detail.



First, we will see simultaneous changes in input data. The sensitivity analysis information in computer output assumes that only one coefficient changes. It is assumed that all other coefficients will remain as stated in the original problem. The range analysis for the objective function is a range of optimality coefficients, and the constraint on the right-hand side is only applicable for changes in a single coefficient.

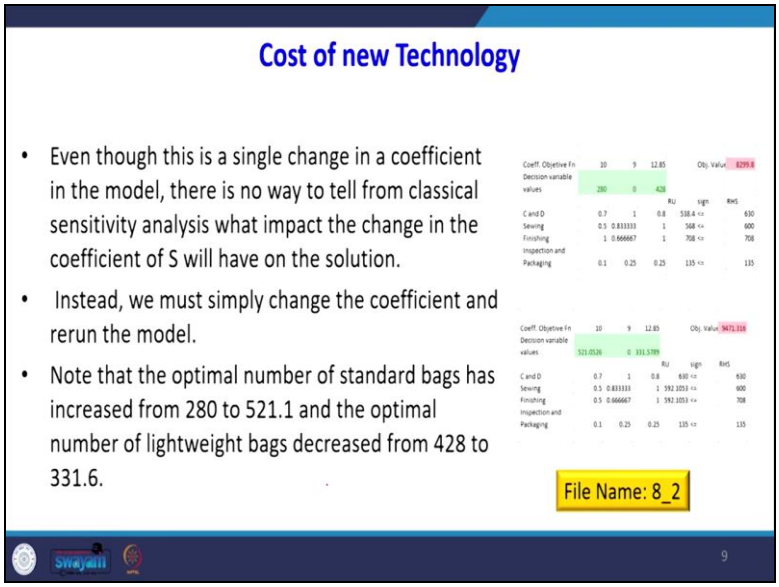
We have seen there are 2 points. One is a range of optimality. The second one is the range of feasibility. So, whenever this range of optimality and feasibility we are considering one value changes and its corresponding effect. If there are 2 values changing more than 2 values simultaneously, we cannot see the value which the result we got with the help of solar output cannot be interpreted. We must resolve the whole problem.



Now we will see that is the case. Second one the second limitation was changes in the constrained coefficient; classical sensitivity analysis provides no information about changes resulting from a change in the coefficient of variable in a constraint. Suppose we are considering the adoption of a new technology that will allow us to finish the standard bags more efficiently.

See the previous coefficient of this one is finishing time for finishing and finishing time it was 1S + 2/3D plus 1L <= 708. Assume that we have adopted a new technology due to which the time is decreased from one to off. This effect we cannot see in our sensitivity analysis problem. So, this has come due to technology. This technology is dedicated to standard bags and would increase the finishing time on a standard bag from its current value of one to half an hour.

The technology would not impact the finishing time of deluxe or lightweight bags there would the new technology which has brought it there will not be any impact on the D and L. So, the new finishing constraint under the new scenario is (½) S + (2/3)D+ 1L <= 708.



We cannot interpret if any change in the coefficient of constraint is happening. However, we can see the cost of new technology in the problem that we discussed in the previous slides. So, we adopted new technology, and because of that, the coefficient of S is reduced by 1 to (1 / 2).

We will see how to make use of that change in the coefficient of the constraint. Even though this is a single change in a coefficient in the model, there is no way to tell from classical sensitivity analysis what impact the change in the coefficient of S will have on the solution.

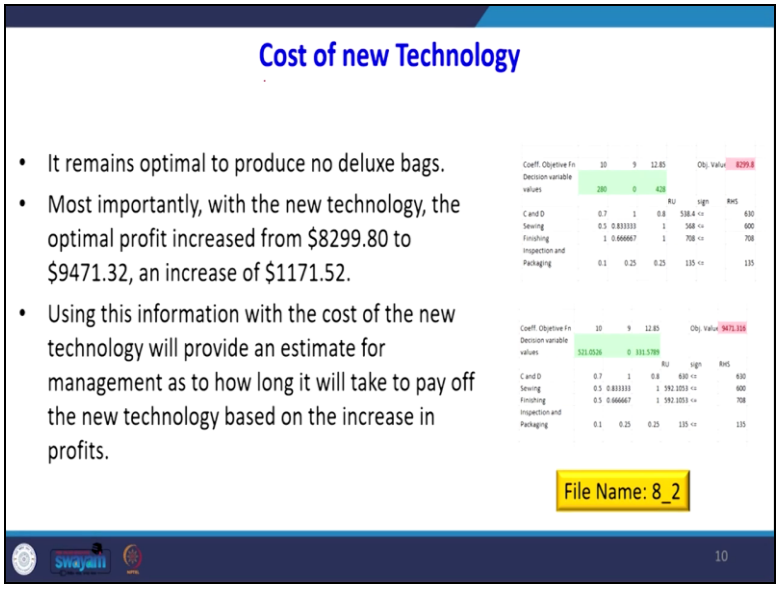
Instead, we must change the coefficient and rerun the model. Now, when we rerun the model, I have brought the output for both models. What is the model when the finishing time was one hour and the finishing time was 0.5 when it was one hour? You see, the value of our objective function is this much: 8300. When the finishing time is reduced to half an hour, our objective function has increased.

So, this difference can be considered the cost of new technology. Otherwise, because of going for new technology, we are getting this much gain in our objective function. So, what has happened is that the optimal number of standard bags has increased from 280 to 521. So, previously, it was 280, now the standard bags because the time it has new technology has helped you to do it at a time so that you are able to produce more standard bags.

And the optimal number of lightweight bags decreased. You see, previously, it was 428, but now it is 331. So, this difference is the benefit of new technology. I will explain with the help of a solver. How are we getting these 2 objective functions, then what is the difference in the values of these 2 objective functions?

Students, now I have taken the problem where the coefficient of S is one, as usual, and I have formulated it when I solve this problem. So, go to data solver. As usual, I entered all the values when I solved it. So, see that the objective function value is 8300. So, I am copying this value now. What I am going to do now is I am going to change this coefficient to 0.5 instead of 1 because of new technology that has reduced the finishing time from 1 to 0.5.

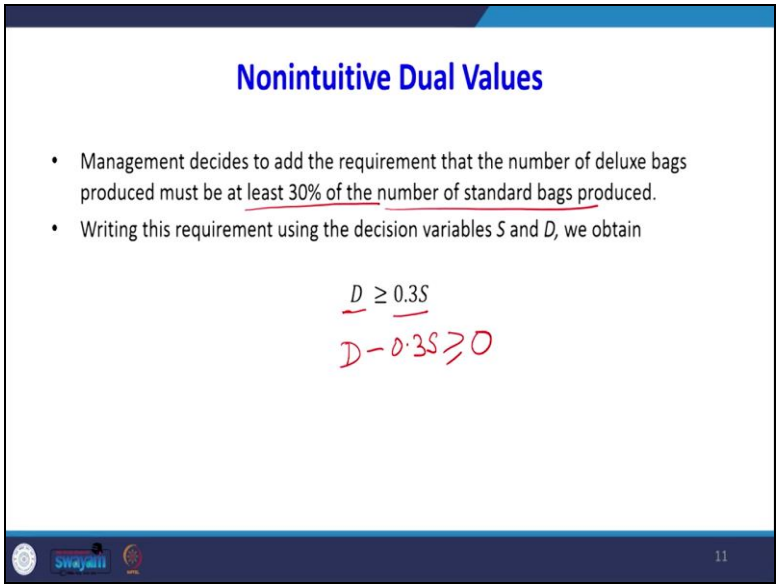
So, I am taking 0.5 again, and I am going to resolve it. So, go to data solver, yes. So, this is a look at our new value for the objective function. So, I am copied here. So, when I put here home paste values, these differences this is: when I have new technology, the value of the object function is this much. When I have older technology, the objective function is this much. This is a difference of 1171 dollars, which may be the cost of going for new technology.



So, we are discussing the cost of new technology. I see this one when we have our value of this S coefficient of S is 1, then when the coefficient of now becomes 0.5, this objective function values increase this much here it remains optimal to produce no deluxe bags. Here, this is S standard light weight S standard light weight. So, we are not producing any deluxe bags because 0.

Most importantly, with the new technology, the optimal profit increased from 8299.8 to 9471.32, an increase of 1171.2 dollars. So, using this information with the cost of new technology will provide an estimate for management as to how long it will take to pay off the new technology based on the increasing profit.

So, this increase in profit is just one year. Maybe assume that this problem is solved in one year. Obviously, when you go for, say, assume that this is 1000. So, the cost of new technology states, say, 10000. So, we can estimate that it will take approximately 10 years to get back the cost of new technology. So, this value will give you a rough idea of how much we are able to recover towards the cost of new technology.

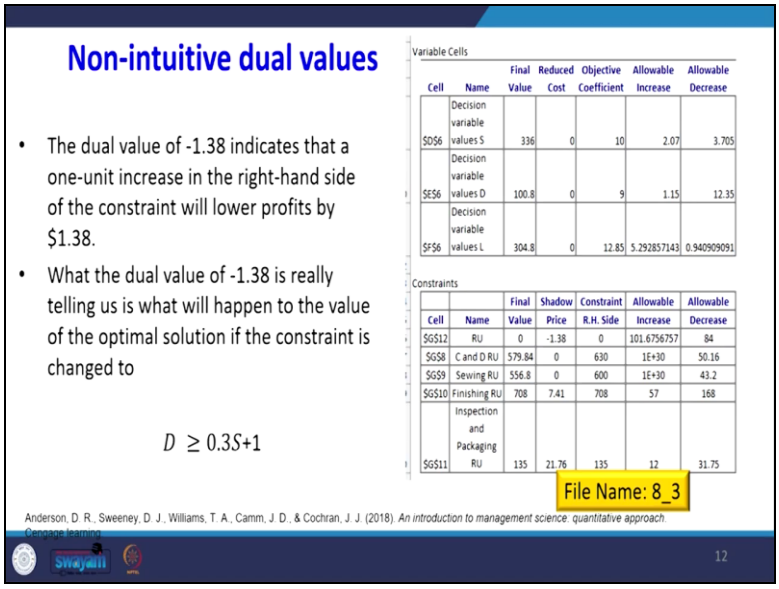


Now, the next limitation is nonintuitive dual values: Now assume that the same problem the management decides to add the requirement that the number of deluxe bags produced must be at least 30 percent of the number of standard bags produced. So, what is a new constraint? So, the number of deluxe bags should be at least greater than or equal to 30 percent of your standard bag.

So, what we can do is add this new constraint, and then we have to resolve the problem. So, what I am going to do I am going to add this new constraint again I go to resolve this problem.

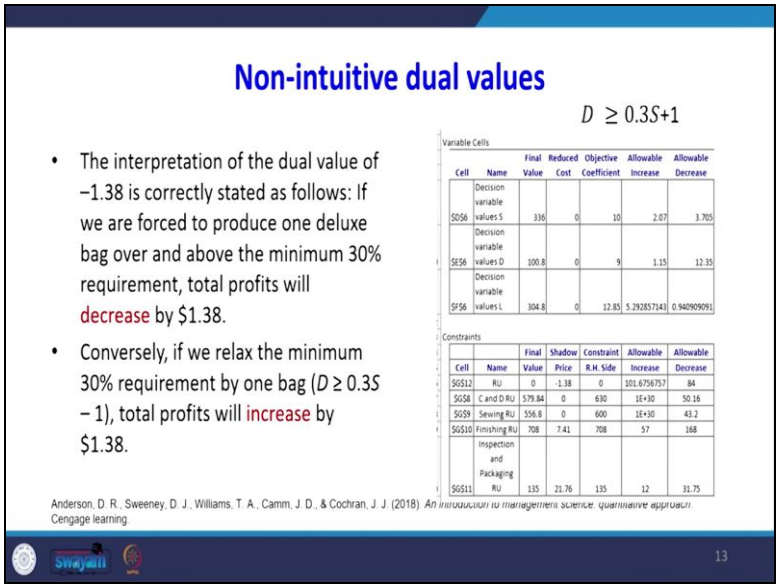
Now, I have introduced a new constraint that constraint is here this one which is – (0.3)S + D >= 0. So, when I solve this, I am solving it. I am going to see the dual value, so as a data solver, I will already have entered all the constraints when I solve it. So, I need these answers and the sensitivity analysis limits. Now that the answer report is okay, I need a sensitivity analysis.

When you look at the sensitivity analysis, this can be formatted. Click there, go to home, yes, look at the dual value. Ok, it is minus 1.38.



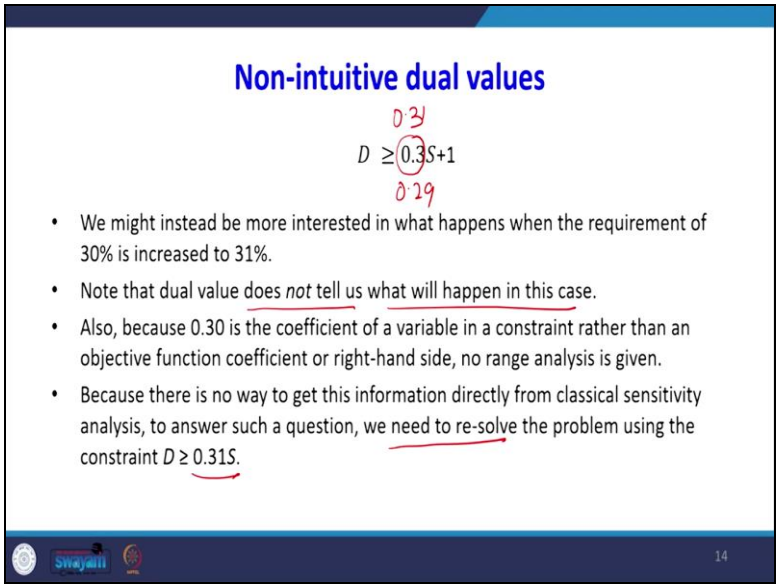
So, I will interpret what the meaning of this minus 1.38 is. Now, this is the dual value for the new constraint that we have added. So, the dual value of minus 1.38 indicates that one unit increase in the right-hand side of the constraint will lower profit by 1.38 dollars because it is negative. What the dual value of -1.38 really tells us is what will happen to the value of the optimal solution if the constraint is changed to (0.3S + 1). That is what our classical definition of dual value is.

What is the definition of our dual value if the right-hand side of the constraint is increased by one unit corresponding to our objective function, which is a dual value? So, this is our constraint: if you add plus one to the right hand side, what will happen? The dual value of our objective function will decrease by minus 1.38. But there is no use of this interpretation this is not required for us. actually, what we think will be useful for us is what we will discuss in the next slide.



So, the interpretation of dual value of minus 1.38 is correctly stated as follows. If we are forced to produce one deluxe bag over and above the minimum 30 percent requirement, the total profit will decrease by 1.38 dollars. Conversely, if we relax the minimum thirty percent requirement by one bag, that is D > (0.3S – 1). So, the total profit will increase by 1.38.

So, if the right-hand side is increased by one unit. So, the profit will decrease by 1.38. If it is decreased by one unit, the profit will increase by 1.38. So, that is what is meaning.

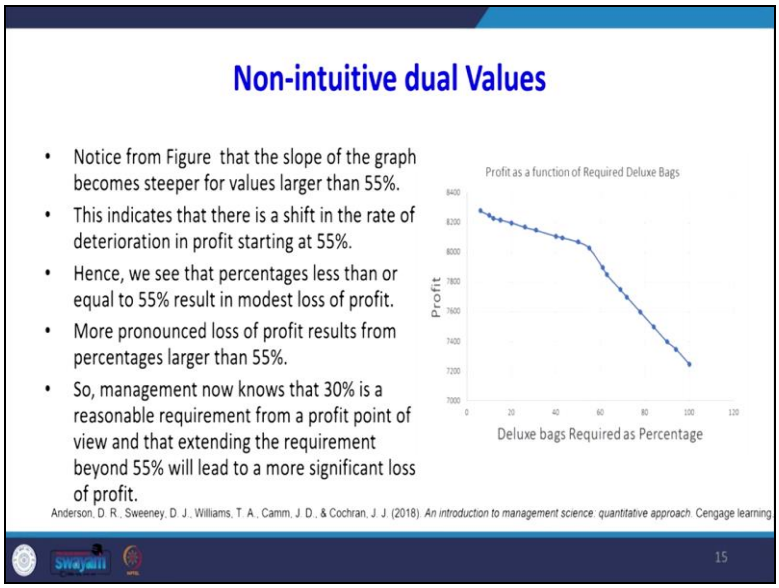


If D > (0.3S + 1), what will happen to our objective function? Instead of interpreting this way, we might be more interested in what happens when the requirement of 30 percent is increased to 31 percent. Here, we are going for one more deluxe bag. instead of this, we put a limitation that it should be greater than 30 percent. Suppose it goes to 31 percent or it comes

to, say, 29 percent. How will our objective function be affected? So, this interpretation cannot be done with the help of our classical dual value.

The only thing is what we can interpret if one unit is added on the right-hand side, then we can see what the effect on our objective function is. But if the value is increased from 0.3 to 0.31 or 0.30 to decrease by a decrease to 0.30 to 0.29, that effect on objective function cannot be captured, which is why it is nonintuitive dual values. This cannot be done. So, what to do we have to resolve. The problem notes that dual value does not tell us what will happen in this case.

Also, because 0.30 is the coefficient of variable in a constraint rather than an objective function coefficient just now we have discussed. If the coefficient of constraint is changed, you have to resolve it because there is no way to get this information directly from classical sensitivity analysis to answer such questions. We need to resolve the problem using 0.31, 0.32, 0.34, and so on.



So, I have resolved it. I brought up the final answer when you looked at this: it starts from 0, 20, 40, 60, 80, 30, then suppose you are 20 to 21. What is the effect on our profit? That is what was captured. So, when we look at our picture. So, what was the problem our D is greater than equal to 0.3S. So, 330 maybe, say approximately here, 30 may be here, 30 may be here. So, when you go at 0.31. So, we have to change this value again. We have to resolve it, then we have to capture what is the profit 0.32 0.34 up to 100.

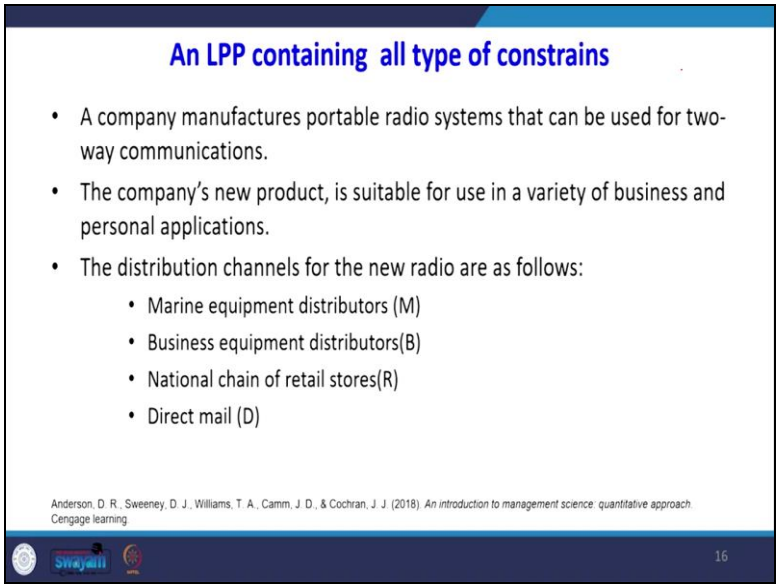
So, this was our final result. There are 2 columns in one column and profit in another column, 0.3 to 100, which means 30% is to 100%. We have captured the corresponding objective function value. So, we will interpret this figure and notice from the figure that the slope of the graph becomes steeper for values larger than 45. You see that when it is 45, the slope becomes steeper. Hence, we see that a percentage less than or equal to 55% results in a modest loss of profit because we got a negative because we got negative slope.

More principled, more pronounced loss of profit results from a percentage larger than 55. So, when your percentage requirement is increased beyond 55, we will be incurring more laws. So, what can the management understand from this figure? So, the management now knows that 30 percent is a reasonable requirement from a profit point of view and that extending the requirement beyond 55 percent will lead to a more significant loss of profit.

We have seen in the previous lecture that if your objective function is greater than or equal to type, it is binding. If you increase one unit on the right-hand side, that means you are tightening the constraint, your expectation is increasing. Whenever your expectation is increasing, obviously, what will happen is that the value of the objective function will decrease. That is why we got the negative dual value.

And another case, if the objective function is less than or equal to type when you add one resource on the right-hand side, that means you are relaxing your constraint. When you relax the constraint, the value of the objective function will increase. So, you will get a positive dual value. Here, in this case, because the right-hand side is greater than or equal to type again, you are increasing the right-hand side value, which means you are very strict, which means your expectation is more.

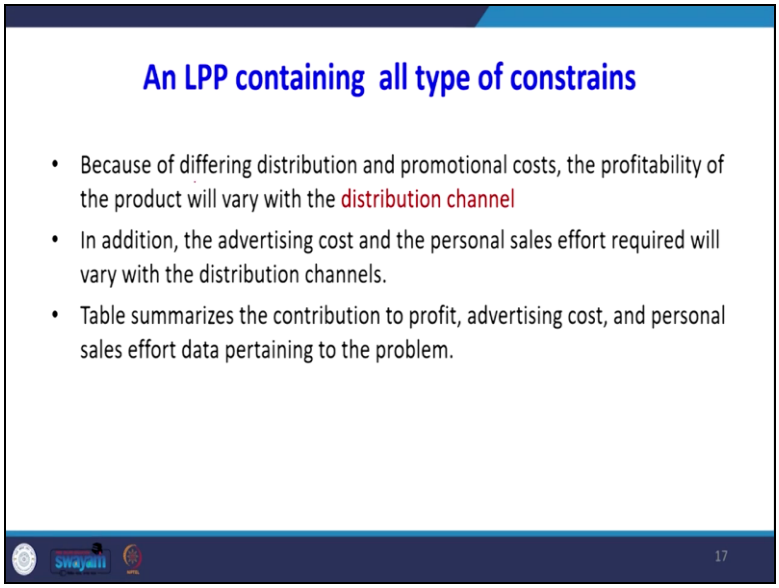
So, the value of dual will become negative. That is the interpretation that the manager can infer from this figure.



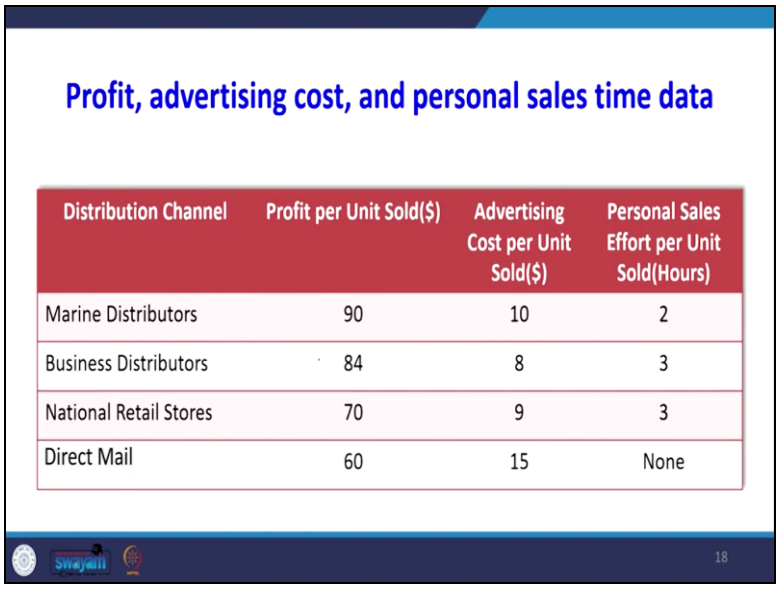
Students, we have seen three limitations of our classical sensitivity analysis. Next, we will take one problem that the problem will cover, or that problem will contain all types of constraints. What is the meaning of all types of constraints? There will be an equal to sign; there will be less than or equal to sign, and there will be greater than or equal to sign. So, we will see if there are all types of constraints is there.

If you are able to interpret this problem, then any problem in which we have a formula that we are going to formulate in the future can be interpreted very easily. So, this is a hypothetical problem that he has taken from Anderson Sweeney Williams, as you should. The company is a semiconductor company that manufactures portable radio systems that can be used for 2-way communications.

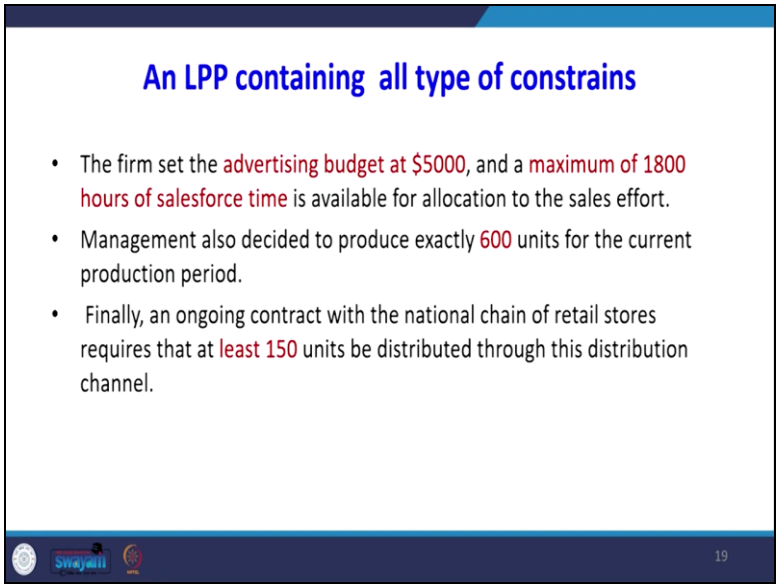
The company's new product is suitable for use in a variety of business and personal applications. The distribution channel for the new radio or as follows. Now, the company is thinking of different types of distribution channels and how to allocate our resources to different types of distribution channels. The company is considering four types: marine equipment distributor, business equipment distributor, national chain of retail stores, and direct email.



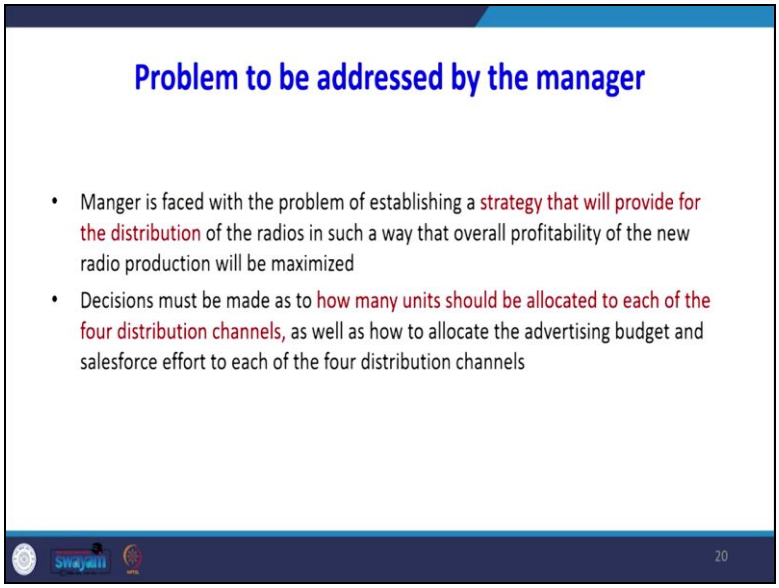
Because of different distributions and promotional cost, the profit the profitability of the product will vary with the distribution channel. In addition, what I see cost and the personal sales effort required will vary with distribution channels. The table which is in the next slide summarizes the contribution to the advertising profit and personal sales effort data pertaining to the problem.



So, what is there? There are four distribution channels for profit per unit. So, the marine distributors when you give the product to distribute via this distributor you can get profit per unit is 90 dollar advertising cost. For example, direct mail need to be advertised more then personal sales effort per unit this until constraint 233.



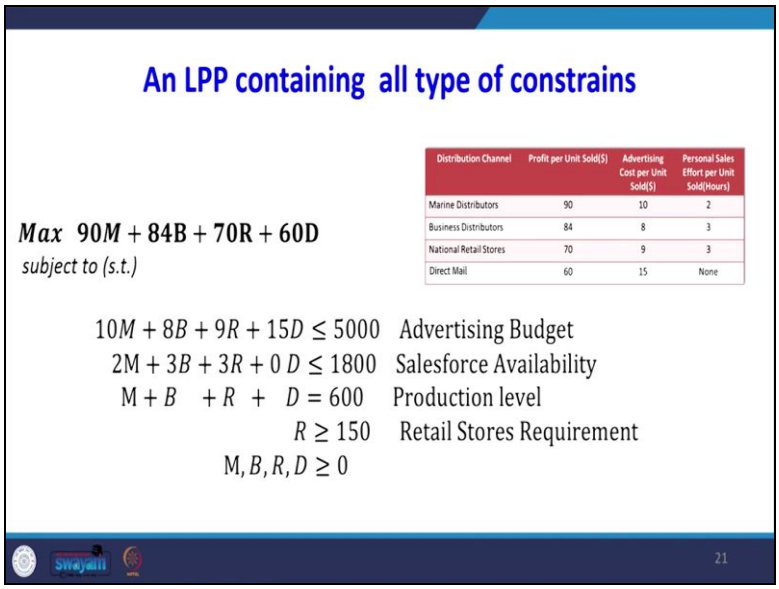
The firm said the advertising budget of 5000 dollars and a maximum of 1800 hours of sales force time is available for allocation to the sales effort. These are the 2 constraints. Management also decided to produce exactly 600 units for the current production period. Finally, the ongoing contract with the national chain of retail stores requires that at least 150 units to be units be distributed through this distribution channel.



What problem the manager is facing now is faced with the problem of establishing a strategy that will provide for the distribution of the radios in such a way that the overall profitability of the new radio production will be maximized. That is decision must be made as to how

many units should be allocated to each of four distribution channels. As well as how to allocate the advertising budget and sales force effort to each of the four distribution channels.

We have four distributions on how to allocate our product into the four distribution channels. So that our profit is maximized by satisfying all our constraints.

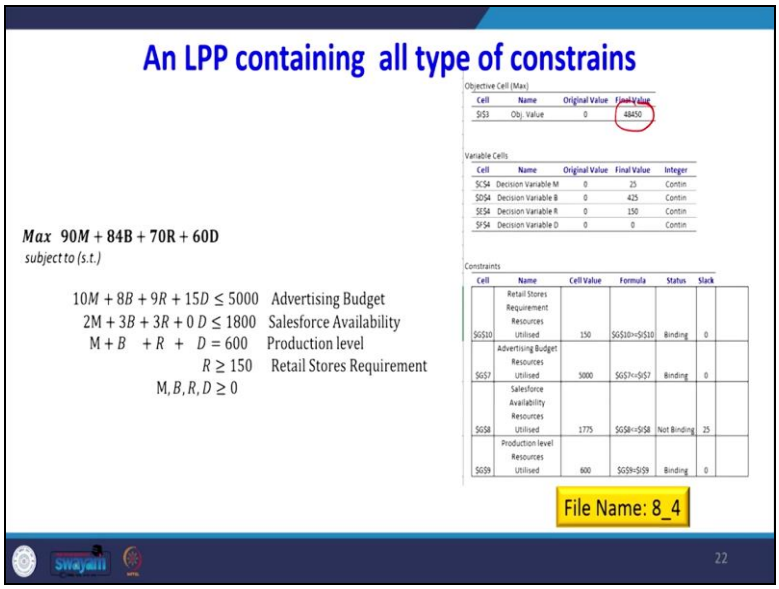


So, the problem which we have explained is formulated in mathematical form. So, 90M, M represents marine, business, retail, and direct mail. We have taken 90M, 84 of this value. So, advertising budget 10 is taken from this value and should be less than or equal to; then sales force availability this one is taken from this one. Then, there is a production constraint. So, all the products should be equal to 600. There is another constraint: there should be greater than or equal to 150 retail stores' requirement.

Now, when you look at this problem, there is lesser equal to type, is there equal to type, is there greater than or equal to type, you say? Now, we will solve this problem with the help of Excel.

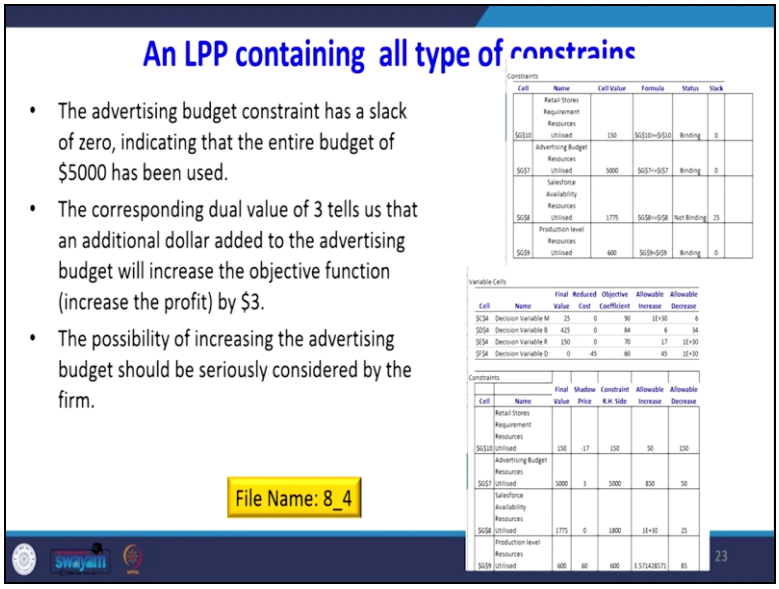
Students, I have brought the problem here in Excel. As usual, I have entered it. So, this is the decision variables coefficient objective function. There are four types of distribution channels I have taken here. I return the objective function. Please look at the formula for the objective function. Please look at the formula for resources utilized. Then the right hand side then I entered all the data into the solver also data solver when I solved it I got this result.

What is this result, for example, answer? See, this is our objective function is 48450 dollars. So, you have to align the final value. So, via a marine distributor, 25 business distributors, 425 retail stores, and 150 direct miles, we are not doing this one. So, there is a sensitivity analysis report is there; there is a limits report there that I will explain now.



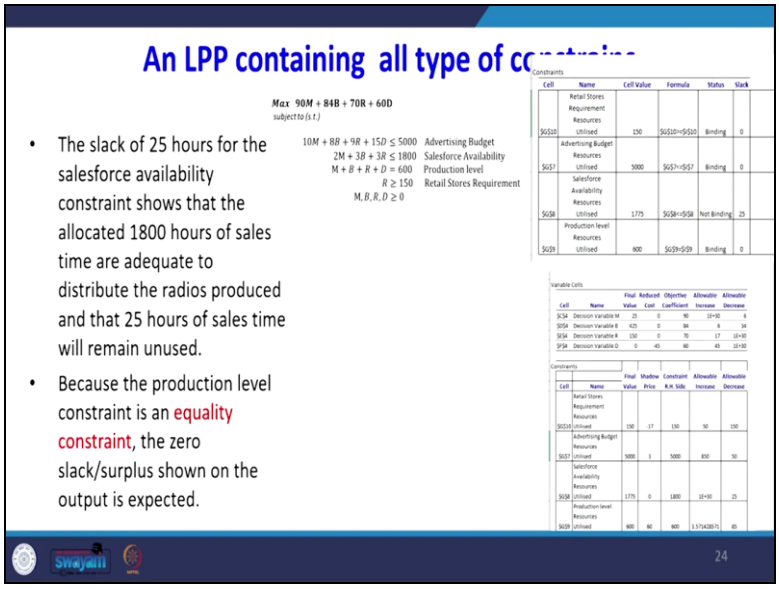
So, the value of our objective function is 48450 dollars. The value of our decision variable is like this: 25, 425, 150, and 0. Then, when you look at the slack column right, there is a slack of 0 because it is greater than or equal to type. So, it is a binding constraint. So, all the resources are fully utilized, and here also, the slack is 0. It is less than or equal to the type of all the resources that are fully utilized.

Now you see that the slack is 25 it is less than or equal to type. So, it is a it is slack variable. This is unutilized resources where the sales force availability resources are unutilized. So, the last one is also a slack variable. It is equal to the type. We cannot say obviously it will be 0. We cannot say either slack or surplus because it is equal to the type constraint.

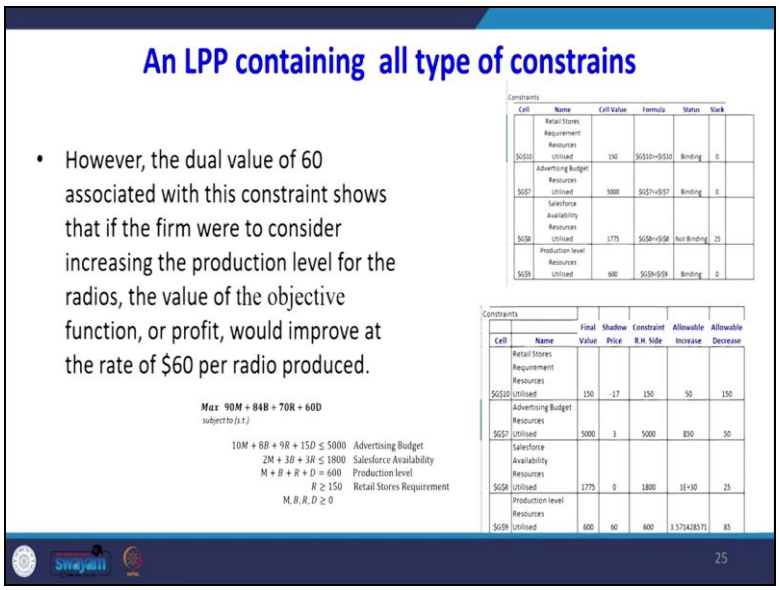


So, how to interpret this Excel output, the advertising budget constraint has a slack of 0 here, a slack of 0 indicates that the entire budget of 5000 has been fully utilized ok has been fully utilized. The corresponding dual value of 3. So, for the advertising budget, looking at this dual value, it tells us that if an additional dollar is added to the advertising budget, that will increase our objective function by 3 dollars.

You see that this objective function is less than or equal to type obviously in positive shadow price. So, that will help to improve our objective function. So, the company's possibility of increasing the advertising budget should be seriously considered by the firm, which is one interpretation or another interpretation.



The slack of 25 when you look at this one is sales availability, sales force availability resources. The slack of the 25-hour sales force availability constraint shows that the allocated 1800 hours of sales time is adequate to distribute the radios produced, and 25 hours of sales time will remain unused. This many hours, the sales force will be idle. Because the production level constraint is an equality constraint, look at this: there is an equality constraint. This one constraint is the 0 slack variables. Here, there is a slack the 0 slack variable 0 slack or surplus shown on output is expected because we put equal to sign.



We can interpret this Excel output in detail. The dual value of 60 is associated with this constraint. What are these constrained dual-value production-level resources? Is it dual value, which shows if the firm were to consider increasing the production level for the radios? The value of the objective function or profit would improve at the rate of 60 dollars per radio produced. So, because this is a positive shadow price, the company is thinking of many things to increase their production level.

The surplus of 0 associated with this retail store distribution channel commitment is a result of this constraint being binding. So, the retail store distribution channel is why we are calling it surplus. Look at the constraint: this is greater than equal to type if it is greater than or equal to type this, instead of slack we should call it as surplus and look at this as usual I am saying when you are tightening if the right-hand side is greater than or equal to type when you again you are adding you see that that you are getting negative shadow price.

So, the negative dual value indicates that increasing the commitment 150 to 151 units will actually decrease the profit. This is the point I emphasized in the previous 2 three minutes also thus, this company may want to consider reducing its commitment if it is constraint is greater than or equal to the type and binding constraint we should not go for increasing when you increase that, you are increasing commitment that will deteriorate your objective function what is our objective function is profit.

So, if it is a profit, your profit will decrease. Suppose the constraint is greater than equal to the type of objective function, which is cost. So, again, increasing on the right-hand side will increase the cost. Ok, we should be very careful. So, this company is considering reducing its commitment to the retail store distribution channel because that will increase its profit. So, a decrease in commitment level will actually improve the profit at the rate of 17 dollars per unit.

In this lecture, we have discussed how to reuse these unutilized resources that are re-skilling the employees. Then, we have discussed three limitations of our classical sensitivity analysis. In the end, we have taken a problem that has all types of constraints less than or equal to greater than or equal to unequal to types. Then we solved it with the help of a solver, and we interpreted the Excel output; thank you very much.