**Decision Making with Spreadsheet**
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Lecture - 54
Multicriteria Decisions - II

Dear students, in this lecture, I am going to discuss a multi-criteria decision-making technique called the analytic hierarchy process. The problem which I have selected is choosing a car by considering multiple criteria. Some steps of the analytic hierarchy process I will cover in this lecture, and the remaining steps I will cover in the next lecture.
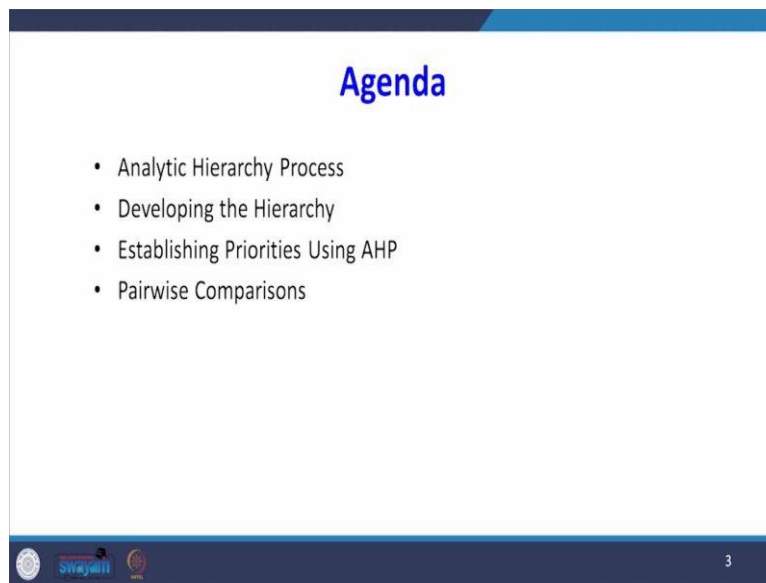
So, the agenda for this lecture is the analytic hierarchy process, developing hierarchy, and establishing priorities using AHP and pairwise comparisons. I will cover the remaining steps in the next lecture.

Analytic Hierarchy Process

- **The analytic hierarchy process (AHP),** developed by Thomas L. Saaty, is designed to solve complex multicriteria decision problems.
- AHP requires the decision maker to provide judgments about the relative importance of each criterion and then specify a preference for each decision alternative using each criterion.
- The output of AHP is a prioritized ranking of the decision alternatives based on the overall preferences expressed by the decision maker.

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So, this technique is a multi-criteria technique. So, in the end, we will get an overall preference for different alternatives.

Alternatives

The example problem that I have taken here is choosing a car and selecting a car. There are 3 types of cars. I call it car A, car B, and car C.

For choosing the car, I have considered four criteria: one is the price of the car, and the second one is miles per gallon. Here, the miles per gallon is kind of a mileage than comfort and style.

The information for the car selection problem is given in this table. This example is taken from the book Anderson et al. What are the criteria? Price, color, miles per gallon, interior, body type, and sound system. What are the three alternatives? Car A, car B, and car C. You see that, with respect to the price of car B, the price is the highest one. So, we can choose a particular car by considering these criteria. Here, the weightage for the criteria will differ from person to person.

Analytic Hierarchy Process

- Data regarding the Price and MPG are provided in Table.
- However, measures of Comfort and Style cannot be specified so directly.
- We need to consider factors such as the car's interior, type of audio system, ease of entry, seat adjustments, and driver visibility in order to determine the comfort level of each car.
- The style criterion will have to be based on decision maker's subjective evaluation of the color and the general appearance of each car.

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In the previous slide data regarding the price and miles per gallon are provided. However, measures of comfort and style cannot be specified so directly. For example, price and miles per gallon can be given directly in the form of absolute numbers. For example, comfort and style are qualitative factors and this cannot be given directly. So, we need to consider factors such as the car's interior, type of audio system, ease of entry, seat adjustment, and driver visibility in order to determine the comfort level of each car.

The style criterion is also qualitative in nature; it will have to be based on the decision makers' subjective evaluations of the color and the general appearance of each car. Because this style will differ from the decision maker's expectation, different decision makers may prefer a different style.
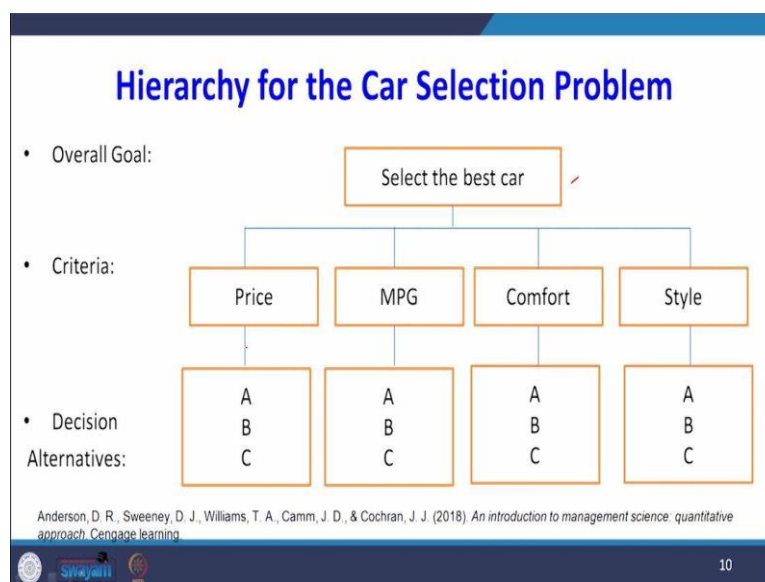

Analytic Hierarchy Process

- Even when a criterion such as price can be easily measured, subjectivity becomes an issue whenever a decision maker indicates his or her personal preference for the decision alternatives based on price.
- Thus, whether the car C is considered "extremely more expensive" than the car A or perhaps only "moderately more expensive" than the C depends upon the financial status and the subjective opinion of the person making the comparison.
- An advantage of AHP is that it can handle situations in which the unique subjective judgments of the individual decision maker constitute an important part of the decision-making process.

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Even when a criterion such as price can be easily measured, subjectivity becomes an issue whenever a decision maker indicates his or her personal preference for the additional alternative based on the price. Even though the price is quantitative in nature, here we are going to consider a kind of a no even though the price is quantitative in nature, but here we are going to consider price in the form of subjective evaluation.

Thus, whether car C is considered extremely more expensive than car A or perhaps only moderately more expensive than C depends upon the financial status. Whether car C is considered extremely more expensive than car A is perhaps only moderately more expensive than C depends upon the financial status and the subjective opinion of the persons making the comparison.

So, with respect to decision makers the concept of expensive will differ. Some decision makers may say extremely more as more expensive, but another person may say moderately more expensive. This is based on their buying capacity. An advantage of AHP is that it can handle situations in which the unique subjective judgment of the individual's decision-maker constitutes an important part of the decision-making process.

So, here, with respect to decision-makers, the priority level of decision makers will taken into consideration while making the decision.

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Here is the hierarchy of the car selection problem; what is the overall goal? There are different hierarchies. The first hierarchy is the selection of the best car; the next level is the criteria price, miles per gallon, comfort, and style. Here, price and miles per gallon, we did

not take the absolute value we have taken. We have considered this as a qualitative nature. Then, decision alternatives are car A, car B, and car C. So, this is the hierarchy of the given problem.

The first step in AHP is to develop a graphical representation of the problem in terms of the overall goal, the criteria to be considered, and the additional alternatives. Such a graph depicts the hierarchy of the problem. Look at the right-hand side; you can see there are different hierarchies: hierarchy 1, hierarchy 2, hierarchy 3. Note that the first level of hierarchy shows that the overall goal is to select the best car.

On the second level, there are 4 criteria: price, miles per gallon, comfort, and style; each contributes to the achievement of the overall goal.

Finally, the third level at each decision alternative A, B, and C contributes to the criterion in a unique way. Using AHP, the decision maker specifies the judgment about the relative importance of each of the 4 criteria in terms of its contribution to the achievement of the overall goal. So, here, the decision maker out of these 1, 2, 3, and 4 criteria needs to consider the priority of whether the price is important while selecting the car.

Or the miles per gallon are important, or the comfort is important, or style is important. So, first, he has to say what is the priority.

Developing the hierarchy, at the next level, the decision maker indicates a preference for each decision alternative based on each criterion. For example, we have price with respect to price. He has to say there are 3 cars, A, B, and C, which is most preferable for him. There is a mathematical process is used to synthesize the information on the relative importance of the criteria. We are going to get there are 4 criteria.

There is a mathematical process behind the relative importance of the criteria and the preference for the decision alternatives to provide an overall priority ranking of the decision alternatives. So, this step I will explain in the next lecture. In the car selection problem, AHP will use the decision maker's personal preference to provide a priority ranking of 3 cars in terms of how well each of the cars meets the overall goal of being the best car.

The priority will change from the decision maker to the decision baker. For example, decision maker A may say something decision maker B may prefer something. So, the priority will

defer the decision maker's personal preference. Another point here is the mathematics behind this AHP is the matrix.

Now, we are going to establish priorities using AHP. In this lecture I will show how AHP uses pairwise comparison expressed by the decision makers to establish priorities for the criteria and priorities for decision alternatives based on each criterion. So, everywhere we are going to do a pairwise comparison. So, we are going to do a pairwise comparison among the criteria, and with respect to each criterion, we are going to do a pairwise comparison for each alternative.

Using the car selection example, I show how AHP determines priority for each of the following: number 1, how do the 4 criteria contribute to the overall goal of selecting the best car? So, we have to prioritize. There are four criteria which are most important. What is your preference? What is your weightage for those four criteria? How do these and the 3 cars compare using the price criterion?

How do three cars compare using the miles per gallon criterion? How are 3 cars compared using the comfort criterion? And the last one is how the 3 cars compare using style criterion.

Establishing Priorities Using AHP

- In the following discussion we demonstrate how to establish priorities for the four criteria in terms of how each contributes to the overall goal of selecting the best car.
- The priorities of the three cars using each criterion can be determined similarly.

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In the following discussion we demonstrate how to establish priorities for four criteria, in terms of how each contributes to the overall goal of selecting the best car. The priorities of the three cars using each criterion can be determined similarly. So, at present I am going to explain how to find the priority among the four given criteria.

Pairwise Comparisons

- Pairwise comparisons form the fundamental building blocks of AHP.
- In establishing the priorities for the four criteria, AHP will require decision maker to state how important each criterion is relative to each other criterion when the criteria are compared two at a time (pairwise).
- That is, with the four criteria (Price, MPG, Comfort, and Style) decision maker must make the following pairwise comparisons:
  - Price compared to MPG
  - Price compared to Comfort
  - Price compared to Style
  - MPG compared to Comfort
  - MPG compared to Style
  - Comfort compared to Style

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Pairwise comparisons form the fundamental building blocks of AHP. In establishing the priorities for the four criteria, AHP will require decision makers to state how important each criterion is relative to each other criterion when the criteria are compared two at a time. So, that is why we are going. We are calling it a pairwise comparison that is with the four criteria: price, miles per gallon, comfort, and style.

The decision maker must make the following pairwise comparison. What are they? Price versus miles per gallon, then price, see price versus miles per gallon, the price versus

comfort, the price versus style, miles per gallon versus comfort, miles per gallon versus style, then finally, comfort versus style.

Here, for the comparison purpose we are going to use a comparison scale. So, the comparison scales for the importance of criteria using AHP. So, what are the comparison scales? The first one we start from the bottom if, it is equally important. If the two criteria are equally important, we call it 1. Moderately, more importantly, we can give 2 or 3. Strongly more important, we can give 4 or 5.

Very strongly more important 6 and 7 extremely more important 8 and 9. Remember, there is no zero here.

In each comparison, the decision maker must select the more important criterion and then express a judgment of how important the selected criterion is. For example, in the price miles

per gallon pairwise comparison, assume that the decision-maker indicates that the price is more important than the miles per gallon. To measure how much more important the price is compared to miles per gallon, AHP uses a scale with values from 1 to 9.

The table, which is shown on the right-hand side, shows the decision makers' verbal description of the relative importance between the two criteria, which is converted into a numerical rating. So, first, you have to see which is more important than how much is more important that numerical value has to be picked from the table which is given on the right-hand side. In the car selection example suppose the decision maker states that the price is moderately more important than miles per gallon.

Where is moderately more important? This one is moderately more important. In this case, the numerical rating 3 is assigned. See that this 3 is assigned; you can select 2 also, but here, it is up to you to decide, but it has to be consistent. If you say moderately more important, you can go for 3 and 2 also, then if you feel that is more moderately important, then you can go for 3, but it has to be consistent.

In this case, a numerical rating of 3 is assigned to the price of miles per gallon in a pairwise comparison.

So, that is why you see that the 3 is entered. So, if in case the price is equally to moderately important, now we are jumping from equally to moderately important, then the numerical scale should be 2. Suppose the price is equally to moderately more important than style.

Again, it is 2 equally to moderately more important. Suppose you say comfort is moderately too strongly more important.

So, from moderately to strongly more important, it has to be 4, and the style is equally to moderately more important than 2. So, where you have to see from which level to which level we are moving.

Summary of Pairwise Comparisons of the Four Criteria for the Car Selection Problem

Pairwise Comparison	More Important Criterion	How Much More Important	Numerical Rating
Price–MPG	Price	Moderately	3
Price–Comfort	Price	Equally to moderately	2
Price–Style	Price	Equally to moderately	2
MPG–Comfort	Comfort	Moderately to strongly	4
MPG–Style	Style	Moderately to strongly	4
Comfort–Style	Style	Equally to moderately	2

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Summary of pairwise comparison of the four criteria for car selection problem. So, this slide is more important because we will pick up this value and use it for further analysis. So, what is the first pairwise comparison? I have compared price and miles per gallon. The decision-maker I asked about which is more important for you says the price is most important, how much more important than miles per gallon? He was saying moderately.

So, if it is moderately control P if it is moderately, it is 3. Then I asked about the price versus comfort again. He said price is important, and how much price is important when compared to comfort equally to moderately 2. So, these values have been taken from the previous table, which is given in the previous slide. So, there are two points I want to emphasize; whenever we do the pairwise comparison, first you have to see which is most important. Then you have to say how much is more important than the other criteria.

As shown, the flexibility of AHP can accommodate the unique preferences of each individual decision-maker. First, the choice of the criteria that are considered can vary depending upon the decision maker. Not everyone would agree that price, miles per gallon, comfort and style are the only criteria to be considered in a car selection problem. Perhaps, you would want to add safety resale value and other criteria if you are making the car selection decision. AHP can accommodate any set of criteria specified by the decision maker.

Of course, if additional criteria are added, more pairwise comparisons will be necessary. In addition, if you agree with the decision maker that price, miles per gallon, comfort, and style are the four criteria to use, you will probably disagree with him as to relating to the importance of the criteria. Using the format of the table, which is given on the right-hand side, you could provide your own assessment of the importance of each pairwise comparison.

AHP would adjust the numerical rating to reflect your personal preferences. For example, if somebody has given 3, you can give it to 4; also, it is up to you to decide. For example, here, somebody is given 2, you can make it 3. Here, instead of 3, you can give 4; instead of 4, you can give 6. So, it will differ from the decision maker to the decision maker.

To determine the priorities for the four criteria, we need to construct a matrix of the pairwise comparison rating provided in the table. Look at the right-hand side. There are four criteria. So, we have to make a table like this using the four criteria. The pairwise comparison matrix will consist of 4 rows and four columns. There are four rows and four columns.

Each of the numerical ratings in the table, the table which is given at the bottom, must be entered into the pairwise comparison matrix. As an illustration of this process, consider the numeric numerical rating 3 for the price and miles per gallon. For example, see the first one:

when you compare price and miles per gallon. When you do a pairwise comparison, the price is most important.

How much is more important? Moderately. What is the score for moderately? It is 3, so 3 have to be entered here. So, the table shows that for this pairwise comparison, the price is the most important criterion.

Thus, we must enter a 3 into the row labeled price. See, we compared prices, and the miles-per-gallon price is more important. So, whichever is more important, you have to pick up that row, and then for corresponding criteria, the price is 3 times more important than miles per gallon, so you have to enter 3. In general, the entries in the column labeled most important here. For example, the price is the entry when we compare price and miles per gallon. So, with respect to price, we have to enter 3.

As another illustration consider miles per gallon and comfort. Where is a mile per gallon here? This is miles per gallon and comfort. Which is more important? Comfort is most important. How much score? It is a 4. So, we have to catch this comfort roll, so when and corresponding miles per gallon, you have to enter this 4. So, the table shows that comfort is the most important criterion for this pairwise comparison, and the numerical rating is 4, which is why we were given 4.

Likewise, we have to do for all criteria. So, we entered four here following this procedure for the other pairwise comparison shown in the table. We obtain the following pairwise comparison matrix, which is on the right-hand side. For example, when you compare style and miles per gallon, you see style, and suppose you are here miles per gallon style, which is more important, style is more important, how much the rating is 4.

So, this 4, suppose you compare miles per gallon and comfort which is more important comfort is more important. So, we have already entered 4. For example, if you compare price and style, price is more important than when you compare price and style, it is 2. So, we entered this table by looking at the table at the bottom.

Because the diagonal elements are comparing each criterion to itself, the diagonal elements of the pairwise comparison matrix are always equal to 1. So, here we entered 1, 1, 1, 1. For example, if the price is compared to the price, the verbal judgment would be equally important with the rating of 1; thus, a one would be placed into the row labeled price into the column labeled the price in the pairwise comparison matrix.

At this point, the pairwise comparison matrix appears as follows because we have entered 1, 1, 1 in the diagonal.

All that remains is to complete the entries for the remaining cells of the matrix. So, far we have entered the bottom one. There are some cells that I am going to fill in that are not filled. Now, to illustrate how these values are obtained, consider a numerical rating of 3 for the price

miles per gallon pairwise comparison. Look at this 3. This rating implies that since already we have entered 3.

So, here we are going to enter 1 by 3 when we compare price and comfort in 2. So, when you compare comfort and price, it should be 1/2. For example, when you compare price and style, it is two. When you compare style and price, it should be the inverse of this 1/ 2, and you see comfort and miles per gallon. It is 4. Then, miles per gallon and comfort should be 1/ 4. You see style versus miles per gallon. It is 4.

Then miles per gallon versus style should be 1/ 4 when you compare stale, and comfort is a 2. When you compare comfort and style, it should be 1/2. So, we are entering the inverse of that value; for example, here, what is the meaning of this 1/ 3? This rating implies that miles per gallon price pairwise comparison should have a rating of 1/3. This is because the decision maker already indicated.

The price is moderately more important than the miles per gallon rating of 3; we can infer that the pair-wise comparison of miles per gallon relative to price should be 1 upon 3. Then only that relationship will be maintained. That is the logic behind we are writing the inverse of each number. What do we understand? If the upper diagonal or the lower diagonal of the pairwise comparison matrix is given, we can fill the remaining portions easily.

Similarly, because of the comfort and miles per gallon, see this: it is four miles per gallon, and comfort should be 1/4. Thus, the complete pairwise comparison matrix for the car selection criteria is as follows. That is given in the table on the right-hand side.

Pairwise Comparison Matrix

* Thus, the complete pairwise comparison matrix for the car selection criteria is as follows:

	Price	MPG	Comfort	Style
Price	1	3	2	2
MPG	1/3	1	1/4	1/4
Comfort	1/2	4	1	1/2
Style	1/2	4	2	1

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This is the complete pairwise comparison, which considers all the inverses of given pairwise comparison values. Dear students, in this lecture so far, I have explained how to do a pairwise comparison for the given criteria. In the next lecture, I will use this pairwise comparison to explain how to get the priority level for the criteria. We have four priorities. I will identify which priority is most important for the decision-makers.

And also, I will explain how to get the overall priority level considering both criteria and alternatives.