Security Analysis & Portfolio Management
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Lecture: 08
Arbitrage Theorems, EMH, Money Markets

Welcome back. So, we were discussing the theorem which is here on your slide, "Portfolios A and B are such that in every possible state of the market at time t=T (which is the investment horizon), portfolio A is worth at least as much as portfolio B and portfolio A is worth more than portfolio B in some states of the world, then at any prior time, t<T, portfolio A is worth more than portfolio B.

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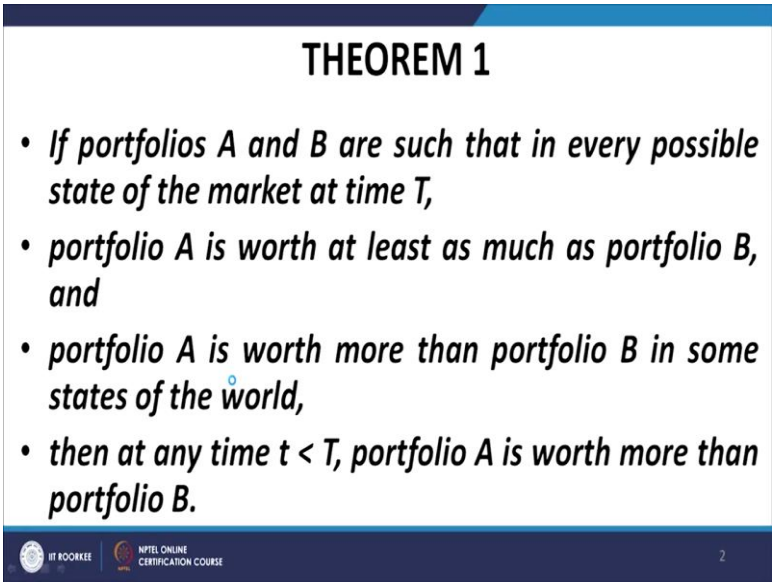

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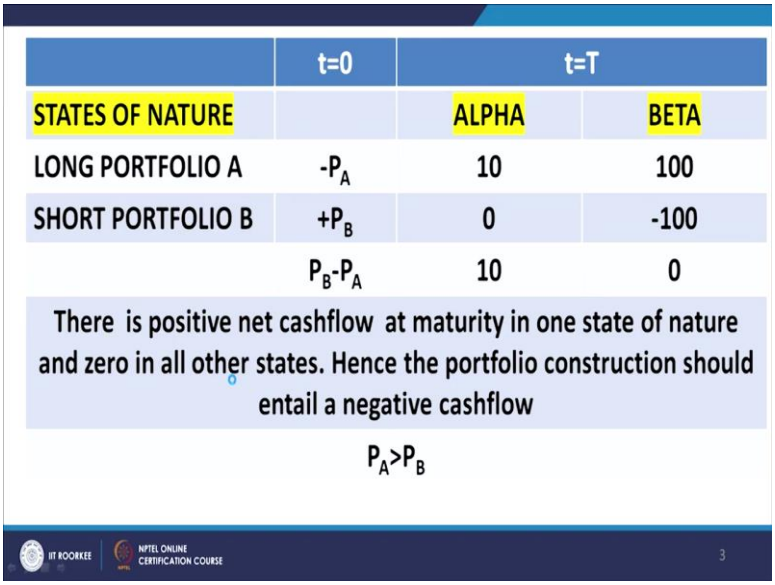

We try to establish this theorem by virtue of an example. We have got two portfolios A and B; we take a long position in portfolio A and a short position in portfolio B. The price of portfolio A is P_A, price of portfolio B is P_B. Because we are long in portfolio A it will entail a cash outflow. Therefore, P_A is negative and is given a negative sign. Similarly, because we are short in portfolio B, we will get a cash inflow therefrom at time t=0, and therefore, we are taking P_B as positive. We assume that in the state Alpha portfolio A gives us a payoff of 10 and in state Beta, portfolio A gives us a payoff of 100. Portfolio B gives us 0 and 100 respectively. Because P_B is short, the payoff would be negative (that is negative of the long payoff). So, it would be 0 and (-)100.

Now, clearly the conditions required by the theorem are satisfied. Portfolio A is as good as portfolio B in all the states. And there is at least one state (which is the Alpha state) in which portfolio A is superior to portfolio B. Now, the net payoffs from the combination of the long and short positions in portfolio A and B respectively, is 10 if Alpha state occurs and 0 if Beta state occurs. Because there is a positive payoff in one of the states and a 0 payoff in the other state, this constitutes an arbitrage portfolio. As a result of this, the cost of establishing this portfolio must be positive or there should be a cash outflow at t=0 for establishing this portfolio. Therefore, $P_B - P_A < 0$ or $P_A > P_B$ as required by the theorem.

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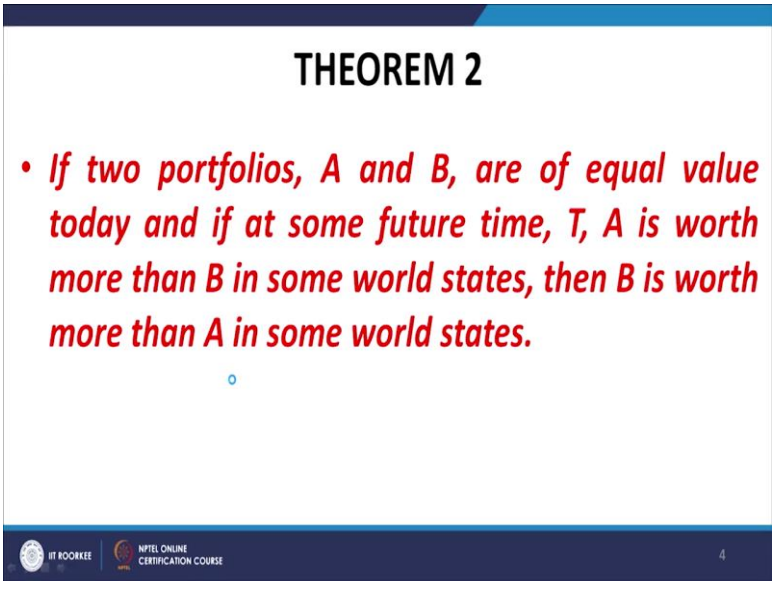

Now, let us look at the second theorem. “If two portfolios A and B are of equal value today (t=0), and if at some future time t= T, A is worth more than B in some world states, then B must be worth more than A in some other world states.” I repeat, “if two portfolios A and B, are of equal value today (t=0), and if at some future time (t=T) A is worth more than B in

some world states, then it must necessarily be that B is worth more than A in some other states.

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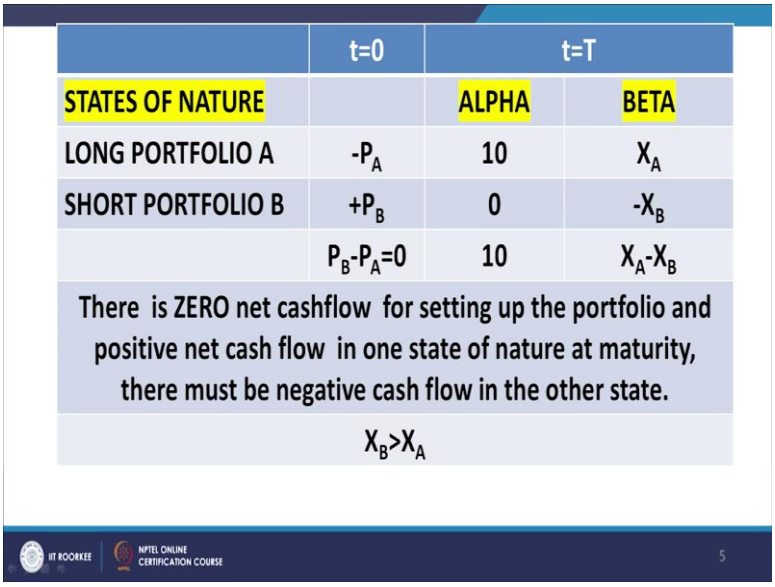

Again, let us try to establish this by an example. We have got two portfolios A & B here and we will take a long position in portfolio A, and a short position in portfolio B (similar to what we did just now). And we pay a price for establishing portfolio A, so P_A is negative and P_B is positive.

If the state Alpha materializes portfolio A gives us 10 and if the state Beta materializes portfolio A gives us X_A. As far as portfolio B is concerned, it gives us 0 if state Alpha materializes, and it gives us (-)X_B if the state Beta materializes. Why minus X_B? Because we are short in portfolio B. So, while P_B is positive, X_B will be negative because we are short in B.

Now, the net cost for establishing this portfolio is clearly P_A- P_B=0 because it is given that the value of the two portfolios is the same at an earlier point in time t=0. Now, the payoff of the combination, long portfolio A and short portfolio B is 10 at t=T, if Alpha state materializes and it is X_A-X_B if Beta state materializes. Because the cost of the portfolio is 0, the expected payoff should be 0. In other words, we must necessarily have is that, because the payoff in the Alpha state is +10, the payoff in the state Beta must be negative irrespective of whatever probabilities we assign to the occurrence of the states Alpha and Beta. However, small or large the probabilities of the occurrence of state Alpha and Beta may be, it must necessarily be that the state Beta must have a negative payoff for the combined portfolio. Otherwise, we will end up with a positive or a negative expected value at t=0 or any earlier point in time

t<T, which is contrary to the assumption of the theorem. The assumption says that the prices are identical. So, in this situation, it must necessarily be that $X_A - X_B < 0$ or $X_A < X_B$ or $X_B > X_A$. This establishes the theorem. Then there is another theorem.

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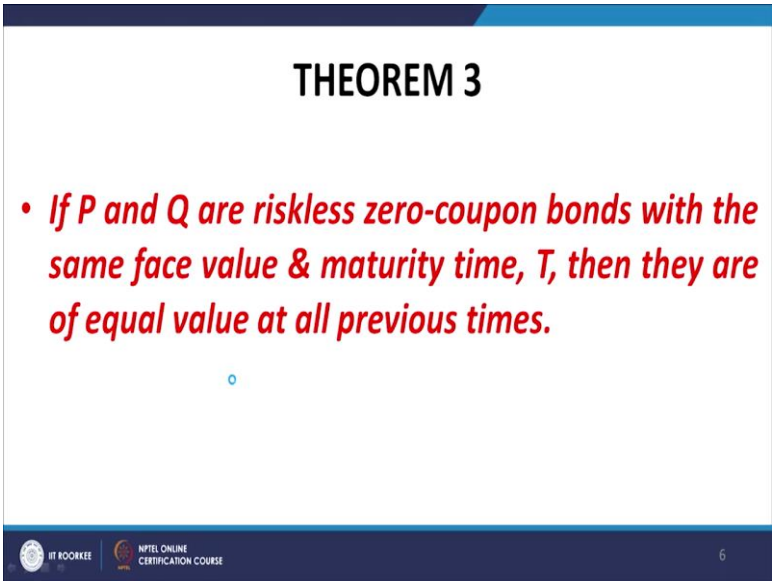

If P and Q are riskless zero-coupon bonds with the same face value and maturity time T, then they are of equal value at all previous times. This is a very interesting theorem, but a very simple theorem to prove.

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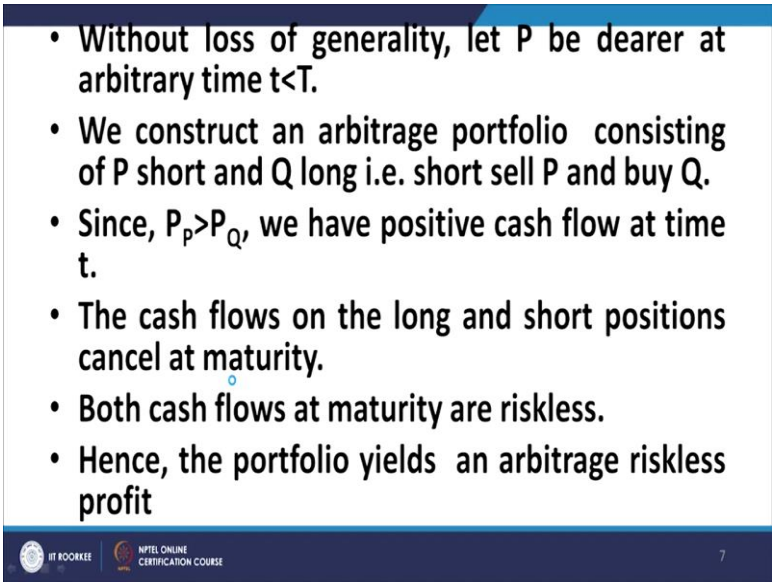

Let us say we have got two bonds P and Q. Let us assume that the bond P is dearer and the bond Q is cheaper. We construct an arbitrary portfolio consisting of short position in P and long position in Q. Because P is dearer and Q is cheaper in the market, when we have a short

position in P and a long position in Q, the net result is going to be a positive cash flow at t=0. Now, clearly the face values of the two bonds are identical. And therefore, because one bond is long and the other bond is short, the maturity payoff of the combination will be 0. The two cash flows that occur on the dates of maturity of the two bonds (which coincide) will annul each other and we will have a 0 payoff at maturity from the portfolio.

Secondly, it is given that both the bonds are riskless bonds. That means, it must necessarily be true that the price of the combination of the two bonds must be 0 at any earlier point in time. That means $P_A = P_B$.

So this theorem is quite straightforward. ***Please note the relevance and the importance of the two bonds being risk-free, because if they have differing risk profiles, then this theorem would not hold. The theorem holds only because the two points are essentially risk-free.*** Because the portfolio has 0 payoff at maturity and the portfolio is riskfree, it must necessarily be that the cost of establishing this portfolio must be 0 at any earlier time. So, we are through with the proof. And again, we get the result from the arbitrage portfolio.

Now, an introduction to the subject of Security Analysis and Portfolio Theory would not be complete without a reference to efficient markets.

So, I will make a quick reference to the concept of efficient markets. I will discuss the relevance of the relationship between efficient markets and arbitrates portfolios briefly at this point in time, we will revisit this particular issue in detail towards the end of this course. So, let us start with a brief review of the postulates of the efficient market hypothesis. There are different forms of the efficient market hypothesis.

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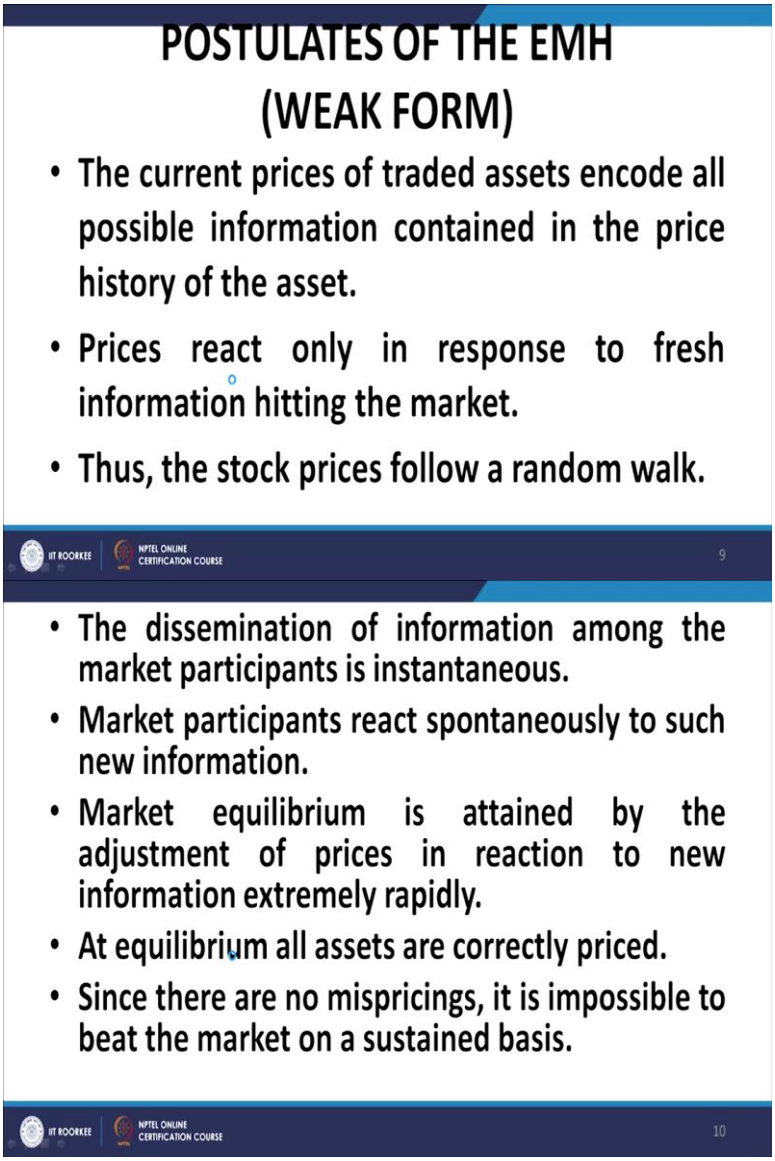

The form that I am discussing at present is the weak form of the efficient market hypothesis. So, the postulates of this weak form of efficient market hypothesis are:

(i) The current prices of traded assets encode all the possible information contained in the price history of the asset. The current price encodes all information that is extractable from the price history of the asset. In other words, all the technical analysis that is done to extract information or to forecast future prices is, in some sense, absolutely futile in relation to this hypothesis. So, the current prices encode everything that is knowable by doing any kind of analysis of the price history of the asset.

(ii) Prices react only in response to fresh information hitting the market. Prices react only if fresh information hits the market. The changes in prices is a response to new information percolating into the market. Therefore, because new information coming to the market is not known to the market participants in advance, it can be given the character of random

information and as a result of which the prices follow a random walk under the weak form of the efficient market hypothesis.

(iii) The dissemination of information throughout the market is instantaneous. This is a very strong assumption and has been questioned. It is true that this assumption does not hold absolutely in the real world. But this assumption is a part of the postulates of the efficient market hypothesis.

(iv) Market participants react spontaneously to this new information and take-up trades in the market immediately. So, the reaction of the market participants to the new information is also spontaneous. So, on the one hand, the information percolates into the markets instantaneously and the reaction of the market participants to take up trades in relation to the new information is also spontaneous.

Therefore, because of these two factors, market equilibrium is re-established very rapidly. Once some fresh news enters into the market, the market equilibrium is disturbed. The process of dissemination of information and the market's reaction thereto is so fast that a fresh equilibrium is very rapidly created and established in the market with the realignment of the prices to reflect the new information. The prices need to realign in the light of new information and that realignment is very rapid.

(v) At equilibrium, all assets are correctly priced. So, the efforts that are made to identify mispriced assets are futile. Further, because there are no mispricings, it is impossible to beat the market on a sustained basis. Now, please note here the use of the word ***"on a sustained basis"***, there may be the occasional situation where an investor beats the market, because we are talking about random processes. So, there may be the occasional situation when you call heads and the market gives you heads. In that situation you may make a profit, but on a sustained basis, if you continue to call heads, it is very unlikely that you will end up making a profit out of your strategy.

So, that is the efficient market hypothesis. The EMH says that because there are no mispricings you may not make abnormal profits on a sustained basis. However, although there are no mispricings, the process followed by prices are random and therefore there could be the occasional situations where what you call turns out to be correct and you make a profit, but there would also be situations where what you call is not returned by the market and as a result of it you end up making losses. So, on a sustained basis, it is not possible to beat the market, this is the inference of the efficient market hypothesis.

Now, what is the relationship between efficient market hypothesis and arbitrage?

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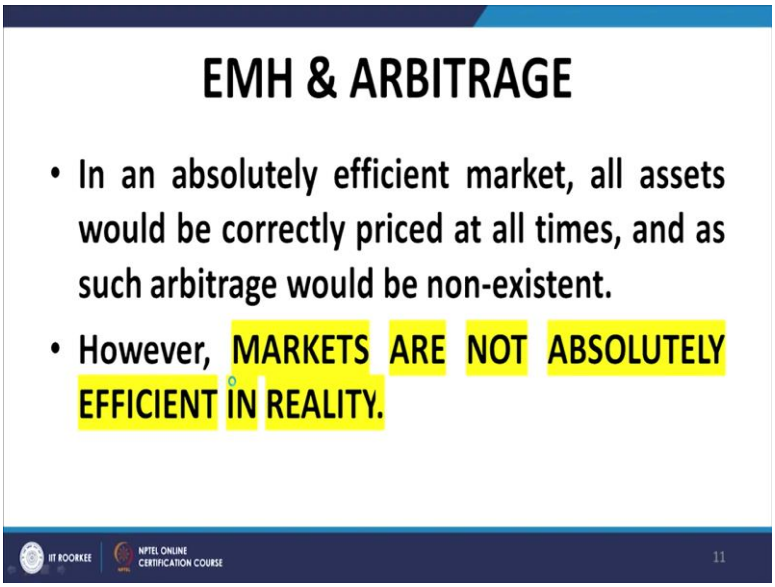

Now, on the one hand, one can straightaway say that if the markets are absolutely efficient, and all assets are correctly priced at all points in time, then there is no possibility of arbitrage in the market. But when we talk about real markets, the percolation or flow of information in the layers of the market is not spontaneous. It is not instantaneous. It takes a finite amount of time. Therefore, at least, during that finite amount of time, mispricings could be prevalent in the market. This will give arbitrageurs the opportunity to come into play. The arbitrageurs take up the trades in order to extract profits from the mispricings and eliminate the mispricings in the process. It is reiterated that these mispricings may arise due to the finiteness of time taken for the percolation of information in the market. The trades of the arbitrageurs in extracting the arbitrage profits as a collateral cause the neutralizing of the discrepancy between the correct price and the mispricings. As a result these mispricings are completely eliminated in the market and the market again returns to an equilibrium where all assets are correctly priced in the market.

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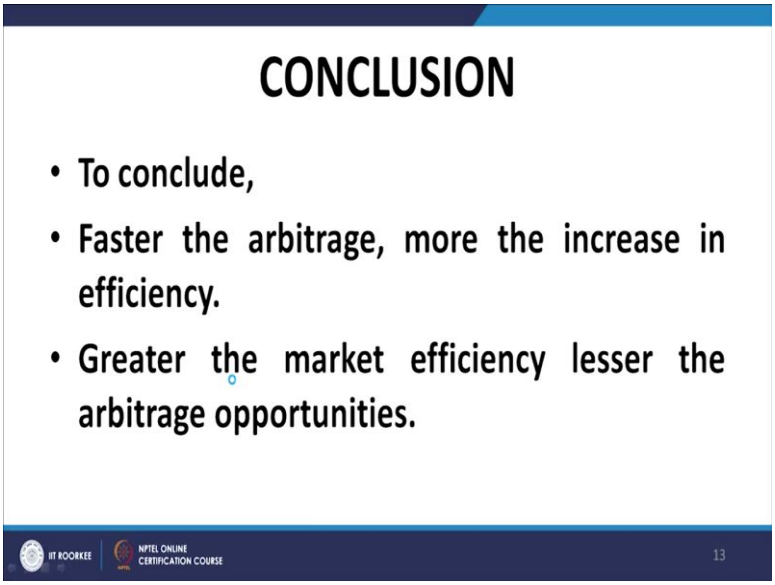

So, in a nutshell, the conclusion is that, "faster the arbitrage, greater is market efficiency". The more rapidly arbitrage take place, if there is any mispricing at any point in time, the faster the mispricings will be corrected. Greater the arbitrage i.e. the more the volume of arbitrage, the more rapid would be the returning of the market to an equilibrium of correctly price assets and therefore, the market should increase in efficiency. On the other hand, greater the market efficiency, the more rapid would be the percolation of information, the lesser chances would be there of assets being mispriced in the market, and as a result of which fewer will be the arbitrage opportunities.

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Now, I will talk about securities market segmentation.

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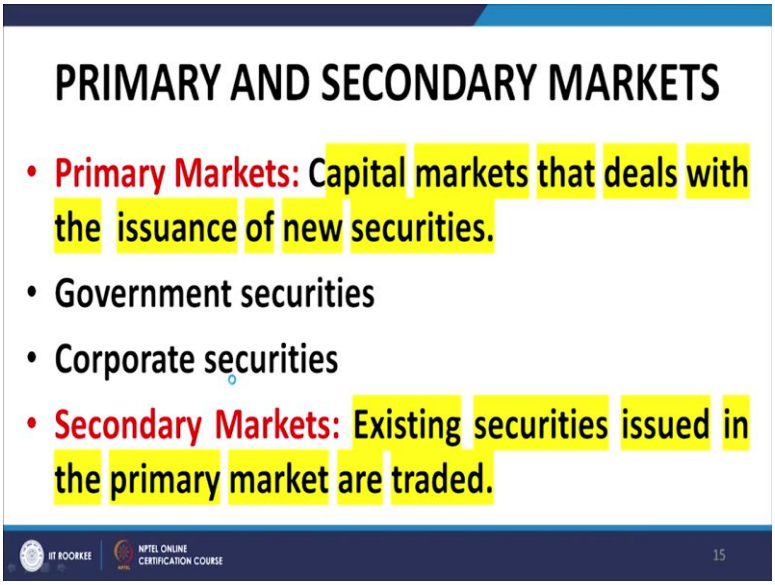

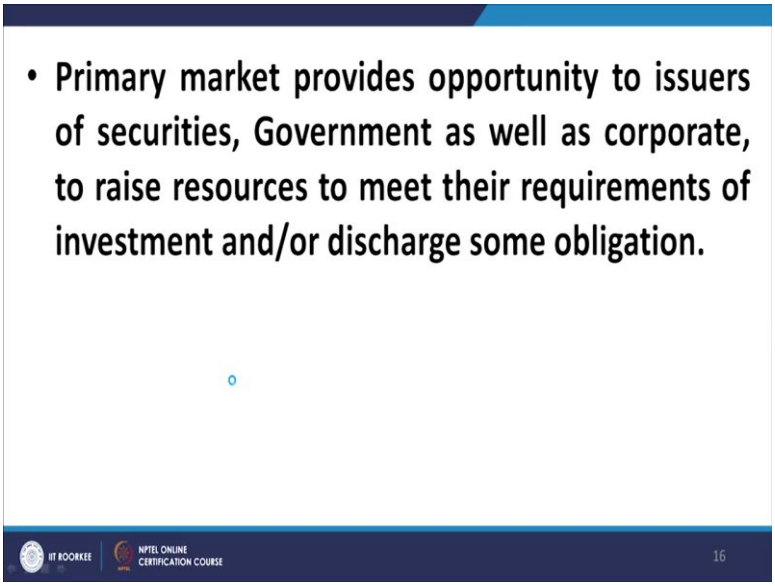

Markets may be segregated into primary and secondary markets. Primary markets are those markets in which the issuer itself markets the securities and the investors take up the positions in the securities directly through an issue by the issuer of the securities. The issuer, of course, can be a body corporate, issuing shares or debentures or bonds or it could be government issuing the bills or bonds. Primary markets are the marketplace, where there is interplay between the issuer of the securities and the potential investors in the instruments which are issued by the issuer.

In the secondary markets, it is the existing securities that are traded. Securities, which have been taken up by the investors either in the primary market or through other secondary market trades can be traded in these secondary markets.

So, primary markets relate to transactions through which you take up securities directly from the issuer and secondary markets relate to trading done between parties, which hold the securities procured from the issuer in the primary market or procured from other parties, who have taken up securities in the primary markets.

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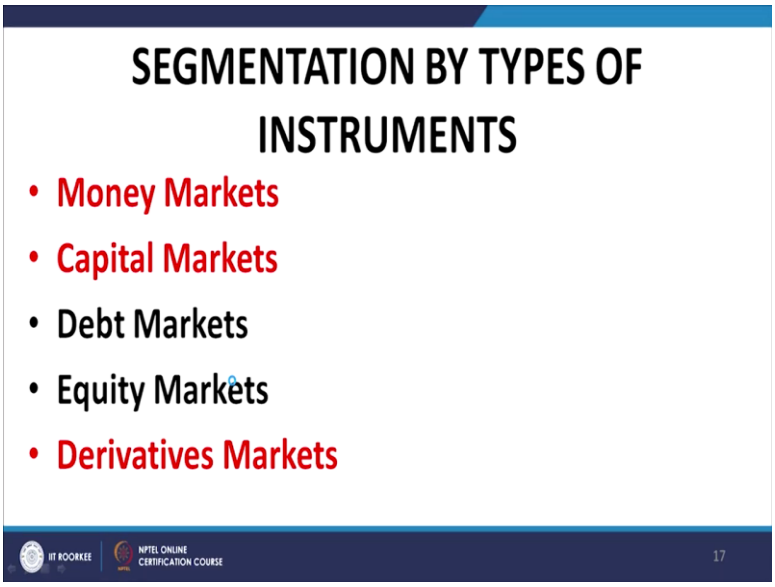

So, this is one form of market segmentation. We can also have market segmentation by types of instruments. For example, we can talk about money markets, which are markets in which short maturity instruments are traded.. We can talk about capital markets. Capital markets involve bond markets and equity markets, and then we also have markets in which derivative instruments are traded, like the Futures and Options segment of the National Stock Exchange (NSE) and the Bombay Syock Exchange (BSE).

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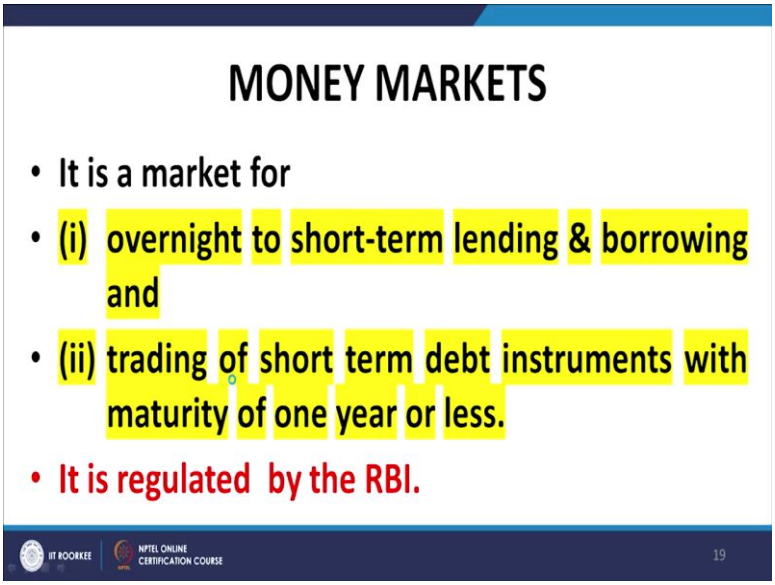

So, now I discuss about money markets. Money markets are markets, in which trading is done for overnight as well as short-term lending and borrowing. Trading of short-term debt instruments with maturities of one year or less takes place in money markets. So, money markets are markets in which money market instruments are traded, that is lending and borrowing instruments of short maturities are traded. To reiterate, these instruments have maturities of one year or less. And this money market is regulated by the Reserve Bank of India (RBI).

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The major participants in the money markets are the Reserve Bank of India, the commercial banks, financial institutions, foreign institutional investors, mutual funds, brokers, and large

corporate houses. Individuals are also allowed to participate in money markets, but the role of individuals is not very significant in these types of markets.

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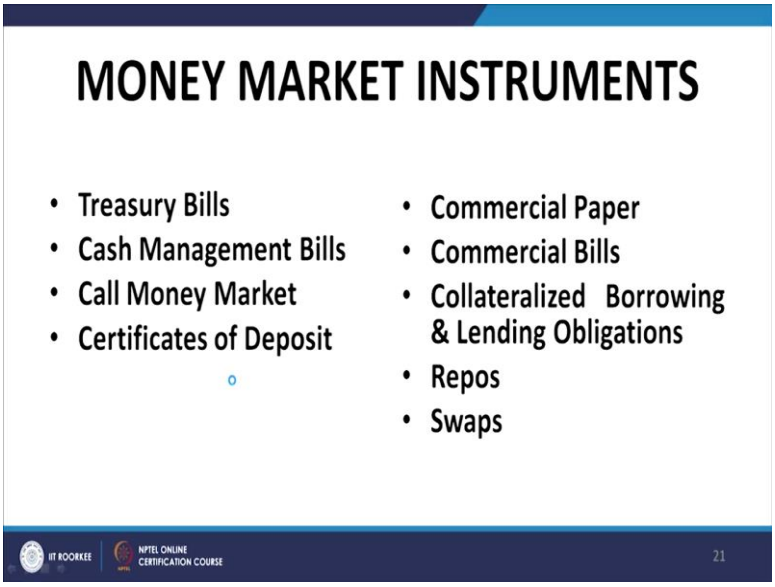

There is a large spectrum of instruments that are traded in the money markets. We have treasury bills, cash management bills, call money markets (which is an interbank market where short-term exchanges of money take place, we will come back to it), certificates of deposits, commercial paper, commercial bills, collateralized borrowing and lending obligations (CBLO), repos and reverse repos and swaps.

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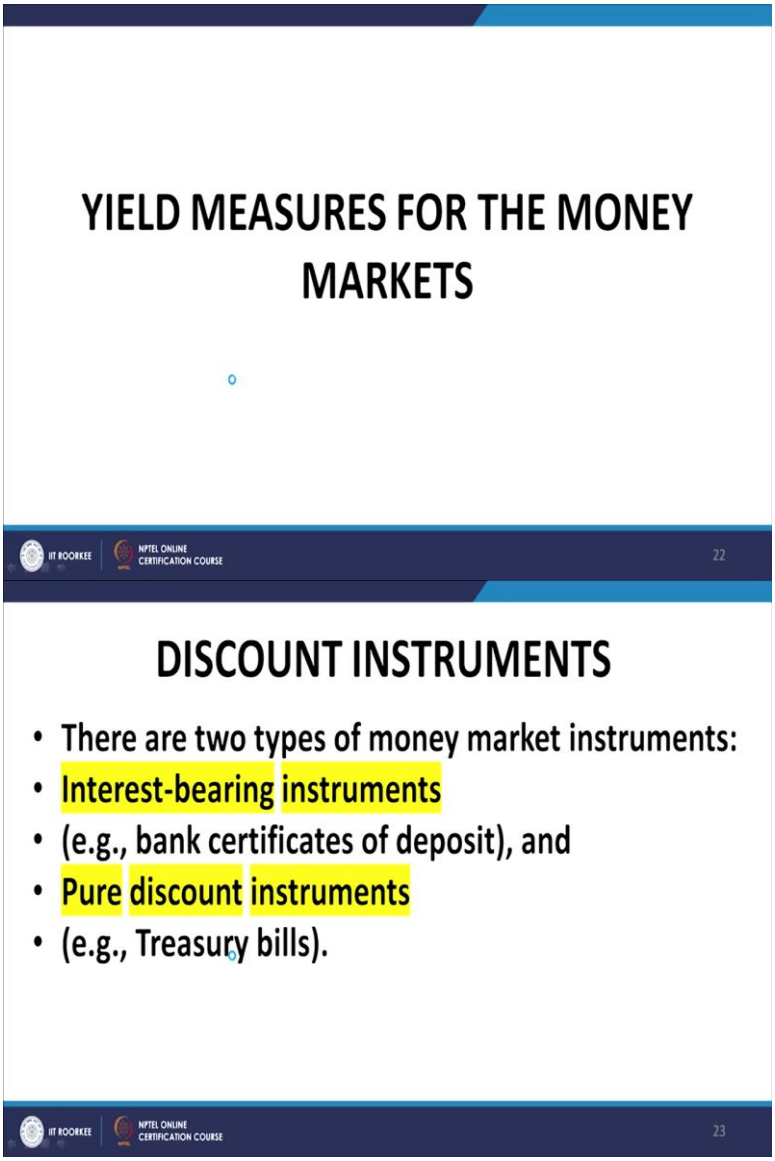

Before I take up these instruments in detail, it is important to discuss the nuances of money markets in terms of the measures of yield that money markets frequently employ. So, in the money markets, two types of instruments are traded:

(i) Discount instruments which are instruments issued at a discount to face value and redeemed at face value. I repeat, the instrument is issued at a discount to face value and is redeemed at face value. The difference between the issue value which is at a discount to face value, and the face value, which is the value at which the instrument is redeemed, constitutes the return to the investor.

(ii) Traditional instrument, which is the instrument is issued at face value and redeemed above face value, that is, at face value plus the interest for the period of issue.

So, we have two types of instruments, discount instruments which are instruments issued at a discount to face value and redeemed at face value, or instruments which are issued at face value and are redeemed at a value above face value, that is, the redemption is at face value plus the interest for the holding period.

Now, the most common measure of yield in the money market is what we call the bankers discount yield (BDY) or simply the discount yield or T-bill yield.

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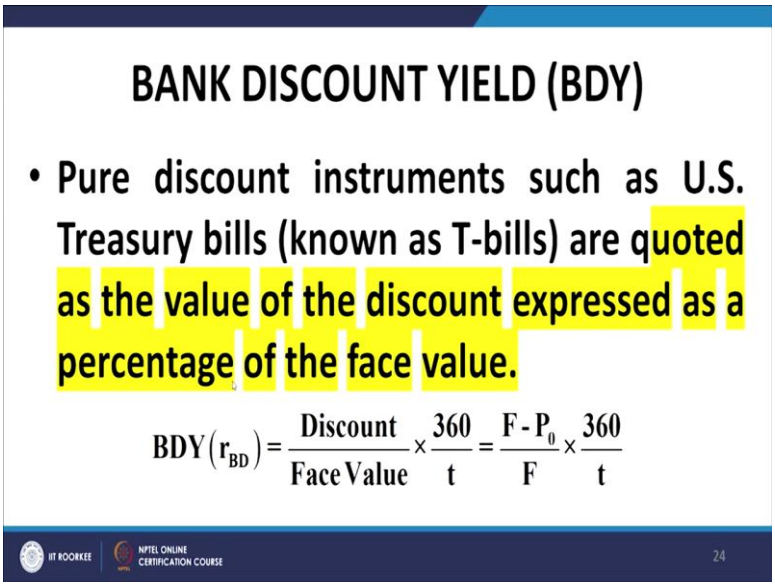

The formula for obtaining this yield is given at the bottom of this slide. There are certain very important features about this particular measure, which makes it considerably different from the traditional yield measures that we are accustomed to in the longer-term markets like the bond markets or equity markets.

The first feature is that the normalization of the return is done with respect to the face value. It is not with respect to the value of the investment, that is, it is not at the value that you expend when you take up the instrument. The normalization is with reference to the value that you receive at the maturity of the instrument. Almost all measures of yield use the investment value as the normalizer for calculating the yield, when we want to work out the return per unit of money i.e. we use the gross return divided by the initial investment to arrive at a return per unit of money. Here, the division is done with respect to the face value. So, the value of the return is expressed as a percentage of face value, not as a percentage of the initial investment to get the return per unit of money and also the yield (yield is the return per unit of money per unit of time).

Secondly, the normalization with respect to time for the conversion to annual yield is done with respect to 360 days here, not 365 years, which is the convention in the bond markets and the longer-term instruments. So, this is the second feature.

Thus, the first feature is that the normalization with respect to money is done with respect to face value and not with respect to the initial investment. Secondly, the normalization with respect to time is with respect to a 360-day year, not with respect to 365 days a year.

The third important feature is that it is quoted on a simple interest basis. There is no compounding involved in the calculation of this yield.

So, these are three salient features, three nuances of the banker's discount yield, which set it apart from the other measures of yield that we are going to talk about, or the other measures of yield that we usually encounter in long term investments.

Then we have the holding period yield.

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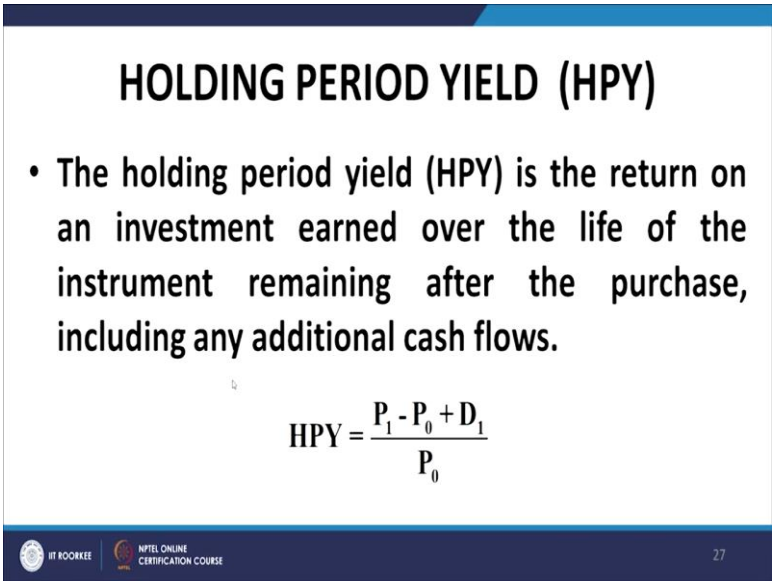

The holding period yield is significantly different from the discount yield. The first difference is that the normalization is done with respect to the initial investment. As you can see here, the denominator is P_0. P_0 is the amount or the price that we pay for entering into the investment i.e. taking up the investment. It is the initial price. This is the way it should be. It is the cash outlay on which we are earning the return. In the discount yield, this feature is absent as the normalization is with respect to final value of the investment.

In the holding period yield, the initial investment value is correctly used as the normalizer for working out the return per unit of money.

The second point is that in this measure, there is no normalization with respect to time. In fact, holding period yield is the return or the yield over your holding period. It is the yield over your avtual holding period. It is not on a per annum basis. I reiterate, HPY is not on a per annum basis. It is calculated on the basis of the period for which you hold the investment.

Thirdly, it includes, as it should, other cash flows on account of interest or dividends that accrue to the investor during the holding period. So, it is the total value (including cash inflows on account of interest, dividends as the case may be and the closing price or the exit price of the investment) less the initial price divided by the initial price.

But please note the important flaws with this yield measure is that it is not normalized with respect to time. It is not expressed on a per annum basis.

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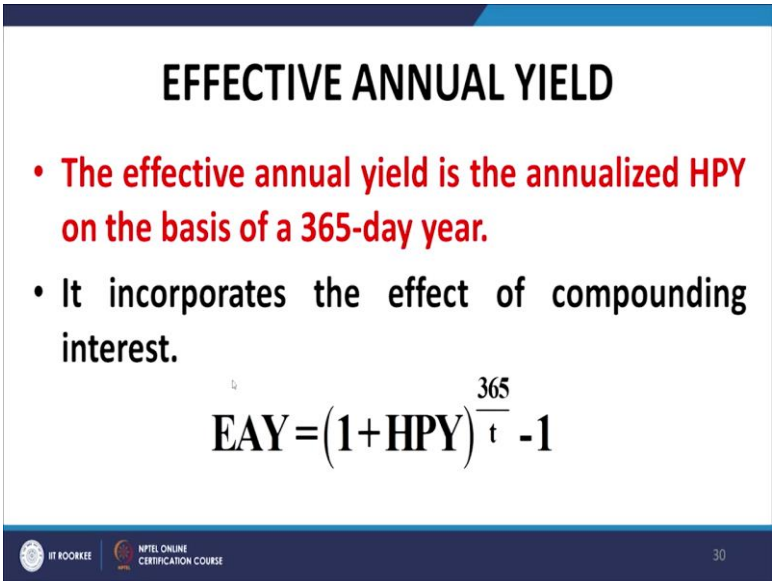

Now, effective annual yield is the most correct form of yield. It does away the shortcomings of both the holding period yield and the bankers discount yield or the T-bill yield. The formula, as you can see here, involves normalization with respect to 365 days and also normalization with respect to the initial investment. So, both the issues with the BDY are taken care of. EAY is not normalized with respect to face value but with respect to initial investment. The time normalization is with respect to 365 days. Thirdly, the yield involves compounding also, so the flaw of using simple interest has also been done away with in this particular measure of yield.

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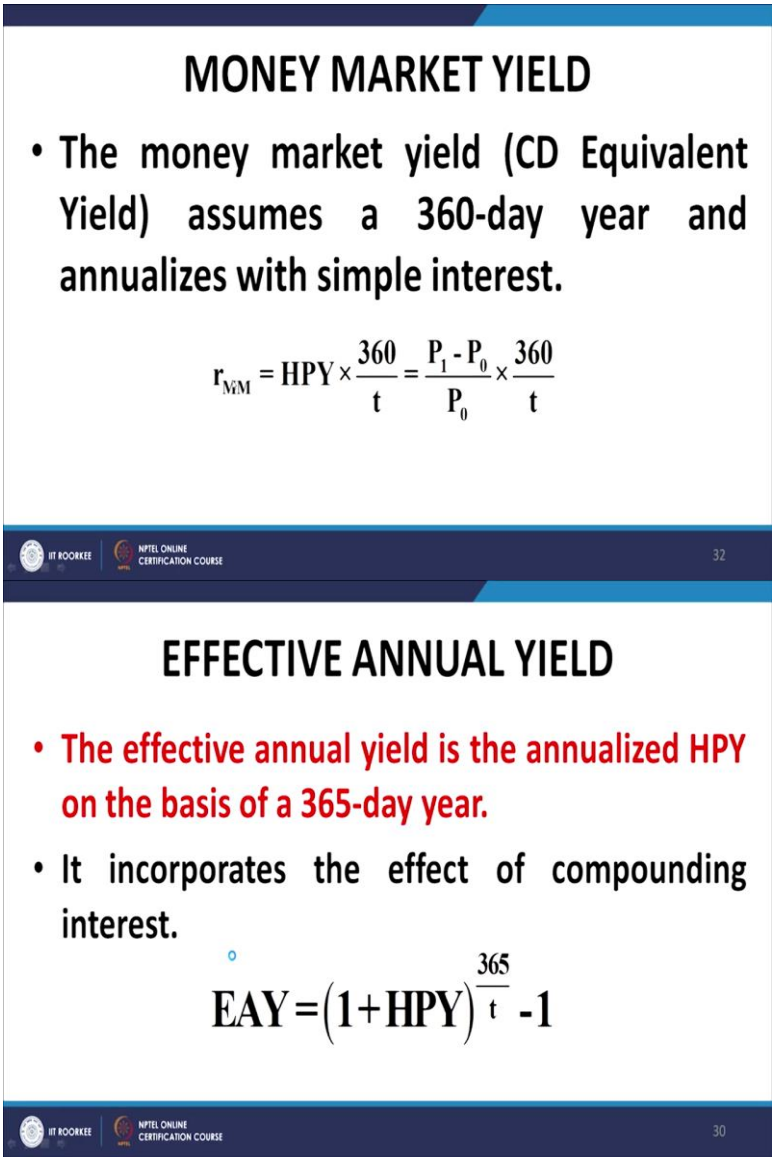

Then there is another measure of yield which is exclusive (as is the bankers discount yield) to money markets. The money market yield is the annualized form of holding period yield. It is the annualized version of holding period yield. However, the annualization is with respect to 360 days which is similar to the T-bill yield.

So, the effective annual yield is also annualization of holding period yield. It takes care of the compounding as well as of the 365-day year. In the case of money market yield, we extrapolate the yield to the per annum basis by using a 360-day year. And secondly, just like the T-bill yield, this is also a yield that is based on simple interest, it involves no compounding.

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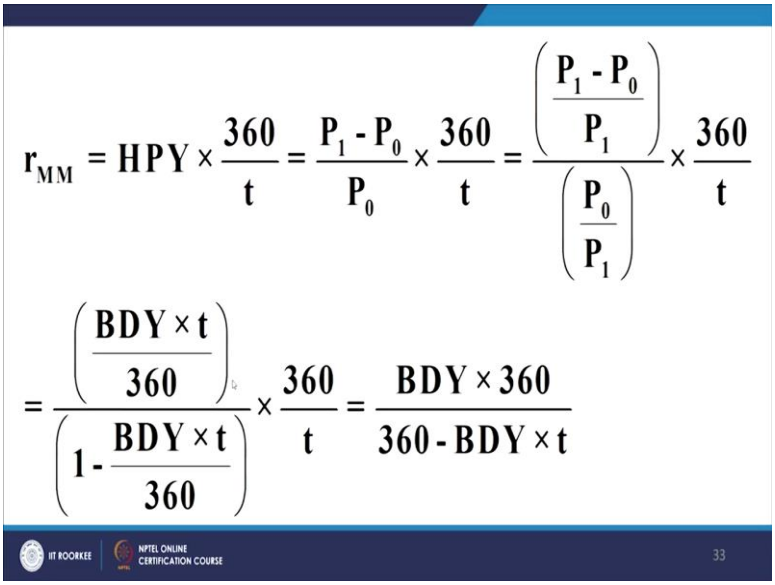

This slide gives you the relationship between money market yield and the bankers discount yield. The derivation is quite straightforward. I will not go through it step by step, but it is simple algebra. So, you can easily work it out. The money market yield is equal to bankers discount yield into 360 divided by 360 minus bankers discount into the holding period.

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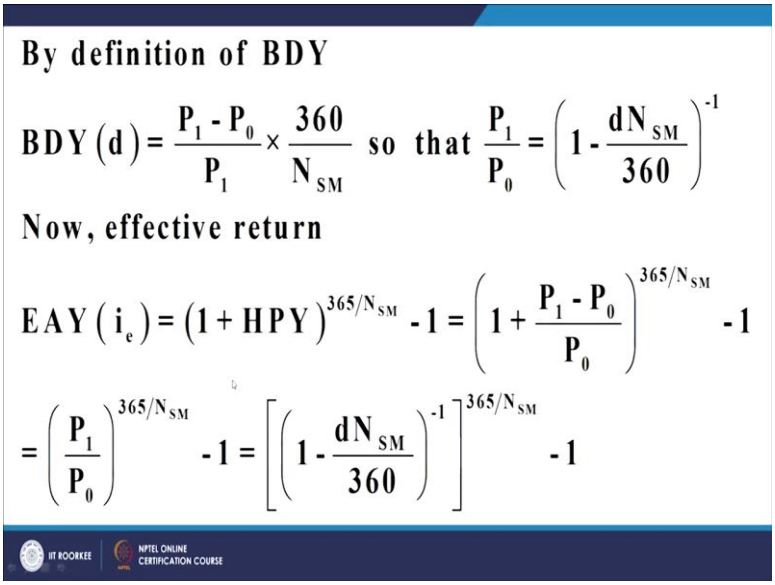

Then this slide gives you the relationship between the bankers discount yield and the effective annual yield. This is a very important formula. But again, the working is quite straight forward, simple algebra nothing more. The bankers discount yield is given by P_1 minus P_0 upon P_1 into 360 upon N_{SM}, N_{SM} is the holding period. From this, we obtain the value of P_1 upon P_0 as the expression that is given in the right-hand upper corner of your

slide. The effective annual yield is given by P_1 upon P_0 to the power 365 upon the holding period minus 1, which gives you the final result here on the bottom of your slide.

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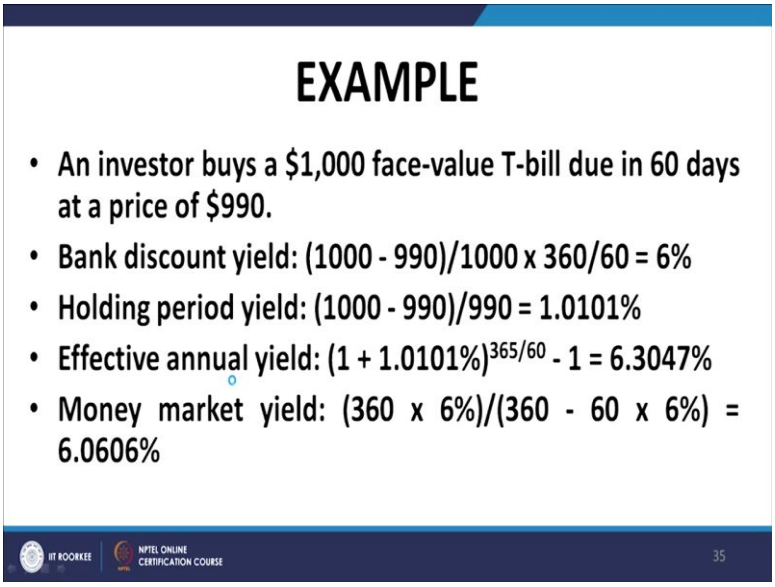

Let us take up an example: An investor buys USD 1,000 face value T-billwith a maturity of 60 days at a price of USD 990.

The bankers discount yield is quite straight forward, [(1000-990)/1000]*(360/60). This is important, divided by 1000. This 360 is the second feature. The third feature is that there is no compounding as we can see from the formula. So, we get the result of 6% p.a. as the bankers discount yield or the simply the discount yield or the T-bill yield.

Then we come to the holding period yield. In the holding period yield there is no time normalization. So, we have (1000-990)/990 i.e. 1,000 minus 990 divided by the price of entry that is 990. This gives us 1.0101%. But please note this is the yield over 60 days. It is not the annualized yield.

Then the effective annual yield which is the correct measure of yield, is worked out on the basis of P_1 upon P_0 to the power (365/60) minus 1 and that gives us 6.3047% p.a..

As you can see here, the difference between the bankers discount yield (which is 6% p.a.) and the effective annual yield (which is 6.3047% p.a.) is significant. The issue is of compounding and the number of days (that is the day count method). The day count approach is different in both cases. As a result of these differences the effective annual yield turns out to be more than the bankers discount yield.

The money market yield has the normalization with respect to 360 days, but it is annualized and it gives us 6.06 percent. But please note here, the denominator of the normalization with respect to money is with respect to the initial investment, the entry amount, and therefore, we get it as 6.06 percent. Here, the denominator is 990. Recall that when we worked out the bankers discount yield, the denominator was 1,000.

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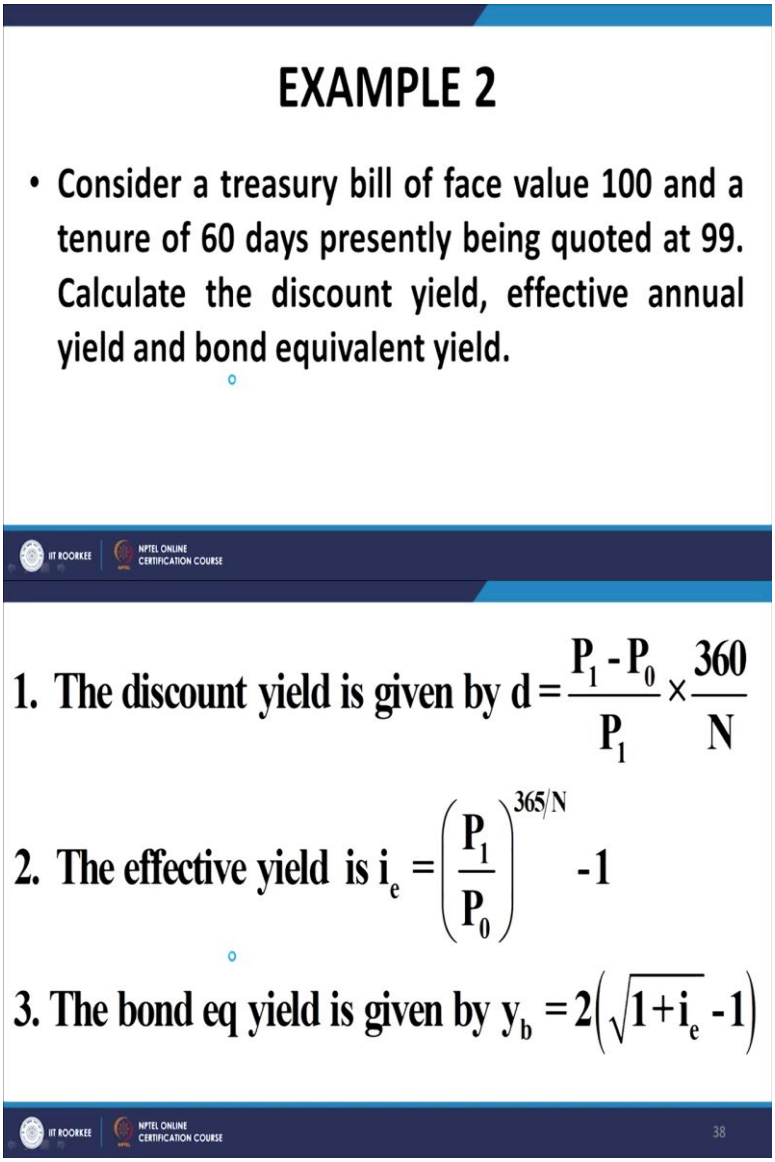

Another example here. We have a treasury bill of the face value of 100 and it has a tenure of 60 days remaining. It is quoted at 99. We need to calculate the various yields. It is quite simple. The discount yield is given by P_1 minus P_0 divided by P_1 into 360 upon the holding period. P_1 is 100, P_0 is 99, holding period is 60 days. The effective annual yield is given by 100 divided by 99 to the power 365 upon 60 minus 1. The bond equivalent yield I will talk

about when I discuss the long-term bonds and so on, which would be our next topic. Thank you.