Security Analysis & Portfolio Management
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Lecture 07
Risk & Arbitrage - II

Welcome back. So, let us continue from where we left off in the last lecture, but before that a quick recap of what we have done so far. In the last lecture, I introduced the concept of preference shares and explained the relevance of preference shares. Why are they called preference share? Because they have a preferential right as to the payment of dividend and as to the repayment of capital in the event of liquidation of the company. Then, I explained the rationale behind calling preference shares as hybrid instruments.

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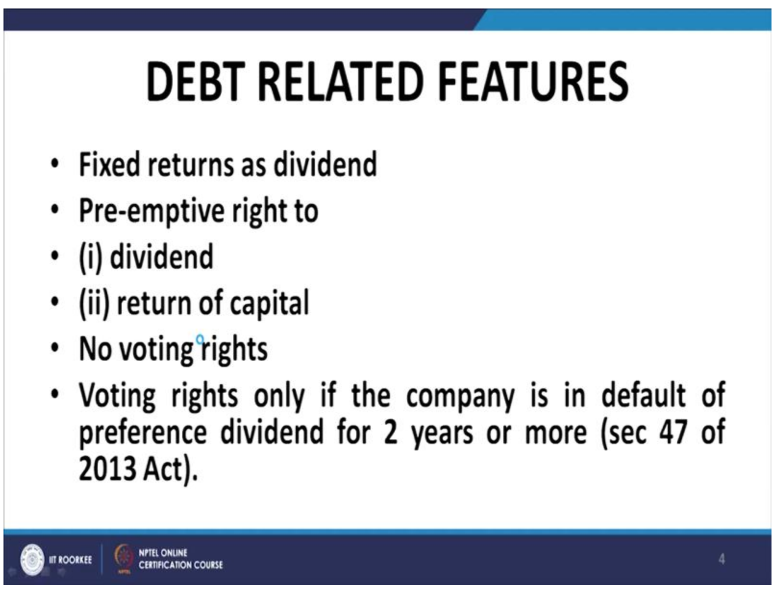

I explained the debt related features of preference shares. They usually carry a fixed rate of dividend, have preemptive rights just like the rights of lenders over equity shareholders in so far as payment of returns (that is dividend or interest as the case may be), and also rights in respect of the repayment of capital in the event of winding up of the company. Further, preference shares have no voting rights in the normal course, as in the case of lenders.

However, if the preference dividend is in arrears for two years or more, then the preference shareholders get a right under Section 47 of the 2013 Companies Act to vote on resolutions, on which equity shareholders are entitled to vote.

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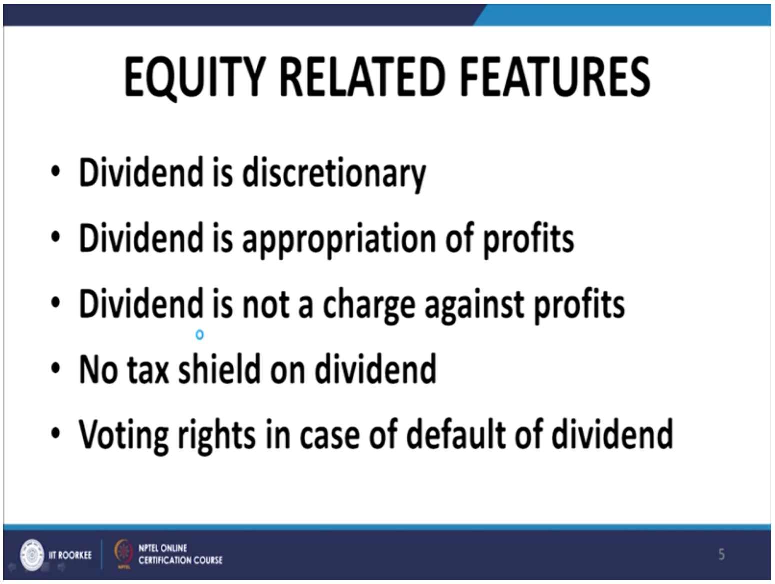

As far as the equity related features of preference shares, dividend on preferred shares is discretionary. This is a very important feature. Just like equity dividend, preference dividend is an appropriation of profits and therefore, a the issuing company does not get any tax shield on dividends. Secondly, preference dividend (like equity dividend) is not a charge against the profits.

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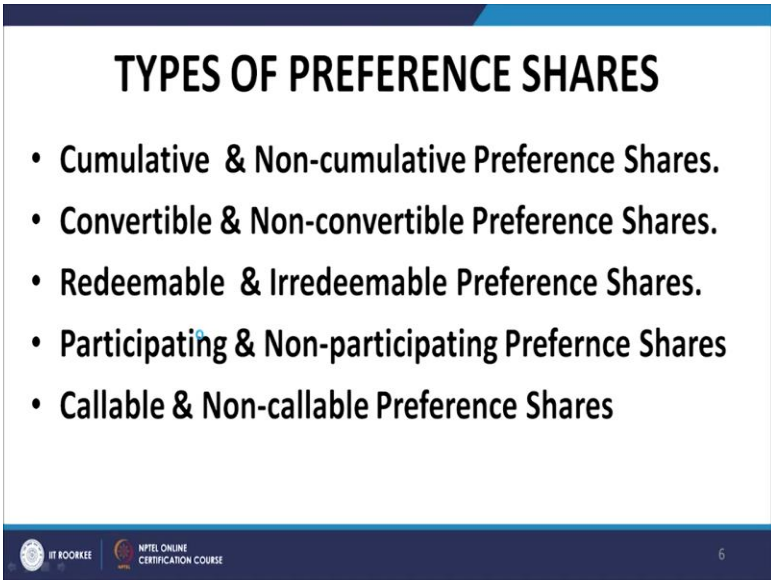

Then, I discussed the types of preference shares. I explained cumulative and non-cumulative preference shares, convertible and non-convertible preference shares (which can be converted to equity shares as per the terms of issue). Cumulative preference shares, incidentally, are those shares on which, if dividend for a particular year is not paid, then that is carried forward to the subsequent years. In other words, the arrears of dividends are not extinguished in the year to which they relate, but they get carried forward. I talked about redeemable preference shares and irredeemable preference shares. I also discussed the rationale behind the abolition of the issue of irredeemable preference shares in India.

I also discussed participating and non-participating preferences shares. Participating preference shares are those that are entitled to participate in the profits of the company and the resources of the company in the event of winding up over and above the fixed rate of recovery of their dividend and paid-up capital as the case may be. In other words, they become part of the resources that belongs to the equity shareholders in the event of payment and declaration of dividends as well as redemption of capital on winding up of the company. Callable preference shares are shares which can be called up by the issuer, as per the terms of issue at the discretion of the issuer. That is what the callability property signifies. It relates to the call option property, which is the right to buy an asset. Callable preference shares accordingly, carry the right on the part of the issuer to exercise that right and call back the capital at its discretion. So, they are called callable preference shares.

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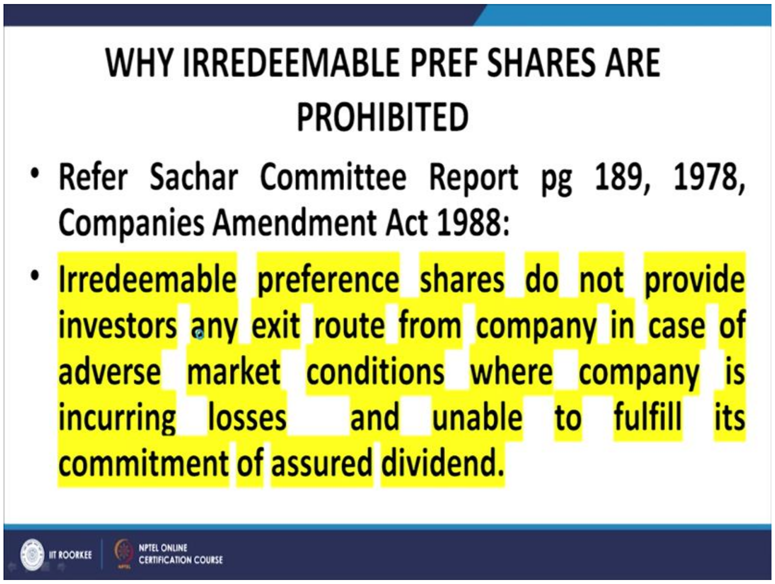

Why irredeemable preference shares are not allowed to be issued to the public in India? Well, this issue was examined by the Sachar committee and it was decided that these shares (keeping in view the level of financial literacy in our country) should not be allowed to be issued by corporates in India. It was felt that such irredeemable preference shares do not provide an exit route to the investors in the event of the company being in bad shape, and the rate of return on these shares may be completely out of phase with the environment to which they relate,

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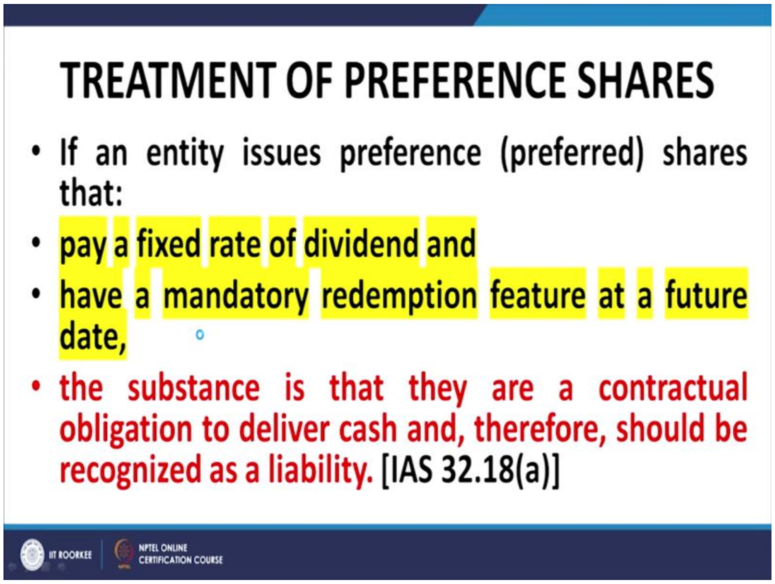

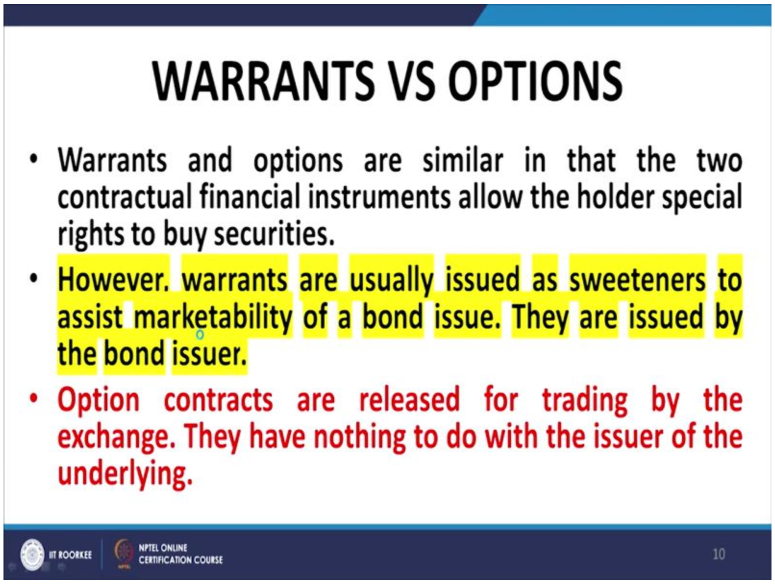

Insofar as treatment of preference shares in accounts is concerned, if the preference shares carry a fixed rate of dividend and have a fixed maturity, then they are to be considered as part of liabilities as per the IFRS provisions. If these conditions are not a part of the issue, then they would be treated as part of equity.

Then I discussed the difference between warrants and options, warrants are sweeteners that are usually attached added either to preference shares or to debentures by a company, which are tradable parts of the primary instrument and they can be traded in their own rights. Warrants entitle the holder of the instrument to buy one or more equity shares in the company at terms that are specified in the issue document at a price which is called the exercise price and on or before a date, which is called the expiry date. So, warrants are instruments which are issued by the company itself (the issuer company) and the holder of the warrant is entitled to subscribe to the equity shares of the issuer company at the exercise price and on terms as to the expiry which are contained in the issue document.

As far as options are concerned, options are tradable contracts which are usually traded on appropriate derivative exchanges. And they are issued by the relevant exchange for trading. They are released for trading by the relevant exchange. They really do not have anything to do with the issuer company of the underlying asset. So, that is the fundamental difference between a warrants and options that we discussed in the last lecture.

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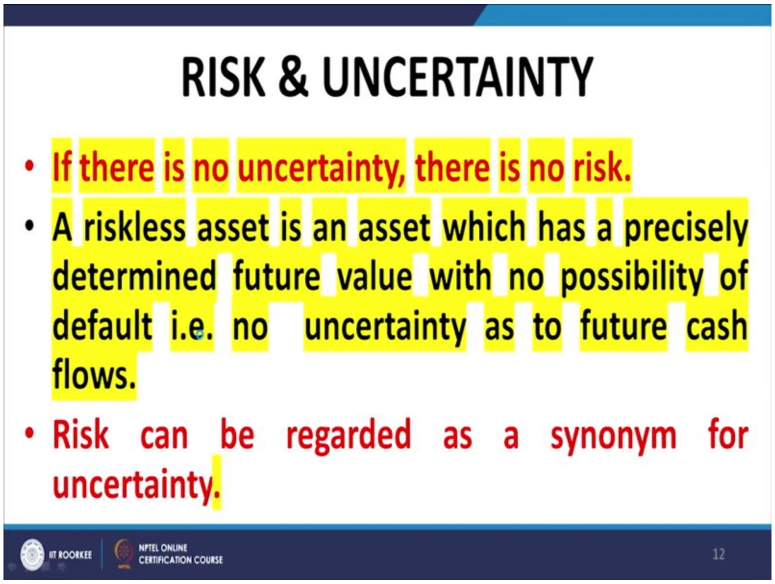

Then I moved on to risk and arbitrage. I defined risk in terms of uncertainty and we agreed that if there is an instrument, which guarantees the payment of a certain amount or a certain value at a future date, then obviously, there is no risk attached to that investment, if the guarantee is infallible i.e.if the guarantee will not be defaulted upon. And therefore, I said that if there is absolute certainty as to the final value of our investment then there is no risk. Putting it in other words, one could say that risk arises from the uncertainty in the future value of investment. If the investment can take one of two or more values at the future date, then that results in risk at the point of investment. So, risk was treated as synonymous with uncertainty.

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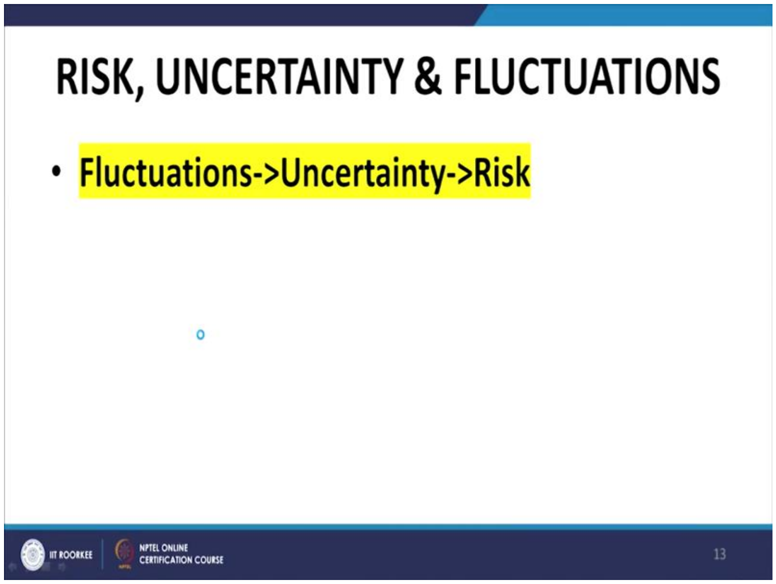
) 

And then I said that it is only when the value of the investment on the date of maturity or the final value of the investment has the possibility of fluctuating between two or more values uncertainty is there and therefore risk is there. So, the fluctuations in the final value of an investment results in the creation of uncertainty as to which value the investment would take on date of its maturity. This results in creation of risk in the mind of the investor.

So, that is the flow that normally takes place. The fluctuations or the possibility of fluctuation between different values of the asset result in uncertainty and uncertainty results in risk in the mind of investors.

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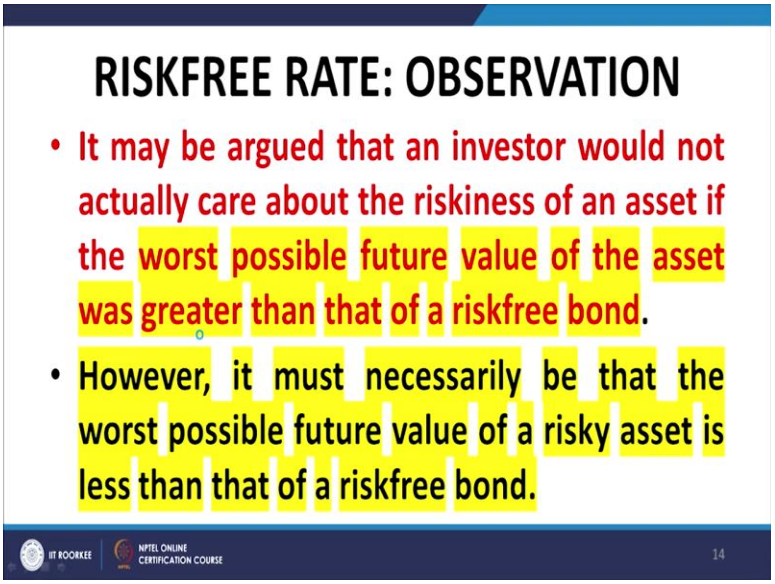

Then, we discussed an important property, that the worst possible future value of a risky asset must necessarily be less than the corresponding value of a risk-free asset. What, essentially, it means is that if there is a risk-free asset trading in the market and there is a risky asset trading in the market at the same price, then the worst possible outcome of the risky asset must necessarily be worse than the outcome of the risk-free asset. I repeat, if there are assets trading in the market at the same price, one is a risk-free asset and the other is a risky asset, then the worst possible value that the risky asset can take must be less than the value that the risk-free asset can take. This is dictated by the grounds of requirements of no arbitrage.

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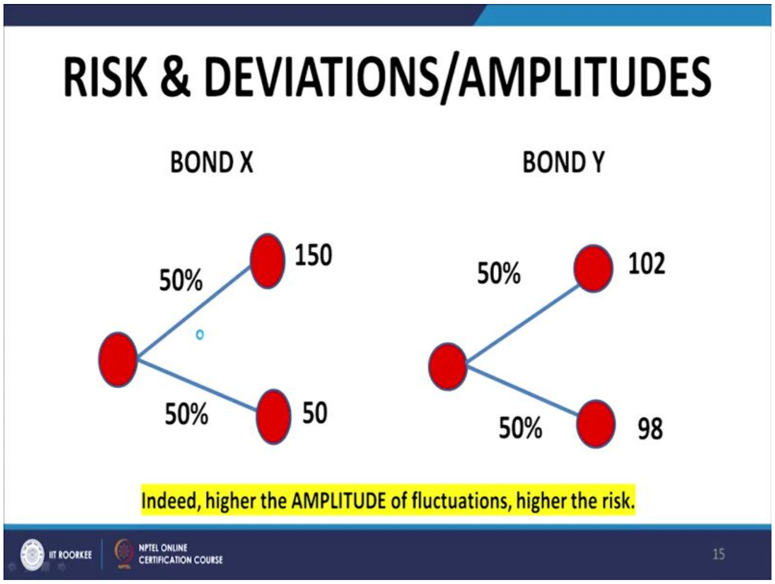

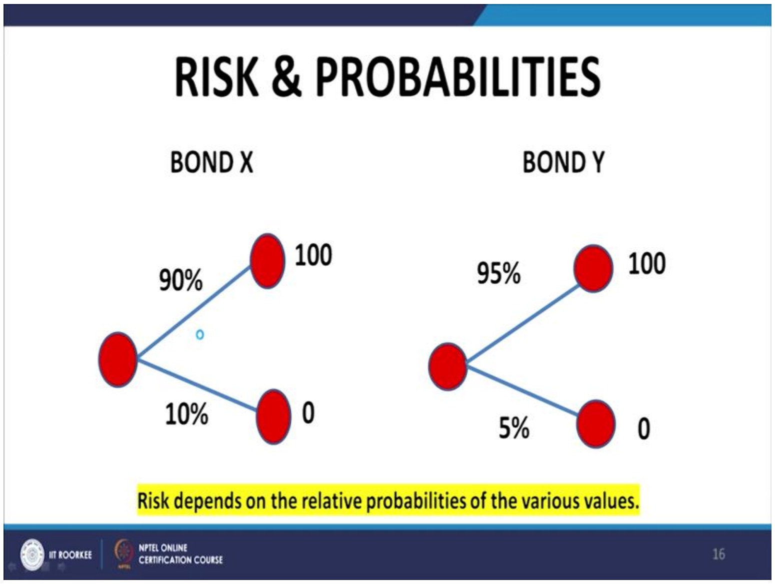

Then I discussed the relationship between risk and deviations or risk and amplitudes and we agreed that greater is the amplitude of fluctuations, greater is the deviation of the future value of the asset from the mean value, then greater is the risk embedded in the particular investment. Similarly, I talked about probabilities, the probabilities of the various possible final values also do have a strong say in the riskiness of assets. For example, we discussed two bonds X and Y. Bond X had a 90% probability of success and a 10% default probability. Bond Y had a 5% default probability. So, it was obvious at the outset that bond X is riskier than bond Y.

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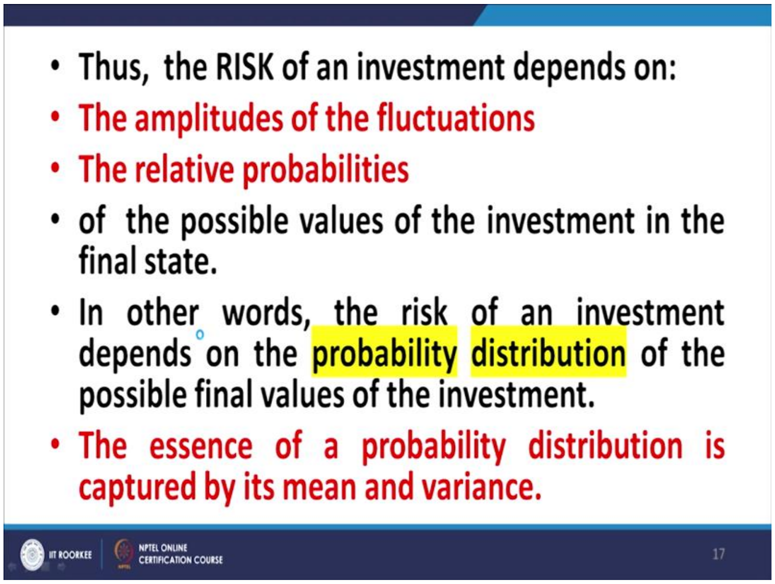

Both these properties, the deviations or amplitudes of fluctuations as well as the probabilities of the occurrence of those deviatiions or amplitudes are captured by the probability distribution of the values of the asset or the investment in its final state. So, I said that, it is basically the probability distribution of the values of the asset on the future date of the investment that would determine the riskiness of the asset.

And whenever we talk about probabilities, as I mentioned, there are certain probability distributions which are entirely captured by the expected value and the variance for example, the Gaussian distribution. In any case, the expected value and variance of a distribution gives us significant information about the distribution. The first and the second moments of the distribution capture, to a large extent, the essence of the distribution.

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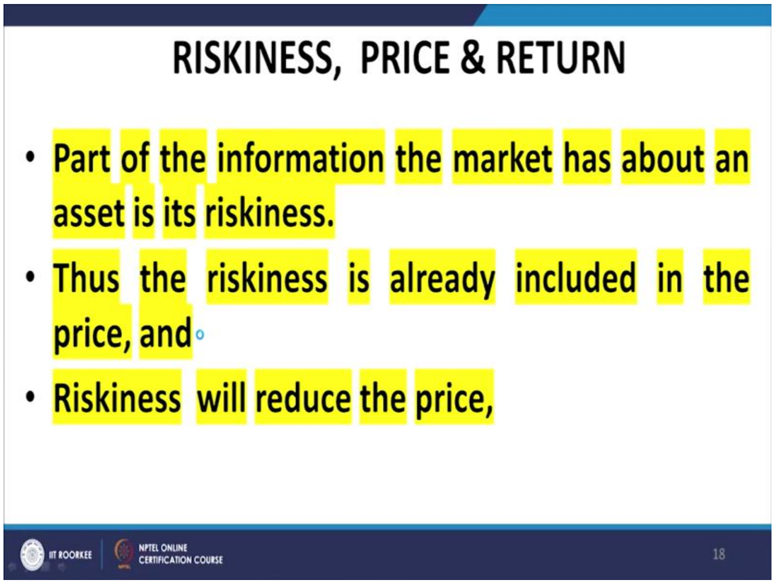

And then I discussed the issue of riskiness, price and return and we agreed that, given three assets, which are likely to return the same final value, but with different default probabilities, one being a risk-free asset with zero default probability, the second being a low risk asset and the third being a high-risk asset with highest default probability, the asset that has the least risk would be traded at the highest price and the asset that has the maximum risk of default would be traded at the lowest price.

Putting it the other way around if there are three assets, which are being traded at the same price, then naturally the expected future value of the least risky asset would be lowest and the future value of the most risky asset would be the maximum. In other words, the expected return would be a function of risk and the most risky asset will give you the highest expected return and the least risky asset or the risk free asset will give you the lowest return.

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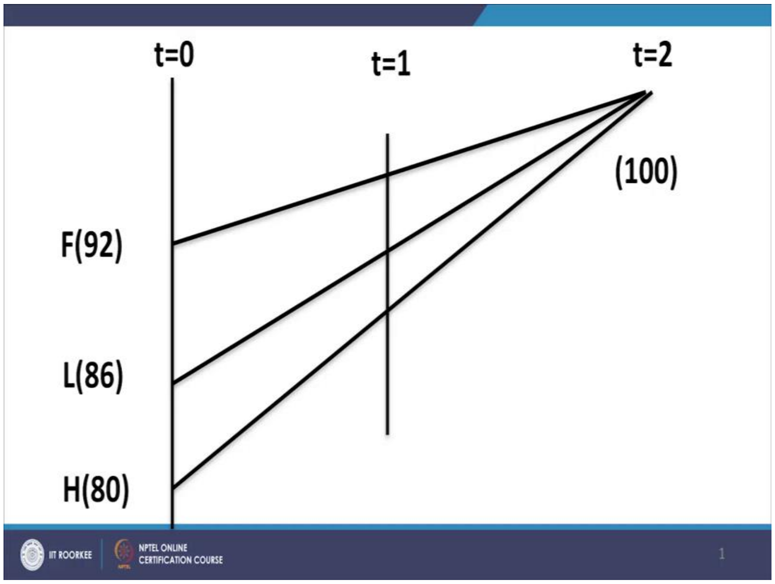

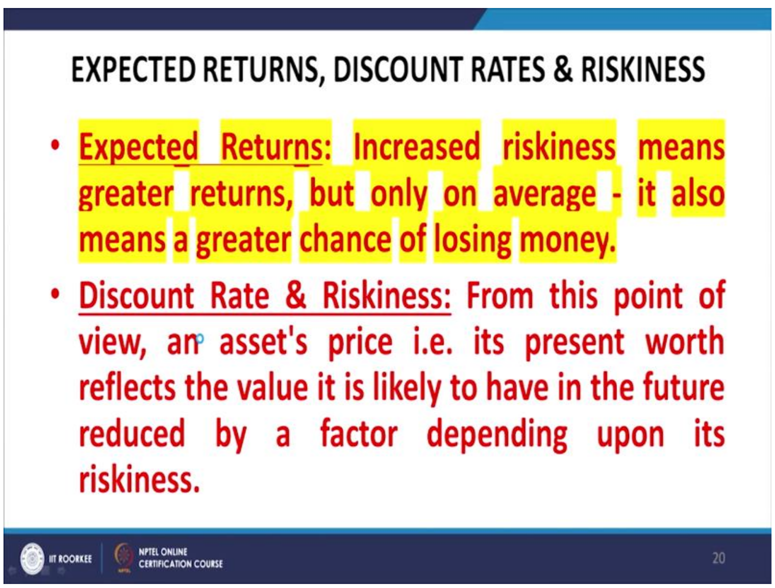

This is depicted in the above diagram, which we discussed in the last lecture. The outcome was that increased riskiness results in increased return, ***but that increased return is on the average.*** I emphasize this point strongly. I emphasize it again, ***that we are talking about expected returns, we are not talking about real returns. Increased riskiness means increased expected return, but at the same time, because the asset is more risky, there is a greater possibility of non-achievement of that expected return.*** So, that is the important part, we need to look at the higher expected return in conjunction with the riskiness of attaining or achieving that particular return.

Now, again, putting it the other way around, what we find is that if there are three assets having the same maturity value, and different levels of risk than the asset that is having the least risk or the risk-free asset, as the case may be, would be traded at the highest price. **In other words, the discount rate would be the lowest in the case of the risk-free asset and the discount rate would be the highest in the case of the risky asset.** Thus, we can conclude that the discount rate that we use for discounting future values to present values or future values from one future date to an earlier future date, is a function of the riskiness of the asset. It very much varies with the riskiness of the asset, higher the riskiness of the asset, higher is the discount rate that we would use for coming to the present value of the asset.

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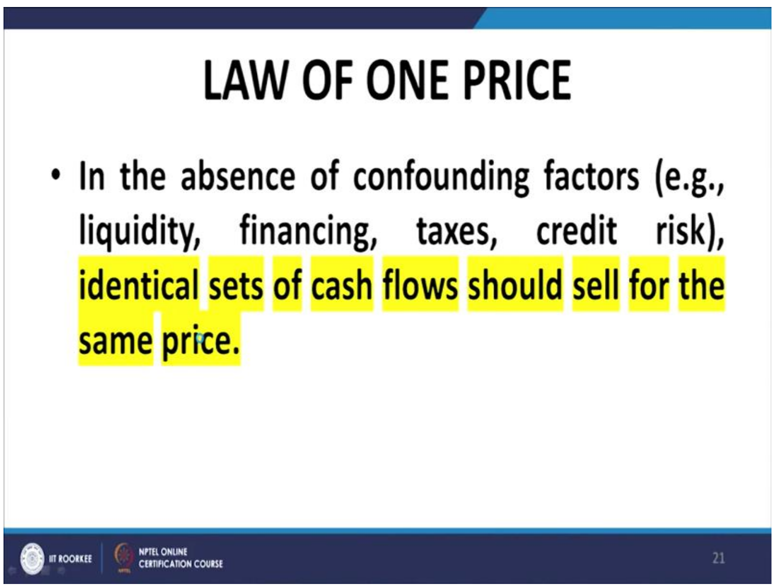

Then I moved on to arbitrage, and I introduced the concept of one price, the law of one price and I said that in the absence of confounding factors like liquidity, financing, taxes, credit risk and so on, ***identical sets of cash flows would sell at the same price.*** I emphasize that we are talking about cash flows, we are not talking about profitability. The reason for this, we shall talk about later on, but for the moment we need to take note of this fact that we are talking about identical sets of cash flows and not identical sets of profitability. So, identical sets of cash flows should sell at the same price.

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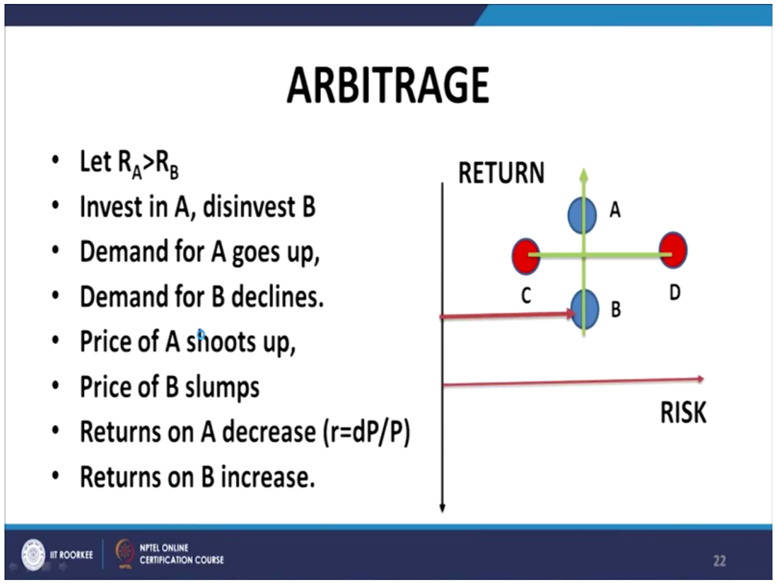

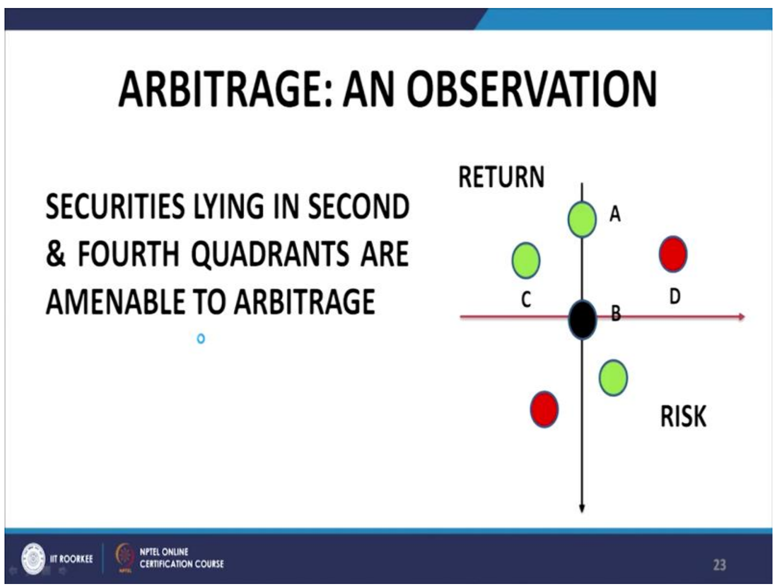

Then I discussed the diagram of arbitrage and we agreed that, for example, if we have assets A and B, asset A is providing you a higher expected return compared to asset B for the same level of risk and therefore, this would not be sustainable in equilibrium. People would sell off asset B, people would buy asset A that would increase demand for A that would decrease demand for B and resulting in the price of A increasing and the price of B decreasing and therefore, the return on A decreasing and the return on B increasing until they converge.

Similar situation will be with asset C and D. But we cannot say much about assets B and D because asset B has a lower expected return than asset D, but then asset B also has a lower risk than asset D, I repeat if you look at assets B and D, we cannot say much about them because here the risk return trade off of the market player would come into play. How much incremental risk he is willing to take up for a unit of incremental return or vice versa?

So, here because we are not aware of the risk return tradeoffs of investors, we cannot say much about the relationship between asset B and D, asset B has a lower expected return and a lower risk, asset D has a higher expected return and higher risk than B and therefore, the issue of risk return trade off comes into play.

An interesting observation here. I have shifted the origin from the previous figure, I have shifted the origin to the security B and what I find is that as far as those securities are concerned which lie either on the axis or lie in the second quadrant or the fourth quadrant arbitrage is possible. For example, asset C has a higher expected return and a lower level of risk compared to asset B and therefore, arbitrage between B and C would take place. Similar is the case between the asset B and the asset that lies in the fourth quadrant, because an asset in the fourth quadrant has higher risk than B and a lower expected return than B. Therefore, again arbitrage would take place between B and this particular asset. However, for those assets, which lie in the first quadrant, and which lie in the third quadrant, arbitrage is unlikely to happen. For example, if you look at assets B and D, arbitrage would not happen as I explained just now.

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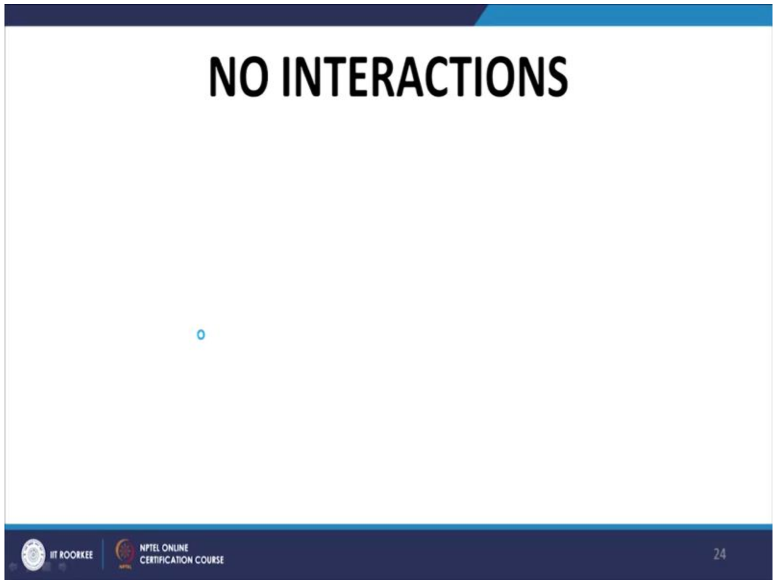

Now, in discussing the theory of arbitrage, I have not alluded to the issue of interactions between securities, I have very much been talking about the securities on standalone basis. The issue of interaction between securities has not been addressed. It is true that at the macro level, when we talk about the security prices, they would incorporate information about mutual interactions between the securities. And therefore, the theory that I have propounded so far, would very much hold even in the presence of interactions. But at a micro level, I need to emphasize that there may be situations where an investment, which, prima facie, will not be justified on a standalone basis, may be strongly justified because it contributes effectively to the singular portfolio of securities of the investor in a positive manner. The investor may be willing to take up that security at a price which is not justified by the market conditions. I would like to emphasize that there may be situations where a security with a negative expected return could be invested in by an investor because the introduction of that security into his existing portfolio may result in significant reduction in risk. This is very much at the my micro level. At the macro level, one may expect that the security prices incorporate information about mutual interactions as well.

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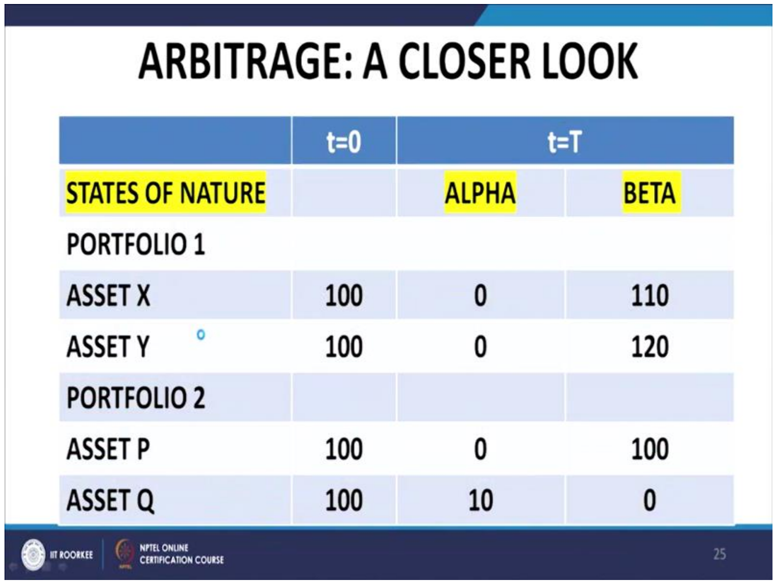

ARBITRAGE: A CLOSER LOOK

	t=0	t=T	
STATES OF NATURE		ALPHA	BETA
PORTFOLIO 1			
ASSET X	100	0	110
ASSET Y	100	0	120
PORTFOLIO 2			
ASSET P	100	0	100
ASSET Q	100	10	0

Now, we will look at certain examples of arbitrage. We will take a closer look at the examples of arbitrage to make the exposition absolutely crystal clear.

Let us look at the case of two assets X and Y. There are two possible states of nature at a future date. I simply named them as Alpha and Beta. They could be anything, say boon and recession or high rainfall and low rainfall and the like. As a general nomenclature, I have named them as Alpha and Beta. Both the assets X and Y are priced at 100 as of today (t=0). As at the end of the investment horizon (t=T) of the investor, (i) if the state Alpha materializes, both of the assets give us 0 payoff and (ii) if the state of Beta materializes then asset X gives 110 while asset Y gives 120.

So, in the two possible states of nature Alpha and Beta, (i) if alpha materializes, there is 0 payoff from both the assets and (ii) if state Beta occurs, X gives 110 while Y gives 120. If both the assets are being priced at 100, then it is quite natural that there would be arbitrage between them because Y has a clearly superior payoff in one state while in the other state the payoffs are equal. Thus, asset Y is definitely superior to asset X and therefore, we can safely conclude that in this sort of scenario arbitrage will operate. The price of X will decline, the price of Y will increase until they achieve equilibrium returns.

Let us talk about assets P and Q. This is very interesting. Both of them are priced at 100. (i) If the state Alpha materializes, the asset P gives 0 the asset Q gives 10 in units of money, whatever the unit of money may be, (ii) if the state Beta materializes, then P gives 100 and Q gives 0. Prima facie it may seem that asset P is definitely superior because it has a payoff of 100 if the state Beta materializes, but a closer look reveals otherwise.

In this situation, it is really not unambiguously correct that asset P is the superior asset. Why? We cannot say so. Why we cannot say that asset P is superior? Consider, for example, a situation where the probability of happening of state Alpha has been estimated as 99.99% or 0.9999 and the happening of state Beta carries a probability of 0.0001. What happens in that situation? Clearly the expected value of Q turns out to be higher than the expected value of P.

So, until and unless we have further information about the occurrences of state Alpha and Beta, we cannot prima facie conclude that just because the payoff of an asset P is higher in one of the states, there should be arbitrage between P and Q and the relative prices should change. It is quite likely that the current market prices actually reflect the relative probabilities of the occurrence of state Alpha and state Beta and therefore, account for the differential payoff in the two states.

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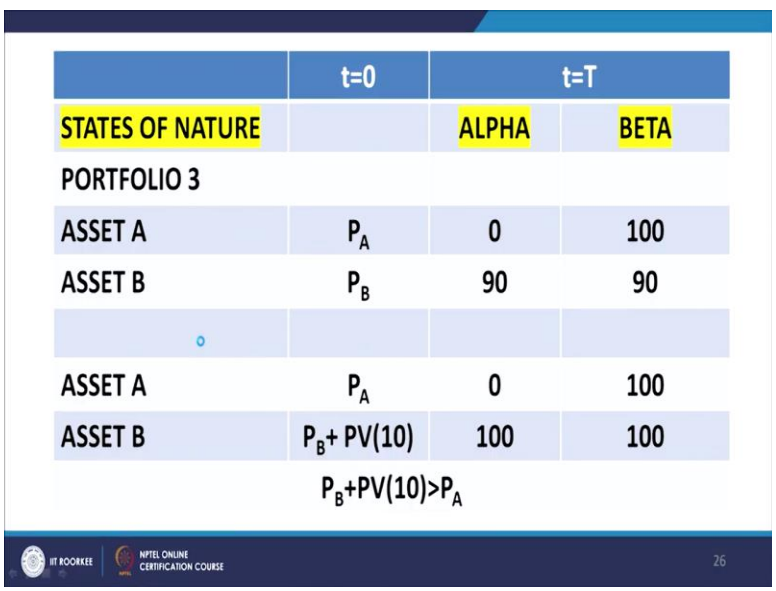

	t=0	t=T	
STATES OF NATURE		ALPHA	BETA
PORTFOLIO 3			
ASSET A	P_A	0	100
ASSET B	P_B	90	90
ASSET A	P_A	0	100
ASSET B	P_B+ PV(10)	100	100
	P_B+PV(10)>P_A		

Let us look at some more examples. Consider assets A and B. Now, the prices are open. The price of asset A I have represented by P_A and the price of asset B by P_B at t=0. Again, we have two states of nature Alpha and Beta. The asset A gives you 0 if Alpha state materializes and 100 if Beta state materializes. Asset B is giving you 90 in both the states, 90 if state Alpha materializes, 90 if state Beta materializes. Prima facie there seems to be no connection between P_A and P_B.

And we cannot say much, if anything at all, about the possibility of arbitrage between A and B, because again one payoff is not clearly dominant over the other in both the states. Asset A is giving you a higher payoff in Beta state, asset B it is giving you a higher payoff in Alpha state. However, you can arrive at some conclusion about the relationship between P_A and P_B indirectly.

Let us say, we introduce a risk free asset into our problem. We introduce a risk free investment at t=0 equal to the present value of 10 i.e. PV(10), computed at the risk free rate naturally in combination with asset B. So, what would be the payoffs in that case? In that case, the payoff of A would remain unchanged at 0 and 100 corresponding to Alpha & Beta respectively. And in the asset (B+PV(10)), or asset B with the riskfree investment the payoff will be 100 in both the states. This is because investment in the riskfree asset when liquidated on the date of maturity of the investment would give you 10 units of money and therefore, the total payoff from B plus the risk free investment would be 100 in either state. So, now, it is quite clear that the portfolio comprising of asset B and the risk-free asset is definitely superior to the portfolio comprising of asset A alone and therefore, we must have $P_B + PV(10) > P_A$, P_B plus the present value of 10 is greater than P_A. So, in this way, some bounds can be derived for P_A & P_B.

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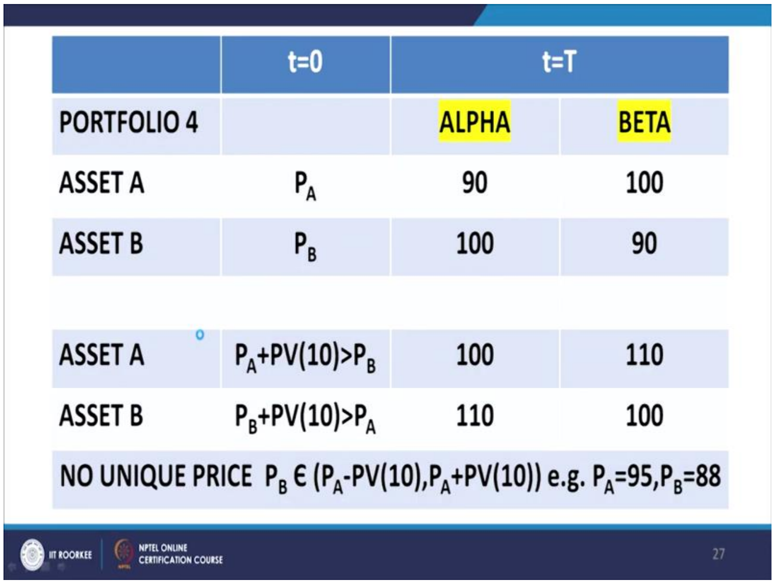

	t=0	t=T	
PORTFOLIO 4		ALPHA	BETA
ASSET A	P_A	90	100
ASSET B	P_B	100	90
ASSET A	P_A+PV(10)>P_B	100	110
ASSET B	P_B+PV(10)>P_A	110	100
NO UNIQUE PRICE P_B ∈ (P_A-PV(10),P_A+PV(10)) e.g. P_A=95,P_B=88			

Similarly, we can work out the bounds in respect of the last set of assets A & B. A and B are priced at P_A and P_B, A gives 90 and 100 in the two states Alpha and Beta respectively, B gives 100 and 90 in the two states Alpha and Beta. Prima facie, again I must emphasize, that there is no information which enables us to take up arbitrage exercise among A & B.

However, we can work in the same manner in which we did in the previous case and arrive at bounds for P_A and P_B. P_B must lie between P_A- PV(10) and P_A+ PV(10). And by symmetry, we get the same bounds for P_A as well. So, this I leave as an exercise. It is absolutely similar to the previous case that I discussed just now.

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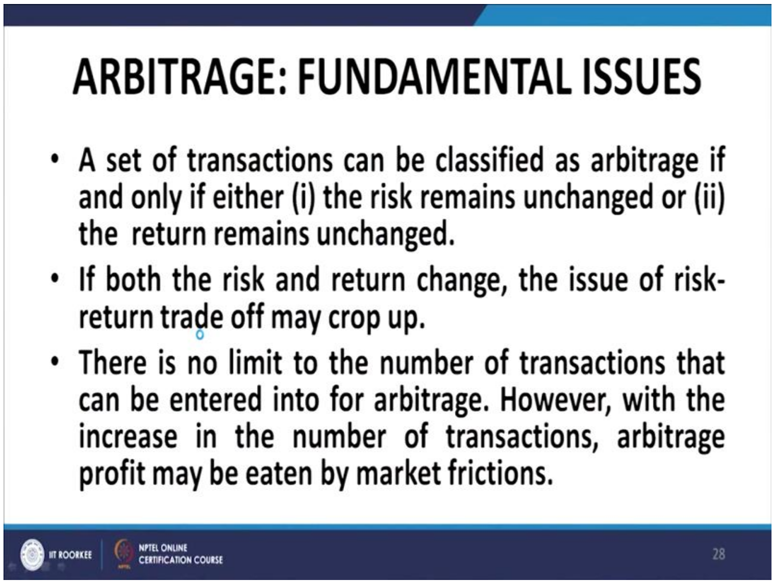

So, let us recap the fundamental issues relating to arbitrage. A set of transactions can be classified as arbitrage if and only if either the risk remains unchanged or the return remains unchanged. If both risk and return change as it happened between the asset B and the asset D in our diagram, the issue of risk return trade off crops up and therefore, we cannot conclude that there is an arbitrage opportunity.

Now, there is another important point that I must emphasize. There is no limit to the number of transactions that can be entered into for arbitrage. For example, a typical example, you could convert INR into USD, USDs into GBP and GBP back into INR. It is not necessary that we must confine arbitrage to two assets say INR to USD and simultaneous reconversion of USD to INR. That is not at all necessary. We can have as many transactions as we like.

For example, not only two pomt, three point arbitrage, you can have even more point arbitrage, n point arbitrage, but the problem is that as the number of transactions constituting the arbitrage increases, the frictional cost (that is the transaction cost of buying and selling, brokers commission and so on), that is incurred in the entire process tends to eat away into the arbitrage profits. So more the number of transactions, greater are going to be these frictional costs that are there in the real market. And as a result of these, the profits may be less.

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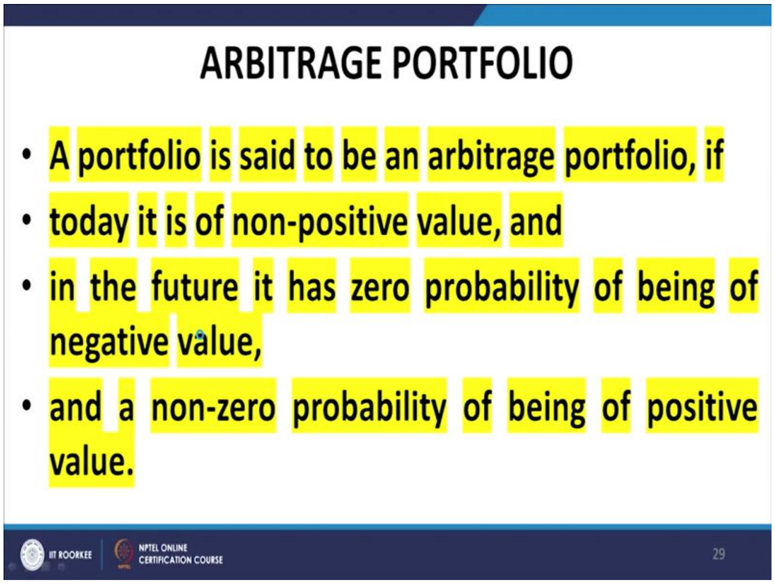

Now, there is an important definition. We are going to use this in the pricing of a number of assets that we are going to discuss, bonds as well as derivatives. A portfolio is said to be an arbitrage portfolio if today (t=0) it is of non-positive value, and in the future (t>0), it has zero probability of being of negative value, and a nonzero probability of having a positive value.

In other words, today it has a non-positive value, that is, it does not involve a cash outflow for setting up. But at any future point in time, it has a zero probability of being of negative value, but a nonzero probability of having a positive value. In other words, there is a possibility of a positive cash flow and there is no possibility of a negative cash flow in future then it is called an arbitrage portfolio.

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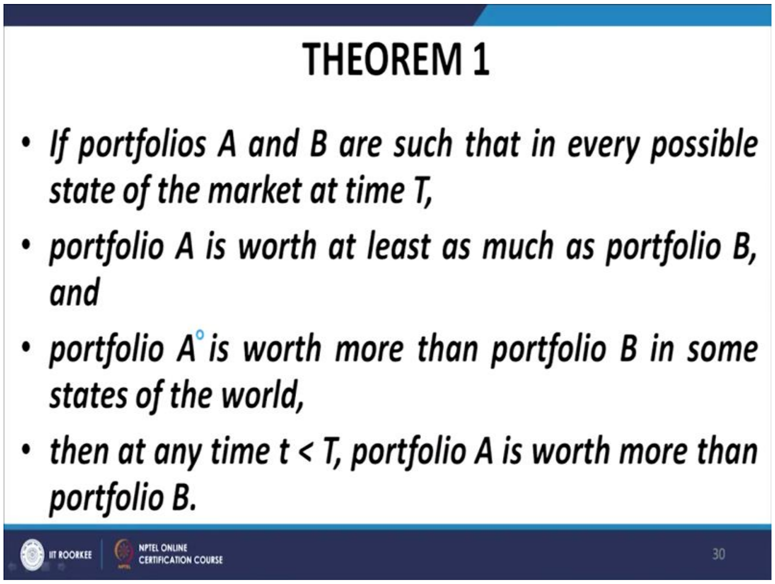

Let us now take up three simple theorems. The first theorem states that if portfolio A and portfolio B are such that in every possible state of the market at time T, that is the end of the investment horizon, portfolio A is worth at least as much as portfolio B and portfolio A is worth more than portfolio B in some states of the world, then at any earlier time t<T, portfolio A is woth more than portfolio B. It says that, if in every state of nature, that could possibly occur on the date of maturity of the investment portfolio, A is worth at least as much as portfolio B, but there is at least one state in which portfolio A is worth more than portfolio B. So, in all states, A must at least be equal to B and at least some state must be there in which the value of A exceeds the value of B, then at any previous time, portfolio A is worth more than portfolio B.

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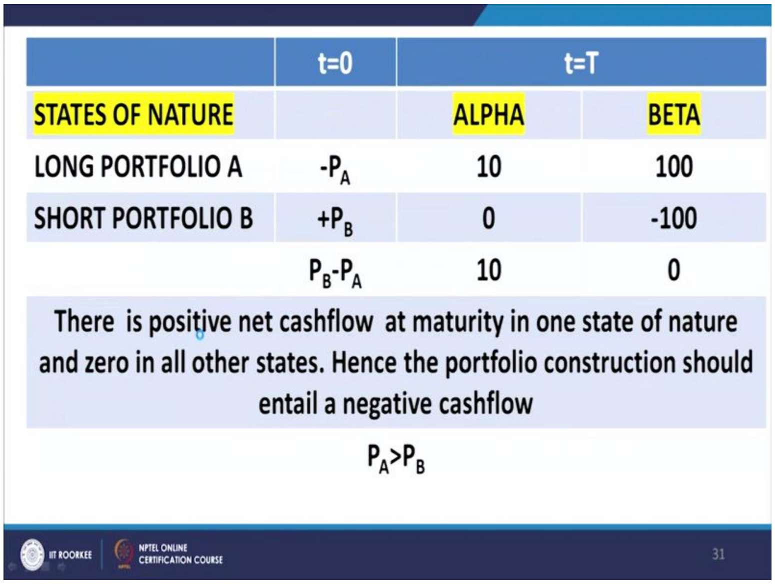

	t=0	t=T	
STATES OF NATURE		ALPHA	BETA
LONG PORTFOLIO A	$-P_A$	10	100
SHORT PORTFOLIO B	$+P_B$	0	-100
	P_B-P_A	10	0

There is positive net cashflow at maturity in one state of nature and zero in all other states. Hence the portfolio construction should entail a negative cashflow

$P_A>P_B$

I establish this theorem by an example. Again, let us consider two states of nature, state Alpha and state Beta. We have a long portfolio A that costs us P_A. It is a cash outflow. So, we write it as minus P_A. We short portfolio B and because shorting results in inflow of cash, we take plus P_B. Now, let us assume that in state Alpha, the portfolio A gives me 10, the portfolio B gives me 0 and in state Beta the portfolio A gives me 100, the portfolio B gives me 100.

In other words, in both the states Alpha and Beta, A pays off more than or equal to portfolio B and there is one state Alpha in which portfolio A pays off more than portfolio B. Let me repeat, in both the states portfolio A is at least as good as portfolio B, but there is one state Alpha in which portfolio A is superior to portfolio B.

Now, please note this minus sign here in the Beta column for portfolio B. This is because we are short in portfolio B. Because we are short in portfolio B, so the payoff would be negative of whatever the payoff on the long portfolio is, and therefore, we write it as (-)100. So, the net result of combining these two portfolios, long portfolio A and short portfolio B is that we get a payoff of 10 if Alpha state happens and a payoff of 0, if Beta state happens.

Clearly this is a non-negative payoff, the payoff is 0 in one state and positive in the other state and therefore, the cost of establishing this portfolio must be positive. In other words, there should be a cash outflow at t=0 for establishing this portfolio therefore, P_B -P_A should be negative. Thank you, we will continue after the break.