Security Analysis & Portfolio Management
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Lecture 06
Risk & Arbitrage - I

Okay, so let us continue from where we left off before the break. Before the break, I discussed in detail the concept of preference shares and why they are classified as hybrid instruments. Those were general features of the hybrid instruments, and preference shares form the prototype of hybrid instruments. There is a huge variety of other hybrid instruments. I shall talk about some more of them later on in this course, as we move into greater detail of these topics. For the moment, let us leave it at that. And let us move on to a very interesting concept called risk and talk about what exactly we mean by risk in financial parlance.

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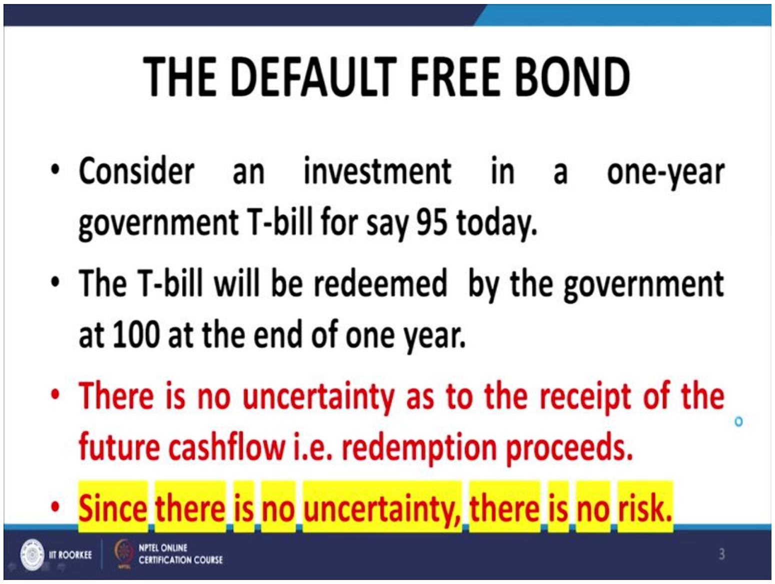

To motivate the discussion, let us consider the case of a bond, which is, let us say, being sold at 95 as of today, t=0. At the end of one year from now this bond promises (without any possibility of default) 100 units of money. So, let us say, a bond is trading at 95 as of today, and at the end of one year from today, t=1 year, the bond promises an amount of 100 and the important thing is that the bond is guaranteed by a very powerful economic institution like the government of the United States, for instance, and the possibility of default is nil.

Now, if I invest in this bond, clearly, I am certain to get 100 at the end of one year from now, and therefore, I may classify my investment as a risk-free investment or a riskless investment. In other words, if there is absolutely certainty about the future value of an investment, then there is no risk attached to that investment. Let me repeat this very fundamental, very important sentence, "if there is absolute certainty, about the future value of an investment, then there is no risk attached to that investment". Nevertheless, "absolute certainty", is a very important term which is, in its an absoluteness, not realizable in practice. But still we will talk about it to motivate the discussion.

If there is absolute certainty about the future value of an asset, then there is no risk attached to your investment in that particular asset. So, that is the fundamental premise of what we mean by risk. It implies as a corollary, that, if there is uncertainty in the future value of an investment, then you are exposed to a certain amount of risk. Continuing with this example, if suppose, the instrument gives you 100 units of money at t= 1 year, but there is a semblance of a possibility that the issuer may default on the payment of the amount of 100 at t=1 year, then you cannot classify that instrument as being risk free.

Therefore, to some extent, we may say that risk is synonymous with uncertainty. Uncertainty of what? Uncertainty in the future value of your investment. If there is only one value that the investment can take at that future time, there cannot be any uncertainty and hence there cannot be any risk. This also implies that the future value of your asset must take more than one value with non-zero probabilities in order that it may be risky. If the asset was to take only one value, then obviously, that value would be realized are there would be no risk. But if the asset was to be able to take any one of more than one value, say two values, then the possibility of the asset ending up at value X_1 or value X_2 at time t=T would invoke a certain amount of uncertainty and hence, a certain amount of risk in your mind when you make this investment. This is what is the meaning of risk in its most general form.

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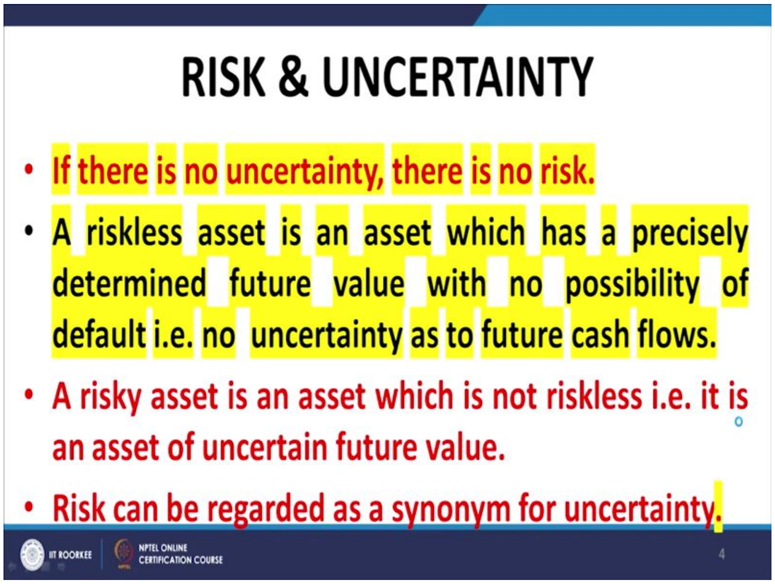

We now move to the issue of how risk relates to the final values that the investment /asset in which you have taken a position. The first thing we conclude, as I mentioned just now, is that if there is no uncertainty, there is no risk. This is the first and foremost and the most important takeaway.

The second thing that we take away is that, if there is uncertainty in the future value of an asset, the investor who has an open position in that asset is exposed to a certain amount of risk. The risk may be small, may be large, we are not at the moment addressing the issue of quantification of risk. We are not addressing that issue, we are simply trying to understand the meaning of the concept of risk.

Now, as I mentioned, if the asset can take more than maturity value, then and only then uncertainty exists in the realization of these values at the point of maturity of the investment and therefore, there is a certain amount of riskiness, certain amount of uncertainty in the mind of the investor as to which value the asset will take. This means that there should be a possibility of the asset fluctuating in terms of its value on the date of maturity, only then the investor is exposed to risk. If there is no fluctuation, if the value is fixed, then there is no risk. If there is a possibility of the asset taking more values than one i.e. fluctuations in the maturity value are possible, then the investor is exposed to risk.

So, risk arises from uncertainty. Uncertainty in the final value of the asset. Uncertainty arises from the possibility of the asset fluctuating in its value. So, fluctuations gives rise to uncertainty, uncertainty gives rise to risk in the mind of the investor.

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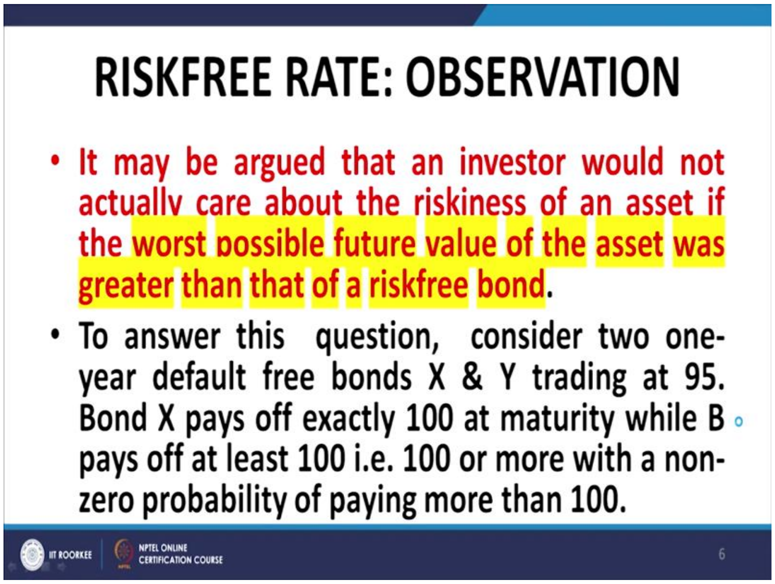

Now, there is an important observation here regarding the risk-free rate. Let me read it out, "it may be argued that an investor would not actually care about the riskiness of an asset, if the worst possible future value of the asset, was greater than that of a risk-free bond." Let me repeat, "it may be argued that an investor would not actually care about the riskiness of an asset, if the worst possible future value of the asset was greater than that of a risk-free bond."

What does it mean? Let us take an example. Let us say we have got two bonds, bond X and bond Y. Both the bonds are trading at 95 as of today (t=0) and bond X promises exactly 100 with certainty at t=1. So, bond X is a risk-free bond, it promises exactly 100 at t=1. Bond Y, on the other hand, promises 100 or more with absolute certainty at t=1.

The argument that this particular axiom propounds is that the investor with an asset Y would not care about the riskiness because he is guaranteed the risk-free rate which is 100. Now, the question is: Can we have this situation in reality? If you carefully examine the payoffs of these two bonds, bond X promising 100 with certainty and the bond Y promising 100 or more with certainty, you obviously find it advantageous to invest in the bond Y.

Why? Because in the bond Y, you are assured of getting 100 but you also entertain the possibility of getting more than 100. So, naturally, bond Y has a superior payoff compared to the payoff of A as per the perception of any investor. What would that translate to? This would translate to the price of bond Y at equilibrium not being equal to that of bond X. In other words, bond X would be overpriced, bond Y would be underpriced, if the prices were to be equal. As a result of this, the demand supply dynamics would operate. The demand for bond Y would increase, the demand for bond X would decrease and we would end up in a situation where the price of bond Y would go above the price bond X. Why? Because Y guarantees a superior payoff at maturity compared to the bond X. In other words, if the two bonds were to be priced at 95 i.e. if the bond X and the bond Y were both to be priced at 95 at equilibrium, then it must necessarily be that the bond Y must promise a threshold amount of lower than 100. For example, it could promise an amount of 98 or more, but it cannot promise 100 or more.

Because otherwise the equilibrium would be disturbed. The bond Y would be underpriced and as a result of this, the prices would realign themselves with the price of Y becoming more than that of X. On the other hand, if the prices are to remain the same, at least one value of the spectrum of values that Y could take must be below 100. This is a very important outcome arising out of arbitrage considerations. We will talk more about arbitrage very soon.

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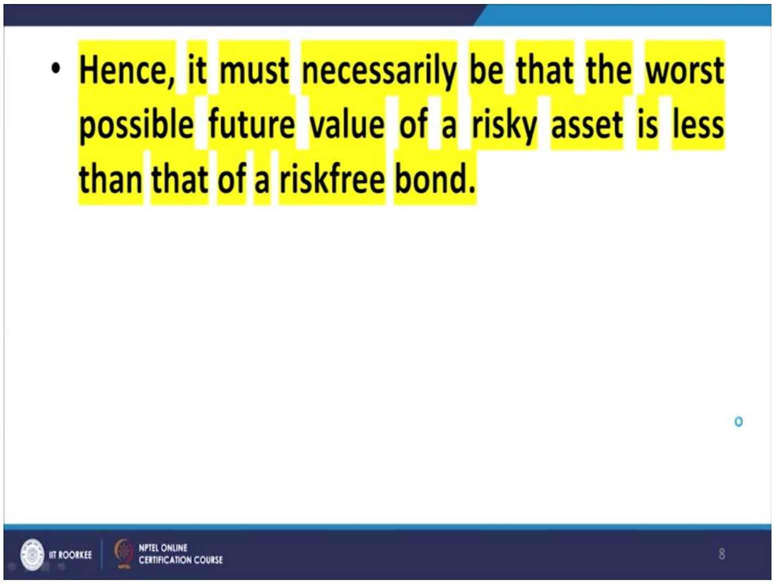

But for the moment, this is an important observation. It must necessarily be that the worst possible future value of a risky asset is less than the future value of a risk-free bond. Because if the worst possible value of a risky asset is equal to the value of the risk-free bond at maturity and the risky asset being risky would promise more than that as well, then the two cannot command equal prices at equilibrium bond. The risky bond would be underpriced in that situation. And obviously, the prices would realign. So, it must necessarily be that the worst possible future value of a risky asset be less than that of a risk-free bond. The worst possible outcome of the future value of risky asset must be less than that of a risk-free bond.

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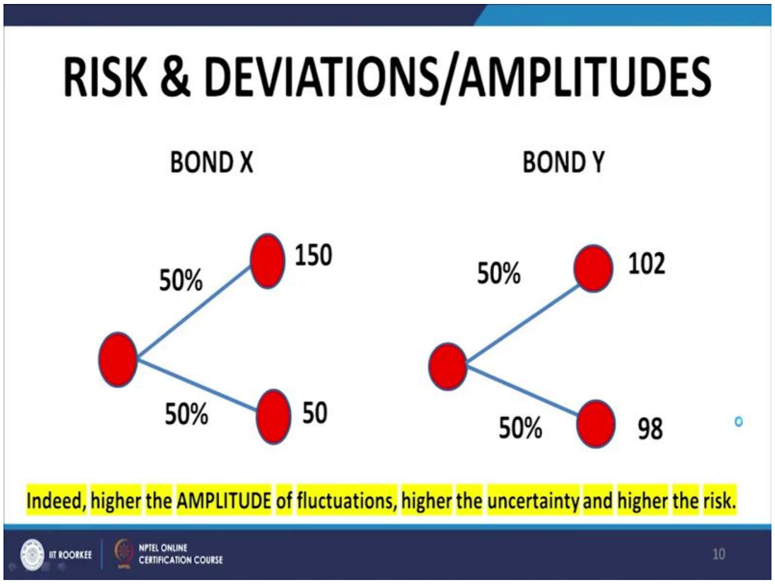

Now, risk and probabilities. Let us look at two bonds, bond X and bond Y. Bond X has a payoff of two possible values. Its spectrum of payoffs consists of two values 150 and 50, each with probability ½ i.e. 50%. In other words, bond X could realize either 150 or 50 with equal probability after one period, whatever that period may be, one year, one month or whatever we chose. It does not really matter. So, the bond X promises an amount of 150 or 50 with probability of ½ in each case. Bond Y is similar in terms of probabilities, but it promises either 102 or 98. Thus, bond Y promises 102 or 98, eeach with ½ probability. Obviously the events are mutually exclusive. Now, the question is, which is more risky & which is less risky?

It is obvious that because the amplitude of fluctuation is lesser in the case of bond Y, it seems to be lessrisky than X. If you invest in bond Y, the loss that you could expect at most is only of 2 units of money whereas if you invest in the bond X, the maximum loss that you could incur is 50. Although in terms of expected values, both bonds have the same expected payoff value. Both the bonds have expected payoff value of 100.

So, the amplitude of fluctuations about the expected value i.e. the deviations from the expected value are an important constituent of the riskiness of an instrument. The lesser are the deviations or the lesser is the amplitude of fluctuations about the mean value, the lesser is the risk of the instrument.

For example, let us go back to the default free bond. The default free bond promises 100. There is no other value that it can take. That means the deviation is 0, the amplitude of fluctuation is 0 because there is no fluctuation at all. And we agree that that bond is a risk-free bond. The bond has a zero level of risk. So, the takeaway that we have from this is that riskiness of an instrument is related to the amplitude, the deviation of fluctuations, about the expected value.

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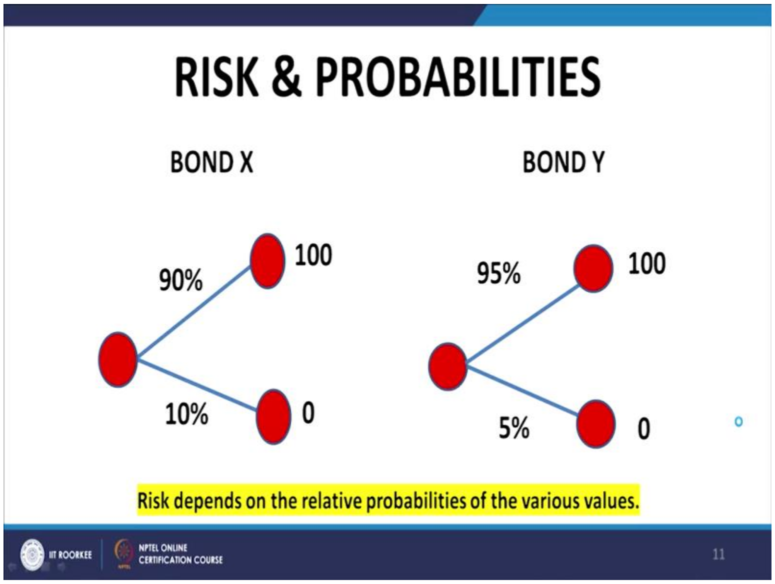

And the second thing, let us again examine two bonds X and Y. Bond X promises you either 100 or 0, the probability of getting 100 is 90%, the probability of getting 0 is 10%. In other words, the bond X has as a default probability of 10%. The default probability of bond X is 10%. The default probability of bond Y is only 5%. In other words, bond Y is likely to give you 100 with a probability of 95% percent. Bond Y is going to default and give you 0 with a probability of 5%.

Now, if I addressed the same question to you. Which is more risky? It is quite obvious that bond X is more risky. So, the outcome or the takeaway from this is that the probabilities of the default of the bond, the probabilities, let us make it slightly more general, of the various values that the instrument could take as its final value in the spectrum of values, determines or is related to the riskiness of the bond. Let me repeat, the probabilities of the various values that the bond could take in a sample set, or in its spectrum of possible values on the date of maturity is related to the riskiness of the bond.

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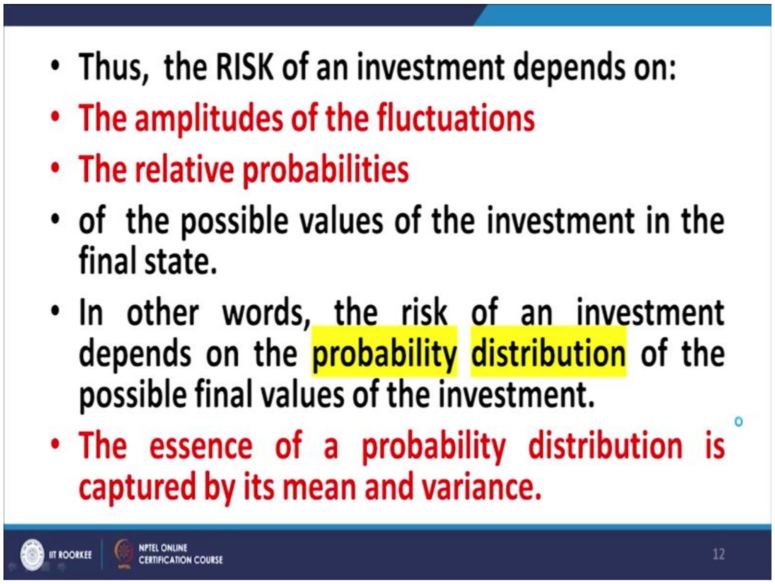

What have we concluded? We have concluded two very important points: the riskiness of an investment depends on (i) the amplitude or the deviations of the fluctuations from the expected value and (ii) the relative probabilities of the occurrence of those deviations.

If there is no deviation, the bond has no risk. If the deviations carry a high probability, then obviously the bond would be more risky. If the deviations carry low probabilities, the bond would be lesser in terms of risk. Now, both these factors, the amplitudes of the fluctuations, and the probabilities are captured by the probability distribution. If we know or if we are able to gauge the probability distribution of the spectrum of prices/values of our investment at a future date, then we are able to, in some sense, assess the riskiness of the investment. In other words, another takeaway, which combines the takeaways of the two previous slides, is that the riskiness of a bond depends or can be assessed or gauged from the probability distribution of the final values of the investment.

And once we talk about probability distributions, the immediate fallout is that probability distributions, in essence, are captured by mean and standard deviation. Some of the probability distributions are completely captured (normal distributions) while others may not be completely captured, but largely so. Significant information about the distribution is contained in the mean and variance of the distribution. So, the mean and the variance constitute significant characteristics of an investment. We shall be dealing with this issue in a lot of detail when we talk about risk and return in the context of portfolio management. For the moment, the conclusion that we arrive at is that mean and variance constitute significant parameters for the assessment of risk of an investment, which directly depend on the probability distribution of the final values of the investment.

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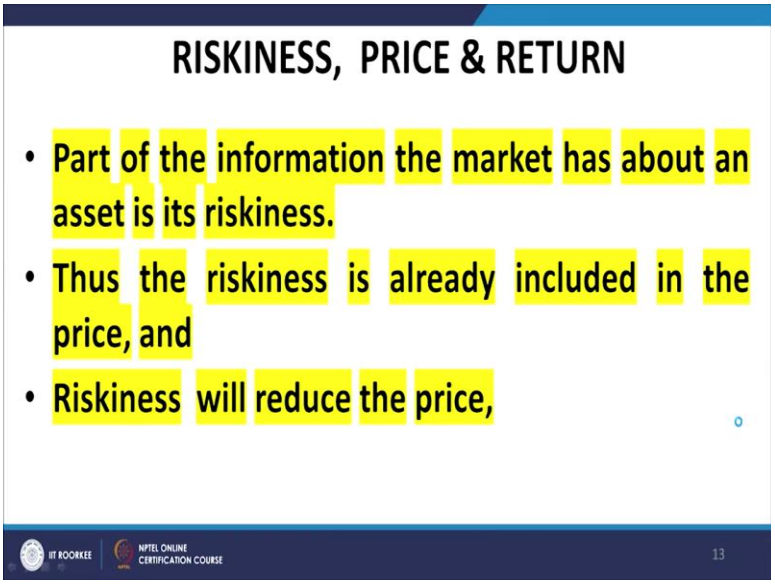

Let us examine another issue relating to risk. Now, before I talk about an example, let me give you a prologue of this. We operate, for the moment, in the realm of efficient markets. What exactly efficient market means, we shall be discussing later on in this course. But for the moment, let us take efficient markets as markets where information transmission is spontaneous, all assets are deemed to be correctly priced and the information content available to all investors is the same. So, the important thing is that the dissemination of information is spontaneous and as a result of this, only new information, when it enters the market, would influence the prices. Prices at a particular point in time capture all the information that is accessible, available in the public domain to the investor. Let us work on this. Now, part of the information that the efficient market (that we are talking about) has about the asset is the riskiness of the asset. This is because players in the market would naturally assess the riskiness of the asset. Whenever we talk about assessment or the evaluation of an investment, we talk about the evaluation, fundamentally, in terms of two parameters, the expected return and the riskiness.

How we measure the two is a separate issue. Let us not address that, but we use two parameters to characterize an investment viz. its expected return and its riskiness. So, naturally, the players in the market would be aware about the riskiness relating to all the assets that are being traded in the market. Therefore, riskiness is incorporated as a part of the price at which the assets are being traded in the market. So, riskiness as well as expected return are both embedded, incorporated in the current market price at which a particular financial asset is being traded. This is a fundamental inference of the efficient market.

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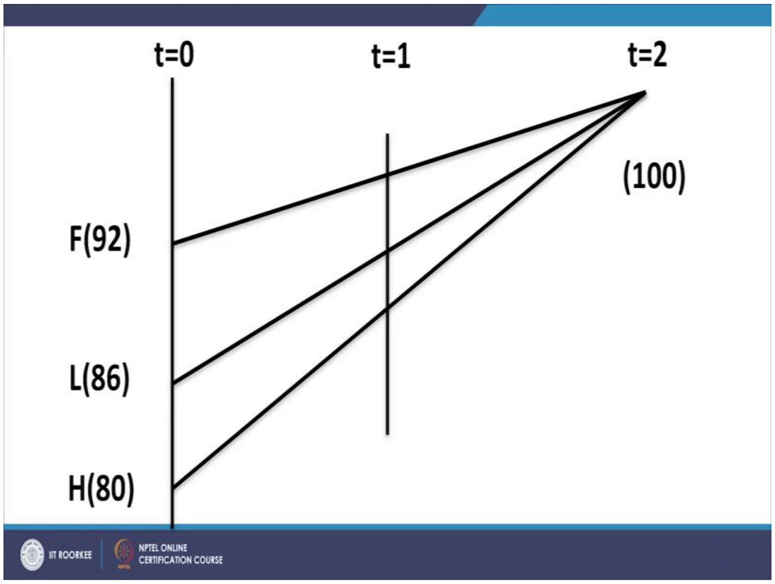

Now, let us consider three assets. Let us call the first asset F, the second asset L and the third asset H. These are three financial assets, which are available for investment. F is a risk-free asset, L is a low risk asset, and H is a high risk asset. F is a risk-free asset and, thus, has default probability of zero. Let us assume that L has adefault probability of 2% and H has a default probability of 10%.

All the three assets, subject to these probabilities of default, promise to give you 100 units of money at maturity, let us say at the end of two years. Now, obviously, anybody in the market sharing these perceptions of default probabilities, would be willing to pay the maximum for the risk free asset F, lesser than that for the low risky asset L, and even lesser than that for the high risk asset H. Let us assume that the risk free asset F carries a price of 92 (guaranteeing 100 at the end of two years), the low risk asset L has a price of 86 and the high risk asset H is trading at 80.

All three of these assets will provide 100, subject to default probabilities as above, at the end of two years. Recall that the ;east risky asset will be priced the highest and vice versa. Now, let us rescale this problem. Let us say there are three assets, F, L & H being priced at the same amount, say 90. Then it must necessarily be that the final value of asset F would be the lowest, the final value of asset L would be in-between and the final value of asset H would be highest.

In other words, what I am trying to say is that the highest risky asset would be expected to give you the highest return and the least risky asset gives you the least expected return (we measure expected return as the incremental rate of value/price change expressed as a percentage of the initial investment).

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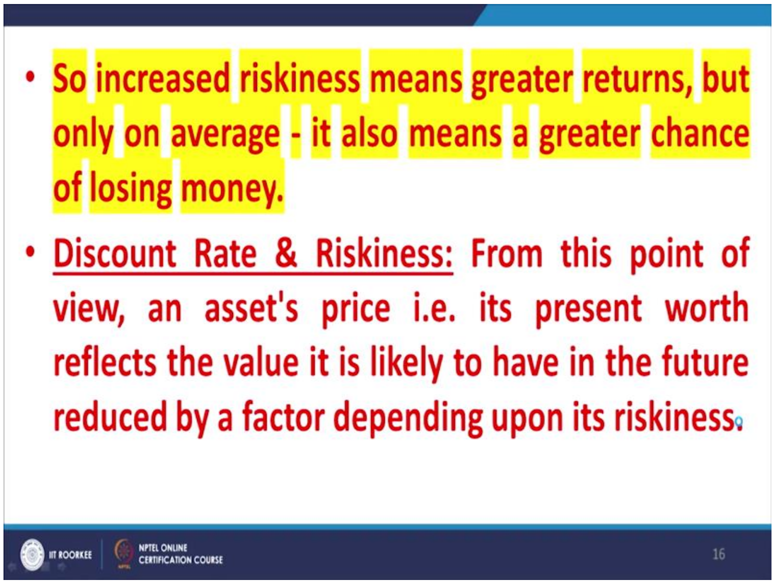

So, this is a very important fallout, and that is, increased riskiness means greater returns, but it does not mean greater certain returns. Please note this. Because when we talk about more risk, we are talking about the higher possibility of default in the occurrence of the expected return, that is why the asset is more risky. So, to say that a higher risky asset gives you higher returns is a wrong statement, it is a wrong connotation of a principle, we must say that the higher the risk, higher is the **"expected"** return, because the return is the expected return and if the risk is higher, the possibility of realization of that expected return is lower.

In other words, you may realize the expected return, you may not realize the expected return and the higher the riskiness of the asset, higher is the chance that you do not realize the expected return. So, we must always use the term **"higher the risk, higher the expected return"**. We cannot use the sentence ~~*"higher risk, higher the return"*~~. That is wrong. So, let us read it again. **"Higher riskiness, increased riskiness means greater returns, but only on the average".**

This is very important, **only on average.** High riskiness also means a greater chance of losing money. It also means a greater chance that you will not realize that higher returns. So, that is very important. So, another fallout of this, this is an important takeaway, is that **"higher the risk, higher is expected return"** ~~not the return itself~~. See, once the return is realized, then there is no riskiness attached to it, because the event has already occurred and hence, there remains no uncertainty attached to its outcome. There is no uncertainty attached to it.

So, when we are talking about future returns, we must talk about the riskiness versus expected returns. Higher is the riskiness, higher is the expected return, but higher is the chance that you will not realize that expected return. So, we must use **"expected"** word whenever we talk about returns in the context of the risk return trade off of an investor or of a security, for that matter.

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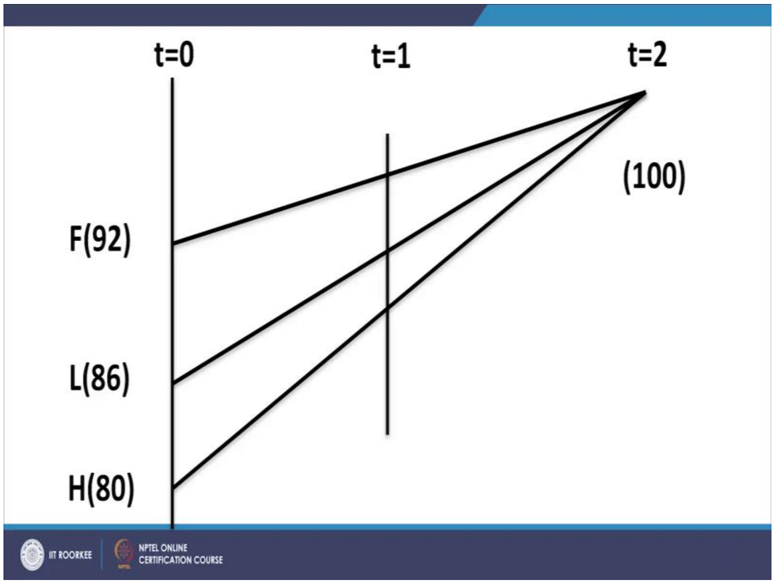

Another point arises out of the above diagram. Suppose I want to work out the value, the current value of the asset, given the future value of the asset. In other words, now, I move in the backward direction. I am given that each of the three assets, F, L & H have a maturity value of 100 (F is the risk-free asset, L is the low risk asset and H is the high-risk asset). Now, I want to work out the value at say t=1 year.

How will I work it out? I will work it out by discounting this final value of 100 for one year. Now, the numerator is the same in all cases, but what I find is that the value of F is the highest, because it is the least risky, the value of L is relatively lower and the value of H is lowest. What does this mean? This means that I must discount the final value at a higher rate for the highest risky asset, at an intermediate rate for the less risky asset and at the lowest rate for the risk free asset. So, in other words, when we talk about discounting of an investment, the investment should be discounted at the risk adjusted rate and the risk adjusted rate will be higher for a high risky asset, relatively lower for a low risky asset and the lowest for the risk-free asset. This is another fallout of this discussion.

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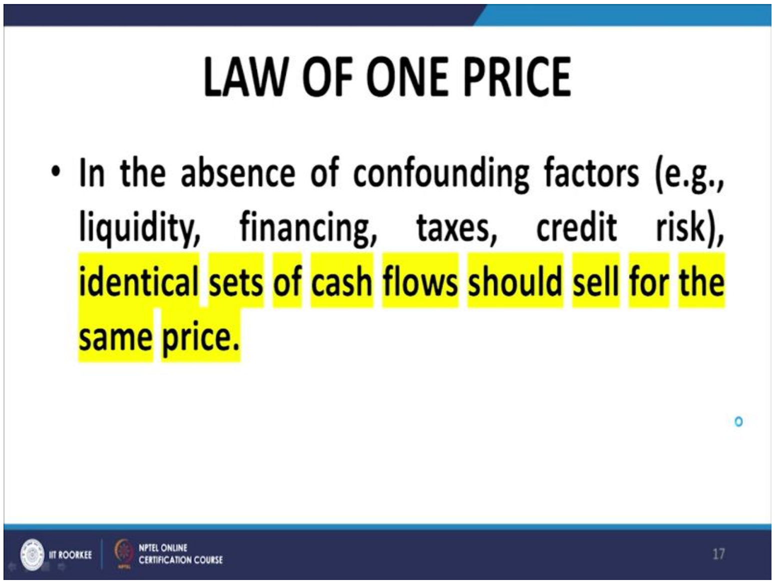

Now, I talk about arbitrage. Arbitrage, as a principle, is based on the law of one price. What is the law of one price? It is given on this slide. The law of one price says that ***"in the absence of confounding factors like liquidity, financing, taxes, credit risk, etc. identical sets of cash flows should sell for the same price."*** This is one of the most fundamental laws of finance.Please note the word used is "cash flows", it is not "~~profits~~". Identical sets of cash flows should sell for the same price. This is a fundamental law and whenever there is a violation of this law of one price, market dynamics will operate and will bring equilibrium back to a situation where this law of one price holds. This process is called arbitrage.

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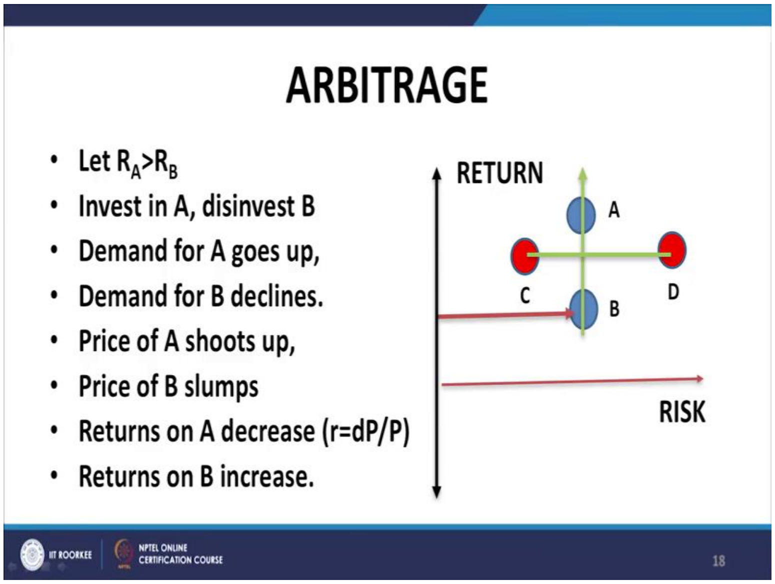

Let us look at an example. This is the risk-return space. Now, please note I have used the word risk and return, because at this point, it is not necessary to quantify either of the two in terms of some specific parameters. What parameters can exactly be used for the purpose of measuring risk or for the purpose of measuring expected return are not required to be explicitly stated for the moment. I shall talk about that in a much later section of this course. For the moment, let us keep the discussion abstract, let risk be quantified by some particular parameter and let that parameter be designated along the X axis and the return along the Y axis.

Let us say, we have four assets whose risk return characteristics are plotted on this two dimensional space, risk along the X axis, return on the Y axis. We have assets A, B, C and D. Let us look at the relationship between asset A and B. It is very clear that both the assets provide provide you the different returns, but have the same level of risk. Both assets A and B have the same level of risk, but are providing you different returns. Is this situation sustainable in an equilibrium market? It is quite obvious, this situation is not sustainable.

Asset A provides a higher return for the same level of risk as B. This would mean that investors will flock in numbers to buy asset A and at the same time those that have asset B will try to liquidate their positions and take up positions in asset A. What would this result in? This will result in the demand for asset A increasing in the market and the demand for asset B diminishing in the market and that would mean that the price of asset A would increase and the price of asset B would decrease.

What would that translate to? That would translate to the returns of asset A diminishing and the returns on asset B increasing. Gradually, as equilibrium is attained in the market, the difference between the two assets' returns will narrow down and the returns on both these assets will tend to converge. So, that is how the dynamics of arbitrage takes place in the market. This is how the principle of arbitrage operates.

A similar example can be given in terms of assets C and D. If you look at assets C and D, they are giving you the same return, but asset C has a much lower level of risk compared to . In other words, you are much more certain of achieving the return that is promised by these two instruments in case you invest in asset C compared to your investment in asset D.

So, what would happen? Again, the same dynamics will operate. The demand for asset C will increase, the demand for asset D will decrease. As a result of this, the prices will realign and again, we will have a situation where the two points in risk return space represented by C and D tend to converge.

Another example, what about asset C and B. If you look at the assets C and B, asset C is providing you higher expected returns at a lower risk compared to asset B. So, it is doubly superior to asset B. In other words, it is superior in the context of expected returns,.it is also superior in the context of riskiness. It has a lower risk. In other words, if you invest in asset C, you are likely to get higher returns than asset B with a higher probability of the realization of those returns. So obviously, arbitrage would again operate between assets C and B with a stronger force of convergence. And again, these two assets will tend to converge in risk return space.

Now, what about assets A and D? What can we say about asset A & D? Asset A has a higher return and a lower risk, asset D has a lower return and higher risk. So, the situation would pretty much be similar as in the case of asset B and C.

What about B and D? Asset B has a lower return and a lower risk compared to asset D, which has a higher return and higher risk. Can we have arbitrage between asset B and asset D? Well, we cannot have arbitrage between assets B and D. Why? Because asset B is promising you a lower return, but it is also having a lower risk or conversely asset B has a lower risk, but it is also giving you a lower return compared to asset D which is having a higher risk, but it is also promising you a higher expected return. So, we cannot say prima facie that asset B is superior in all states of nature compared to asset D and therefore, we cannot say that arbitrage would operate between asset B and asset D.

The choice of an investor between asset B and asset D would depend on the risk return characteristics of or the risks return tradeoffs of the investor as based on or as depicted by his indifference curve. So, this is, in essence, the phenomenon of arbitrage. We will continue from here in the next class. Thank you.