Security Analysis & Portfolios Management
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Lecture 53
Capital Asset Pricing Model - III

Welcome back. So, let us continue with a recap of the key takeaways of the capital asset pricing model. Now, as explained in the last lecture, the capital asset pricing model is a statistical risk return relationship and can be expressed in the form of equation number 1.

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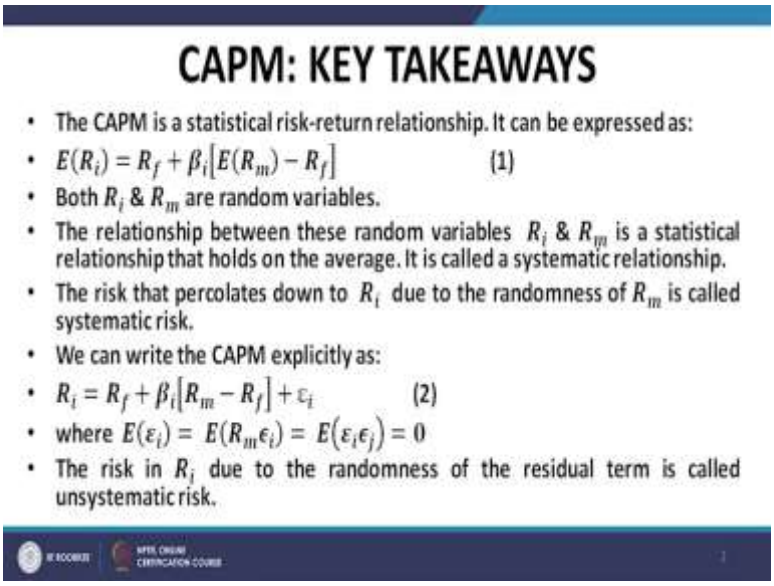

That is the expected return on a security i is equal to the risk free rate plus beta times a beta of the security times, the excess return of the market over the risk free rate. So, now here in this particular equation, equation number 1, both Ri and Rm are random variables. This kind of relationship between two random variables which is represented in equation 1 is and which is statistical in nature is called a systematic relationship between the 2 variables. In other words, this equation 1 represents a systematic relationship between the random variables Ri and Rm that is the returns on the security and the returns on the market.

Now, we discussed this term in more detail in an earlier section dealing with this single index model. The risk that percolates term you see, Rm is also a random variable, the market returns are also random variable. Now because there is a relationship between the market returns and the security returns as the fluctuations in the market returns also manifested themselves as fluctuations in the security returns.

So, the variance that arises or the fluctuations that arise or the randomness that is carried forward that percolates down from the market returns onto the security returns is called this systematic risk in line with the systematic relationship. I reiterate a systematic relationship is a relationship between two random variables that holds on the average. Each and that relationship may not hold for each and every observation, but on the average over a sustained number of observations, the relationship predicts the functionality between the two random variables.

As far as the CAPM is concerned, we can write the CAPM relationship explicitly in the form of equation 2 where we have introduced a residual term which captures the portion of the return that is not derived in relation to the excess market returns or that is not derived due to the systematic relationship between Ri and Rm. And this is the random term epsilon i. Where as in the case of the single index model we assumed that the expected value of epsilon is 0 the covariances between Rm and epsilon i and epsilon i and epsilon j are all 0.

Let me repeat; the expected value of epsilon is 0 and the covariance between Rm and epsilon i is 0 which holds in any regression process. And the covariance between epsilon i and epsilon j is 0 that is we are ignoring other effects that may cause a direct association between the random returns in the security i and the security j. In other words, the relationship between security i and a security j is the manifestation purely and solely of the relationship between the security i and the market and the security j and the market. There is no direct relationship between the security i and the security j.

Any co-movement between the security i and the security j as returns arises out of the relationships or the systematic relationships between the security i and the market and security j and the market. So, the risk in Ri that is due to the randomness of this residual term epsilon i is called the unsystematic risk. So, the randomness that percolates down to Ri due to the randomness of Rm is called the systematic risk and the randomness that percolates down to Ri to the residual term epsilon i is called the unsystematic risk.

And please note, as in the case of the single index model, these 2 sources of risk are orthogonal to each other.

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Thus, the total risk of a stock or a portfolio can be segregated into two orthogonal comp1nts. Number 1 the market risk or that arises from the market fluctuations which is called the systematic risk and the singular risk or the unsystematic risk that is specific to a particular security. Now, the important takeaway, the most important takeaway of the capital asset pricing model is that market does not price total risk, market prices only the systematic risk.

In other words, if you construct a portfolio with a significant composition or with the significant content of unsystematic risk, you are not likely to be rewarded by an equivalent or related expected return on account of the taking of that excess or substantial unsystematic risk. The expected returns that you could derive from the market would be in direct tandem with the extent or the content of systematic risk that you have taken in the portfolio.

The unsystematic risk will not be rewarded by the market. The premise is that the market participant should be astute enough to neutralize their unsystematic risk by the process of diversification. So, that is why the market believes that the real risk that is to be rewarded by additional or incremental expected return is the systematic risk and not the unsystematic risk.

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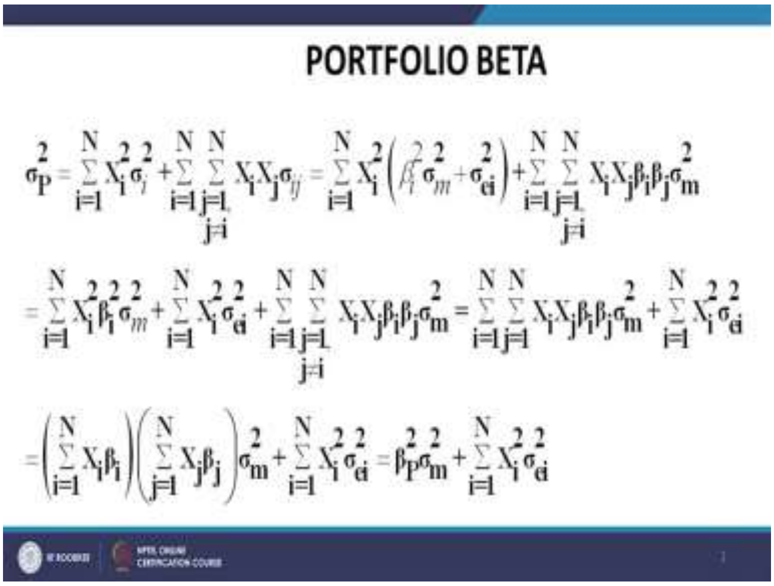

And as far as the portfolio beta was concerned, I briefly touched upon this in the last lecture, what we find is that the portfolio beta is the weighted average beta of the constituents. The portfolio beta is the weighted average beta of the constituents. Now, this is very interesting and very important because this scaling or this behavior of the beta is in tandem with an A is in line with the behavior of expected returns. In the sense that the expected returns of a portfolio are also the weighted average of the expected returns of the constituent securities.

So, the two measures that is the measure of expected return and the measure of systematic risk or the risk or relevant risk, let me call it the relevant risk is both scale in the same manner in o far as the evaluation for a constitution or a combination of securities is concerned. If you have a portfolio of securities, both the risk measured by beta and the expected return measured by the arithmetic return scale in the same way. Now, there is another important observation that I need to bring to the learners.

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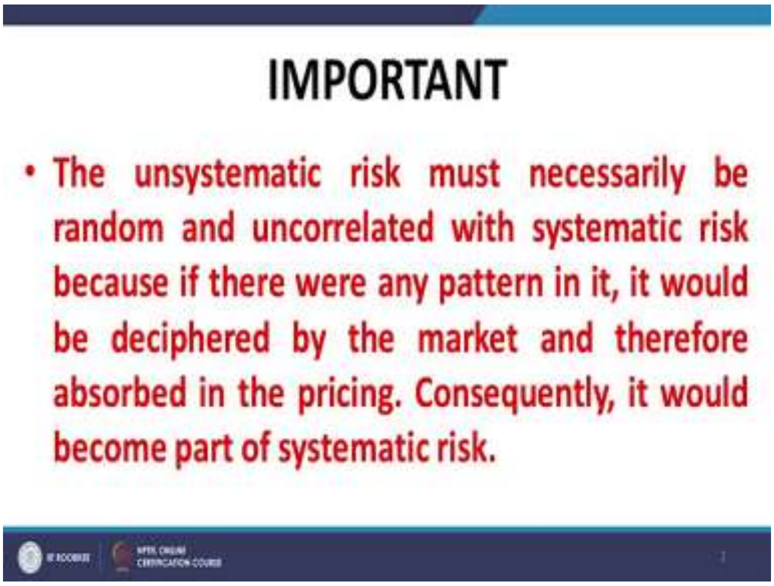

The unsystematic risk must not necessarily be random and uncorrelated with the systematic risk, why it is so? Because, if there were any pattern in it, if there was any decipherable pattern in it, it would be deciphered by the market and therefore, absorbed in the pricing and consequently it would become a part of the systematic risk.

So, it is important to know that the unsystematic risk comp1nt is totally independent and random. If there was any observable pattern in the behavior insofar as the unsystematic risk is concerned, that pattern would be captured by the market and incorporated into the prices and therefore, the unsystematic risk would no longer be remain as unsystematic risk it would become systematic risk.

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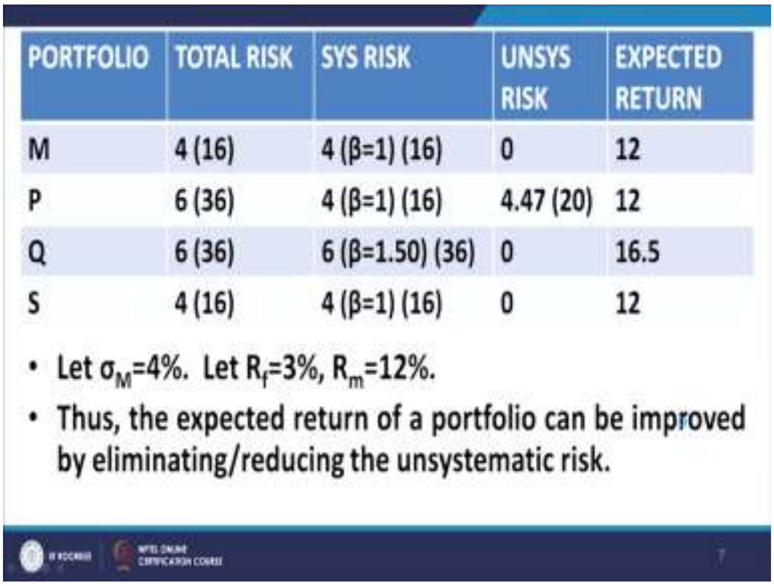

PORTFOLIO	TOTAL RISK	SYS RISK	UNSYS RISK	EXPECTED RETURN
M	4 (16)	4 (β=1) (16)	0	12
P	6 (36)	4 (β=1) (16)	4.47 (20)	12
Q	6 (36)	6 (β=1.50) (36)	0	16.5
S	4 (16)	4 (β=1) (16)	0	12

And then, we moved over to this example, if you recall, we had at the market portfolio which had a total risk of 4 percent or 16 percent square depending on your measure and the systematic risk was also 16 percent square, why? Because the market portfolio is an efficient portfolio invariably under the CAPM model and therefore, it would have no unsystematic risk as you can see on the slide as well. And on the premise of Rf being 3 percent, we arrive at a expected return of 12 percent on the market portfolio.

Then we had an arbitrary portfolio P which had system total risk of 36 percent square and which had systematic risk of 16 percent square and unsystematic risk of 20 percent square. And because it had a systematic risk equal to the market, it was giving you the same expected return of 12 percent.

What the example attempted to convey to the learners is that, if now, we construct a portfolio Q that has the same total risk as portfolio P but that eliminates the unsystematic risk comp1nt in favor of the systematic risk comp1nt. In other words, a substitutes the unsystematic risk by the systematic risk also retaining the same level of total risk then the portfolio Q is able to earn a higher expected return that is 16.5 percent

In other words, for the same level of total risk provided that total risk is manifest as the as entirely of systematic risk, then the returns are magnified, the returns are increased from 12 percent to 16.5 percent. Similarly, if you have another portfolio S which has a total risk of 16 percent square. But with the entire amount is or the entire systematic, entire total risk I am sorry is the systematic risk. In other words, systematic risk is also 16 percent square and unsystematic risk is 0, then you end up with the expected return of 12 percent.

So, what do we observe between portfolio P and portfolio S? Portfolio P and portfolio S have the same expected returns. But as you can see here, the portfolio P has a higher total risk compared to portfolio S. A portfolio S has a total risk of 16 percent square against the total risk of 36 percent square in the case of portfolio P. So, here again we find that we can improve upon the position or the worthiness of a portfolio by replacing the unsystematic risk by systematic risk.

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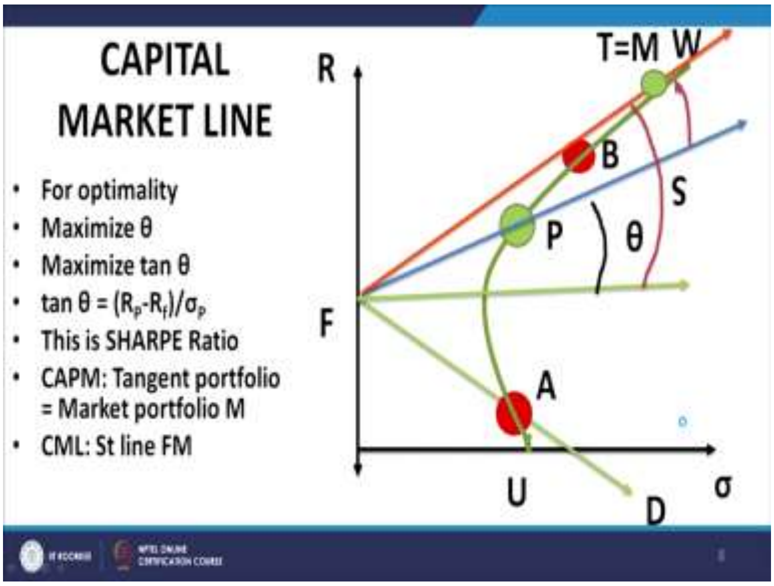

Now, we move over to the concept of capital market line. The learner should be familiar with the diagram that appears in the right hand panel of the slide. However, there is one small variation from the diagram that you have seen earlier and that variation is that under the assumptions of the capital asset pricing model, the tangency portfolio T now becomes the market portfolio under the, I repeat under the assumptions of the CAPM model, because each and every investor in the market under the assumptions of the CAPM model holds the same proportion of risky assets. That proportion of risky asset constitutes a market outstanding.

And therefore, this is equivalent to saying that the tangency portfolio becomes the market portfolio. The straight line FM then constitutes the capital market line, the straight line FM is called the capital market line. Clearly, all the efficient portfolios, we have discussed about that, that FT is the efficient frontier in the context of mean variance optimization. When you have both risk free lending and borrowing and short sales of securities both allowed. In that situation FT extended, it constitutes the efficient frontier.

And now because T is equal into the portfolio M which is the market portfolio, the efficient frontier becomes FM. And all investors in the market would hold combinations of the market portfolio together with risk free lending and borrowing as represented by any point on the straight line FM. This straight line FM is called the capital market line. We need to distinguish the capital market line from another similar term, which is called the security market line.

We will come back to it but please note one thing that this capital market line is in sigma R space that is the risk is represented by the total risk that is the standard deviation and the return is represented by expected return as usual. So, let us now move to this security market line.

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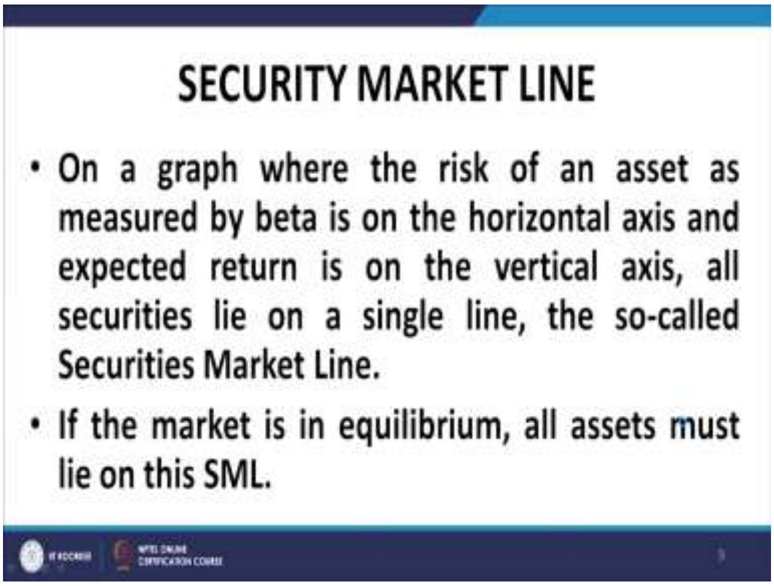

On a graph, where the risk of an asset as measured by its beta is on the horizontal axis and the expected return is on the vertical axis, all securities lie on a straight line, this is called the security market line. So, if you plot a graph between beta representing risk that is systematic risk representing the risk of the security or the portfolio along the X axis and the expected return along the Y axis. Then the CAPM model guarantees that what you will get as a plot is a straight line. Of course, with a slope of RM minus RF and the ERM rather expected return on the market minus RF, the excess return on the market and with a y intercept of RF.

So, this line on which all the securities would like irrespective of whether they are efficient or not efficient, I repeat this point all the securities in the market will lie on the effect on the security market line corresponding to their respective betas. The, please note this significant contrast between the capital market line and the security market line, only efficient securities oblique portfolios lie along the capital market line. However, all securities or portfolios will lie along the security market line.

Now this is important and if the market is in equilibrium, all assets must lie on this security market line as I just mentioned.

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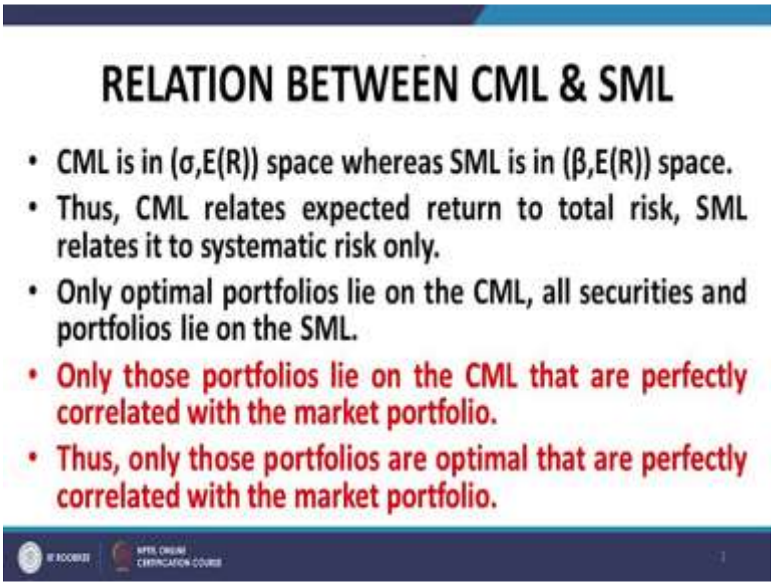

So, now, let us compare the two terms, the capital market line and the security market line. The capital market line as I have mentioned a few minutes back is in sigma expected return space, the risk is measured in terms of the total risk sigma and the expected return is the expected return, it is usually measured as the arithmetic return. And the security market line is in beta ER space. So, I repeat capital market line is in sigma ER space, the security market line is in beta ER.

So, when we are talking about the security market line, we are talking about the relationship between expected return and the systematic risk as indicated or represented by the CAPM model. And when we are talking about the capital market line, we are essentially referring to the mean variance optimization in conjunction with the CAPM model.

The capital market line relates expected return to total risk, security market line relates expected return to systematic risk. Only optimal portfolios lie on the capital market line as I mentioned, whereas all securities and portfolios lie on the security market line. Because in deriving the or in arriving at the CAPM equation that CAPM equation that we have arrived at the i that we have in the CAPM equation Ri bar is equal to RF plus beta i into RM bar minus RF.

And the i that we have applies to all the securities, the risk return relationship manifest in the CAPM equation holds for all securities whether they are part of efficient portfolios or they are simply isolated securities. So, I repeat the CML or the capital market line plots only the efficient portfolios whereas the security market line represents all the securities and portfolios. So, only those portfolios lie on the capital market line that are perfectly correlated with the market portfolio.

Because it is a straight line comprising of the risk free asset and the market portfolio with no contribution to the covariance coming from the risk free asset. What happens is that all the portfolios that lie on this straight line FM, that joins the risk free asset and the market portfolio will be perfectly correlated with the portfolio M because the risk free assets makes no contribution to the covariance. And, thus only those port portfolios are optimal, that are perfectly correlated with the market portfolio.

Because all portfolios that lie on the capital market line are efficient. And all portfolios that lie on the capital market line are perfectly correlated with the market portfolio. So, the combination of these two statement gives us that all efficient portfolios are perfectly correlated with the market portfolios and they are represented along the straight line FM extended. This figure represents a two straight lines.

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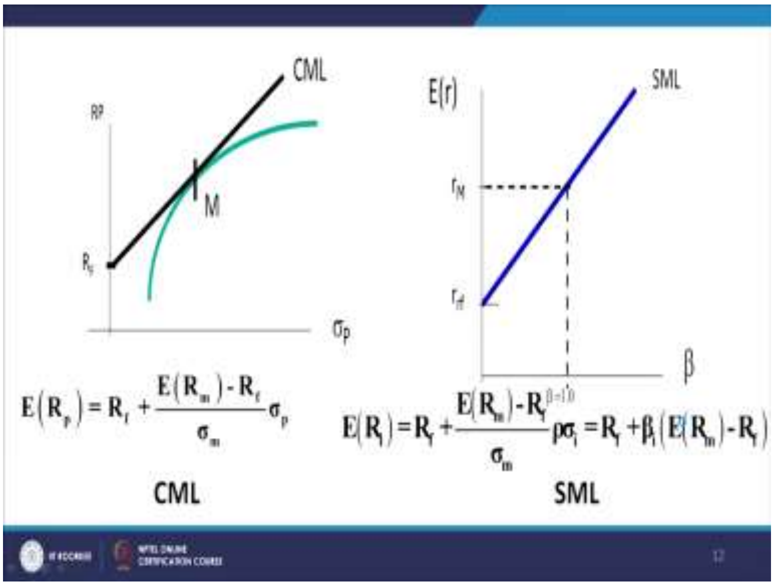

$$E(R_p) = R_f + \frac{E(R_m) - R_f}{\sigma_m}\sigma_p$$

$$E(R_i) = R_f + \frac{E(R_m) - R_f}{\sigma_m}\rho\sigma_i = R_f + \beta_i(E(R_m) - R_f)$$

You can see the capital market line on the left hand panel and you can see the security market line on the right hand panel. You can see that the capital market line is in sigma R space and the security market line is in beta R space. And the equation for the security market line is the CAPM equation, as you can see here right at the bottom right hand side of this slide.

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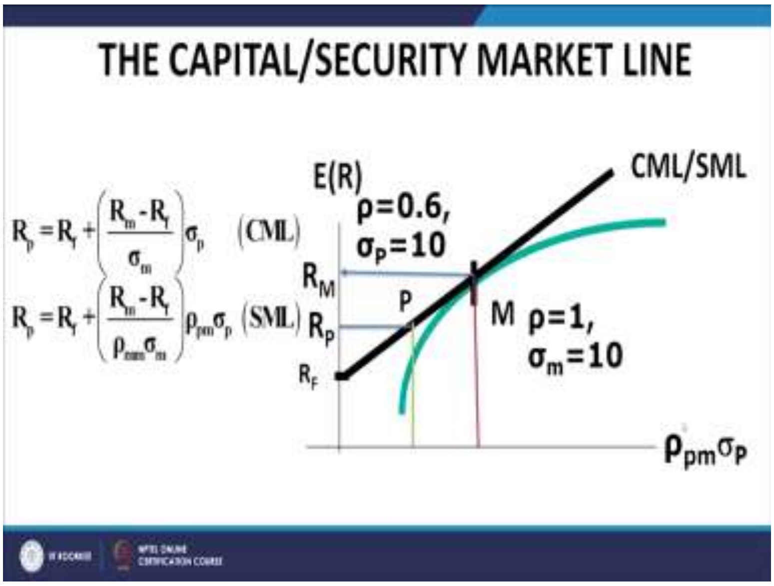

Now, there is an interesting relationship between the CML and the SML. In fact, you can represent both these lines on a single graph, what you need to have is in that case is that you as usual you plot the expected return along the Y axis. But along the X axis, instead of plotting either beta or sigma, what you plot is rho sigma i. So, instead of using sigma or beta as the measure of risk, if you measure use the measure of risk rho sigma i, then you can plot both this CM and SML on the same plot, same graph. And with the Y axis representing the expected return.

This is so why? This is so because of the reason that as far as the portfolios that lie on the capital market line, the rho is equal to 1, the covariance between any portfolio that lies on the capital market line, the correlation between any point that lies on the capital market line and the market portfolio is 1. So whether you write in the CML, whether you write a sigma P or rho sigma P, it would amount to the same thing because rho is equal to 1 for any portfolio that relies on the capital market line.

Therefore, you can represent both the lines on the same space provided you rescale the measure of risk as instead of sigma or beta but rho sigma. So, if you are using rho sigma as a measure of risk, then you can plot sigma and beta, I am sorry, you can plot the capital market line and the security market line on the same graph. As you can see here the portfolio M is a portfolio that lies on the security market line, it has a rho which lies on the capital market line, I am sorry, it has a rho equal to 1.

Our another portfolio with rho equal to 0.6 would lie on the security market line, but it would give you a lower return because rho sigma would be lower. So, both the securities can be represented on the same graph. Although of course, they would have different points depending on the correlation coefficient.

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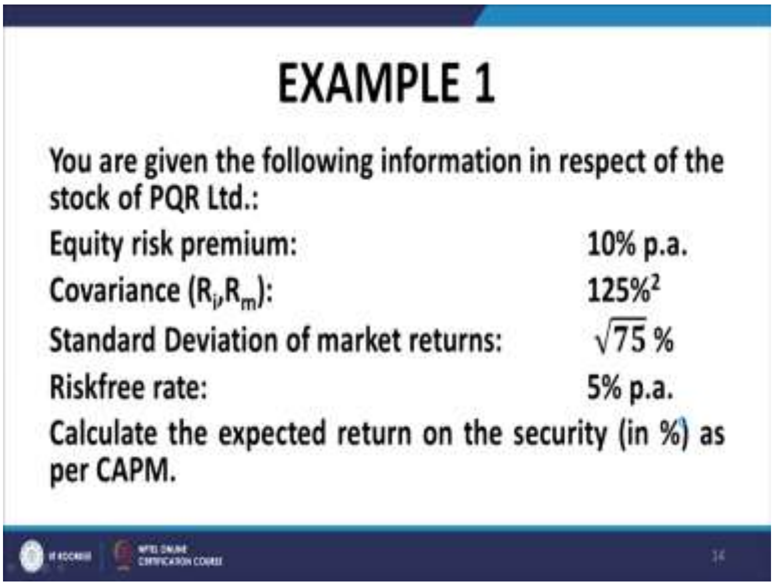

RISKFREE RATE	5
EQUITY RISK PREMIUM	10
COVARIANCE	125
MARKET VARIANCE	75
BETA	1.66666667
EXPECTED SECURITY RETURN	21.6666667

Now, let us do some examples. You are given the following information in respect of the stock PQR limited. Expected risk premium is 10 percent, covariance between RI and RM is 125 percent squared, standard deviation of market returns is under root 75 percent and the risk free rate is 5 percent. You are required to calculate the expected return on the security in percentage as per this CAPM model.

So, in this, this is a very straightforward problem, you are given the covariance between the security and the market and you are given the standard deviation of market returns. This enables us to determine the beta directly, beta is equal to what? Beta is equal to the covariance between Ri and Rm divided by the sigma m squared.

So, using this expression, what we get is beta is equal to 1.66667. And using beta, using the risk free rate and using the equity risk premium, we can easily find out the cap and return which turns out to be 21.67 percent.

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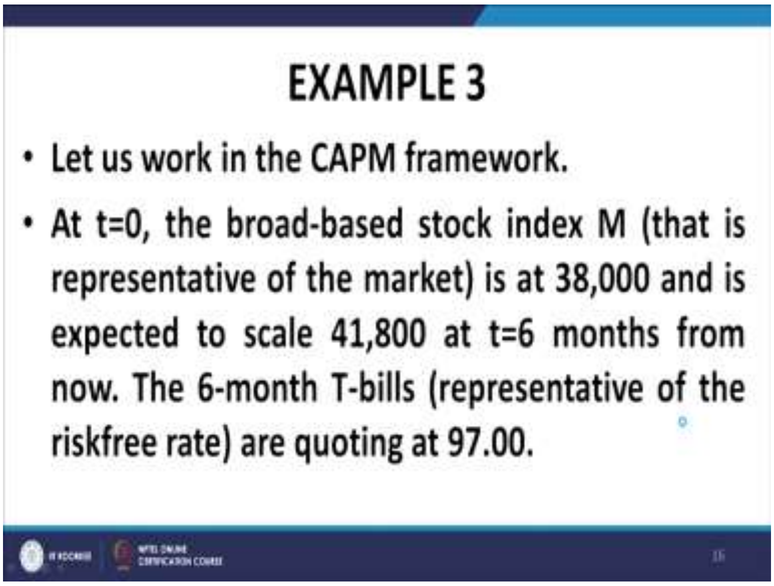

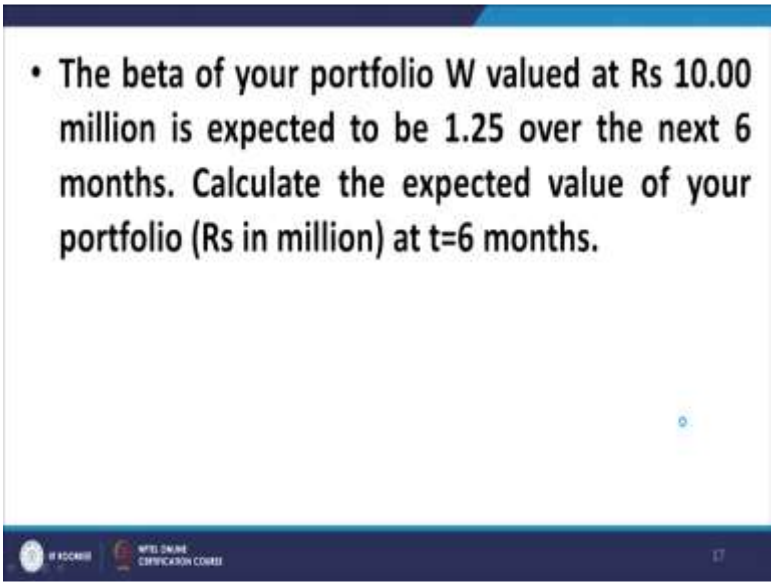

Now, we will look at another example, we again work in the CAPM framework. At t equal to 0 the broad based stock index m that is representative of the market is at 3800. And is expected to scale to 41,880 equal to 6 months from now. The 6 month tables rate which is representative of the risk free rate is quoting or the tables are quoting at 97 that is hundred rupee face value table is quoting at 97.

The beta of your portfolio W valued at rupees 10 million is expected to be 1.25 over the next 6 months. Calculate the expected value of your portfolio rupees in million at t equal to 6 months. So, this is the question, let us read it once again quickly, let us work in the CAPM framework. At t equal to 0 the broad base stock index M that is representative of the market is at 38,000 and is expected to scale 41,800 at t equal to 6 month. That means after 6 months the market index is expected to rise from 38,000 where it is today at t equal to 0 to 41,800.

The 6 month t will rate or the 6 month t will quote rather is at 97. That means at the end of 6 months if you get 100 on the table today you have to pay 97. This this gives you a measure of the risk free rate valued over 6 months. The beta of your portfolio W valued at rupees 10 million is expected to be 1.25 over the next 6 months. Calculate the expected value of your portfolio rupees in million at t equal to 6 months.

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CURRENT MARKET INDEX	38000	
PROJECTED MARKET INDEX AT T=6	41800	
EXPECTED RETURN ON M OVER INV HOR	0.1	
INVESTMENT HORIZON	0.5	YEARS
QUOTE OF 6-MONTH T-BILL	97	
RISKFREE RETURN OVER INV HOR	0.03092784	
BETA	1.25	
CAPM RETURN ON W	0.11726804	
CURRENT VALUE OF W	10	
PROJECTED VALUE OF W	11.1726804	

So, the current market index is given us 38,000 which is given in the problem. The projected or the expected market index at t equal to 6 is given as 41,800. So, the expected return on the market portfolio over the inverse investment horizon of 6 months is equal to 10 percent that is 0.1. And the investment horizon is 0.50 years. The quote of a 6 month table is given as 97. That translates to an expected return of 3.09 to 7 percent over 6 months.

Please note this is not analyzed this is the return over 6 months. The beta of the portfolio is given as 1.25, the CAPM return on W, therefore turns out to be using that CAPM formula Ri is equal to Rf plus beta i into Rm minus Rf, everything is available with us. Rm is available with us, the expected return on the market, the beta is given to us and Rf is also given to us. So, we simply plug in the values in the CAPM equation in the security market line and what we get is the CAPM return on the portfolio W is equal to 11.7268 percent and because the current market value of the portfolio is 10 million, the projected market value or the expected market value is equal to 11.1726 million.

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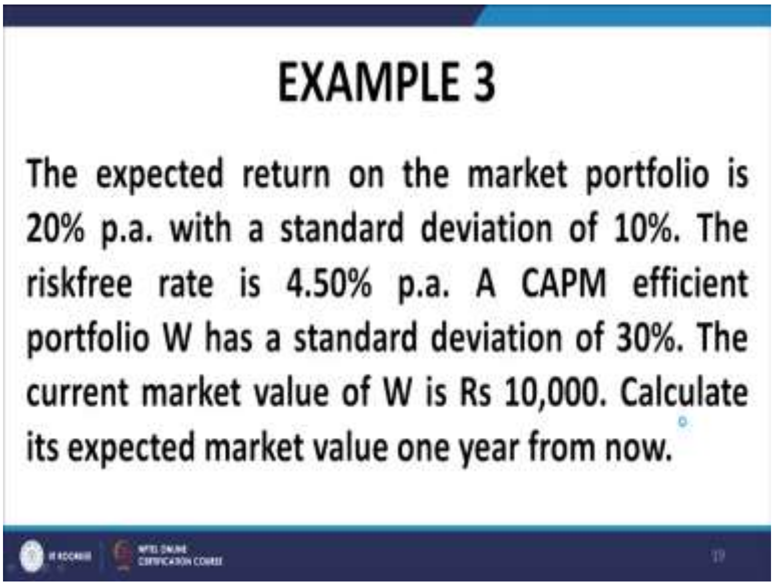

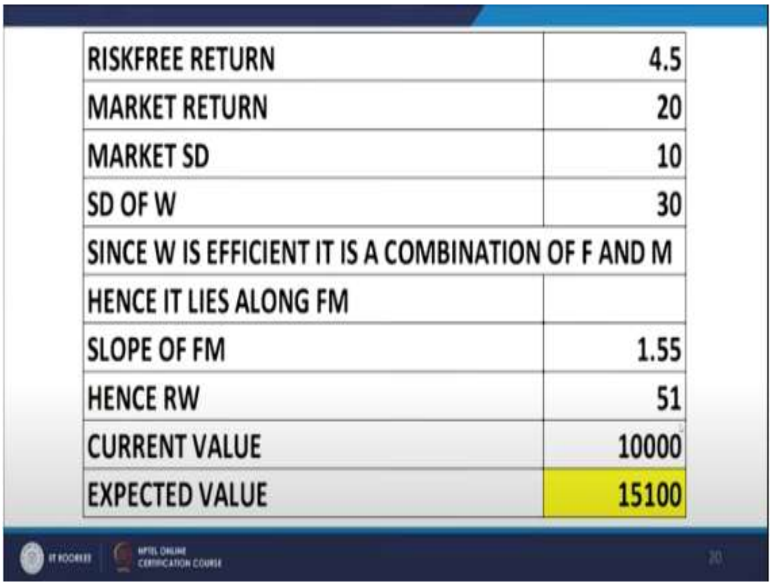

Then, another example, the expected return on the market portfolio is 20 percent per annum with a standard deviation of 10 percent. I repeat, the expected return on the market portfolio is 20 percent per annum with a standard deviation of 10 percent. The risk free rate is 4.50 percent per annum. Please note this point is this 4.50 percent per annum.

A CAPM efficient, CAPM efficient portfolio W, this is very important, it is a efficient portfolio under the CAPM model. So, CAPM efficient portfolio W has a standard deviation of 30 percent. The current market value of the portfolio is 10 thousand, calculate the expected market value 1 year from now. Now, please note this very important point it is a CAPM efficient portfolio and therefore, it would lie on the capital market line on the CML.

And, we know that the slope of the CML is what? The slope of the CML is given by RM or point m that is the expected return corresponding to the point m minus RF divided by sigma m. So, what do we have here? We have the risk free return that is given to us, we have the market return that is given to us that is expected market return, the market standard deviation is given to us.

Therefore, and the standard deviation of the portfolio W is given to us. Since W is efficient, it is a combination of F and M premium particularly because it relies on the capital market line. And all portfolios that lie on the capital market line are linear combinations of F and M are linear combinations of F and M. Therefore, W also lies on the capital market line FM. We know the standard deviation of W that is given a 30 percent and we know the slope of the capital market line, we know its y intercept.

So, we know the equation of the capital market line and we can find out the expected return corresponding to this standard deviation of W which is thirty percent and the expected return turns out to be 51 percent. And using this expected return of 51 percent, we can calculate the value of our portfolio, the initial value the t equal to 0 value of the portfolio is 10,000. Therefore, the t equal to 1 year value would be an additional increment and appreciation of 51 percent and hence the t equal to 1 year value of our portfolio will be 15,100.

Now, we start a new topic what that is arbitrage pricing theory. This is another model that relates to the risk return trade off portfolios. The most common model that we use at least at the graduate level of the paper security analysis and portfolio management is this CAPM model. This is an extension or a generalization rather of the CAPM model. And it is the contemporary framework of relating risk and return of financial assets. So, let us now get into it.

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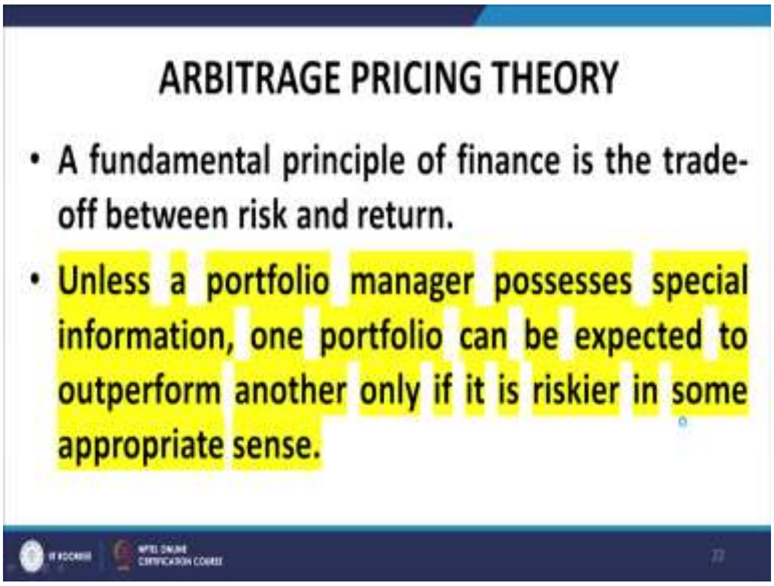

A fundamental principle of finance is a trade-off between risk and return. Unless a portfolio manager possesses special information, one portfolio can be expected to outperform another portfolio only if it is riskier in some appropriate in some sense. In some sense, the portfolio that is doing better or that is expected to do better has to be riskier than the portfolio that is not doing so well. Because there is a positive relationship between risk and return, irrespective of how we measure risk and return.

One thing is very clear, higher the risk in a portfolio higher would be the expected return and converse will also hold.

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So, what is the appropriate measure of risk that is a fundamental question that actually academicians and practitioners have been trying to answer and it is a very difficult question because there are a lot of heterogeneous factors that go into that contribute to the total risk that is rewarded by the market. This CAPM model in essence identifies only one single factor which is the market returns or the market risk. A CAPM model, as I have emphasized again and again is based on the premise or results in the premise rather that the market rewards only this systematic risk.

Market rewards only that risk which is related to the market or which arises from the fluctuations of in the market. All other idiosyncratic risks are not rewarded by the market. But the as I mentioned, the APT model, I am sorry is a generalization of the CAPM model in some sense, although not exactly a generalization, but it is an extension of the CAPM model in what manner we shall soon see. Many attributes might be related to an assets risk like market capitalization, dividend yield, growth, price earnings ratio.

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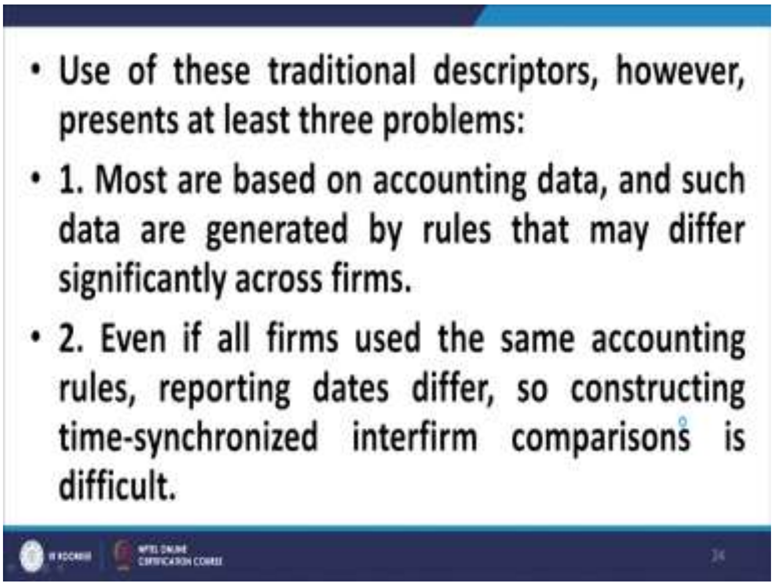

Use of this traditional descriptors, however, poses some problems, what are those problems? The problems are that most of these descriptors are based on accounting data and such data are generated by rules that may differ across firms. So, interfirm comparison becomes difficult on this count. Not only on this count, interfirm comparison becomes difficult also, because even if the same accounting rules are adopted by the firms forming the comparable set, the reporting dates may differ. And therefore, in constructing time synchronized interfering comparison becomes a problem.

Then, thirdly, there is an issue at the fundamental level at the very basic level. And that is that no rigorous theory tells us how traditional accounting variables should be related to an appropriate measure of risk. There is no direct Nexus as explained by some underlying theory, which is which is very important for a model to be to be universally acceptable, that it should have a sound underlying theoretical underpinnings on which it is built up and so that it can be accepted across the fraternity.

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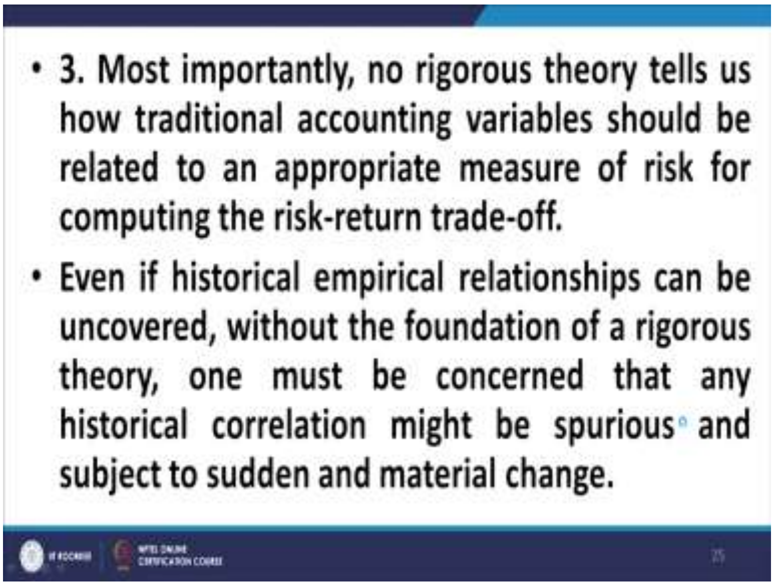

So, the no rigorous theory tells us, how traditional accounting variables should be related to an appropriate measure of risk for computing the risk return trade off, even if historical empirical relations this is important. Even if historical empirical relationships can be uncovered without the foundation of a rigorous theory, one must be concerned that any historical correlation might be spurious and subject to sudden and material change.

Now, the current scenario of risk return trade off theorists, as I mentioned, we have 2 fundamental models that attempt to explain the risk return relationships. The first is the capital asset pricing model that we have discussed in a lot of detail. And the second is the arbitrage pricing theory, which we are in the process of discussing.

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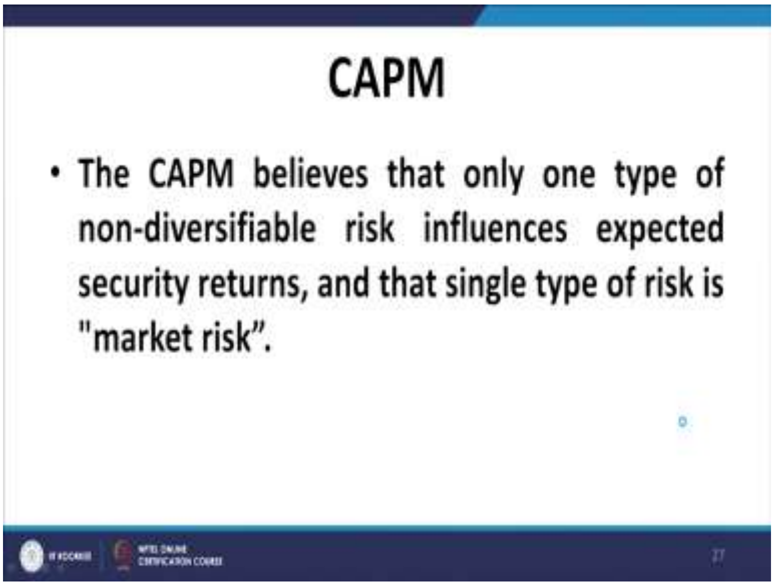

Now, the CAPM model as I mentioned, believes that only 1 type of non-diversifiable risk that is systematic risk influences expected security returns and that single type of risk is a market risk. We have discussed this in a lot of detail while talking about the CAPM model. The arbitrage pricing theory is slightly more extensive.

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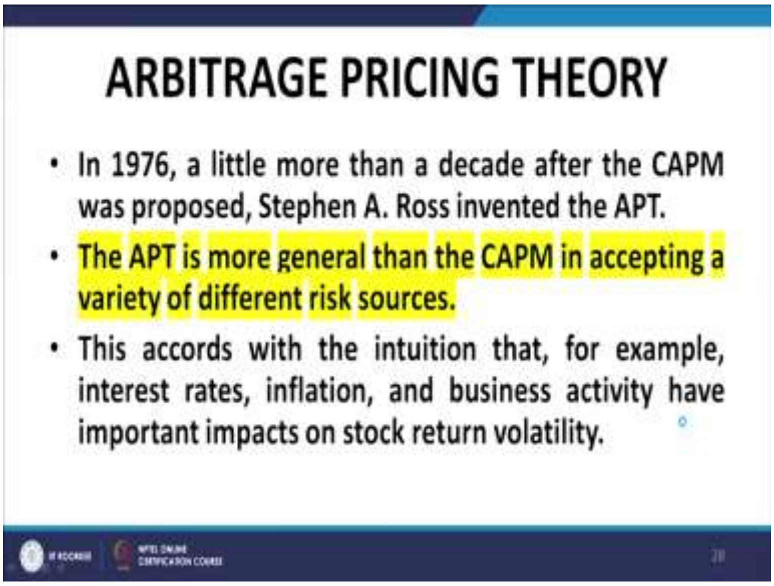

And it was proposed by Stephen Ross in the year 1976. About a decade after the CAPM model was propounded by William Sharpe. The APT is more general than the CAPM in accepting a variety of different resources. This accords to the intuition that for example, interest rates, inflation and business activity have important impacts on stock return volatility.

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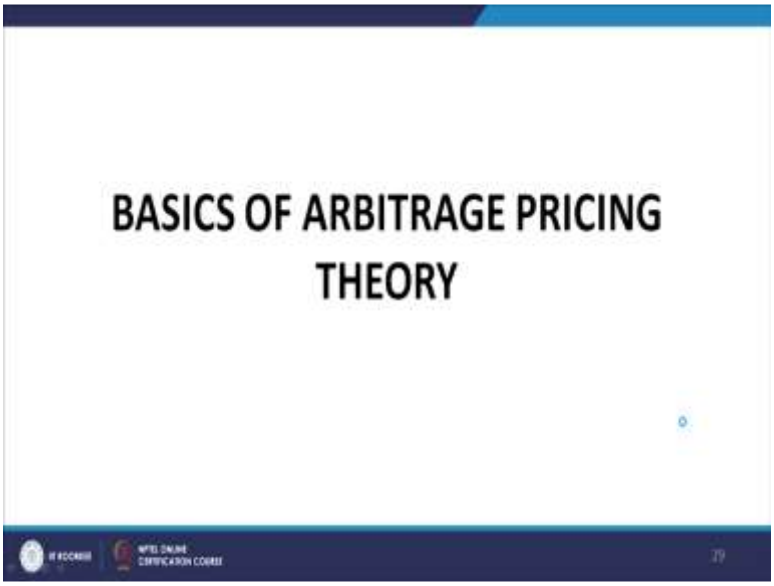

So, from here we shall take up after the break, we shall delve deep into the arbitrage pricing theory. Thank you.