

Security Analysis & Portfolio Management
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Lecture No. 51
Capital Asset Pricing Model I

Welcome back. So, let us continue from where we left off in the last lecture, but before that a quick recap about the concept of systematic and unsystematic risk, which is a very important issue in the context of the single index model, as the CAPM, which we are going to discuss in today's lecture. So, let us start with the expression for the variance of an N security portfolio, the standard expression for the variance of the N security portfolio, which is given in equation number 1 here, on this slide.

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SYSTEMATIC & UNSYSTEMATIC RISK

The variance of an N security portfolio is :

$$\sigma_p^2 = \sum_{j=1}^N (X_j^2 \sigma_j^2) + \sum_{j=1}^N \sum_{k=1, k \neq j}^N (X_j X_k \sigma_{jk}) \quad \text{--- (1)}$$

If the portfolio is equally weighted
in all securities, then

$$\sigma_p^2 = \sum_{j=1}^N \left(\frac{1}{N}\right)^2 \sigma_j^2 + \sum_{j=1}^N \sum_{k=1, k \neq j}^N \left(\frac{1}{N}\right)^2 \sigma_{jk} \quad \text{--- (2)}$$

Now, if now I consider a portfolio comprising of equally weighted securities, N securities equally weighted, then I can write x1 is equal to x2 is equal to 1 by N and the above expression simplifies to the expression number 2 that is given here, at the bottom of your slide.

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$$\sigma_p^2 = \left(\frac{1}{N}\right) \left(\frac{\sum_{j=1}^N \sigma_j^2}{N}\right) + \left(\frac{N-1}{N}\right) \left[\frac{\sum_{j=1}^N \sum_{\substack{k=1 \\ k \neq j}}^N \sigma_{jk}}{N(N-1)}\right] \quad (3)$$

$$\sigma_p^2 = \frac{1}{N} \bar{\sigma}_j^2 + \left(\frac{N-1}{N}\right) \bar{\sigma}_{jk} \Rightarrow \text{Lim}_{N \rightarrow \infty} \sigma_p^2 = \bar{\sigma}_{jk}$$

Now, this expression 2 that I have here can be modified or can be rewritten in a slightly different form which makes it enable amenable to future, future manipulations algebraic manipulations. I write it in the form that is given in equation number 3. What I have done is simply I have taken a pre factor of 1 upon N and then within the round bracket the second round bracket, I have summed up all the variances and divided by N.

Similarly, I have taken a pre factor N minus 1 upon N in the second term and I have summed all the co-variances over all the securities i equal to 1 to N, j equal to 1 to N or j equal to 1 to N, and k equal to 1 to N rather, and I have divided the sum of the variances, the sum of all the variances by N into N minus 1. Of course, out of these two sigma ij will always be equal to sigma ji, so we have N into N minus 1 divided by 2 distinct co-variances, although the total number of co-variances will be N into N minus 1. But pairs of variances sigma i j and sigma j i will be equal to each other.

So, let me repeat what we have here, in the first, in the second round bracket here of the first term, we have the sum of all the individual variances divided by the total number of securities. And the second term we have the sum of all the co-variances, divided by the number of co-variances.

So, the first term or the first expression that is summation sigma i square divided by N in some sense, gives us the average variance for security over the portfolio of securities. And

the second term that is double summation $\sum_{j,k} \sigma_{jk}$ divided by $N(N-1)$, gives us an assumption, the average covariance over every pair of securities constituting the portfolio. So, that is how the equation has been written in the next equation, that is let us call it equation number 4.

σ_i^2 is, σ_j^2 is the average variance over the set of securities constituting the portfolio. And σ_{jk} is the average covariance over the pair of securities, any pair of securities constituting the portfolio. Now if I take the limit N tending to infinity what I get is the first term tends to 0, why because, σ_j^2 would be, would be a finite number and if you divide it by N , you end up with the limit, in the limiting case the value tending to 0 when N tends to infinity.

However, the second term that is the covariance term, does not go to 0 completely. If you find that one part of the covariance term that is covered by $\frac{1}{N} \sum_{j,k} \sigma_{jk}$ will go to 0 as N tends to infinity. However, the first term that is $\sum_{j,k} \sigma_{jk}$ will not go to 0 and that will be a residual left over when we take the limit N tending to infinity. And this is what $\sum_{j,k} \sigma_{jk}$ represents the systematic risk of the portfolio and the first term that is arising out of the variances the intrinsic variances of the securities represents the unsystematic risk.


So, the outcome of this as I explained in the earlier lectures also, is that firstly the, the total variance of a portfolio cannot be made 0, it will tend to a finite nonzero value even in the case when we have a massive diversification, when we have infinite diversification. The total risk will tend to a finite non-zero risk, the amount of risk that can be diversified away, arises from the intrinsic variances of the securities and the, the amount of risk that cannot be diversified away, that is retained or that survives the diversification arises from correlations between the securities.

So, this is the outcome of the general mean variance framework, let us now see, how this, this approach gets modified in the domain of the single index model. So, in the single index model, we can write the variance of a single security variance of a single security.

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SYSTEMATIC & UNSYSTEMATIC RISK (SIM)

$$\begin{aligned} \sigma_p^2 &= \sum_i X_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} X_i X_j \sigma_{ij} \\ \sigma_p^2 &= \sum_i X_i^2 (\beta_i^2 \sigma_m^2 + \sigma_{\epsilon_i}^2) + \sum_i \sum_{j \neq i} X_i X_j \beta_i \beta_j \sigma_m^2 \\ &= \sigma_m^2 \sum_i X_i^2 \beta_i^2 + \sigma_m^2 \sum_i \sum_{j \neq i} X_i X_j \beta_i \beta_j + \sum_i X_i^2 \sigma_{\epsilon_i}^2 \\ &= \sigma_m^2 \sum_i \sum_j X_i X_j \beta_i \beta_j + \sum_i X_i^2 \sigma_{\epsilon_i}^2 \\ &= \sigma_m^2 \sum_i (X_i \beta_i) \sum_j (X_j \beta_j) + \sum_i X_i^2 \sigma_{\epsilon_i}^2 = \beta_p^2 \sigma_m^2 + \sum_i X_i^2 \sigma_{\epsilon_i}^2 \quad (9) \end{aligned}$$


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In terms of two parts, one part that is related or that arises from the market risk or the market randomness that is captured by beta i square sigma m square and the other part that is intrinsic to the security, that is captured by the second term that is sigma e i square which is the residual variance. So, that is how I have written here the second equation on the slide, and then I do some algebraic simplifications, nothing more than this, and I can write this in the form of.

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$$\begin{aligned} &= \sigma_m^2 \sum_i X_i^2 \beta_i^2 + \sigma_m^2 \sum_i \sum_{j \neq i} X_i X_j \beta_i \beta_j + \sum_i X_i^2 \sigma_{\epsilon_i}^2 \\ &= \sigma_m^2 \sum_i \sum_j X_i X_j \beta_i \beta_j + \sum_i X_i^2 \sigma_{\epsilon_i}^2 \\ &= \sigma_m^2 \sum_i (X_i \beta_i) \sum_j (X_j \beta_j) + \sum_i X_i^2 \sigma_{\epsilon_i}^2 = \beta_p^2 \sigma_m^2 + \sum_i X_i^2 \sigma_{\epsilon_i}^2 \quad (9) \end{aligned}$$

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For an equally weighted N security portfolio

$$\sigma_m^2 = \sigma_m^2 \sum_i \sum_j X_i X_j \beta_i \beta_j + \sum_i X_i^2 \sigma_{\epsilon_i}^2$$

$$= \sigma_m^2 \sum_i (X_i \beta_i) \sum_j (X_j \beta_j) + \sum_i X_i^2 \sigma_{\epsilon_i}^2 = \beta_p^2 \sigma_m^2 + \sum_i X_i^2 \sigma_{\epsilon_i}^2 \quad (9)$$

$$= \sigma_m^2 \left(\frac{1}{N} \sum_i \beta_i \right) \left(\frac{1}{N} \sum_j \beta_j \right) + \left(\frac{1}{N} \right) \left[\left(\frac{1}{N} \sum_i \sigma_{\epsilon_i}^2 \right) \right] \xrightarrow{N \rightarrow \infty} \bar{\beta}_i^2 \sigma_m^2$$

— (6) —

Equation number 5, which I write here this is equation number 5 and this is how we can write this particular equation, which, which represents the total variance of a portfolio in the context of single index model or within the domain of the single index model. Now, let us look at a equally weighted and security portfolio, let us look at a equality and security portfolio. Then xi will be equal to 1 upon N for all i and even, we make this substitution we arrive at the expression which is given as an equation number, let me call it equation number 6.

So, if you look at equation number 6, the first term which I underline now, is what, it is the summation of all the betas of all the all the securities that constitute the portfolio divided by the number of securities. So, in some sense it represents the average beta per security of the Security Set that form the portfolio. Similarly, when you sum the same thing over beta j you will get again the same expression that is the average beta over the set of securities constituting the portfolio.

So, what we have as the complete first term is, sigma m square beta bar i beta bar i that is sigma, that is sigma m square beta bar i whole squared. And what about the second term, and the second term if you look at this, what happens to the second term, it is the sum of all the residual variances of all the securities divided by the number of securities and that is in some sense the average residual variance, variance over the set of securities that constitutes a portfolio.

That is the expression that that is within the square brackets but, there is a pre factor of $1/N$ also so, when I take the limit N tending to infinity, this the expression that is within the square bracket together with the pre factor will approach 0 why, because the expression within the square bracket is finite and when you are dividing by N and taking the limit N tending to infinity, this expression will go to 0. And what we will be left with is, the expression that survives in is the first expression and that can be written in the form $\beta_i^2 \sigma_m^2$.

So, in other words what we are getting at is that, this expression the first expression that is $\beta_i^2 \sigma_m^2$, represents the systematic risk and the residual risk that is captured in the single index model by the random error term, represents the unsystematic risk. So, that is one inference, the second inference is to look at the comparison between what we had in the earlier slide let me go back to it, σ_{jk} which was the expression that we arrived at, in the context of the general mean variance framework, where no assumption as to the pattern of risk return, risk return trade-off was made as is some case in the single index model.

So, we have the systematic risk being captured by the co-variances or the average co-variances of the portfolio. However, in the case of the in the case of the single index model, we find the expression is slightly different as is captured by $\beta_i^2 \sigma_m^2$.

In other words, the, the presumption or the assumption of the single index model, that any mutual interaction between the securities arises only on account of or any co-movement between securities arises only account of their relationship of individual, the relationship of individual securities with the market portfolio. And there is no direct interaction between the securities i and j that is vindicated by this expression which, which is the outcome of the single index model for systematic risk.

You can see here, that we have $\beta_i^2 \sigma_m^2$ so there is a this expression, clearly shows that the systematic risk is only on account of the relationship or the interaction or the systematic relationship between the security i and the market. And there is no direct interaction between any pair of securities i and j , that is the pre that is indeed the premise of the single index model, so that is vindicated by this expression as well.

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INFERENCES

- There is some part of the total risk of a portfolio that can be diversified away.
- This component of risk arises from the intrinsic variances of the individual securities.
- The component of risk that is non-diversifiable emerges from mutual co-relations between securities.

The slide is titled "CARDINAL ASSUMPTIONS OF MARKOWITZ MEAN VARIANCE FRAMEWORK". It contains two bullet points in red text. The slide also features a dark blue header and footer with logos for "IFSCORSE" and "NPTEL ONLINE CERTIFICATION COURSE".

CARDINAL ASSUMPTIONS OF MARKOWITZ MEAN VARIANCE FRAMEWORK

- Expected return is a function of risk.
- Risk is measurable by standard deviation.

Let us now move forward this, this inferences I have already discussed in an earlier lecture and I highlighted them a few minutes back as well. So, now we move over to the capital asset pricing model, now in the mean variance framework, in the mean variance framework of Markowitz that we have discussed in a lot of detail in the earlier lectures.

We had made two fundamental assumptions number 1 that expected return is a function of risk and number 2, risk is measurable by standard deviation. Number 1 expected return is a function of risk and number 2 risk is measurable by standard deviation. These were fundamental assumptions, basic assumptions of the Markowitz mean variance model.

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WILLIAM SHARPE & CAPM

- William Sharpe asks: What are the implications for asset prices if everyone adopts the Markowitz framework?
- He reasons that "In equilibrium, all assets must be held by someone."
- Therefore, For the market to be in equilibrium, the expected return of each asset must be such that investors collectively decide to hold exactly the supply of shares of the asset.
- He investigates:
 - Whether standalone risk (standard deviation) is an appropriate measure of risk?
 - Does market "price" standalone risk??



William Sharpe asked the question, what are the implications for asset prices, if everyone adopts the Markowitz framework, if all the players in the market, all the investors adopt the Markowitz framework for the optimization of their portfolios, then what would be the implications for the market prices.

You see the point is, as far as the Markowitz theory is concerned, it is investor centric in the sense that everything has been developed from the perspective of the investor, how the investor should manage his portfolio, how it should distribute the various assets at its command or at its disposal to arrive at an optimal mix of the various securities of the various assets.

So, now William Sharpe looks at it from a different perspective, he looks at it from the perspective of collective wisdom, he looks at it from the perspective of the market. If everybody in the market was to adopt the market, the Markowitz model what would be the implication, on the market prices of the various assets that are traded, that are being traded in the market and that are being used by the constituents of the market, in order to arrive at their optimal portfolios.

How will the market respond to that, how would the market prices adjust themselves in a situation, where the market may be deemed to be in equilibrium. So, he reasons that in equilibrium.

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WILLIAM SHARPE & CAPM

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- Therefore, For the market to be in equilibrium, the expected return of each asset must be such that investors collectively decide to hold exactly the supply of shares of the asset.

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All assets must be held by someone, and that means what, that means that in order that the market be in equilibrium, the expected return on each asset, this is fundamental, the market return on each asset the expected market return rather on each asset, must be such that investors collectively decide to hold exactly the supply of shares of the asset. And this is the most important part of this particular proposition, pronounced by William Sharpe.

That, what he says is that, if the if everybody does the Markowitz framework if everybody follows the Markowitz framework for designing his optimal investment policy, and how the prices of the various assets would react. In other words, what would be the risk return representation in the market and because, you see at the end of the day, it is necessary that if the market is in equilibrium, the demand supply must be matched, demand supply of each asset must be matched.

So, if the demand supply of each asset must be matched, then, the what would be the implications on the returns on each security, the returns on its securities has to adjust themselves in such a way that the demand supply is matched and every asset that is available in the market is held by somebody. That was the line of reasoning that was adopted by William Sharpe in deriving his capital asset pricing model. So, I mean given this background to you, you know why did William Sharpe come up with these questions, let us try to understand that.

The quest, the issue was whether standalone risk that is the standard deviation that is measured by the standard deviation, is an appropriate measure of risk number 1 and number 2, does market price market embed standalone risk in other words if a security has higher standalone risk, will it command a higher expected return, that was the question these were two questions which the which William Sharpe went on to investigate, number 1 where the stand alone risk is the appropriate measure of risk, number 2 does market price stand alone risk, in other words does the expected return flow in tandem with standalone risk, is the standalone risk the sole determinant of the of the expected return on a particular security.

So, if you take a higher standalone risk, are you entitled or are you allowed by the market to obtain, higher expected returns. Please note, please note the word expected I have discussed this in an earlier lecture also, we are talking about expected returns, expected returns means returns, which are to be realized in future. And if there is a higher risk attached to higher expected returns, then that implies that the realizability or the probability of realization of those returns is also reduced. Because, the risk is more so, while that is expected return may be higher the possibility of probability of the realizability of that expected return may be reduced, that is the concept of risk and return.

So, anyway coming back to this, this was these were the questions that, that William Sharpe went on to investigate. Whether stand alone risk is the correct measure of risk and number 2, does market price stand alone risk. Let us, try to understand why these questions arose in the mind of William Sharpe, let us take an example to motivate the discussion.

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STANDALONE RISK APPROPRIATE?

- Consider two shares A,B with standalone risk $\sigma_A = \sigma_B = 6\%$. Let the shares be so priced that expected returns on both shares is $R_A = R_B = 10\%$. Let the correlation between A,B, $\rho_{AB} = 0.50$. Then $R_p = 10\%$, $\sigma_p = 5.2\%$.
- If the market was to "price" the σ risk, then the portfolio P should necessarily provide a lower expected return than A & B because it has a lower risk (in terms of lower σ).

Consider two shares A and B with standalone risk, that is a standard deviation of variance you might call, you may use either sigma A is equal to sigma B is equal to 6 percent, consider two shares A and B with standalone risk sigma A is equal to sigma B is equal to 6 percent. Let, the shares be so priced because, they are having the same standalone risk and on the premise that the market prices stand alone risk, they should come on the same expected returns, so the shares would be priced so, that expected returns on both the securities are the same.

Let us, assume that the expected return is 10 percent for A and 10 percent for B because they are having the same standalone risk. And we are working on the premise that standalone risk is the correct measure of risk and the market prices standalone risk. Now, so far so good there is no issue so far now, let us form A portfolio comprising of A and B in equal proportions and let us assume, let us assume that the correlation, coefficient between A and B is 0.50.

Let us assume that the correlation between A and B is 0.50 that is ρ_{AB} is equal to 0.50, then what we find is, because the portfolio A and B comprises of two securities having the same expected returns. Any linear combination of the two securities, would still give me the same return that is 10 percent. So, if we are taking half of A and half of B, then we will still have r_P is equal to 10 percent.

But, what about sigma p, if you work out the sigma P using the standard formula for the variance of a pair of securities, using x_A is equal to x_P . And what we find is using x_A is equal to x_P is equal to 1 by 2, what we find is that sigma P is equal to 5.2 percent and this is very interesting.

What does it convey, it conveys that Sigma P the combined portfolio of A and B we have done just nothing, we have simply diversified from one security to taking two securities, the two securities are identical in terms of standalone risk and in terms of expected returns, we have combined them to form a 50-50 portfolio. And we find that for lower rational whatsoever, the the risk of the combination is less than the risk of either A or the risk of B and this is very interesting.

And yet the anomaly is that notwithstanding the fact that the risk of A B let us call the combined portfolio P; the risk of the combined portfolio P is lower and yet it is giving you the same level of return, a same level of expected return as A or B. This is obviously a contradiction. If the market is pricing standalone risk, then because the portfolio P has a lower risk, it should get a lower expected return, but it does not happen and the math does not, does not agree with this.

The math says that return on the portfolio P will also be the same as that on security A and security B notwithstanding the fact that the portfolio P has a lower level of standalone risk or aggregate risk. So, there was some issue there was some problem and this is what motivated William Sharpe to get into the into these issues in depth to investigate these issues and he came up with the capital asset pricing model.

So, if the market was to price the sigma risk that standalone risk the standard deviation risk, then the portfolio P should necessarily provide a lower expected return than A and B because, it has a lower risk in terms of lower Sigma. So, that was the basic issue, market cannot be in equilibrium in this situation which is manifest by using the concept of standalone risk as a measure of risk in the market. So, that is what motivated the development of the CAPM model.

Now, where does the problem lie in so far, as the math is concerned, the problem lies in the fact that while expected return on a portfolio is the weighted average of returns of its

constituents, as we all know, that the expected return scales according to the weights of the constituent securities in the portfolio. r_p or er_p or \bar{r}_p is equal to summation of $x_i r_i$ so, that means what, that means there is linear scaling.

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THE PROBLEM'S SOURCE

- While expected return on a portfolio are the weighted average of returns of its constituents,
- Standard deviation does not scale in this linear manner.
- The standard deviation of a portfolio also depends on the correlations between the constituent securities forming the portfolio.

The expected return on our portfolio is equal to the weighted average of the expected return of its constituent. However, the standard deviation does not scale that way that is the problem, the variance does not scale that way the variance also depends on the correlations or the co-variances that exist between the various securities that form the portfolio. The variance does not simply scale as the weights of the constituent securities in the portfolio, so this difference in the behavior in the aggregate behavior of expected return and variance or standard deviation is the root cause of the math, that is behind this problem.

So, let us now go with the CAPM assumptions, let us quickly run through the various assumptions that are made by the capital asset pricing model, they are quite radical let me tell you at the outset nevertheless, the test of a model does not really lie on the assumptions that form the model but on how good that model fits reality and it is observed that the Kappa model fits reality to a reasonably good extent. And that is why it has become so very popular that it forms the cornerstone of most of the valuation exercises that we do in corporate manage or financial management.

So, that that being the case let us quickly run through the assumptions, and let us not get carried away by the unrealness of these assumptions because, as I reiterate the test of a model, is not in terms of the assumption it makes, but in terms of how good it, it is describing reality which the Kappa model seems to be reasonably good.

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ASSUMPTION 1

- **No transaction costs.**
- **There is no cost (friction) of buying or selling any asset.**
- **Given the size of transaction costs, they are probably of minor importance.**



So, the first assumption is, there are no transaction cost there is no cost or friction of buying or selling any asset given the size of transaction cost they are probably of minor importance. So, in any case transaction costs nowadays because of the, because of the electronic trading and so on, is a very small proportion of the volume or value of transactions. And therefore, they do not form a significant constituent in the analysis nevertheless we assume that, there are no transaction costs.

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ASSUMPTION 2

- **Assets are infinitely divisible.**
- **This means that investors could take any position in an investment, regardless of the size of their wealth.**



Assets are infinitely divisible that is the second assumption, this means that investors could take any position in an investment regardless of the size of their wealth. So, this is an the second assumption.

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ASSUMPTION 3

- **Absence of personal income tax.**
- **This means, for example, that the individual is indifferent to the form (dividends or capital gains) in which the return on the investment is received.**



Absence of personal income tax is the third assumption this means for example, that the individual is indifferent to the form, whether he gets the return on his investment in the form of dividends or he gets the return on his investment in the form of capital appreciation, in

either case he is not concerned with the source of the increment or the accretion in wealth and the income tax does not differentiate between these two sources of returns.

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ASSUMPTION 4

- **An individual cannot affect the price of a stock by his buying or selling action.**
- **This is analogous to the assumption of perfect competition.**
- **Although no single investor can affect prices by an individual action, investors in total determine prices by their actions.**

Assumption number 4, an individual cannot affect the price of a stock by buying or selling action, this is similar to the assumption that we make in the context of perfect competition in economics. So, an individual's own actions cannot influence the various parameters in the market like the price of an individual security, he cannot buy his individual actions influence the price. Nevertheless, although no single investor can affect prices by individual action, investors in total determine the prices by their actions.

So, it is the collectiveness, collective behavior or the collective wisdom of all the market players that determines the prices by equating supply to demand at the equilibrium the demand and supply of each and every security that is there in the market must be equal and, and the, the interaction of this demand and supply yields of particular or a unique price which is called the market price.

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ASSUMPTION 5

- **Investors are expected to make decisions solely in terms of expected values and standard deviations of the returns on their portfolios.**



Investors are expected to make decisions solely in terms of expected values and standard deviations of the returns on their portfolio. So, as I mentioned William Sharpe took the Markowitz model as given, and he proceeded there from or thereafter so, the Markowitz model assumes that the investors optimize their portfolios, optimize their holdings on the basis of a mean variance framework.

So, that is what this assumption iterates in the context of the capital asset pricing model investors are expected to make decisions solely in terms of expected values and standard deviations of returns on their portfolios.

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ASSUMPTION 6

- **Unlimited short sales are allowed.**

ASSUMPTION 7

- **Unlimited lending and borrowing at the riskless rate is allowed.**



Unlimited short sales are allowed, and there is no restriction on the use of the proceeds of short sales. Unlimited lending and borrowing at the riskless rate is also allowed, these are fundamental assumptions.

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ASSUMPTION 8

- **Investors are assumed to be concerned with the mean and variance of returns (or prices over a single period), and all investors are assumed to define the relevant period in exactly the same manner.**



Assumption number 8 investors are assumed to be concerned with mean and variance of returns, as I mentioned just now are prices over a single period, and all investors are assumed to define the relevant period in exactly the same manner. So, they are all investors define their relevant period in the same manner and they are concerned with the means and

variances of the various securities and co-variances of course, over that single period as defined by them uniformly.

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ASSUMPTION 9

- All investors are assumed to have identical expectations with respect to the necessary inputs to the portfolio decision.
- These inputs are expected returns, the variance of returns, and the correlation matrix representing the correlation structure between all pairs of stocks.



All investors are assumed to have identical expectations, with respect to the necessary inputs to the portfolio decision, this is quite fundamental this is a very important assumption that all the estimates that go into the Markowitz framework that go into the mean variance framework are estimated by, by the various constituents of the investor community as the same.

So, I repeat this assumption once again all investors are assumed to have identical expectations, with respect to the necessary inputs in the portfolio decision, what are the necessary inputs the expected values of the various securities that form the feasible set or the set of securities available to form portfolios, then the variances of these securities and then the co-variances between these securities.

All the estimates of these securities are assumed to be uniform across the entire investor community. These inputs are expected returns as I mentioned the variance of returns and the correlation matrix representing the correlation structure between all pairs of stocks.

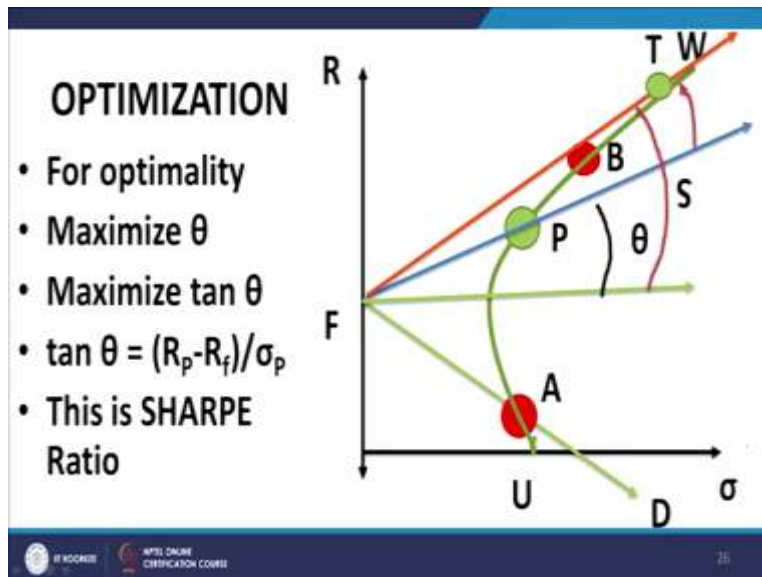
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ASSUMPTION 10

- All assets are marketable.
- All assets, including human capital, can be sold and bought on the market.

Then assumption number 10 all assets are marketable all assets including human capital can be sold and bought in the market. Now, given these assumptions now we come to the real CFM given these assumptions.

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But before we do that a quick recap on how the mean variance optimization was carried out, you can see here in this diagram which we have handled or which we have observed a number of times earlier for optimality what do we do, we maximize the angle theta we maximize the angle that the line F P P is the risk free point or the point representing the risk

free rate r_f is any arbitrary portfolio. And we maximize the angle that the straight line $r_f P$ makes with the x axis.

So, in other words we maximize the angle means we maximize $\tan \theta$ because we are in the first quadrant and what is $\tan \theta$ given by it is given by $r_P - r_f$ divided by σ_P you can see from this diagram and this is a very important quantity in portfolio analysis, it is given a name and that the name that it is given is Sharpe ratio. So, at the end of the day what we do is in order to arrive at the optimal portfolio, within the mean variance framework, each investor, each investor does this exercise what does it do, he maximize the Sharpe ratio.

So, each investor under the Markowitz framework is concerned with maximization of the Sharpe ratio in order to arrive at the optimal portfolio, for office choice this is the premise this is this is the input, that the Markowitz model gives us for the CAPM model. So, improving the Sharpe ratio the objective of introducing securities or shorting securities in a particular portfolio.

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IMPROVING THE SHARPE RATIO

- Thus, optimization is achieved by maximizing the Sharpe ratio.
- If the addition of a security S to a portfolio P increases the Sharpe ratio of the revised portfolio T , then it should be so added.
- Let us assume that a small fraction in value (x) of S is added to P by riskfree borrowing of equal amount to get portfolio T . Then,

Thus, optimization is achieved by maximizing the Sharpe ratio, now if the addition of a Security S , now we talk about the derivation of the CFPM if the addition of a security S to a portfolio P , we have a portfolio P , we are considering adding a security adding a small amount of security S to this portfolio P . For this purpose we need money what, what we do is

we do some riskless borrowing, we do a little bit of riskless borrowing and using that borrowed amount, we buy security S and we added to our portfolio which is portfolio P to arrive at the composite portfolio comprising of P and s which we call portfolio t.

So, if the addition of a security S to a portfolio P increases the Sharpe ratio of the revised portfolio t, then it should be so added that is the principle, that you are going to maximize the Sharpe ratio you are that is what Markowitz tells you maximize Sharpe ratio, you will arrive at the optimal or the efficient frontier. So, that is what we are doing we are trying to add security S to our portfolio P, by borrowing a certain amount and by that what we are trying to do is, we are trying to improve our Sharpe ratio.

Let us assume that a small fraction in value small x of s is added to P by riskless borrowing of equal amount to get portfolio t. So, we add to portfolio P a small amount of security S which we, which we purchase by borrowing the required amount from a bank at the risk free rate, and we get the portfolio t. Now, the expected return on portfolio t will be the weighted average returns of the portfolio P, and the excess return over the cost of borrowing for of security S.

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$$\begin{aligned} \bar{R}_T &= \frac{1}{1+x} \bar{R}_P + \frac{x}{1+x} (\bar{R}_S - R_f) \quad (1) \\ \sigma_T &= \left(\frac{1}{1+x} \right) (\sigma_P^2 + x^2 \sigma_S^2 + 2\rho x \sigma_P \sigma_S)^{1/2} \\ &= \left(\frac{1}{1+x} \right) (\sigma_P^2 + 2\rho x \sigma_P \sigma_S)^{1/2} \\ &= \sigma_P \left(\frac{1}{1+x} \right) \left(1 + 2\rho x \frac{\sigma_S}{\sigma_P} \right)^{1/2} = \sigma_P \left(\frac{1}{1+x} \right) \left(1 + x\rho \frac{\sigma_S}{\sigma_P} \right) \quad (2) \end{aligned}$$

That is what is given in equation number 1 here. So, r bar t your expected return on portfolio t is equal to 1, 1 upon 1 plus x into r bar P plus x into 1 plus x where x is the small amount that I have used for the investing in security S. So, plus x upon 1 plus x, r bar s minus r f, r f

is the cost that I have incurred for getting this additional amount of 1 is small x , so this is equation number 1.

What is σ_t , now σ_t is given by this expression which is the, which is the immediate use of the standard formula that we have for a two security portfolio, the variance of a two security portfolio with the weights $\frac{1}{1+x}$ and $\frac{x}{1+x}$. So, please note this risk-free borrowing will not contribute to variance in any way because, there is free borrowing has no variance in itself, and has no covariance with any other risky asset.

So, the fact that we borrow at the risk free rate is not contributing, in any way to the, to the variance of the security t of the portfolio t that comprises of P and s . And when we simplify this expression we simplify it a bit first of all because x is small we ignore the second order term here, this second order term in x that is $x^2 \sigma_P \sigma_S$ is ignored because x is taken to be very small, infinite decimal.

And then we use the binomial expression and curtail to first order and we arrive at equation number 2, for the variance of or the sorry the standard deviation of portfolio t , so the standard deviation of portfolio t is represented by equation number 2 here. Now, what we do is we work out the Sharpe ratio of portfolio t , the new portfolio that we have formed by a combination of portfolio P and security S .

So, the outcome the Sharpe ratio defined by the excess return to per unit of the standard deviation, using the expressions for \bar{r}_t that we derived in the earlier slide, equation number 1 and σ_t that we derived in equation number 2 and doing some simplification, what we get is this equation that is let us call it equation number 2A.

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$$\Theta_T = \frac{\bar{R}_T - R_f}{\sigma_T} = \frac{\bar{R}_P + x\bar{R}_S - (R_f + xR_f)}{\sigma_P \left(1 + x\rho \frac{\sigma_S}{\sigma_P} \right)} \quad \text{--- (A)}$$

$$\Theta_T > \Theta_P \Rightarrow \bar{R}_S - R_f > \rho \frac{\sigma_S}{\sigma_P} (\bar{R}_P - R_f) = \beta_S (\bar{R}_P - R_f)$$

$$\text{At equilibrium } \bar{R}_S - R_f = \beta_S (\bar{R}_P - R_f) \quad (3)$$

So, now if we are to require that the Sharpe ratio of the new portfolio t, that we have found by adding an infinite decimal amount of security S to security P or to portfolio P, what we require is theta 3 where Theta is the sharp ratio so, what we require is theta t should be greater than theta P. Now, if we use this expression and if you simplify using Theta t is equal to the expression given by equation number 2A, and theta P of course is equal to rP or erP minus rf divided by Sigma P and if you simplify, what we get is the expression that I am underlining.

So, so long as and what is our decision criterion, our decision criterion is that so long as theta t is greater than theta P we will continue adding Security S to our portfolio P. So, in other words so long as r bar s, that is expected value of return of s minus rf is greater than Rho sigma s upon sigma P into r bar P minus rf, we will continue adding security S to the portfolio P, to arrive at portfolio t.

This is the decision criterion so, so long as this process continues, we will do that. So, what will happen in equilibrium now, now please note please note one more thing if, if suppose on analysis what we find is, that r bar s minus rf is less than, rho sigma s upon sigma P, into r bar P minus rf. Then what we could do is instead of long instead of longing in security S what we could do or we could achieve an increase in our Sharpe ratio by shorting S.

So, what happens even I, if this $r_{s} - r_f$ is greater than $\rho \sigma_s$ upon σ_P , $r_P - r_f$, I will long more and more of security S borrow at risk free rate and invest more and more in security S and if this the left hand side is less than, the right hand side, what I will do is, I will do the reverse process, I will short security S and invest in the risk-free asset.

So, the bottom line is, irrespective of whether this is greater than or less than is market dynamism will take place and we will not achieve equilibrium. For the purpose of equilibrium, in other words at equilibrium the left hand side must be equal to the right hand side it is only then that the equilibrium would be achieved, and no more longing or shorting of security S will take place, because you are at the at the equivalence level, should continue from here after the break, thank you.