Security Analysis & Portfolio Management

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Lecture 23

Term Structure of Interest Rates

Welcome, back. So, in the last lecture, I discussed the concept of immunization in a lot of detail. And then I went on to take up the concept of key rate duration and bucket exposures. Today I start with the final topic in bond analysis, and that is the term structure of interest rates. Now, this particular term I have already introduced a couple of times during the discussion on bond analysis, bond valuation rather.

It represents the functional relationship between the maturity of the instrument, maturity of the underlying instrument, or the interest rates and the interest rate themselves. In other words, if you go to a bank and you make a deposit of one year, you get a different interest rate from the interest rate that you would get if you make a deposit of say 10 years. So, this phenomenon, whereby the interest rates vary in relation to the maturity of the bond is called term structure of interest rates.

The allied concept of yield curves is the graphical depiction of this term structure of interest rates. In other words, the yield curve is simply a representation of the term structure. It presents the various types of interest rates, maybe spot interest rates, maybe par yields, maybe the forward yields but it presents the yield for different maturities.

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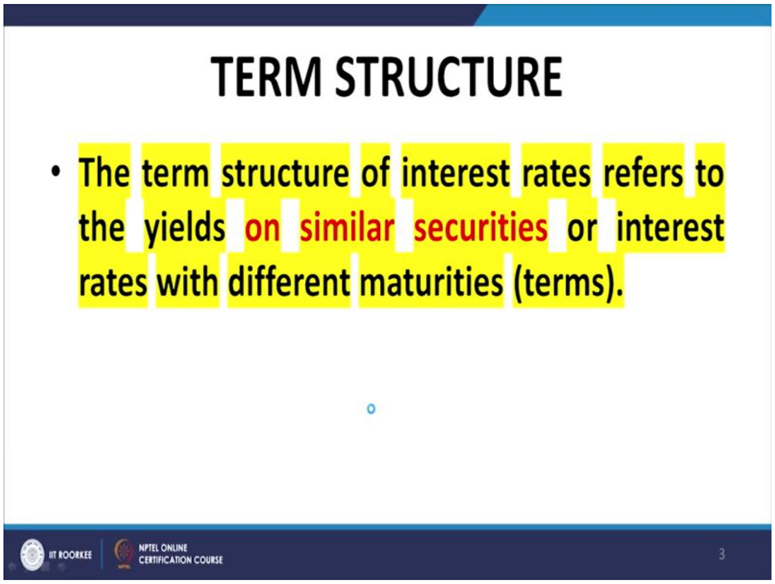

So, we will take the concept in detail now. The term structure I have already introduced, the term structure of interest rate refers to the yields on similar securities or interest rates with different maturities. So, that is called the term structure, the functional relationship between the maturity and the corresponding interest rate is called the term structure.

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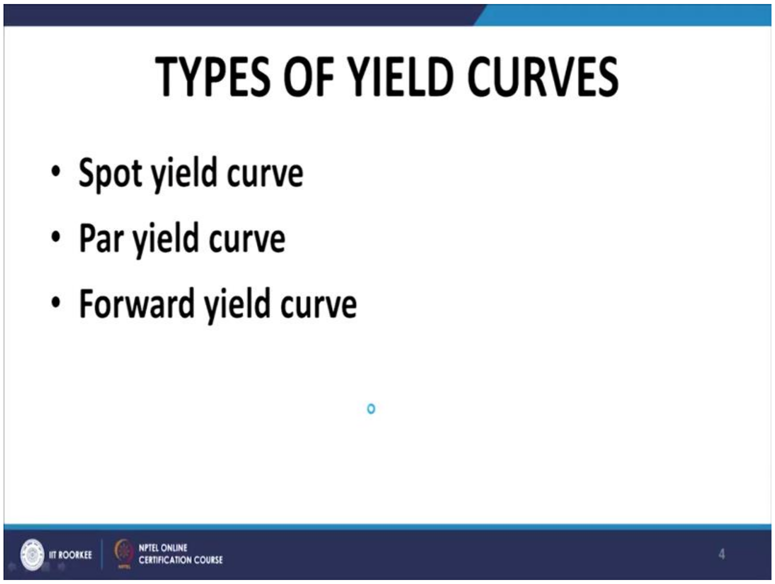

Now there are different types of yield curves, we have this spot yield curve, the par yield curve and the forward yield curve. Each of these yield curves represents or is a diagrammatic representation of the interest rates corresponding to different maturities.

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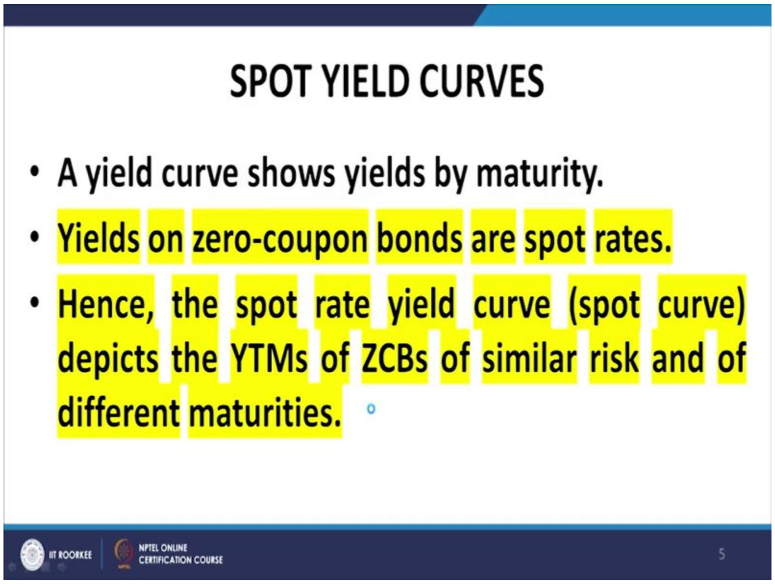

So, now we talk about this spot yield curve, we know the spot rates how do we define these spot rates? We define these spot rates as the YTMs on zero coupon bonds of the relevant maturity, I repeat, we define these spot rates as the yield to maturity on zero coupon bonds of the relevant maturity. Therefore, this spot rate yield curve or which is also called the spot curve, abbreviated as a spot curve depicts the YTM of zero coupon bonds of similar risk and of different maturities.

So, we take the maturities along the x axis and take the yields along the y axis. And we calculate the yields corresponding to different maturities on the basis of the current prevailing prices of those instruments, zero coupon bonds and then we plot them against the maturity to get the spot yield curve.

I repeat we, from the price information of zero coupon bonds of different maturities we work backwards, we work out the YTM and then using that YTM and the maturity to which that YTM relates, so the underlying instrument relates, we work out a curve, this is called the spot yield curve.

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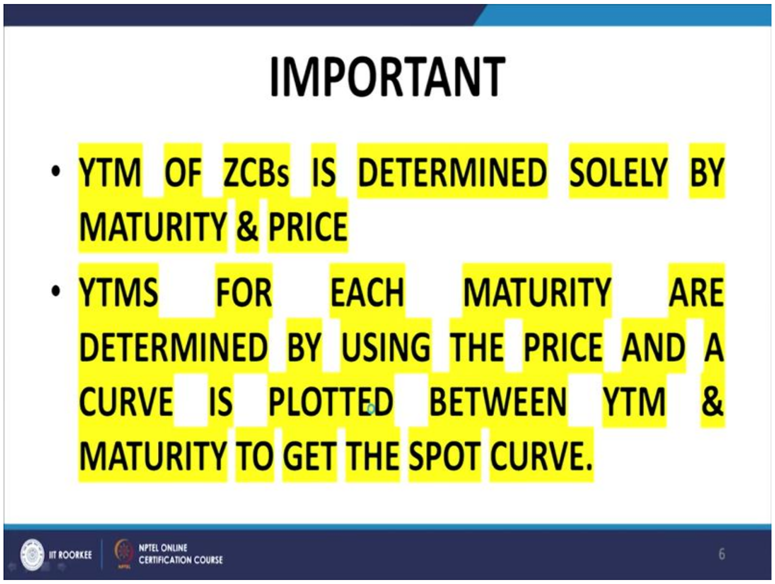

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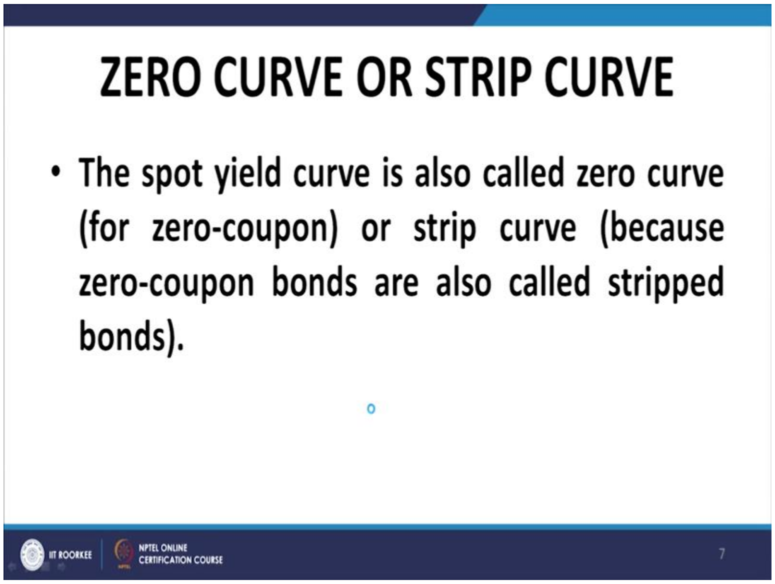

Now, the YTMs of zero coupon bonds is determined solely by maturity and price. Because there are no intermediate coupons, there is no coupon payment, by the name itself one can infer that zero coupon bonds involve no coupon payments as only a single payment at maturity. Either the bond may be quoted at discount to face value and redeemed at face value, or it may be a (quote) represented at face value and redeemed at a premium to face value.

In any case, it does not really matter. The YTM of that bond will be determined by the maturity of the bond and its current prevailing price. So YTM for each such security is determined by using the price as I mentioned just now, using the price we work backwards, we work out the YTM and then we use the YTM to plot a curve between the YTM and the maturity to which that YTM relates. This curve is also called the zero curve or the strip curve, because the zero coupon bonds are also called sometimes called the stripped bonds.

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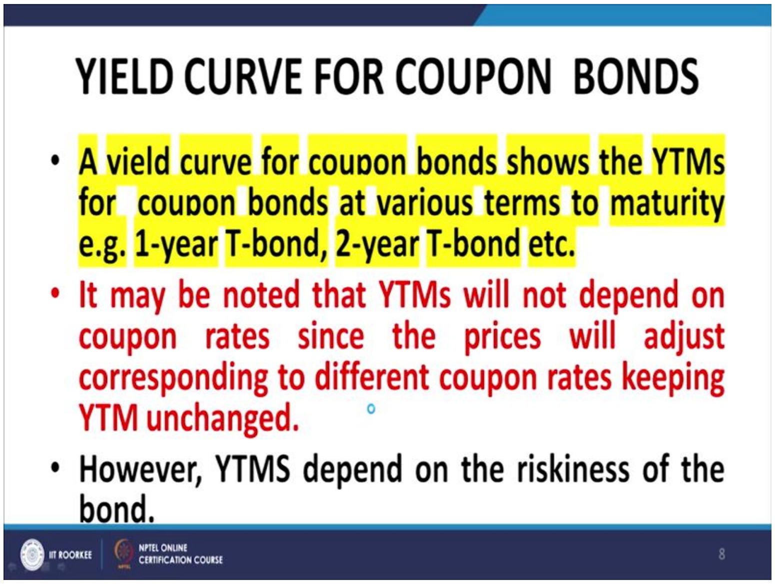

Similar to the spot curve for zero coupon bonds, we can have the yield curve for coupon bonds, in which case, a yield curve is plotted between the YTM of the coupon bonds and the corresponding maturities. You take the YTMs of various bonds, say maturity 1 year, 2 year, 3 year, 4 year these are coupon bonds.

And on these coupon bonds, we work out the YTM on the basis of the current market price, currently prevailing market price. And then we plot a curve between the YTM and the maturities. For example, we have the 1 year T bond, the 2 year T bond and so on. So, this is another example of a yield curve. But here we are using coupon bonds, instead of the zero coupon bonds that I talked about earlier, and which is usually called the spot yield curve or the spot curve.

Now, it is important to mention here that the YTMs will not depend on the coupon rate, since the prices will adjust corresponding to different coupon rates keeping the YTM constant. Actually, the YTM is determined by the riskiness of the bond, it is the market's perception of the riskiness of the bond, which manifests itself in the YTM, a more risky bond, where the cash flows from the bond are risky or the realizability of the cash flows of the bond is suspect.

We would quote at a higher YTM whereas, bonds such as treasury bonds or government bonds, would be quoting at a lower YTM because they are very close to being riskless. Therefore, they would, the risk premium they are on would be minimal. So, that is the main thing, the coupon rate themselves will not determine the YTM.

The YTM is determined by the riskiness of the bond and if the differential in coupon rate for example, if you have a 10 percent coupon bond and a 20 percent coupon bond of the same risk level, then the prices will adjust themselves in such a way that the YTM of both the bonds turns out to be the same.

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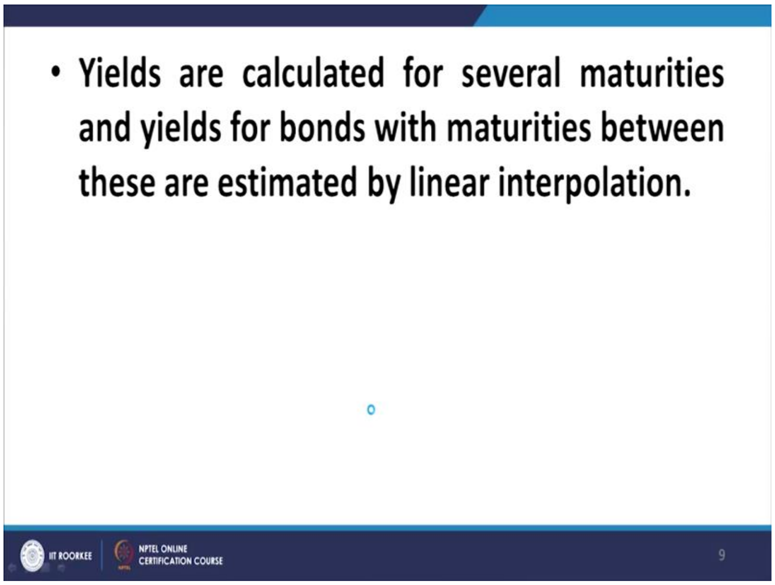

So, yields are calculated for several maturities and yields for bonds with maturities between these bonds are estimated by linear interpolation. So, this is a procedural issue, we have discrete points for different maturities and if you want to work out the YTM for an intermediate maturity say 3.5 years or 3.6 years, you do it by linear interpolation.

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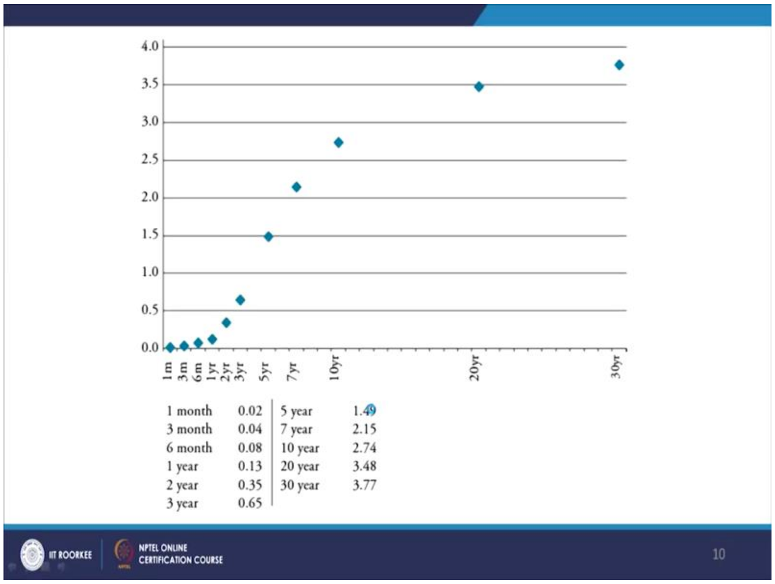

This is the example of a yield curve. You can see here it is flat at the near end and or and again it flattens out in the far end, in between it is rapidly sloping or highly sloping, otherwise it is flat at the both the extremes, the near end and the far end. In other words, what we infer is that for short maturities, the fluctuation in interest rate with maturities are small.

And similarly, for very long maturities, the same is the case, the fluctuation in interests, the changes in interest rates are very small compared to or with reference to the maturities. However, for intermediate maturities, we find that the interest rate change rapidly with the increase in maturity and the interest rates increase with the increase in maturities.

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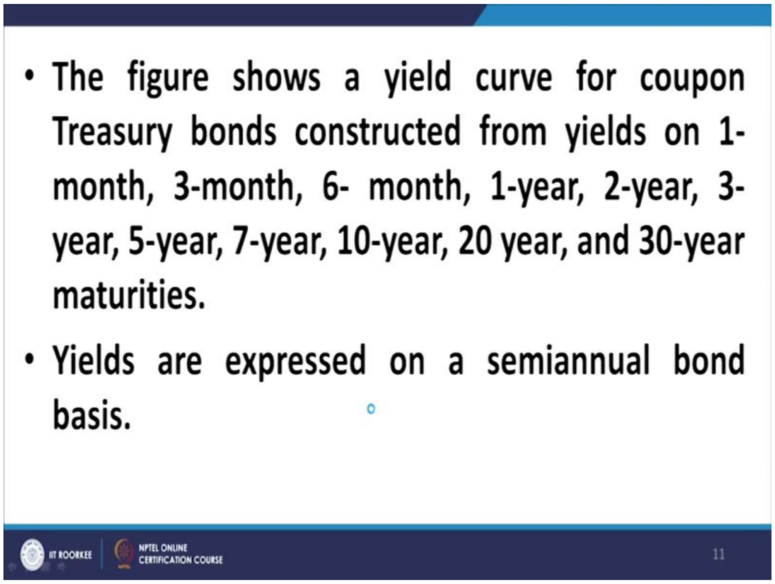

So, this figure represents, the previous figure represented a yield curve for a coupon treasury bonds constructed from yields on 1 month, 3 months, 6 months, 1 year, 2 year, 3 year, 5 year, 7 year, 10 year, 20 year and 30 year securities. These yields are expressed on a semiannual bond basis. So, this is what is this particular figure represented, it is a yield curve of treasury bonds, coupon treasury bonds.

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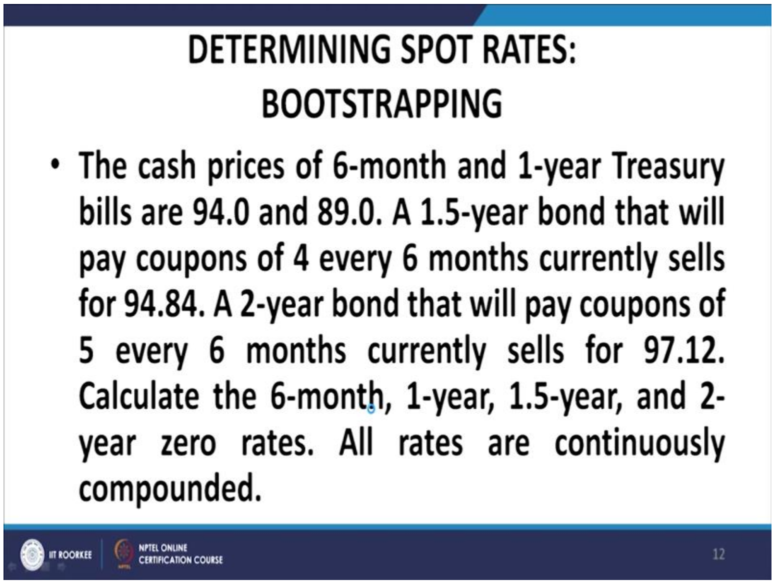

Now determining spot rates. This is an interesting topic, see I mentioned just now that what we do is we use the price, we take the price of the zero coupon bond the currently traded price of the zero coupon bond and on that basis working backwards, what we do is we compute the YTM of the bond which is then plotted against the maturity of the bond. So, that gives us the various points along this spot yield curve.

Now, it may so happen that zero coupon bonds of certain maturities like say, 3 years or 5 years or 10 years may not be existent or may not be trading in the market. In that situation we have to extract the spot rates, we have to extract the spot rates out of the, out of the quotations or the prices for the relevant coupon bonds. This process is known as bootstrapping. And I will explain it with an example here.

Let us do this example. This will illustrate the process of bootstrapping, whereby we determine the spot rates of various maturities on the basis of the prices. If the zero coupon bonds of the relevant maturities are non-trading, then we work out the corresponding spot rates on the basis of the practice of coupon bonds using this process of bootstrapping. So let us look at this example to illustrate this phenomenon of bootstrapping, with the process of bootstrapping.

The cash prices on 6 month and 1 year treasury bills are 94.0 and 89.0, the face value is 100. A 1.5 year bond that will pay coupons of 4 units every 6 months, currently sells at 94.84, a 2 year bond, that will pay coupons of 5 units every 6 months, currently sells at 97.12, we have to calculate the 6 months, 1 year, 1.5 year and 2 year zero rates, all rates are continuously compounded. So, this is the problem. Let us read the problem once again.

The cash prices of 6 months and 1 year treasury bills at 94 and 89, face value is 100, 1.5 year bond again face value 100. That will pay coupons of 4 units every 6 months, currently sells at 94.84. And a 2 year bond again face value 100 that will pay coupons of 5 every 6 months currently sells for 97.12. We have to calculate the various spot rates.

Now, as far as the 6 month and the 1 year rate is concerned, it is a straightforward exercise, we can work out from the T bill quotes because these are zero coupon bonds. So, we do not have to manipulate anything, we can simply use these prices working backwards, we can calculate the YTM on these bonds. And on the basis of this, we work out the spot rates for 6 months and the spot rate for 1 year. This is done here on this slide.

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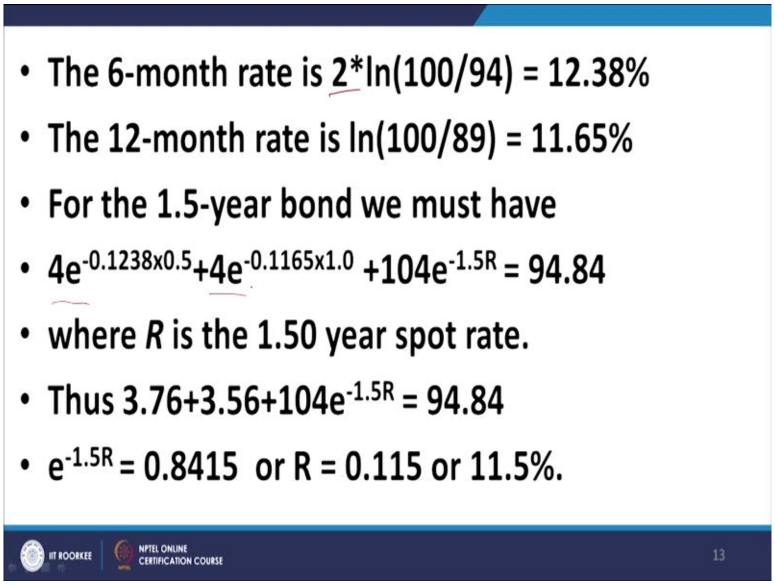

The 6 month rate is because there is two factor appears are to account for the annualization. This two factor here appears for annualization because we work out the 6 month rate and we multiply it by 2 to get the annual rate. So, log of P1 upon P0, that gives me R for the, for the 6 month period I multiply it by 2 to get the R or the YTM for the 1 year period on a bond equivalent basis that comes to 12.38 percent for the 6 month bond which is quoted at 94.

Similarly, the 12 month or the 1 year spot rate is given by, now because this is a 1 year rate, so we do not have to multiply by 2 here, log of 100 upon 89 which comes to 11.65 percent. For the 1.5 year, now please note here, we do not have this spot rate for 1.5 years because we cannot have it directly because we do not have a zero coupon bond with a maturity of 1.5 years. We do not have a say treasury bill or a treasury bond, which is a zero coupon bond with a maturity of 1.5 years.

So, we have to use the information encoded in the coupon bond and extract the 1.5 year spot rate, let us see how we do it. For the 1.5 year bond we have the following equation, we will, what will we receive from this 1.5 year bond, assume the face value to be 100, we will receive 4 units of money at the end of 6 months, we will receive another four units of money at the end of 12 months and then we will receive 104 units of money at the end of 1.5 years.

So, we have to discount these three cash flows and we are given that the current market price of the bond is 94.84. And therefore, we equate it to 94.84. We assume that the 1.5 year spot rate is

R and, on that basis, we discount all these three cash flows, we know the 6 months spot rate, we know the 12 months spot rate but we do not know the 1.5 years spot rate we assume it to be R.

And then we discount all the cash flows 6 months, 12 months and 1.5 year cash flows and equate the aggregate thereof to 94.84 and calculate the value of R, that is precisely what is done here in this slide. The first four figure is discounted for 6 months, the second four figure is discounted for 1 year. Please note the rates are already available with us, the 6 month rate is 12.38 percent, the 1 year rate is 11.65 percent.

So, we have the 6 month rate of 12.38 percent, that first cash flow of 4 is discounted at 12.38 percent. The second cash flow is discounted at 11.65 percent and the third cash flow of 104 which occurs at t equal to 1.5 years is discounted at R and when we equate this to the current market price and solve the expression for R what we get is R is equal to 11.5 percent.

So, this process is called bootstrapping, where we extract the spot rates from the information given for the various coupon bonds, which are traded in the market. You see what may happen is important thing is what may happen is that, we may not have zero coupon bonds trading for all the maturities that are required for construction of the yield curve.

And therefore, we have to whatever zero coupon bonds are available, as in this example, we have the 6 month bond and the 12 month T bills we use directly and work out the spot rate. But for the instruments of maturity is more than that or for which we do not have zero coupon bonds, we make use of this process of bootstrapping, we extract them from the, from the coupon payments or the timing of the coupon payments using the spot rates which are available and, on that basis, we work out the unknown spot rate by equating it to the current market price.

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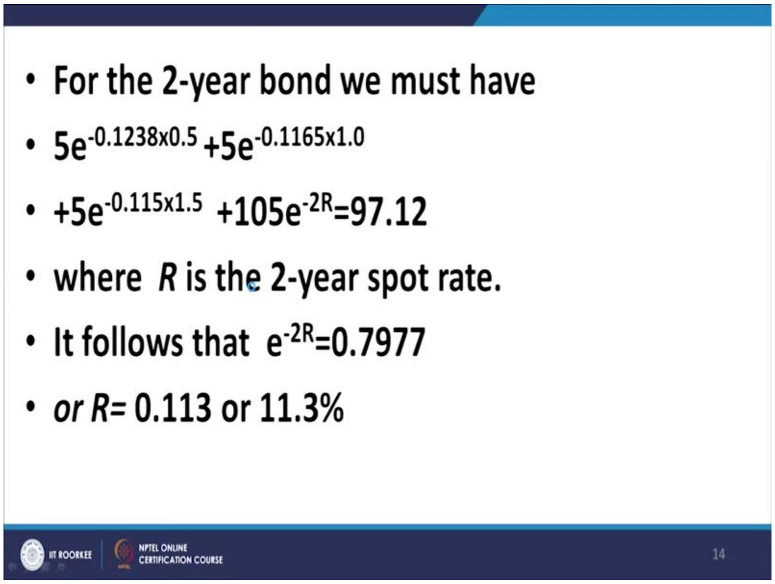

The same exercise is done for the 2 year bond. Now, we have the information about the 6 month rate, we have the information about the 1 year rate and we have the information about the 1.5 year rate. So, again so, equating the present value of future cash flows, assuming the 2 year rate to be R to the current market price of 97.12 we work out the value of R and we find that it is 11.3 percent.

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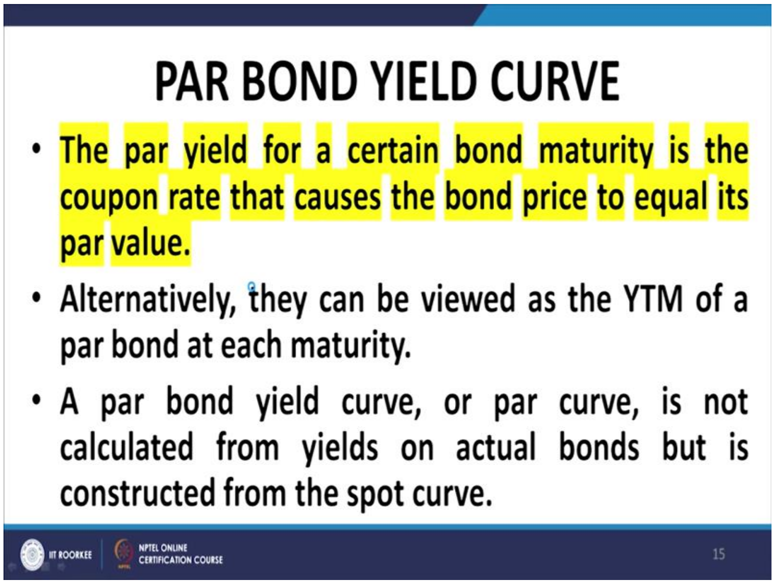

Now, we come to another concept which is the par bond yield curve, what is par yield? Now, we need to understand what is par yield. Now, par yield is the coupon rate on a bond let us say we are given a bond and given maturities say 5 years, then the par yield is that coupon rate at which the bond, the DCF value or the intrinsic value of the bond or the theoretical market price of the bond would be equal to its par value.

Let me explain again. If we have a bond, which has a maturity of five years, then what will, how will we work out the DCF value of the bond? We shall discount the coupons that are been paid at let us say it is an annual coupon bond, we shall discount the coupon payments at t equal to 1, 2, 3, 4 and 5 plus the principal at 5 at the relevant spot rates.

And now, we assume that coupon to be unknown, we use the given spot rates and we equate this to the face value of the bond that is the par value of the bond. And on that basis, we get an equation for the coupon rate and the expression that we get for the coupon rate, the number that we get for the coupon rate is called the par yield of the bond.

So, because for a par bond, the coupon rate must equal the YTM. So, we can alternatively state this par yield as the YTM of a bond, of a par bond of a bond that is quoting at par for each maturity. Both these definitions are equivalent because for a par bond, for a par bond, which is a level coupon bond redemption at face value, then we must have C is equal to Y that is the coupon rate must be equal to the YTM.

So, whether you define the par yield in terms of the YTM or you define the par yield in terms of the coupon, it comes to the same thing. So let me repeat once again, the par yield on a bond is that coupon rate such that the quote of the bond or the market price of the bond is worked out on the basis of the discounting of future cash flows at the relevant spot rates works out towards par value. So, a par bond yield curve or par curve is not calculated from each on actual bonds but is constructed from the spot curve which I showed the formula just now.

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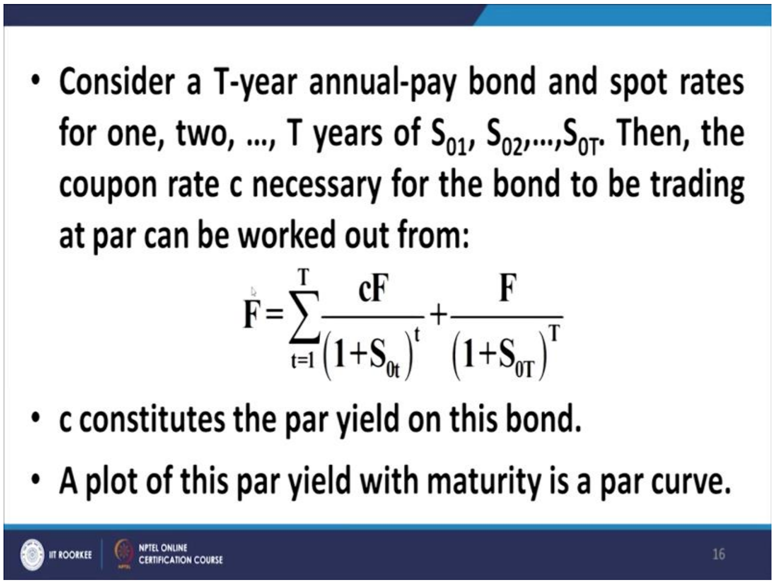

This is the formula here, we have a bond with a maturity of capital T, it is an annual pay bond. And these spot rates for 1, 2 and T years maturity are respectively S 01 S 02 and S 0T then coupon rate c which solves this equation, which makes the present value of future cash flows equal to the par value is called the par yield.

As you can see here, in this expression, the only unknown is c, the coupon rate which equates the right-hand side that is the present value of all future cash flows discounted at the relevant spot rates equal to its par value. Now, I repeat because for bond quoting at par the coupon rate is equal to the YTM. We can also say that the, it is the YTM of a par bond of the corresponding maturity.

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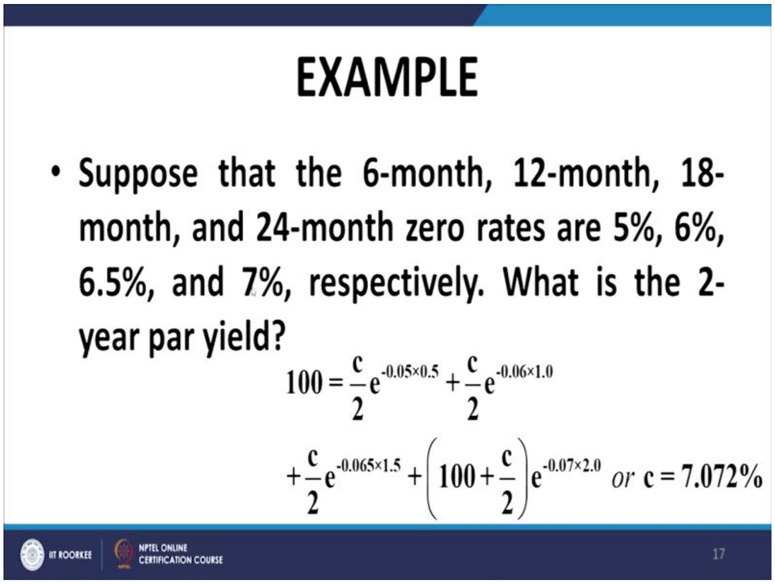

Let us do an example to illustrate this concept, suppose the 6 month, 12 month, 18 month and 24 month zero rates that is the spot rates are 5 percent, 6 percent, 6.5 percent and 7 percent respectively, what is the two year par yield? Let us assume now, please note in this case the coupon payments are semiannual.

So, if the coupon rate is c, the coupon payments are c upon 2. So, we shall receive c upon 2 at the end of 6 months, c upon 2 at the end of 1 year, c upon 2 at the end of one and a half years and then we shall receive the face value that is 100 plus c upon 2 at the end of 2 years. And accordingly, all these figures are discounted at the corresponding spot rates, we are assuming continuous compounding here.

So, we discount all these cash flows of c upon 2 at one point, at 0.5, 1, 1.5 and 100 plus c upon 2 at t equal to 2 we are discounting all these at the relevant spot rates, 5 percent for the 6 month discounting, 6 percent for the 1 year discounting, 6.5 percent for the 1.5 year discounting and 7 percent for the 2 year discounting and we equate this to the par value of the bond which is 100, we are assuming it.

See the par yield is independent of the face value of the bonds, so we assume it to be 100 and, on that basis, we find the value of c to be 7.072 percent, on solving this equation we find that c is equal to 7.072 percent. So, this is what is called the par yield on this particular bond.

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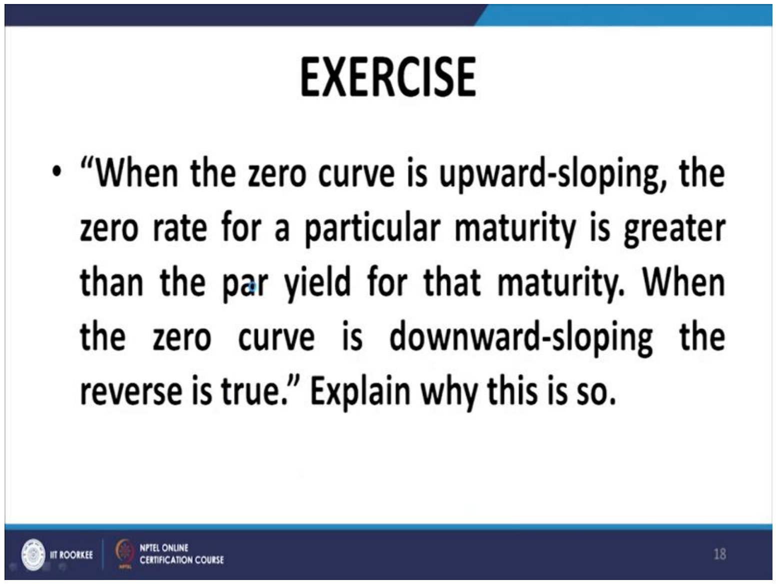

Now this is an interesting exercise when the zero curve is upward sloping, upward sloping means, as you increase the maturity of the bond, the interest rate increases, as you increase the maturity of the bond the interest rates increase. The zero rate for a particular maturity is greater than the par yield for that maturity.

Let us assume that we have a bond of a maturity of 2 years, then what it says is that the spot rate for 2 years will be greater than the par yield of for 2 years. In other words, it would be greater than the YTM of a bond with a maturity of 2 years and which is quoting at par. So, we have a bond which is quoting at par and we have to show that the YTM of that bond is less than the spot rate for 2 years assuming that the curve is upward sloping, the yield curve is upward sloping.

Now, the inference is quite straightforward. I have shown in an earlier lecture, that the YTM of a bond must lie between, must be sandwiched between the minimum spot rate and the maximum spot rate, here because the yield curve is upward sloping, the maximum spot rate will correspond to the higher maturity compared to the minimum spot rate which will correspond to a lower maturity.

So, the YTM must necessarily be between these two figures, the maximum spot rate and the minimum spot rate. And I repeat, because of the upward sloping nature of the curve, the maximum spot rate will correspond to the higher maturity and therefore, the YTM will be less than the spot rate for the higher maturity, the inverse would be in the case of a downward sloping curve.

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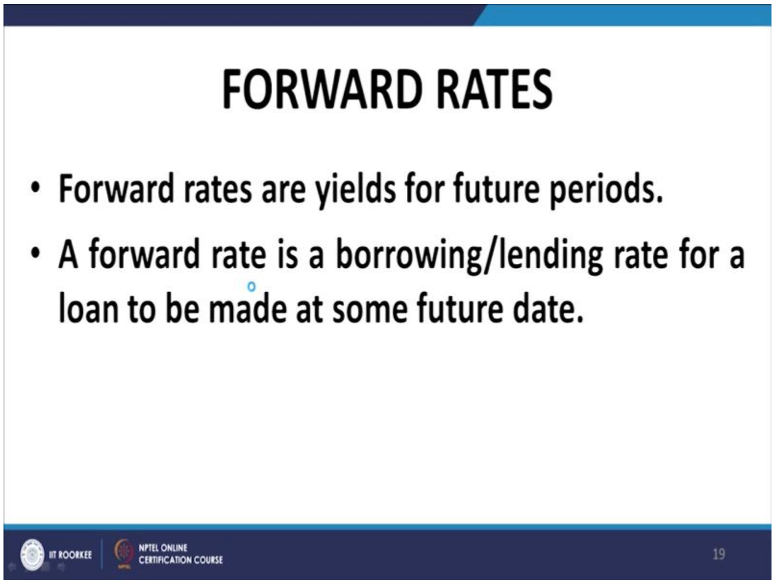

Now we talk about forward rates, now forward rates are yields for future periods. Let me explain it briefly. Suppose today's t equal to 0, please recall the definition of a forward contract, what was the forward contract? A forward contract was a contract which was agreed upon at t equal to 0 which was negotiated at t equal to 0 as to all the relevant terms of delivery, that is the price, the date of delivery, the manner of delivery and whatever the case may be.

But the actual delivery, payment of price and all the issues relating to the settlement of the contract take place at a future date, which is also predefined at t equal to 0. We know the date of settlement at t equal to 0 and we also know the price and all other relevant things that are to occur at t equal to the maturity of the contract, which is the date on which the contract is finally settled.

So, we have two dates, t equal to 0 where the negotiation of the contract takes place. And we have a date at which the goods are delivered or the underlying is delivered and the price is paid. And but please note this time period t equal to capital T which is called the majority of the forward contract is also predetermined. So, all the things are predetermined, all the things that are agreed upon at t equal to 0 so that there is no ambiguity as to the settlement of the contract. So, that is what is the forward contract.

The example that I have given you is of a commodity or a stock on which a forward contract is written, so that on the date of maturity, you buy, you get the or a currency, you get the currency, U.S. dollars, or a stock or a commodity, and you pay the price at that particular point in time, in the future.

Now, what happens if instead of this underlying being a commodity or a stock or exchange a currency, what happens if instead of these things, we have a loan as the underlying or the interest rate as an underlying or in other words, what I am trying to say is, if as a t equal to 0, I agree with a banker, that at t equal to 1 year I shall borrow say 1 million U.S. dollars or 1 million rupees for 1 year from t equal to 1 to t equal to 2.

But I want to protect myself against any change or any increase in interest rate during the intervening period between t equal to 0 and t equal to 1 year when the loan will start, I go to my banker and my banker agrees with me after negotiation that he would charge a certain rate which is agreed upon at t equal to 0 for a loan that is to be disbursed at t equal to 1 year and that would let us say, let us say it would have a maturity of another 1 year or 2 years whatever the case may be, again that will be negotiated at equal to 0.

So, all the terms relating to the loan including the interest, the maturity, the repayment whatever the case may be, are agreed at t equal to 0 and this is called a forward interest rate. The interest rate that is agreed upon as a t equal to 0 for a loan that is to be disbursed at a future date is known as a forward rate. So, forward rates are yields for future periods, a forward rate is a borrowing or lending rate for a loan that is to be made at some future date, as I explained just now.

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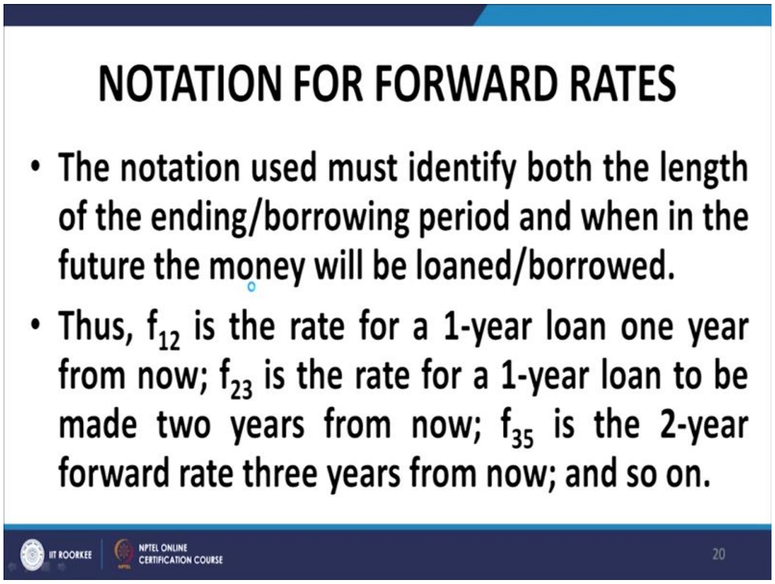

Now the notation, one can go through let us say I use the small f12 that means, this rate corresponds to a loan that is going to take place at t equal to 1 and that is going to be of one period and it would be returned or repaid at t equal to 2.

Similarly, f15 would be a loan which is initiated at t equal to 1, the loan would be dispersed at t equal to 1 but the rate is determined at t equal to 0, please note this, in all cases, the rate is determined at t equal to 0 or the loan is initiated, the loan is disbursed at t equal to 1 and the loan would be repaid at t equal to 5 and so on. So, this is the notation that we shall be using consistently in this course.

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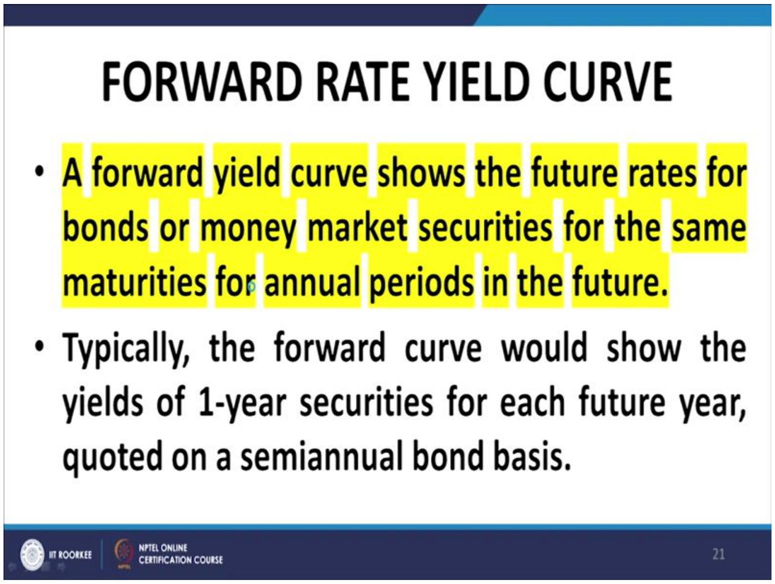

The forward rate yield curve, the forward rate yield curve shows the future rates on bonds or money market securities for the same maturities for annual periods in the future. So, suppose you take a maturity of 1 year, the yield, the forward rate yield curve will give you the rates for loan say from the t equal to 1 year to t equal to 2 years and then from t equal to 2 years to t equal to 3 years, t equal to 3 years to t equal to 4 is and so on, this would be plotted, the rates would be plotted corresponding to these years.

Now, here I have taken the majority of the loan to be 1 year, you can have forward yield curve for maturities of 2 years, but they would be quoted at 1 year intervals. So, you can have forward yield curve for a loan of maturity 2 years, the loan initiates at t equal to 1 and carries on up to t equal to 3 years, then you can have the rate if the loan initiates at t equal to 2 and carries on up to t equal to 4 years, you can have a rate for the loan initiating at t equal to 3 carrying on up to t equal to 5 years and so on.

So, you can have different forward rate yield curves, depending on different maturities of the underlying forward loan. And normally the quotations are on annual basis. So, even if it is a 5 year interest rate, it would be quoted upon a 1 year, 2 year, 3 years in the future. If the loan initiates at 1 year, if the loan initiates at 2 years if the loan initiates for 3 years and so on. Typically, the forward curve, typically the forward curve will show the yields on 1 year securities for each future year quoted on a semiannual bond basis.

So typically, forward curves would show the yield for one year securities, I have said it is usually 1 year it is not necessarily 1 year, maybe 2 years and maybe 5 years, but the quotations are on year to year basis. So, 1 year mature, 1 year initiation, 2 year initiation, 3 year initiation, whatever the maturity of the loan may be, so, relationship between spot and forward rates.

Now, let us, let me give you an example to illustrate this. Let us say, I initiate a deposit with my bank today at t equal to 0 for a maturity of 3 years. I deposit an amount of money with a maturity of 3 years and they give me a certain interest rate, let us call it S03. Let us say, as an alternative, I make a deposit of 1 year, right now, with the perception at least my mental perception that whatever I get at the end of the 1 year period, I will reinvest and reinvest it for another 2 years, so that I get a certain amount at the end of 3 years.

Now, is it necessary that the interest rates that I get at the end or the amounts that I get at the end of 3 years be the same or is it necessary that no arbitrage would operate in this case? The answer is, no. Why it is no, because I am leaving the position in the second alternative, when I am depositing the money for 1 year and keeping it open that whatever the interest rate prevails at t equal to 1 year, I shall deposit it for another 2 years as on that rate, then I am exposing myself to the fluctuations in interest rate that would occur at t equal to 1 year.

So, in that sense, the riskiness of the two rates of the arbitrary say one is when I have already agreed on a certain rate and that let us say that rate is riskless, the bank is the Government of India, let us say Government of India’s guaranteed bank, let us assume that so, the payment on account of the 3 year rate is riskless. Whereas, in the second case of whatever I get at the end of the first year, I reinvest.

So now that reinvestment, I have not fixed the rate at the point of reinvestment. And therefore, if the reinvestment rate decreases, my yield will decrease, if the reinvestment rate or the rolling over rate rather, the rolling over rate increases, I will get a higher yield. So, in this case, the principle of arbitrage does not hold, the principle of arbitrage does not hold, because I have not fixed the (re) the rolling over rate, that is the rate at which I will reinvest the amount that I get at t equal to 1 in the second alternative.

But what happens if that rate at which I am going to reinvest at the end of the first year is also fixed at t equal to 0, if that rate is also pre-determined with certainty at t equal to 0. And just as I explained in a previous slide, the concept of forward rate suppose we now I undertake a forward contract, I take a forward contract that at the end of the first year, I will reinvest at this particular rate that is agreed upon at t equal to 0, and that agreement is default fee, then the arbitrage principle operates.

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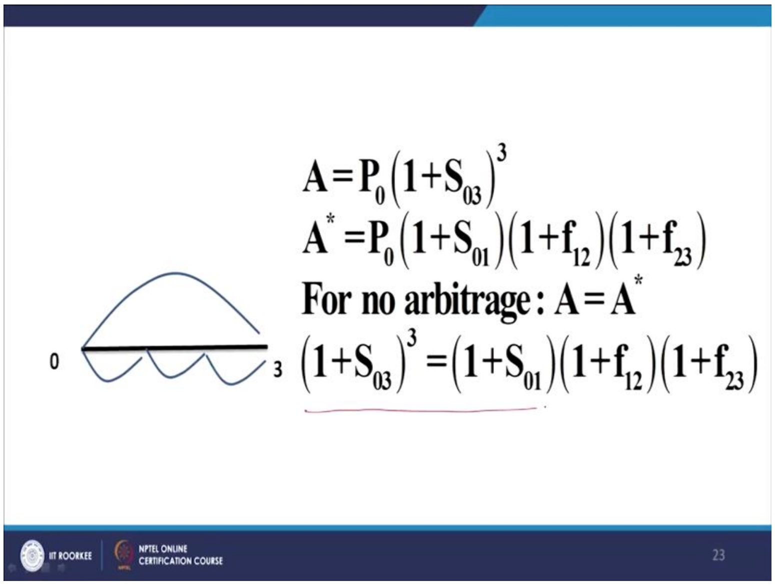

Because in both legs, in the first leg, where I have straightaway invested for three years, and in the second leg, where I have invested for one years, but I also know with certainty, the rate that I am going to get a t equal to 1 year for the future investment, then arbitrage would operate and the amount that I would get at the end of the 2 years, at the end of the 3 year period in both the cases in both the alternatives should be the same by the principle of no arbitrage.

This leads me to the expression which is given here at the right at the bottom of the slide, 1 plus S 03 to the power 3, which is on the basis of this spot rate at that I invest straight away for 3 years, one time investment for 3 years. And then the second, the right-hand side represents the rollover investment, I invest it for 1 year at a time from 0 to 1 year, from 1 to 2 years and from 2 to 3 years.

But the important thing is the rates at which are invested at t equal to 1 year and then t equal to 2 years are already agreed upon or already pre fixed and are default fee and not exposed to any fluctuation and as a result of it, the risk in both cases is zero. And therefore, the arbitrage process will mandate or the phenomenon or the requirement of no arbitrage in efficient markets will mandate that we have this equation, the amounts that I get the at end of the 3 year period must be the same in both cases.

And therefore, you must have 1 plus S 03 to the power 3 is equal to 1 plus S 01, which is the amount I get at the end of the first year investment multiplied by 1 plus f12, which is the amount that I get at the end of the second year, provided the rolling over is done at f12, which is the forward rate agreed upon at t equal to 0 for a loan starting at t equal to 1 and ending at t equal 2 into 1 plus f23, which is another forward rate for 1 year, which starts at t equal to 2 and ends at t equal to 3. So, this is the no arbitrage, this equality is guaranteed by the no arbitrage condition. I shall continue from here. Thank you.