Security Analysis & Portfolio Management
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Lecture 21
Immunization Example

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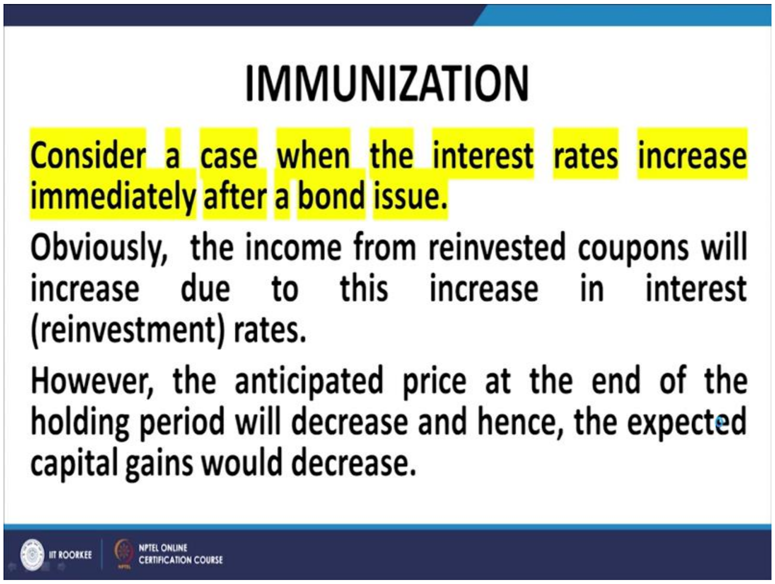

Welcome, back. So, let us quickly recap what we did in the last lecture. In the last lecture, I introduced the concept of immunization. Whenever a person and an investor, invests in a bond, he becomes sensitive, or his returns become sensitive to changes in the market interest rates.

For example, if the market interest rates increase, then the return that he would derive from the reinvestment of his coupons would increase on the one hand, but on the other hand, the capital gains that would accrue to him on the liquidation of the investment would decrease. So, these two forces operate in opposite directions.

For example, as I have just mentioned, if the reinvestment, if the interest rates increase, then the reinvestment income increases, but the capital gains decrease and converses the process if the interest rates decrease. Now the increase in the reinvestment income or the decrease in the capital gains, or the inverse they are of depends on the period of holding of the investor.

If the investor holds upon for a long period, pretty close to its maturity, then what will happen is that the reinvestment income will increase significantly, but the decline in capital gains would be small, because there would be fewer discounting periods remaining to the maturity of the bond. The converse would be the case if the bond is held for a very short period of time. The reinvestment income may increase only marginally, but the decline in the value of the bond, decline in the market price of the bond would be substantial due to an increase in interest rates.

The converse one, inverse phenomenon operates in the case of a decrease in interest rates. So, the bottom line is that these two effects, number one, these two effects tend to annul each other, tend to cancel each other. And number two, the extent of annulment depends on the holding period of the bond.

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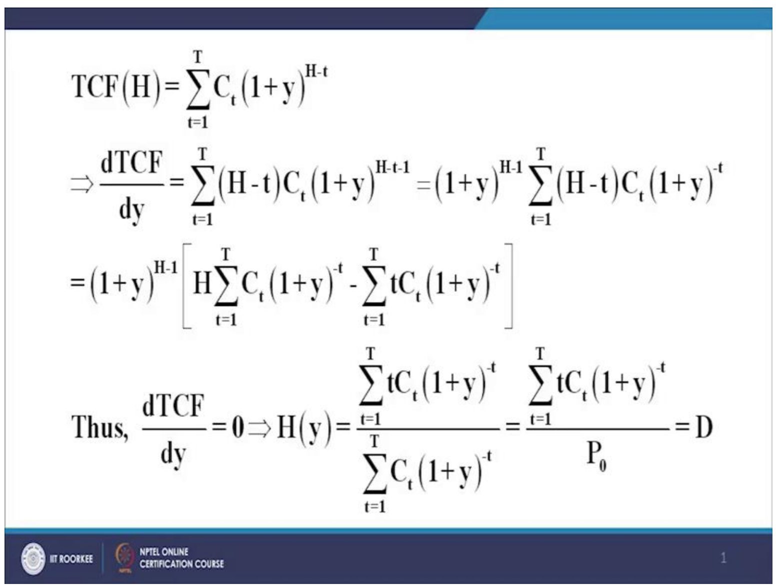

Then we mathematically arrived at a conclusion that if you hold the bond, if the investor holds the bond, for a period, which is equal to quantity defined as the duration of the bond or termed as the duration of the bond, then the change in the reinvestment income exactly nullifies the change in the capital gains. And as a result of it, the overall return to the investor remains approximately independent of these small changes in interest rates. So, that was the basic principle of immunization.

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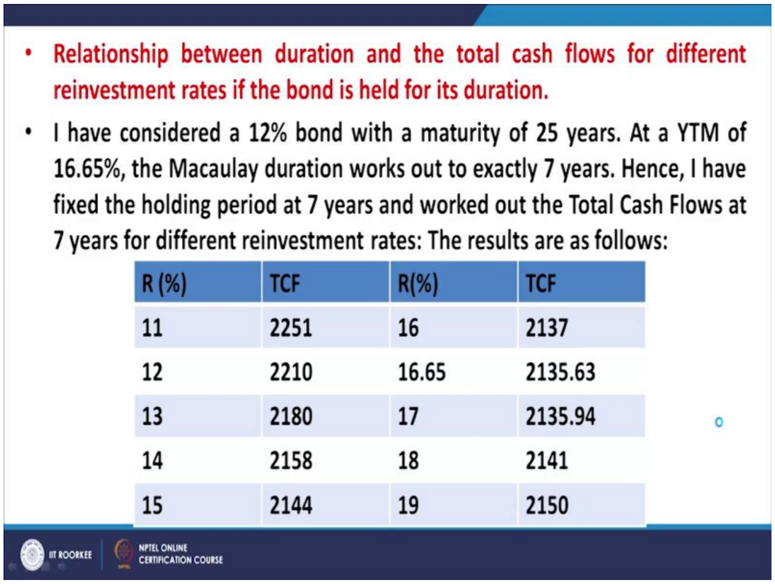

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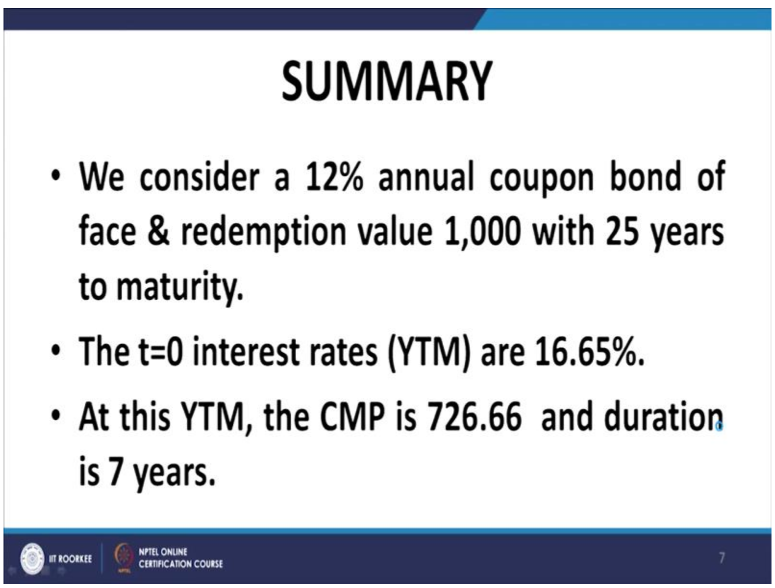

Then I moved over to an example and this was the example that I was talking about, let us recap this example. Once again, or at least a summary there of, it is very instructive. So, let us recap this. So, we are considering a 12 percent annual coupon bond of a redemption value of 1000 which is also its face value and the bond's maturity is 25 years.

The t equal to 0 interest rates YTM are 16.65 percent. At this YTM, the current market price of the bond works out to 726.66 and the duration of the bond works out to almost exactly 7 years.

So, this was the substantive features of the example that we were considering towards the end of the last lecture. Let us continue from here.

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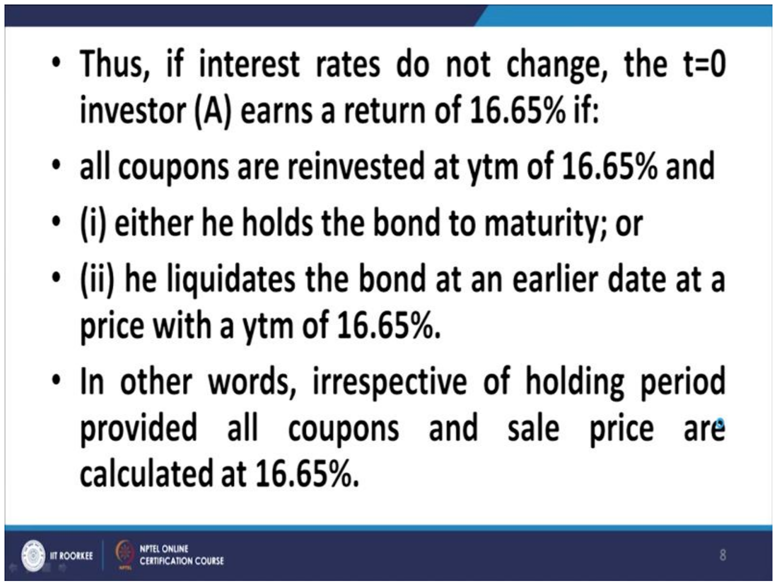

And therefore, if interest rates do not change, and a party say A, gets invested in this bond at t equal to 0 when the market interest rates are 16.65 percent at a price of 726.66. Then and if all the coupons are reinvested at the YTM of 16.65 percent. And either the investor holds the bond that is A holds the bond up to its maturity or if he liquidates the bond prior to maturity, he liquidates the bond at the same YTM of 16.65 percent, then he gets a return of 16.65 percent on his investment.

Let me repeat, if an investor invests in this bond at t equal to 0 when the market interest rates are 16.65 percent and the price is 726.66, as I mentioned just now, and if the coupons, intermediate coupons that the investor receives, that A receives are reinvested at the YTM of 16.65 percent and either A holds the bond up to its maturity or if A does not hold the bond up to its maturity, liquidates the bond at an earlier point in time, then the, he liquidates the bond at a YTM of 16.65 percent.

In either of the two cases, either of the two scenarios, the return earned by A would be 16.65 percent on his holding. So, irrespective of the holding period, provided that he liquidates the bond at a YTM of 16.65 percent and reinvest the coupon at 16.65 percent, the return he will earn is 16.65 percent.

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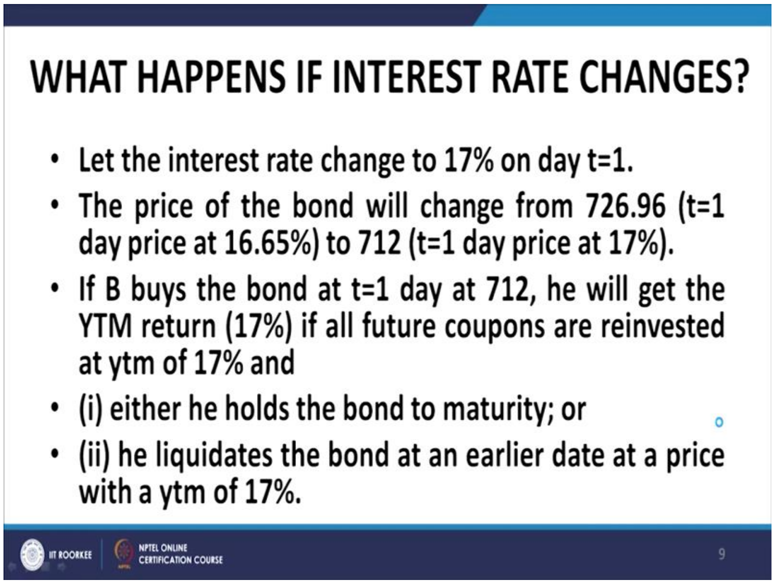

This is provided the interest rates do not change that is he is reinvesting at the rate at which he entered the investment and he is also liquidating the bond at the same rate. So, the rates, in other words, the interest rates do not change and then he gets a yield equal to that YTM at which he entered the investment. What happens if the interest rate changes, that is what is more important from our perspective. So, let us look at this very carefully.

Suppose the interest rate changes to 17 percent on day one, he took the investment, A took up the investment at t equal to 0 and immediately on the next day, the interest rate changed from 16.65 percent which was the rate at t equal to 0 to 17 percent, which is the rate at t equal to 1. Then the price of the bond will be disturbed, the equilibrium will be disturbed and the price of the bond will change from 726.96 which is the t equal to 1 day price at a YTM of 16.65 percent to the price of 712 which is a t equal to 1 day price at the rate of 17 percent.

In other words, because the interest rates have increased the market price of the bond will decrease to enable the YTM to increase from 16.65 percent to 17 percent. Therefore, if B buys the bond at t equal to 1 day, after the YTM is shifted, after the market interest rates have shifted from 16.65 percent to 17 percent then he will get a return.

B will now get a return of 17 percent, 17 percent please note, not 16.65 percent, B will get a return of 17 percent provided he reinvests his coupon at 17 percent and he liquidates the bond at a YTM of 17 percent. So, if both these things, conditions, both these conditions are satisfied or of course, he holds the bond to maturity and he reinvests the coupon at 17 percent, he will get a return of 17 percent.

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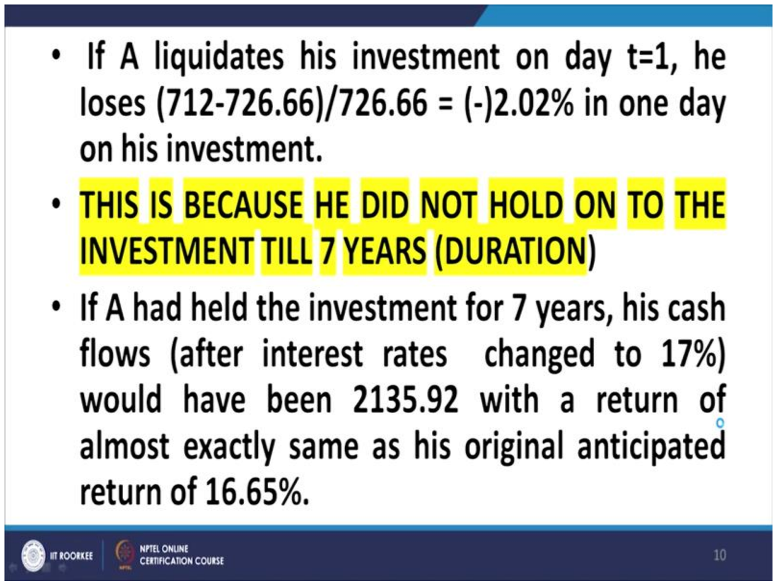

Now, if therefore, now look at the situation of A, suppose B has bought the bond from A. Now, let me repeat, A bought the bond from t equal to 0 when the market interest rates was 16.65 percent at 726.66 and B has bought the bond from A when the market interest rates have changed to 17 percent from 16.65 percent.

Then the return, 1 day return earned by A is a negative 2.02 percent, 712 which is a selling price minus 726.6 which was the price that at which he bought, divided by 726.6 that comes to minus 2.02 percent. This is the 1 day return please note and look at how negative it is or how substantially negative it is. But then what is the reason for this?

The reason for this is, the reason for this is that A has held the bond only for 1 day, had he held the bond for 7 years probably he would not have faced this kind of a catastrophe, the return that he would have earned would not have declined so much, it would have been pretty close to 16.65 percent. Let us investigate that further.

I repeat this substantive fall in return, the substantial negative return that is getting is arising because the holding period of A is only 1 day and therefore the influence or the effect of the increase in interest rates, which manifests themselves due to as a decline in price and substantive decline in price because there is a long, long way to go to the maturity of the bond, it is 24 years and 364 days to the maturity of the bond.

Therefore, the impact of this change in interest rates on the capital gain aspect is massive. The impact on reinvestment income is insignificant because the change is occurring on the very next day of his investments so, he would not have reinvested any coupons or he would not have benefited from the increase in interest rates.

And as a result of it, the disparity between the increase in reinvestment income which is negligible and the fall in the market price of the bond which is massive substantial due to the long remaining life of the bond of 24 years and 364 days. Therefore, they do not annul each other to any significant extent and the return that A earns is hugely negative return.

If A had held the bond for 7 years, then his cash flows after interest rates changed to 17 percent now this is another interesting point. If A had held the bond for 7 years his cash flows after interest rates has changed to 17 percent would have been 2135.92 which is slightly different from the figure where when the market interest rates were 16.65 percent which was about 2135.60.

So, a very small difference has arisen between the two total cash flows, if A had held the bond for 7 years and if A had held the bond for 7 years the return, he would have earned would be close to 16.65 percent as he had originally envisaged.

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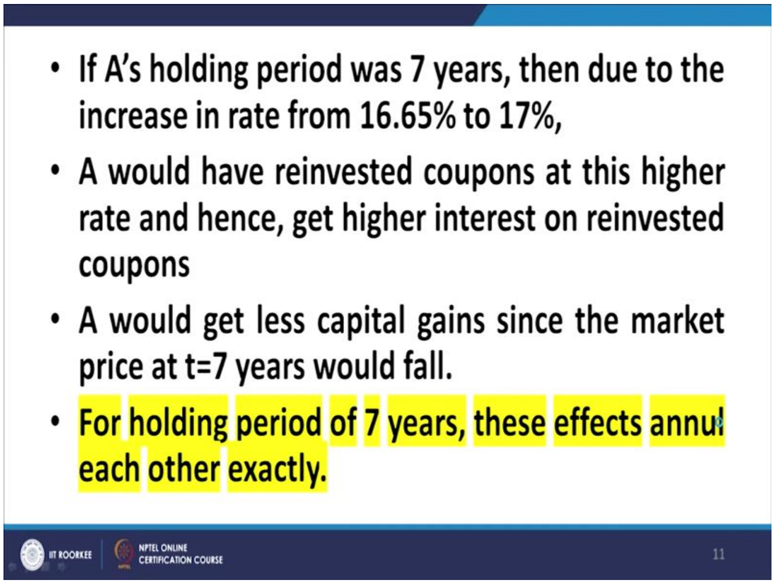

So, if A's holding period was 7 years and due to the increase in rate from 16.65 percent to 17 percent, he would have reinvested coupons at this higher rate of 17 percent and hence got higher reinvested income, A would have lost out a little bit on the capital gain side due to the increase in interest rates and the fall in the market price at t equal to 7 years, because the YTM now has, YTM now on the bond has to return a figure of 17 percent.

So, but the important thing is if A had held the bond for 7 years, these two effects would have almost exactly annulled each other and as a result of which A’s return would have stood at 16.65 percent which was the t equal to 0 return and envisaged by A.

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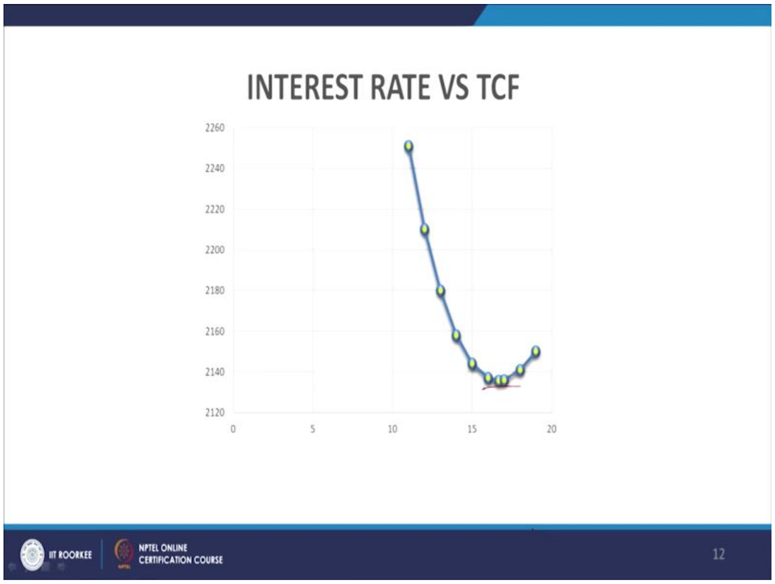

This is the figure now; this figure is very instructive. You look at the point, look at this figure here and right at the bottom of this U, and you find that it is quite flat, it has a flat bed this figure increases on both sides, pretty much like a U curve. And right at the bottom it is substantively flat. And that means that if there are changes in interest rates to either side of 16.65 percent around this flat portion of the curve, then the total cash flows do not change much.

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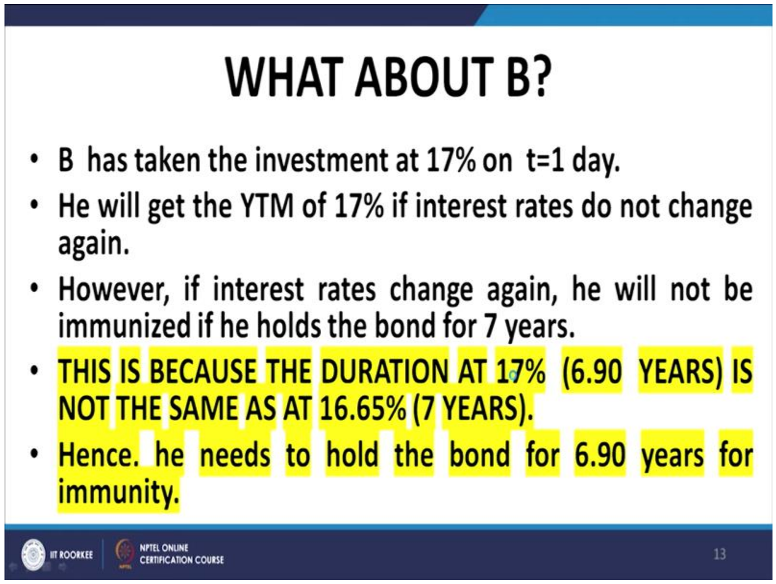

Now, what about B? B is the party who had bought the bond from A at the end of 1 day when the interest rates has changed from 16.65 percent to 17 percent. Now, because B had bought the bond at a YTM of 17 percent, he would get a return of 17 percent provided he (reinvestment) reinvest the coupon at 17 percent and he liquidates the bond at a YTM of 17 percent or he holds a bond up to the maturity of the bond.

In either case, if he reinvest the coupon at 17 percent and either he liquidates the bond at 17 percent YTM or he holds the bond to maturity, he will get a return of seven, return or 17 percent. However, if the interest rates change again, now this is another interesting point. If interest rates change again, he will not be immunized if he holds the bond for 7 years. Why is that? That is because the duration is y dependent, the duration depends on the rate, interest rate at which the duration is calculated.

And therefore, if you work out the revised duration at a rate of 17 percent the revised duration works out to 6.90 years. Therefore, in order that y gets immunized or y immunizes himself from small changes in interest rates, he needs to hold the bond for 6.90 years and not 7 years or maybe 6 years and 364 days. In either case, he would not be perfectly immunized, perfect immunization will occur only if he holds the bond for his duration which is 6.90 years. Therefore, he needs to hold the bond for 6.90 years to be fully immunized.

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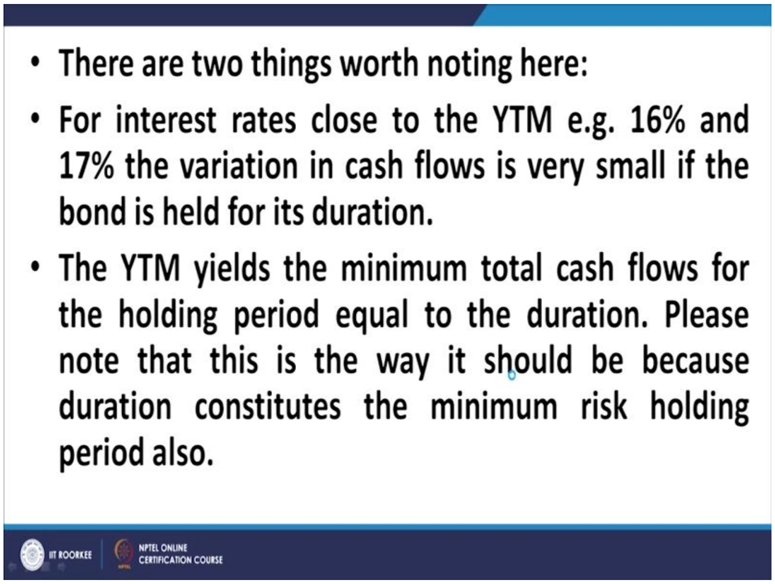

So, there are two things worth noting here. Two takeaways from the above example, for interest rates which are close to the YTM that is 16 percent and 17 percent. The variation is cash flows is very small, if the bond is held for its duration. And the second point I will come back to in a minute, but before I get back to the second point, certain caveats, certain things that we need to be careful about when we talk about the duration or when we use duration for the purpose of management of bond portfolios.

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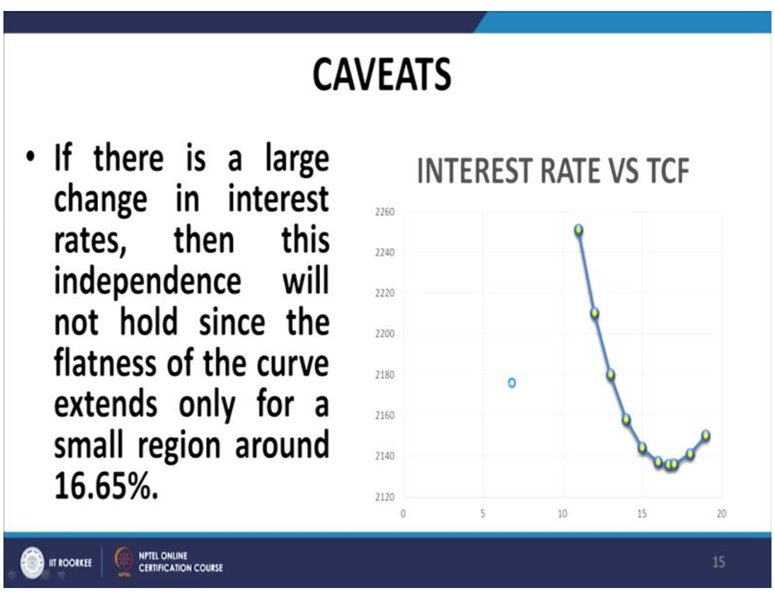

Number one, if there is a large change in interest rates, then the independence or the immunization will not hold. Since the flatness of the curve extends only for small regions around 16.65 percent. You can see this from the adjoining figure on the right-hand side of the screen, that if there is a large change in interest rates, there is protection, this flatness does not hold.

The flatness that is the bed of this particular figure is around 16.65 percent but extends marginally to the left-hand, right-hand side you may see a few basis points to the left and a few basis points to the right of 16.65 percent.

So, the immunization will hold only if the interest rates change in that region. If the interest rate change significantly say by 2 percent or 3 percent, on either side of 16.65 percent the immunization will not hold. And this is because duration is a linear measure. Basically, we have talked about that in a lot of detail.

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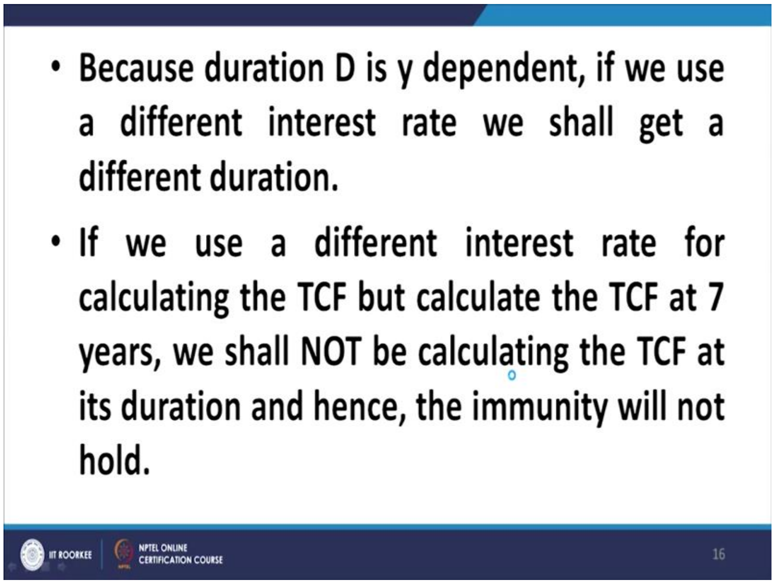

The second thing is because duration is D is y dependent, if we use a different interest rates, we get that different duration. If duration, as I mentioned a few minutes back, duration depends on the interest rate at which it is calculated and for a different interest rate, we get a different duration.

Therefore, if we use a different interest rates for calculating the total cash flows, this statement has to be very carefully understood, if we use a different interest rate for calculating the total cash flows, but calculate the total cash flows at t equal to 7 years, we shall not be calculating the total cash flows at the bonds duration and therefore, the immunity will not hold, now this is very important.

As I reiterate again and again duration depends on the interest rate at which you are calculating it. So, if you are using a different interest rates and calculating duration corresponding to 16.65 percent, you are not calculating the total cash flows at the correct duration, because the duration of 7 years is only when the interest rate is 16.65 percent. If the interest rates are different, let us say if the interest rate is 11 percent, their duration comes to 9.25 years.

So, if you calculate total cash flows for 7 years at the rate of 11 percent, you are not calculating the total cash flows at a point which represents the duration of the bond at the YTM of 11 percent. And therefore, the immunity will not hold, if the interest rates are 11 percent, then the duration is 9.25 years and therefore the total cash flows for immunization need to be worked out at 9.25 years.

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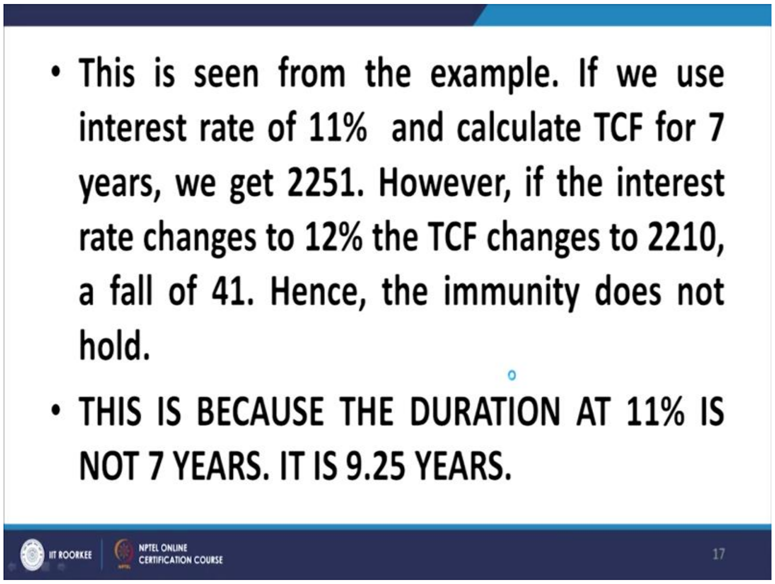

Here is the example, continuation of that example, if we use the interest rate of 11 percent and calculate total cash flow for 7 years, we get 2251. However, if the interest rate changes to 12 percent, the total cash flows change to 2210. So, there is a substantial change in total cash flows of about 41 units, when the interest rate change from 11 percent to 12 percent. And the holding period is 7 years. Why is that?

This is clearly because you are not working, you are not working out the total cash flows at a point in time, which represents the duration of the bond at the interest rate at which you are working out the total cash flows. The total cash flows are worked out at 11 percent, but the duration is not 7 years at 11 percent, it is 9.25 years.

So, you have to work out the total cash flows at 9.25 years and then you will find that the change in total cash flows is very marginal for change in interest rates from 11 percent to 12 percent. So, this is an important feature of duration.

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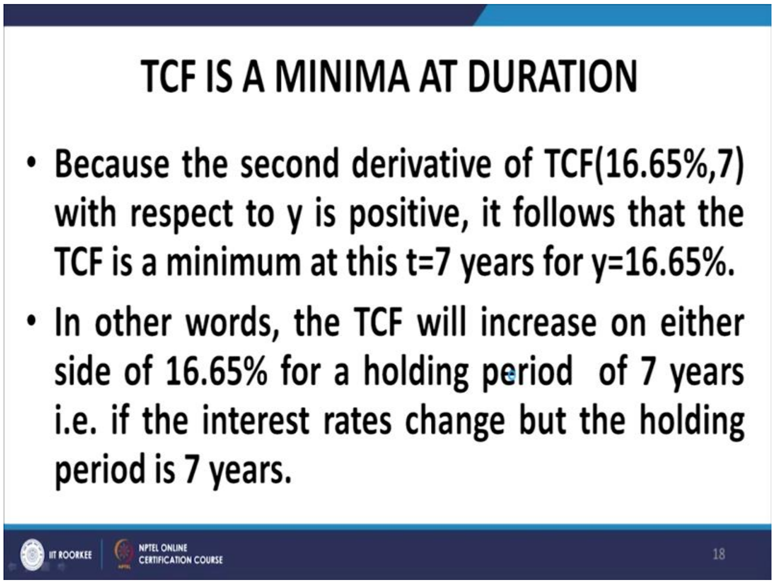

And then there is another feature of duration that when we worked out the duration, when we mathematically as far as the concept of duration, worked out the (concept) definition of duration mathematically, we found that the second derivative of total cash flows with respect to y is positive. And what does that mean, that means that the total cash flows at the point, t equal to duration represent a minima at the given interest rate.

In other words, if I work out the total cash flows of my bond at 16.65 percent for 7 years, this would represent a minima of the total cash flow, the local minima of the total cash flow. I repeat if, therefore the total cash flows will increase on either side of 16.65 percent for a holding period of 7 years.

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That you can see from this figure as well, this is a blow up of the earlier figure. And you can see that as the cash flows moved either to the left or to the right of 16.65 percent, the total cash flow figure tends to increase. This is as I mentioned, this implies that the total cash flows at the point of duration at the interest rate corresponding to that duration or the duration corresponding to that interest rate represents a minima of the total cash flows.

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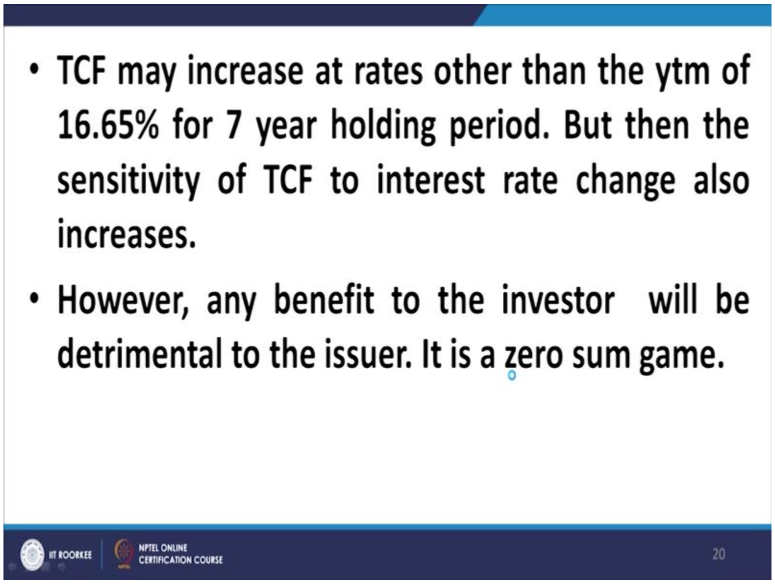

Therefore, total cash flows increase at rates other than the YTM of 16.65 percent for a 7 year holding period, but then the sensitivity of the interest rate change also increase as you saw, when the interest rate change from 11 percent to 12 percent, the change was, a change in total cash flows of the magnitude of 41 units. However, when the interest rate change from 16 percent to 17 percent, the changes in our total cash flows was just about 1 unit.

So, that shows that when you work out the total cash flows at the duration corresponding to the given interest rate corresponding to the current interest rate, you do get immunized. Now there is another point, any benefit to the investor corresponding to the increase in cash flows would obviously operate to the detriment of the issuer of the bond, it is a zero sum game.

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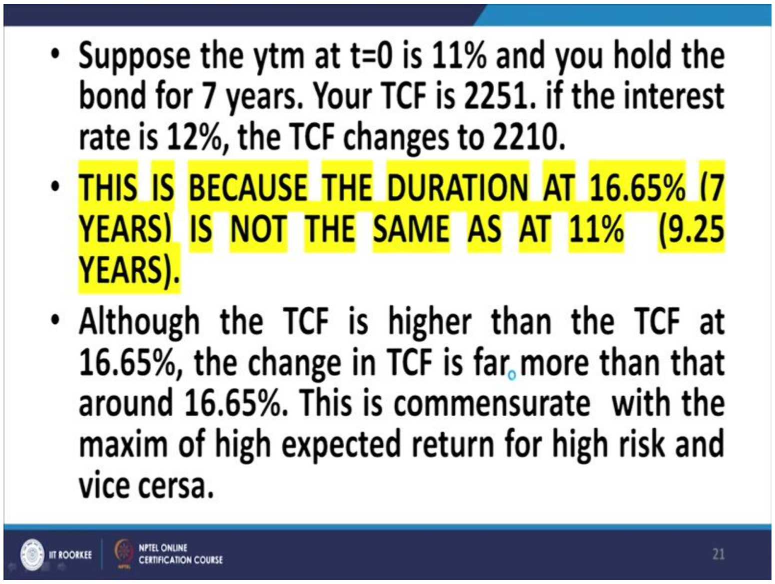

Suppose, the YTM at t equal to 0 is 11 percent and you hold the bond for 7 years, then your total cash flows as I mentioned just now is 2251, and if the interest rate is 12 percent, the total cash flow changes to 2210. This is because the duration at 16.75 for 7 years is not the same as the duration at 11 percent which is 9.25 years. So, when you are working out the total cash flow at 7 years at 11 percent, you are not working out the total cash flows at a duration of 11 percent which is 9.25 years.

So, although the total cash flows are higher, the variation in total cash flows corresponding to marginal changes in the interest rate around that figure of 11 percent is also substantial. Therefore, the sensitivity of the total cash flows is more and the risk element is more and this is compatible with the maximum that expected returns and risk move in tandem, if the risk is, if the expected return is less, the risk is less and vice versa.

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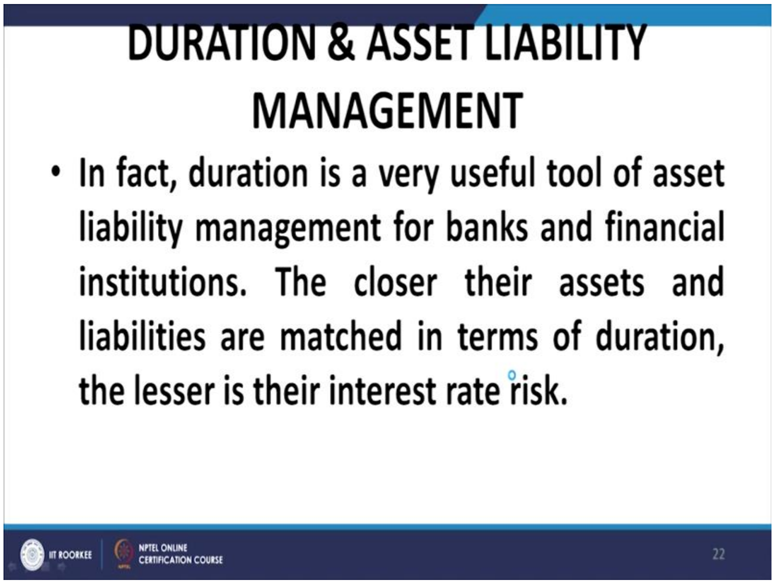

Then duration is a very valuable tool of asset liability management. And it is practiced by most of the institutions, the financial institutions and banks in respect of their asset and liability portfolios. In fact, it is more appropriate for these kind of institutions to manage the duration of their asset and liability portfolios to match the duration of the asset and liability portfolios rather than matching the maturity of those portfolios.

Because by matching the duration of the assets and liabilities, what happens is, if there is a certain unanticipated change in interest rate, market interest rates, then the assets and liabilities would increase by the same percentage and as a result of it the overall damage to a possible detriment, possible damage to the institution may be nullified to a large extent.

So, this duration concept is a very valuable tool for asset liability management of financial institutions. I repeat, if the assets and liabilities of an institution are matched, then what happens is, if there is a marginal increase in interest rates or decrease in interest rates, then the percentage change in the asset portfolio and the liability portfolio would be very close to each other. And therefore, the overall change in the asset liability structure would be minimal.

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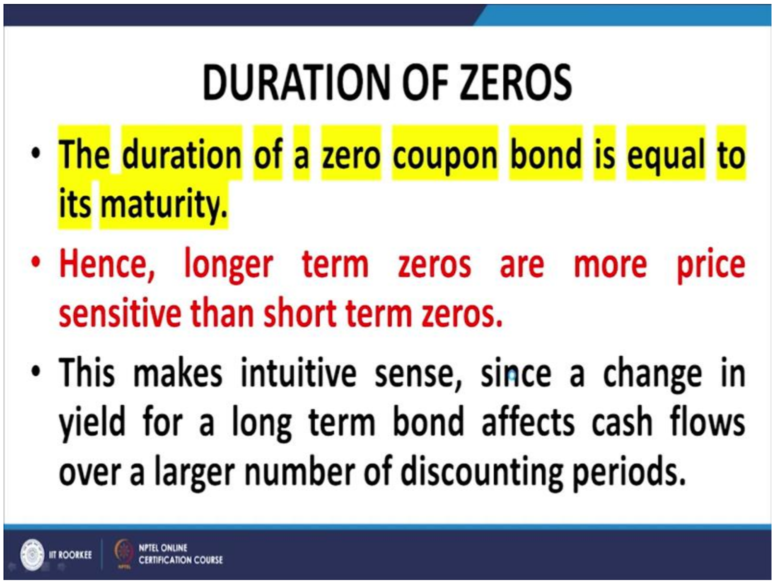

Duration of zeros, now zeros are instruments, zeros or zero coupon bonds or coupon bonds which do not provide any coupon payment, which are either issued at a discount and redeemed at face value or issued at face value and redeemed at a price above face value at a value above face value. In either case, the important thing is that they do not have any intermediate payments, they do not envisage any intermediate cash flows.

And because they do not envisage any intermediate cash flows, one can straightaway infer that the duration of such zeros is equal to the majority of the zeros. And therefore, longer term zeros are more sensitive to interest rate changes compared to short term zeros. And this makes intuitive sense as well, because a change in yield of a long-term bond affects the cash flows over a large number of discounting periods.

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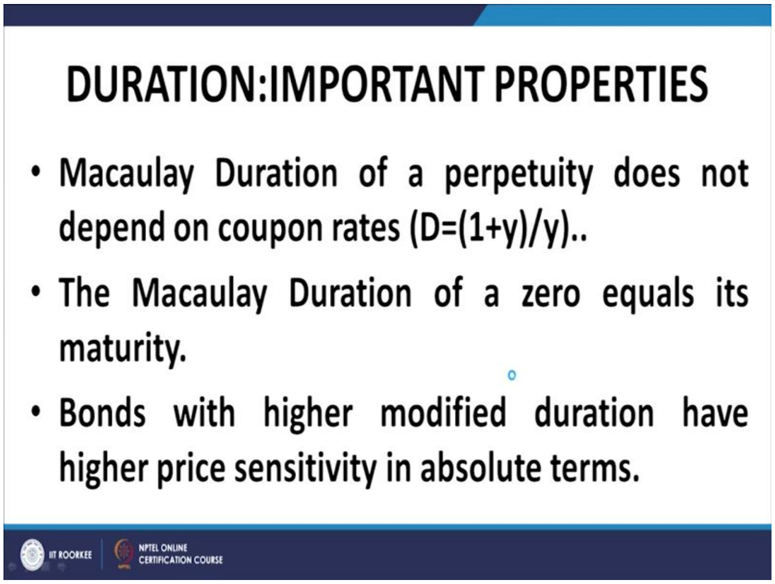

Now, some important properties of duration, the duration of a perpetuity is independent of the component rate. And this is a very interesting result. The derivation of this result is given in this presentation. And it shows that it can be mathematically established that the duration, Macaulay duration of perpetuity is given by 1 plus y divided by y where y is the discounted, y is the YTM and therefore, the duration is independent of the coupon rate. The second Macaulay duration of a zero equals this maturity I have just mentioned that point, bonds of higher modified duration have higher price sensitivity, we will come back to it.

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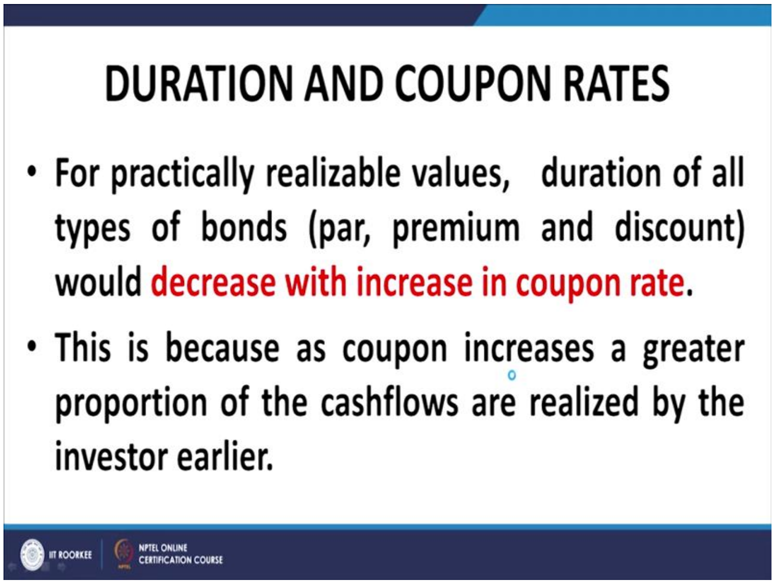

Duration and coupon rates, now, the duration of all types of bonds for practically realizable values, par, premium and discount bonds decrease with the increase in coupon rate. So, other things being kept constant if there is an increase in coupon rate, then the duration of the bond decreases irrespective of whether it is a par, premium or a discount bond.

This is because as the coupon rate increases a greater fraction, greater proportion of the cash flows are realized by the investor earlier. I repeat, because the coupon rate increases the, a greater fraction of the total cash flows is realized by the investor at an earlier date. And therefore, the duration decreases with coupon rate with increase in coupon rate, it is inversely related to the coupon rate, higher the coupon rate lesser is the duration.

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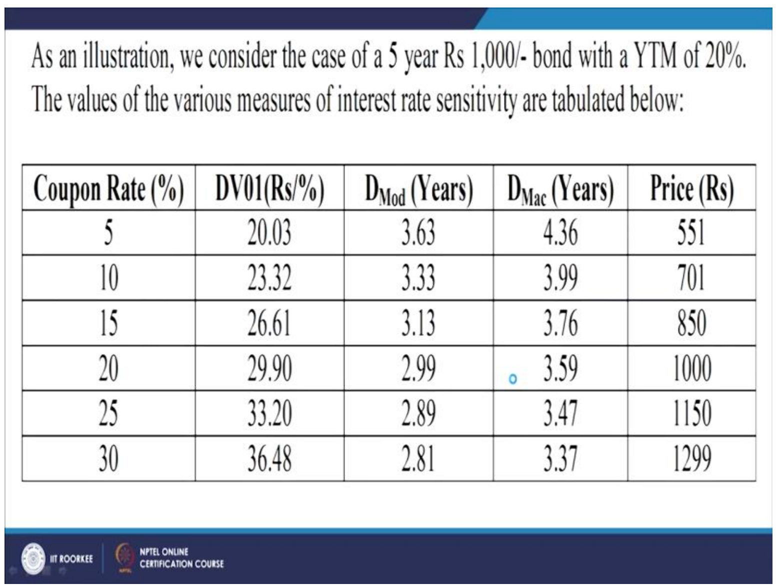
As an illustration, we consider the case of a 5 year Rs 1,000/- bond with a YTM of 20%. The values of the various measures of interest rate sensitivity are tabulated below:

Coupon Rate (%)	DV01(Rs/%)	D_{Mod} (Years)	D_{Mac} (Years)	Price (Rs)
5	20.03	3.63	4.36	551
10	23.32	3.33	3.99	701
15	26.61	3.13	3.76	850
20	29.90	2.99	3.59	1000
25	33.20	2.89	3.47	1150
30	36.48	2.81	3.37	1299

And this is illustrated by this example, where we consider a 5 year, 1000 face value bond with a YTM of (25) 20 percent, the YTM of the bond is 20 percent. It is a 5 year bond with a face value of 1000. Now, the coupon rate is given in the first column, 5 percent, 10 percent, 15 percent, 20 percent, 25 percent and 30 percent. And let us look at the Macaulay duration.

The Macaulay duration for a coupon of 5 percent is 4.36 years, for 10 percent coupon rate is 3.99 years and it gradually goes on decreasing as you can see from the second last column of this table. And we end up with a Macaulay duration of 3.37 years corresponding to a coupon rate of 30 percent. So, as the coupon rate has increased from 5 percent to 30 percent, the duration has decreased from 4.36 years to 3.37 years.

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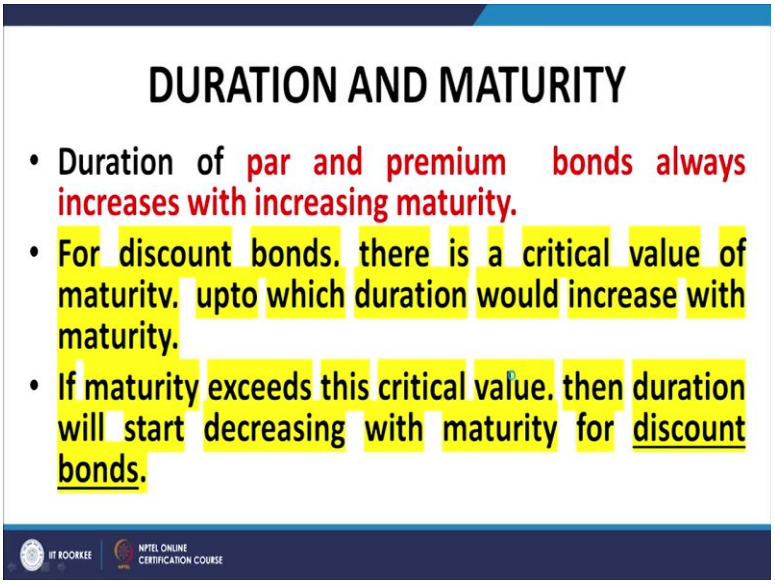

Duration and maturity, now here we encounter certain interesting thing, duration of par bonds and premium bonds increase with maturity. Longer the life of the bond, longer the maturity of the bond, larger is the duration, but that is not the case in the case of discount bonds. In the case of discount bonds, what happens is that up to a certain critical value of the maturity, the duration of the bond will increase and then it will start decreasing until it reaches the limiting value of the perpetuity.

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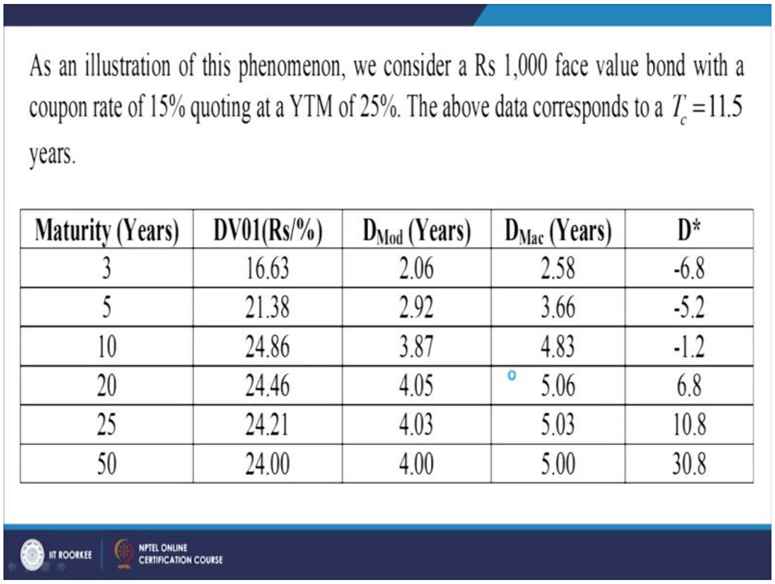

Maturity (Years)	DV01(Rs/%)	D_{Mod} (Years)	D_{Mac} (Years)	D*
3	16.63	2.06	2.58	-6.8
5	21.38	2.92	3.66	-5.2
10	24.86	3.87	4.83	-1.2
20	24.46	4.05	5.06	6.8
25	24.21	4.03	5.03	10.8
50	24.00	4.00	5.00	30.8

So, let us look at this by an example. Now, here we consider 1000 face value bond with a coupon rate of 15 percent, the coupon rate is 15 percent and the YTM is 25 percent. So, clearly since the coupon rate is greater than the sorry, I am sorry, the coupon rate is less than the YTM, coupon rate is 15 percent YTM is 25 percent.

And we find that it is a discount bond it is quoted at a discount and let us see how the duration changes with the maturity. If the maturity of the bond is 3 years, D Mac figure, the Macaulay duration is 2.5 years and duration, a maturity of 5 years gives a duration of 3.66 years. And then up to 20 years it gives a duration of 5.06 years and then the duration starts decreasing.

And at 25 years their duration is 5.03 years and at 50 years their duration comes to be approximately exactly 5 years. So, what we can find here, what we find here is that the duration initially increases with maturity, it reaches a certain critical value at which the duration is maximum, and then the duration starts declining until it reaches the duration of a perpetuity.

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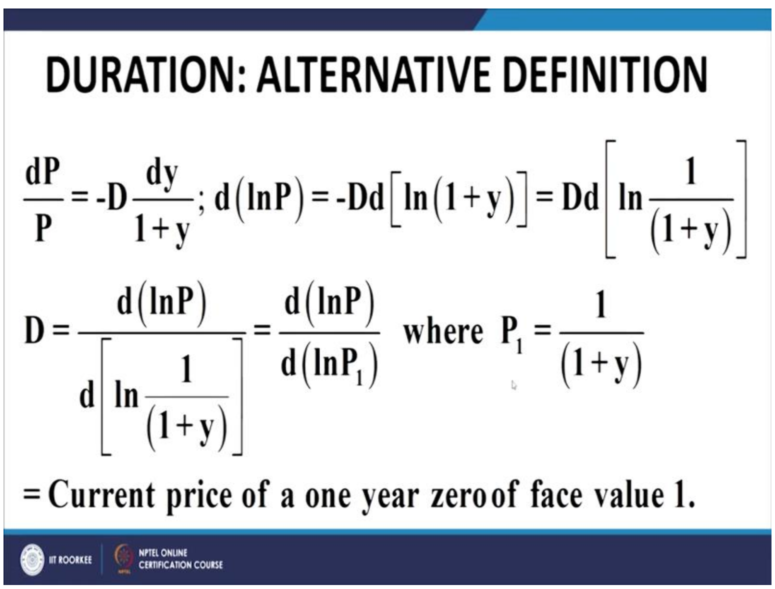

So, this is an alternative definition of duration. I will not spend time on this. This derivation is very straightforward. And it gives the duration as the ratio of the logarithmic price change of the given bond to the logarithmic price change of a single period bond of the face value of 1 unit, so that is what the figure in this box represents. The derivation is straightforward, I will not spend time on it.

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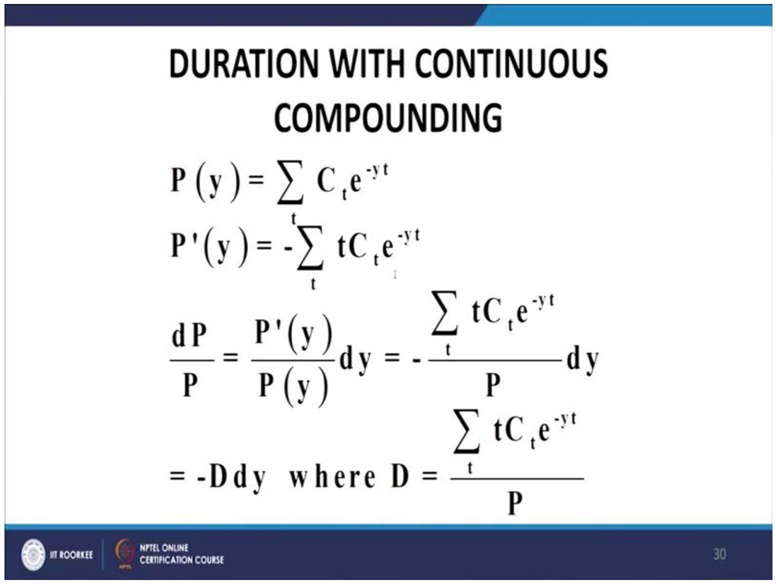

Then we come to duration with continuous compounding. Again, this is a straightforward exercise in algebra. So, again, let us skip the details of this working. And obviously, it is a part of the presentation. And those who are mathematically inclined can work out this duration formula quite easily. And the important thing is very often in mathematical finance, very often we use the concept of continuous compounding.

In fact, the Black-Scholes formula for option valuation also uses continuous compounding. So, it is useful to have an idea of the measure of duration, of the formula for duration, of the expression for duration using continuous compounding. So, that is the utility of this. But the working is straightforward, let us move on.

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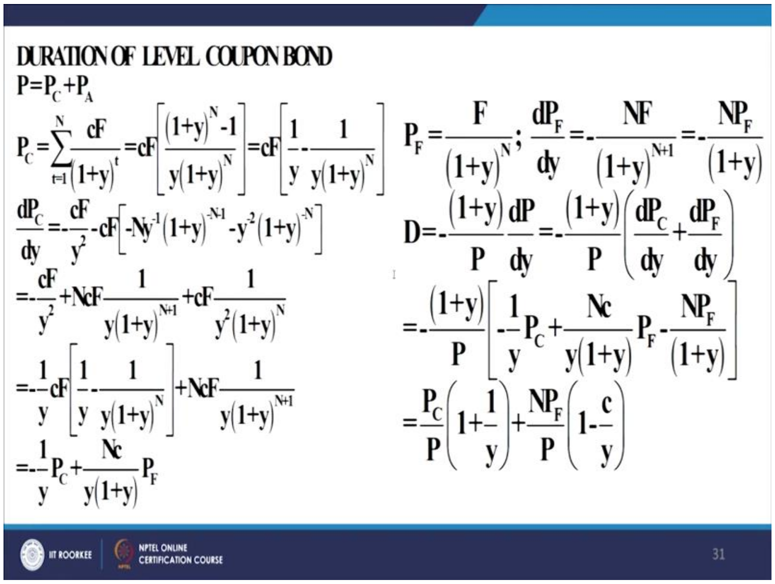

This is the duration for a level coupon bond. Again, the working is quite straightforward. I leave it as an exercise for the students, for the mathematically inclined student. And, again, this is not, there is not much to explain here, simple algebra and elementary high school calculus that is being used for this formula.

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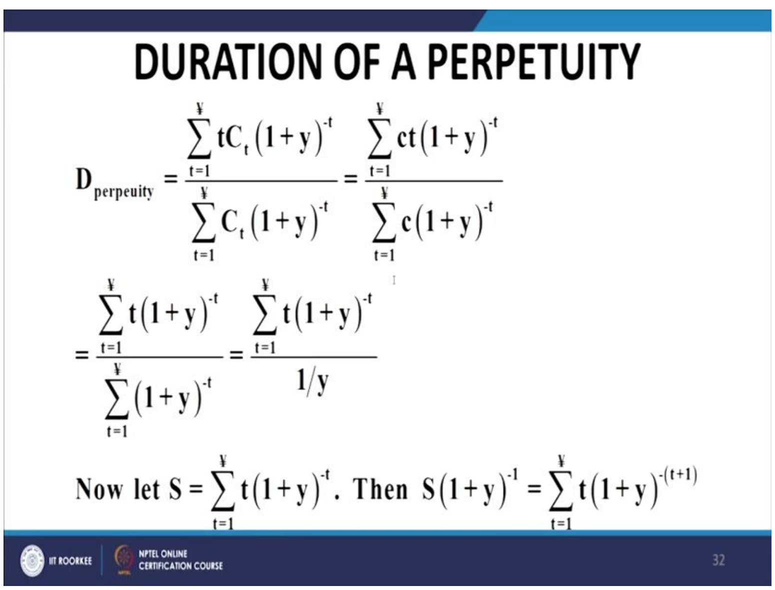

Duration of a perpetuity I have alluded to earlier. This is another exercise in algebra and the summation of infinite geometric series. And again, I leave it as an exercise for the mathematically inclined students.

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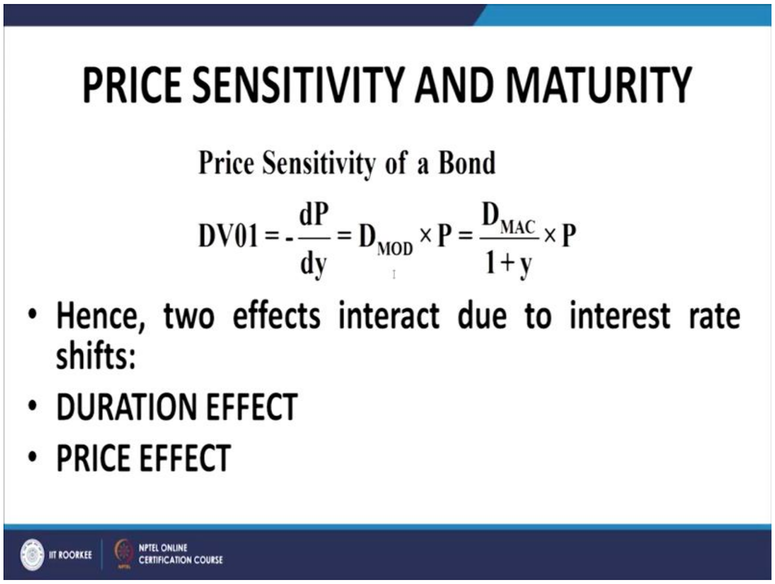

Now, we talk about price sensitivity and maturity. Now, before I move on to the issue of price sensitivity versus maturity, let me recall that price sensitivity of a bond is synonymous with what we refer to earlier as the dollar value per basis point DV01. The mathematical definition is same, it is the negative of the change in price per unit change in the YTM of the bond, it is minus dp by dy.

The slope of the yield price curve at the given point gives you the price sensitivity or the dollar value per basis point minus or the inverse, negative slope, let me call it, the minus sign is there as I mentioned by convention, by choice so, that we return a positive figure, because the price of the bond and the YTM are invariably inversely related.

So, the DV01 figure would otherwise always be negative. So, to avoid that cumbersome use of the minus sign again and again, we introduce a negative sign in the definition itself. So, price sensitivity is equal to DV01, the dollar value per basis point which is equal to the negative slope of the yield price curve at the given point which is equal to y and dp by dy. So, from here we will continue after the break. Thank you.