Security Analysis and Portfolio Management
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Lecture 20
Immunization

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Welcome back. So, quick recap of where we were, we were talking about an investor who is invested in a bond portfolio and has a holding period which is not congruent to the remaining life of the bond. Now, in that case, as I mentioned a change in the interest rate subsequent to his taking up the investment would have two effects, two counteractive effects or two countering effects to annulling effects, one of them would be positive to the investor and the other would be negative to the investor.

For example, if the interest rates increase the reinvestment income of the investor would increase, but the price at which he would liquidate the investment in the market would decrease and the capital gains would therefore decrease the converse would be the case if the interest rates decrease.

I mentioned another point and that was that the extent of annulment between these two effects that is the reinvestment effect and the capital gains effect depends on the holding period of the investor, if he holds a bond for a long point in time very close to the maturity, the reinvestment effect would predominate. And if he holds a bond for a short period of time with a long period of time to remaining to the maturity of the bond, then obviously the capital gains effect would dominate. Let us now look at the, let us now look at quantifying and analysing the situation mathematically.

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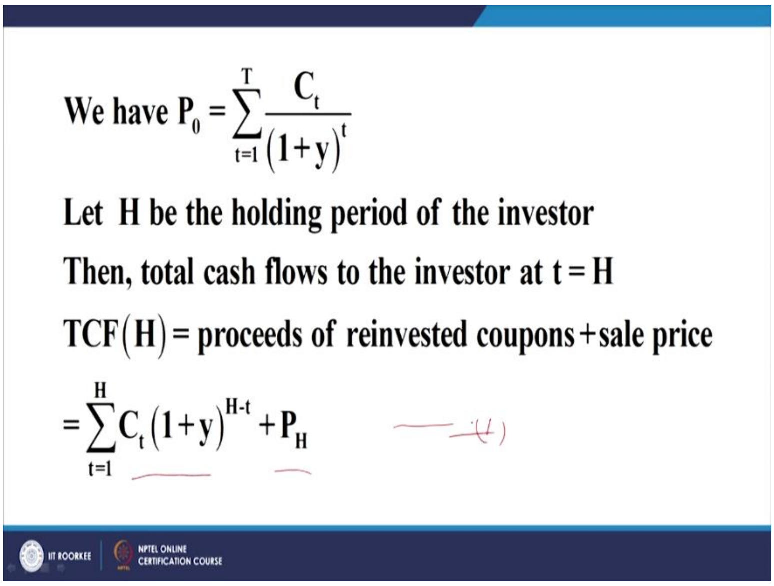

So, for that purpose, we come to this slide we start with the TCF formula, the TCF formula gives us P0 is equal to summation Ct upon 1 plus y to the power t summation over the entire life of the bond up to maturity, P0 is the current market price this is the expression for the YTM.

So, P0 is equal to summation Ct upon y, 1 plus y to the power t summed over all values of t up to the maturity of the bond. We assume that H is the holding period of the world capital H, which is less than capital T, capital T is the maturity of the bond, capital H is the holding period of the bond and we work out the total cash flows at t equal to H at the point at time point in time at which the investor plans to exit the investment to liquidate the investment. Let us work out the total cash flows.

The total cash flows at H would be equal to the proceeds of the reinvested coupons. Whatever coupons he has received, up to the point H he would have reinvested those coupons except of course the last coupon in the event that it coincides with the coupon payment date on which he liquidates the investment. Otherwise, all the earlier coupons he would reinvest and he would reinvest them at the interest rate y, which is the current interest rate at which we are working for the moment.

So, this is one part of the income that he would get and of course, the second part of the cash flows would arise from the liquidation of the investment, selling of the investment in the market, which would be the price of the bond at t equal to H which I write as P suffixed with capital H.

So, the first term here, you can see is the proceeds the first term in this equation, let us call it equation 1, the first term in equation 1 are the proceeds of the reinvested coupons at the rate y up to the time of liquidation of the investment that is t equal to H and the second is the price that the investment would fetch in the market, which at t equal to H which I call us PH.

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$$\text{But } P_H = \sum_{j=1}^{T-H} \frac{C_{H+j}}{(1+y)^j} = \sum_{k=H+1}^{T} \frac{C_k}{(1+y)^{k-H}} = \sum_{t=H+1}^{T} C_t (1+y)^{H-t} \qquad \text{--- (1)}$$

$$\text{Hence, } TCF(H) = \sum_{t=1}^{H} C_t (1+y)^{H-t} + P_H \qquad \text{--- (2)}$$

$$= \sum_{t=1}^{H} C_t (1+y)^{H-t} + \sum_{t=H+1}^{T} C_t (1+y)^{H-t} = \sum_{t=1}^{T} C_t (1+y)^{H-t}$$

$$\text{We have } P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}$$

Let H be the holding period of the investor

Then, total cash flows to the investor at $t = H$

$TCF(H) =$ proceeds of reinvested coupons + sale price

$$= \sum_{t=1}^{H} C_t (1+y)^{H-t} + P_H \qquad \text{--- (4)}$$

Now, what is PH? PH would be what the future value or the present value I am sorry, of all the future cash flows that can be derived from the investment. In other words, the price that the buyer of the bond at t equal to H would be willing to pay as per the TCF presumption is equal to the present value of all future cash flows discounted at the current market interest rates.

So, that is given by the first expression CH flows assuming that the cash coupon payments up to t equal to H are received by our investor and the, after receiving the payments up to time H. He sells the bond in the markets, so whatever coupon payments arise after t equal to H that is H plus 1 and onwards would determine the price of the bond plus of course, the redemption value.

So, these are the cash flows from the coupon from t equal to H onwards after t equal to H onwards that is excluding the cash flow at t equal to H if any. So, that is CH plus j where j starts from 1 that is the H plus 1th coupon or H plus 1th is cash flow as plus 2th with cash flow as 3th as cash flow, up to the final cash flow that is C capital T, which would comprise of the coupon payment as well as the redemption value of the bond.

And of course, they would be discounted at the rate y, therefore, the summation of j here in this expression the summation of j extends from j equal to 1 to t minus H, the last term would be CT the final cash flow from the bond, but it would be discounted up to t equal to H, it would not be discounted to t equal to 0, because we are determining the price at t equal to H. So, the discounting of all future cash flows, that is CH plus 1th plus 2 up to CT would be discounted up to t equal to H.

So, that is 1 plus y would be carried from 1 that is for the discounting of the cash flow at t equal to H plus 1 that is CH plus 1 would be discounted for 1 period CH plus 2 would be discounted for 2 periods. And similarly, CT would be discounted for t minus H periods. So, this is how this formula comes up. Now, we simply redefine the index of summation, instead of j equal to 1 to t minus H, we use k and what is the definition of k?

k is equal to H plus j, we restate this expression in terms of a new summation index k, where k is equal to H plus j. So, obviously, k would be summed up from H plus 1 to capital T and obviously, CH plus j would be Ck and 1 plus y to the power j would be 1 plus y to the power k minus H and that is why we write here k minus H. Again, we simply change the variable k to variable T for convenience, which is, the relevance of which you shall see in the next equation.

So, restating the expression in terms of the new summation variable t, we are summation t equal to H plus 1 to capital T CT and we write 1 plus y to the power t minus H, in the denominator as 1 plus y to the power H minus t in the numerator, I repeat, we have changed k to t and we have written 1 plus y to the power k minus H in the denominator as 1 plus y to the power H minus t in the numerator, substituting k equal to t and taking this 1 plus y to the power k minus H in the denominator to the numerator.

Now, let us revert to our original expression for TCF at t equal to H, the first expression is written as it is and we substitute the value of PH as we obtain from equation number 1, let us call it equation number 1. So, we substitute PH from equation number 1 in equation number 2, equation number 2 is brought forward from the earlier slide.

This is equation number 2 of the earlier slide this equation. So, this equation is carried forward to this slide here, which is equation 2 here. So, substituting for the value of PH from equation 1 to equation 2 what we get is the expression here. And if you look very carefully at this expression, the first term is the summation from t equal to 1 to capital H of what CT 1 plus y to the power H minus T. The second term is the summation from t equal to H plus 1 to T of CT 1 plus y to the power H minus t. In other words, the summands of both the terms are the same CT 1 plus y to the power H minus t.

The first summation carries terms or involves terms from t equal to 1 to t equal to H, the second term involves summation from t equal to H plus 1 to t equal to capital T. So, we can straight away combine these 2 terms and we can write it as summation from t equal to 1 to capital T of CT 1 plus y to the power H minus t. So, this is the value of total cash flows at t equal to H when we liquidate when we exit the investment.

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$$TCF(H)=\sum_{t=1}^{T}C_t(1+y)^{H-t} \quad (1)$$

$$\Rightarrow \frac{dTCF}{dy}=\sum_{t=1}^{T}(H-t)C_t(1+y)^{H-t-1}=(1+y)^{H-1}\sum_{t=1}^{T}(H-t)C_t(1+y)^{-t}$$

$$=(1+y)^{H-1}\left[H\sum_{t=1}^{T}C_t(1+y)^{-t}-\sum_{t=1}^{T}tC_t(1+y)^{-t}\right]$$

$$\text{Thus, } \frac{dTCF}{dy}=0\Rightarrow H(y)=\frac{\sum_{t=1}^{T}tC_t(1+y)^{-t}}{\sum_{t=1}^{T}C_t(1+y)^{-t}}=\boxed{\frac{\sum_{t=1}^{N}tC_t(1+y)^{-t}}{P_0}}=D$$

Let us now take the derivative of this with respect to y. When we take the derivative of this, this is equation 1. When we take the derivative of this formula for the total cash flow at t equal to H with respect to y, what we get is the expression here that I have underlined summation t equal to 1 to capital T, H minus t into 1 plus y to the power H minus t minus 1 or H minus 1 minus t as you may like.

Now, this 1 plus y to the power H minus 1 minus t can be split into 2 parts 1 plus y to the power H minus 1 which is independent of the summation index t and therefore, it can be taken outside the summation and that is precisely what I have done in the second step. I have taken 1 plus y to the power H minus 1, outside the summation and therefore, within the summation what remains is summation t equal to 1 to capital T, H minus t, CT 1 plus y to the power minus t.

Now, I split this second summation term into 2 parts I write the first term is written as it is, the pre-factor is written as it is 1 plus y to the power H minus 1 is written as it is. And as far as the second term is concerned, I take H I multiply the post factor with both the terms H and t and then take H outside the summation, why? Because he is independent of the summation index small t and therefore, I can take it outside the summation in the first expression and in the second expression of course, I cannot do that because it involves the product of CT and T and we are summing over T. So, I cannot take t outside the summation in this case.

Now, when I equate this expression to 0, when I work out dTCF upon dy equal to 0 and simplify, this the pre-factor goes because the pre-factor cannot be 0. So, the pre-factor goes away straight away and therefore, the expression within the square brackets needs to be 0. And when I simplify this expression within the square brackets, what I get is H is equal to H in bracket I have written y because H is a function of y.

So, that is why I have return Hy, but it is H is the holding period basically, H is equal to this expression, which is summation tCt 1 plus y to the power minus t divided by summation Ct upon 1 plus y to the power minus t.

Now, if you look carefully at the denominator, it is the TCF price of the bond it the expression for the TCF price of the bond. So, I have substituted by P0. So, the expression that we get is Hy is equal to the expression that is on the extreme right-hand side. Now, what is the implication of this analysis?

Whatever I done, I have taken the derivative of total cash flows for an arbitrary holding period H and I have equated this to 0 and on that basis, I have arrived at a particular value of

H. So, what does it mean when the derivative of a function is 0, it means that the function is constant in the neighbourhood of the point at which the derivative is calculated.

In other words, if I work out the derivative at a particular point say y equal to y0, then the total cash flows at that point y0 are constant for small changes in y0, if y0 increases by dy, or y0 decreases by dy that would have minimal effect, a very small effect on the total cash flow because the total cash flow curve, if I plot the total cash flow curve versus y0 for a holding period of H, what I will find is that there is a flatness of the total cash flow curve around y0.

And therefore, if I move slightly away from y0, either to the left or to the right, I would get insignificant or very small changes in the total cash flow, that is the meaning of the function of the derivative of a function being 0. In other words, the function is independent of the variable with respect to which it has been there, differentiated in the close proximity of the point at which the value becomes 0.

So, because this value is becoming 0 at t equal to capital H, therefore, at t equal to capital H, what is t equal to capital H? That is the holding period that is a holding period which has a special property that if I hold the investment for t equal to capital H, then I get immunized, I get protected against small changes in interest rates. My total cash flow at t equal to capital H would not change if the cash flows change slightly around the current figure. Sorry, the YTM changes around the current figure at which I have made the investment.

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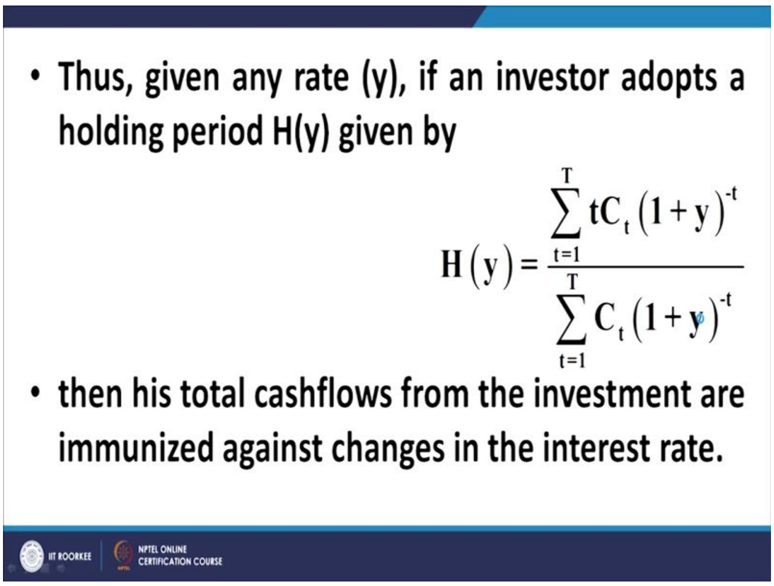

So, thus given any rate y, if the investor adopts the holding period H which is given by this expression. Of course, in the special case that y is equal to YTM, that denominator can be replaced by P0, then its total cash flows from the investment would be immunized against changes in interest rates, precisely what I have explained just now. I repeat, if y this expression holds in general for all interest rates, but for the special case, when y is equal to the YTM of the bond, that denominator becomes the discounted cash flow price of the bond and we can replace it by P0.

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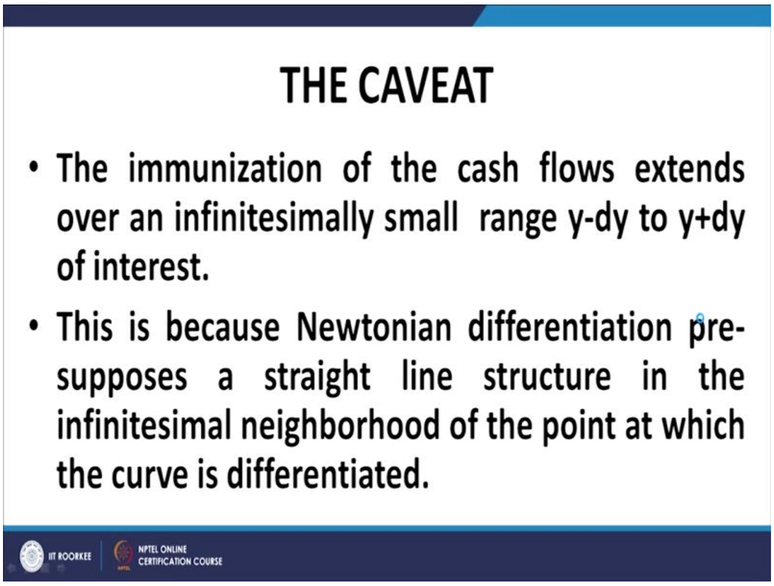

The caveat, what are the caveats to the immunization that I have talked about? Number 1, the immunization of the cash flow extends over an infinitesimally small range y minus dy to y plus dy, there is curvature that is the source of too many problems, so many problems had this yield price curve been a straight line, this immunization would held good over all interest rates around y0. In other words, whatever dy or delta y we take irrespective of the magnitude this immunization would have held, but the curve is not a straight line, it has curvature it is convexity and because of this convexity, the immunization the protection that we have extends over very small delta ys to the left and right of y0.

In other words, the protection is over very small changes in the YTM rates, the independence of total cash flows around the value at which they have been calculated y0 is very, very small, delta y must be very small, very insignificant, and if delta y is large, the curvature effect will come into play and the immunization will break down. This is because Newtonian differentiation presupposes a straight-line structure in the infinitesimal neighbourhood of the point.

So, the point is, we have assumed implicitly when we have calculated the Newtonian differential the derivative that around y0 the curve is a straight line and as long as our approximation of the curve being represented by a straight line being good enough, this immunization will hold but as soon as the curvature effects become significant, because yield price curve is curved. So, as soon as the curvature effects become significant, if you have a large delta y, the curvature effects will be much more significant, if you have a very small delta y, then the curvature effects would be very, very minimal and you can approximate the region around y0 by a straight line and then this immunization would hold good. So, this is the dynamics of it, we shall also discuss it with an example.

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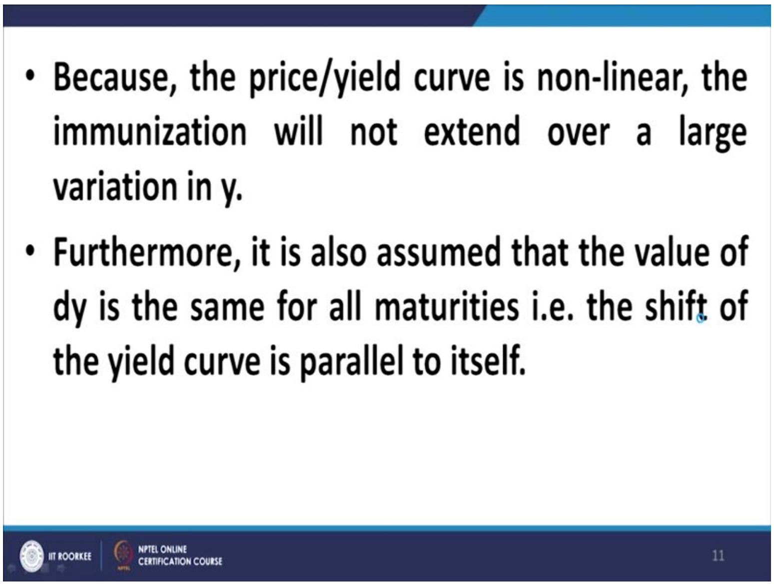

So, this I have already explained. Then there is another assumption that if we are assumed that the value of dy is the same for all maturities, I will come back to this also in a later section, but we have assumed that dy is independent of the maturities and we know that the spot rates corresponding to different maturities are different. So, we have not accounted for this, we have not accounted for the term structure of interest rates.

So, because we have assumed dy to be constant, we assume that our yield price curve when it shifts by dy either upwards or downwards, it shifts parallel to the original version of the curve. That is dy remains constant over all maturities that we are talking about whatever be the spot rates, the shift in those spot rates corresponding to dy or measured by dy is the constant. In other words, all portraits shift by the same amount. This is the underlying presumption of immunization.

If this breaks down again immunization will break down. I repeat this is another important assumption that the yield price curve, the spot yield price curve shifts parallel to itself when we talk about the immunization process due to a shift in the yield price curve.

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Now, if $\frac{d^2TCF}{dy^2} < 0$, we must have

either (i) $t < H$ and $t > H-1$ or (ii) $t > H$ and $t < H-1$

Now, (i) cannot hold because t must be a positive integer and there is no integer between H and H+1.

(ii) is obviously impossible. Hence, $\frac{d^2TCF}{dy^2} > 0$ must hold.

We have $TCF = \sum_{t=1}^{H} C_t (1+y)^{H-t}$ so that

$$\frac{dTCF}{dy} = \sum_{t=1}^{H} C_t (H-t)(1+y)^{H-t-1} \text{ and}$$

$$\frac{d^2TCF}{dy^2} = \sum_{t=1}^{H} C_t (H-t)(H-t-1)(1+y)^{H-t-2}$$

This expression explains the second derivative of the total cash flows with respect to y. And I leave it as an exercise for the mathematically inclined learners, mathematically inclined students.

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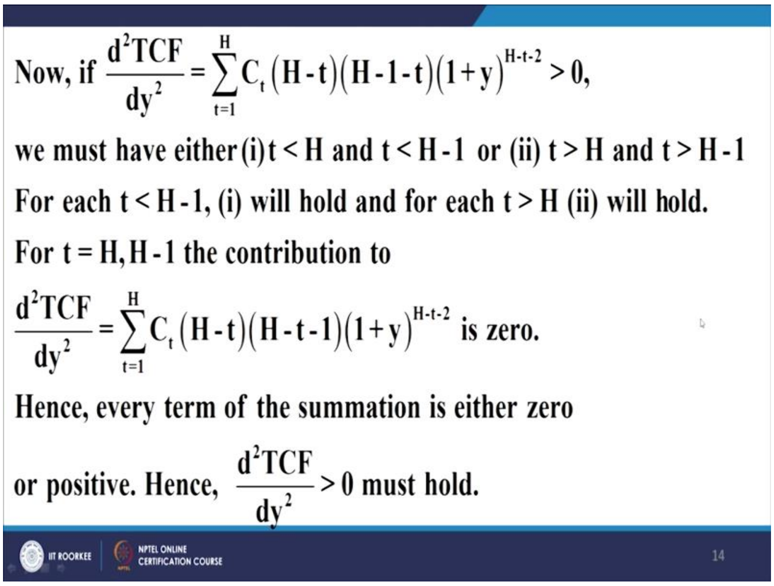

It is easy to show it is easy to show that the second derivative of total cash flows with respect to y is positive and because it is positive what we have arrived at is a minima is the local minima of the fewer total cash flows with respect to y. So, again, I shall talk more about it at a later point in time, but I repeat, the second derivative of the total cash flow with respect to y can be shown to be positive. And the details are in the slide, I leave it as an exercise for the learners.

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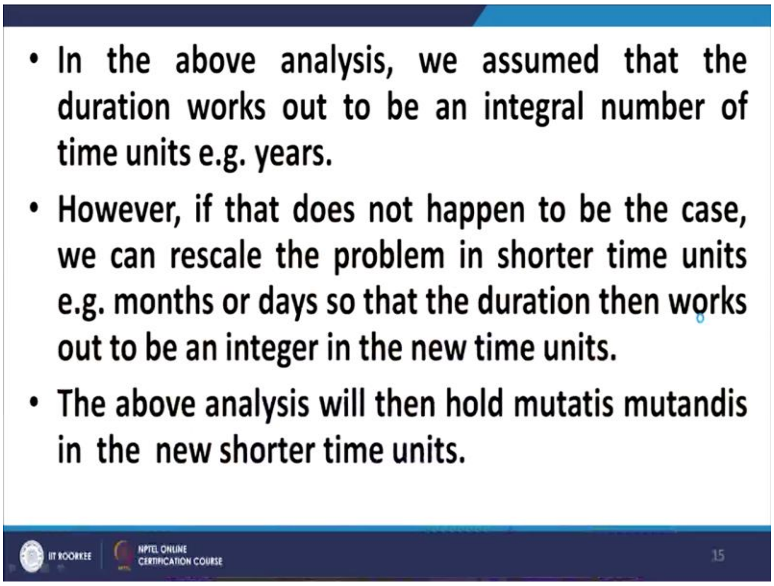

Now, there is another important point, in the above analysis, we assumed that the duration works out to be an integral number of time units that is years. In the above analysis, we assumed that the duration works out to be an integral number of years. However, if that does not happen to be the case, then we can rescale the problem in smaller time units in half years or in quarter or months or days, or even hours, and minutes.

But the important thing is, we can rescale this problem in any appropriate time units in any smaller time units, such that the result that we get for the duration is an integer. So, that can be done that is not an impediment to the above analysis, that does not invalidate the analysis. In other words, what I am simply trying to say is that by assuming the duration to be in years, and thereby, there could be a situation that the duration could be fractions of years, would the above analysis hold, yes, it would hold. The only, and to justify that contention, what we can do is, we can rescale the problem in smaller time units in which the duration turns out to be in integral numbers of that smaller time scale.

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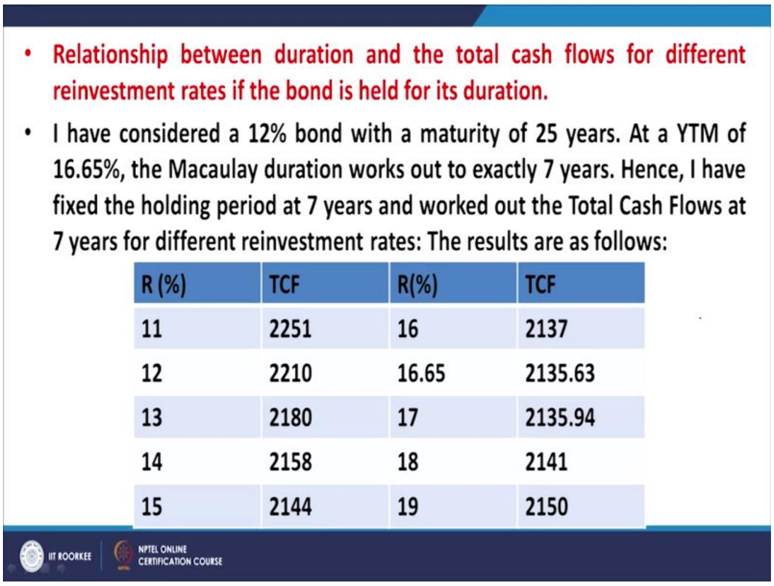

Now, we come to an example of whatever I have spoken, I have tried to illustrate with this example, let us do this example very carefully. I have considered a 12 percent, 12 percent is the coupon rate with a maturity of 25 years. 12 percent is the coupon rate and the maturity is 25 years. At a YTM of 16.65 percent, the Macaulay's duration works out to exactly 7 years. If you work out the duration of this bond, Macaulay's duration of this bond at a YTM of 16.65 percent what you will get is, duration of exactly 7 years.

Hence, I have worked and I have fixed the holding period at 7 years. In other words, I am holding this bond upto its duration activity at YTM is 16.65 percent the coupon rate is 12 percent. These are the parameters of this exercise, this example. And then I have worked out the total cash flows corresponding to different reinvestment rates.

So, I repeat, what I have done is I have taken a bond, the life of the bond is 25 years, the coupon rate is 12 percent. The current yield or the current market interest rate is 16.65 percent. And based on the current yield, if I work out the duration of the bond, I find that it is exactly 7 years. So, what I do? I fix the holding period at 7 years.

And then on that basis, I work out the total cash flows at different reinvestment rates. The table gives us the results of this competition. What we find is if the reinvestment rate is 11 percent, the total cash flows for our 1000 face value bond is 2251 if the reinvestment rate is 12 percent the total cash flows are 2210.

Now and similarly, if the reinvestment rate is 16 percent, the total cash flow is 21.37. If the reinvestment rate is 60.65 percent, which is the YTM at which I have made the investment, the total cash flow is 2135.63. And if the reinvestment rate is 17 percent, the total cash flow is 2135.94.

Now, there are 2 very important observations from this. Number one, if you compare the change in the total cash flow, when the reinvestment ranges from say, reinvestment rate changes from say 11 percent to 12 percent, the cash flow changes by 41 units, the cash flow changes by 41 units. However, if you work out the change in cash flow, when the reinvestment rate changes from 16 percent to 17 percent, the change is hardly of 1 unit, slightly more than 1 unit maybe.

So, this is very interesting. This shows what? This shows that if you fix up your holding period equal to the duration of the bond, then, if there is a small change around the YTM, remember YTM is 16.65 percent if there is a small change in YTM either upwards or downwards, either 16.64 percent goes to 16 percent or 16.65 percent goes to 17 percent let us assume, if it is an upward change or a downward change. The change on the total cash flows is minimal, it is very, very small.

In other words, if you plot the total cash flow curve with respect to y at a duration of 7 years, at a holding period of 7 years, what you find is that the curve becomes significantly flat around the YTM of 16.65 percent. So, if 16.65 percent shifts slightly to the right or to the left, there is marginal change in the total cash flows. The second observation is that the total cash flows at the 16.65 percent at this YTM for this holding period of 7 years turns out to be minimum.

So, again this is very interesting, this provides a testimony of our basic understanding of finance that lower the expected returns, lower the risk or conversely, lower the risk lower the expected returns. Here that is precisely what is happening, we are seeing that, if at 16.65 percent minor changes in interest rates do not affect the total cash flows.

In other words, this is the least risky point on the, at which this investment can be evaluated at which this investment, as far as this investment is concerned, this is the least risky point because if the interest rates shift slightly to the left or to the right, my total cash flows are not changed significantly.

So, this is the least risky point when this is also giving me the minimum returns in terms of the total cash flows, the total cash flows are minimum at this point. And this point has the minimum variability in the total cash flows in terms of the fluctuation in the interest rates. So, again this result, we are testimony to the fact that lower the risk, lower the expected returns.

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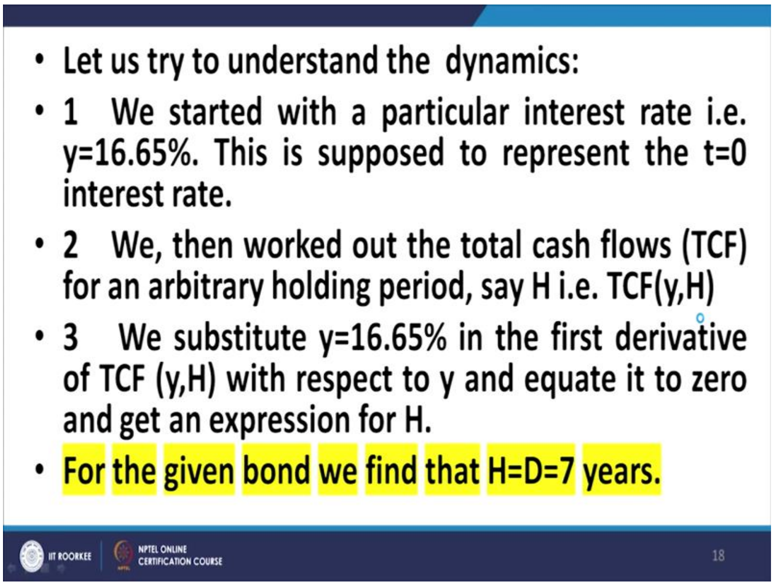

Now, let us try to understand this step by step, this example step by step let us try to understand this. Number one, we started with a particular interest rate that is y is equal to 16.65 percent. This is supposed to represent the t equal to 0 interest rate t equal to 0 market rate, let us assume that we enter the investment at this point. And when we enter this investment, this particular bond, which is a 12 percent coupon bond with a life of 25 years, I work out the duration I find it to be 7 years.

So, then what we did is, we worked out the total cash flows TCF as functions of y and the holding period H. We take an arbitrary holding period just as we did in the derivation a few minutes back and we work out the total cash flows for that arbitrary holding period for this y. And then what we do is, we take the derivative of this total cash flows with respect to y and equate it to 0. On equating it to 0 and putting y equal to 16.65, what we end up with is that the holding period is equal to 7 years which we call the duration, which we find is the duration of the bond.

I repeat, what we do is we first work out the derivative, we work out the total cash flows for this particular bond 25 year 12 coupon bond at arbitrary y and H, we differentiate this with respect to y, we equate this to 0, we put y is equal to 16.65 percent and we solve the expression for H, we get H is equal to 7 years, which constituency duration of the bond.

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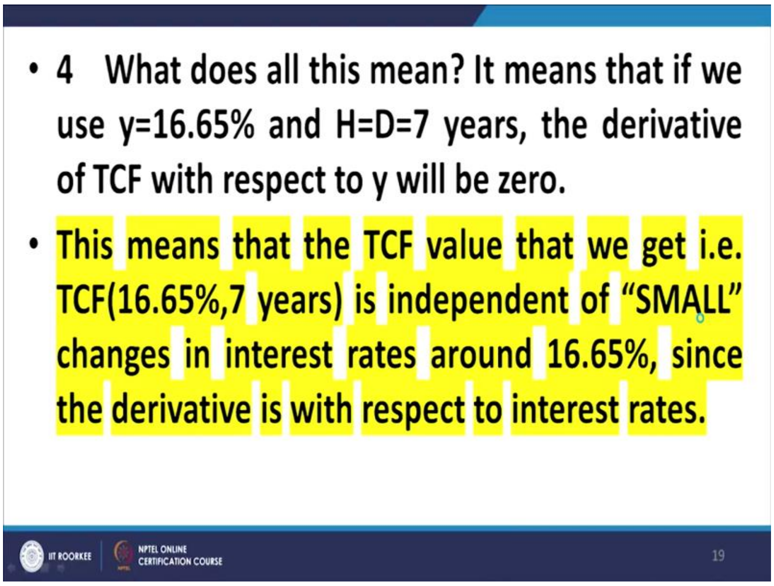

So, what does this mean? It means that if we use y is equal to 16.65 percent and holding period equal to 7 years, which is the duration of the bond, the derivative of TCF with respect to y will be 0. And what does that mean? That means that the total cash value that we get that is TCF 16.65 percent 7 years is independent of the fluctuations in y of the fluctuation in y either to the left or to the right around that 16.65 percent, in the region that we can approximate the yield price curve by a straight line.

That is in for very small fluctuation for very small changes in why to the left or to the right as the case may be the total cash flows at 16.65 percent for 7 years will not be significant. So, will not change significantly that is.

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$$TCF(H)=\sum_{t=1}^{T}C_t(1+y)^{H-t} \quad \text{---(1)}$$

$$\Rightarrow \frac{dTCF}{dy}=\sum_{t=1}^{T}(H-t)C_t(1+y)^{H-t-1}=(1+y)^{H-1}\sum_{t=1}^{T}(H-t)C_t(1+y)^{-t}$$

$$=(1+y)^{H-1}\left[H\sum_{t=1}^{T}C_t(1+y)^{-t}-\sum_{t=1}^{T}tC_t(1+y)^{-t}\right]$$

$$\text{Thus, } \frac{dTCF}{dy}=0\Rightarrow H(y)=\frac{\sum_{t=1}^{T}tC_t(1+y)^{-t}}{\sum_{t=1}^{T}C_t(1+y)^{-t}}=\boxed{\frac{\sum_{t=1}^{N}tC_t(1+y)^{-t}}{P_0}}=D$$

Now, let us investigate further, but before I get to that, let me recap 1 point that perhaps I have missed. If you look at this expression, when I differentiate it TCF with respect to y and equated it to 0, I got a value of Hy, I got a value of Hy which is this expression which is expression which is shown here in the box.

Now, if you compare this with the expression for duration, you find that this is precisely the duration of the bond. Perhaps this escaped my mind at that point in time, this is a very important point here that it means what? It means that if you hold the bond equal to its duration at the given YTM then dTCF upon dy will be 0. That means what? That means TCF will be independent of y.

So, this is important the expression that I get in the box for H which gives me dTCF upon dy equal to 0 is nothing but the duration of the bond. This formula is precisely the formula that we worked out earlier for the duration of the bond that coincide absolutely. So, this is equal to D.

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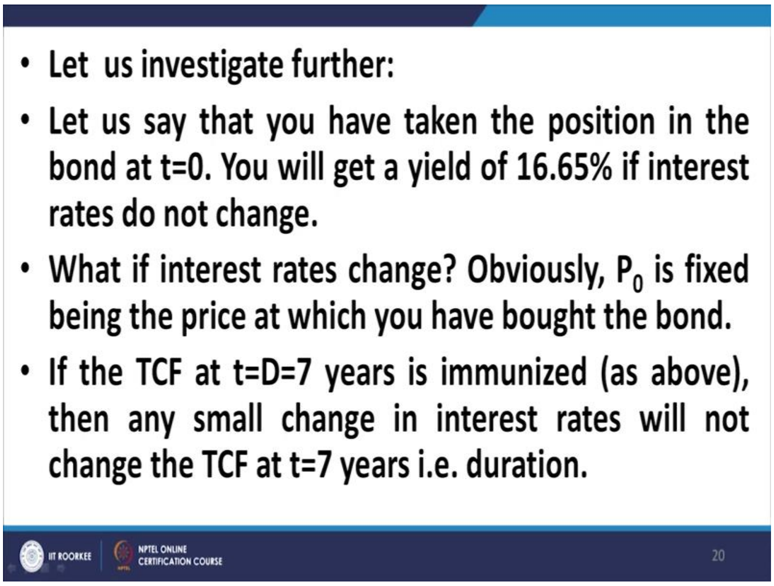

So, let us investigate this further, let us say that you have taken this position in the bond at t equal to 0. You have invested in the bond at equal to 0. I will go through this slowly, this is very interesting, let us say that you have taken the position in the bond at t equal to 0, then you will get a yield of 16.65 percent provided that the interest rates the market interest do not change, then you are entitled to a return of 16.65 percent on your investment in the bond at t equal to 0 at this YTM.

However, what happens if the interest rates change, then obviously, P0 is fixed, that is the price at which you have taken the bond at t equal to 0 is obviously fix you have already done that, that is in some sense, that is a fixed cost. Then we also find that if you hold the bond for 7 years, not to the maturity, maturity is 25 years, but if you decide to hold the bond up to 7 years, then what happen?

Then the total cash flows that you are going to get, at the end of 7 years are protected against changes in interest. In other words, if there is a change in interest rate between t equal to 0 and t equal to 7, then that change in interest rate is not going to affect your total cash flows at t equal to 7 provided you hold the bond not to its maturity, but to its duration which is 7 years. So, because the total cash flow at t equal to 7 years is immunized against changes in interest rate, changes in market interest rates, your cost of entering the investment is known.

So, what does it means? It means the return that you have between t equal to 0 and t equal to 7 years is immunized, is protected against changes in market interest rates provided of course, the condition of smallness of the changes is accepted. If the changes in interest rates are small, then the return that you derive between t equal to 0 and t equal to 7 years is fixed, because of what because the total cash flows at t equal to 7 are immunized against changes in interest rates.

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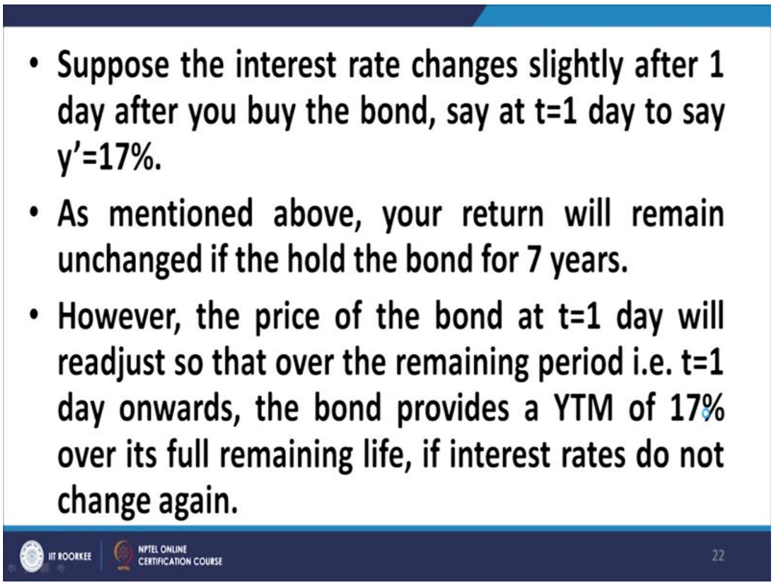

Now, we come to a further analysis. Suppose the interest rates change slightly after 1 day after you buy the investment, let us say you bought the investment A at equal to 0 as I explained just now. Let us assume that the interest rates at t equal to 1 day t increased to 17 percent.

Now, what will happen? What will happen is, that while your return would remain at a predetermined level, provided you hold the bone for 7 years provided you carry on holding the bond irrespective of the chain ignoring this chain, you continue to hold upon up to 7 years, then your return on the bond will remain at the figure that you earlier arrived at on the basis of which you made the investment at t equal to 0.

However, what will happen to the price of the bond in the market? The price of the bond in the market at t equal to 1 day is now in disequilibrium it will realign itself, it will realign itself in such a way that an investor who enters this investment at t equal to 1 day after the interest rates have changed from 16.65 percent to 17 percent. This, the new investor that has entered the market will now get a return of 17 percent provided the interest rates do not change again.

In other words, the price will fall and if a new person enters into this investment at t equal to 1 day after the change in the interest rates have taken place, he will buy the bond or the market price of the bond would be such that you would be able to enter the investment and at such a price that he would get a YTM of 17 percent, he would get a return of 17 percent, the interest rates do not change from 17 percent again over the remaining life of the bond.

Therefore, if you liquidate the bond, instead of liquidating the bond at the end of 7 years, which was your duration. Now if you liquidate the bone at the end of 7 years, then your return is protected. But if you liquidate the bond on the next day after the interest rate changes taken effect, then obviously you will not get that promise return, your return will definitely change. Why?

Because you have not held upon up to its duration, you held up on for just the 1 day and therefore your return is not immunized, immunization will take effect only if you hold the bond up to the duration of the bond. If you hold the phone for a lesser or a greater period than the duration of the bond, then the immunization process will not take place.

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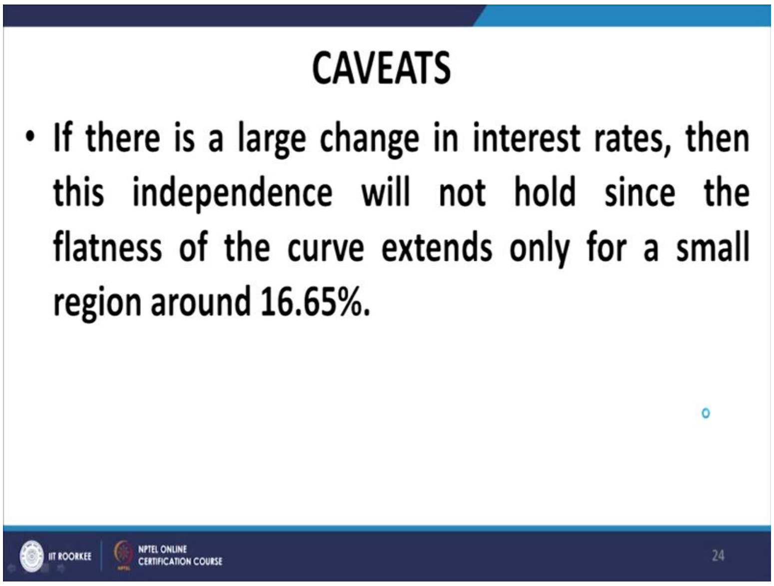

And of course, I have mentioned that if there is a large change in interest rates, then the independence or the immunization will not hold since the flatness of the curve extends only to in finite similar regions to the left and right of the point y0 at 16.65 percent. We will continue from here in the next lecture. Thank you.