Security Analysis & Portfolio Management
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Lecture 17
Clean & Dirty Price, Interest Rate Risk - I

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Welcome back. So, today I propose to take up the concept, measures and management of interest rate risk. Before that, a brief touch up on what we did in the last lecture, a recap.

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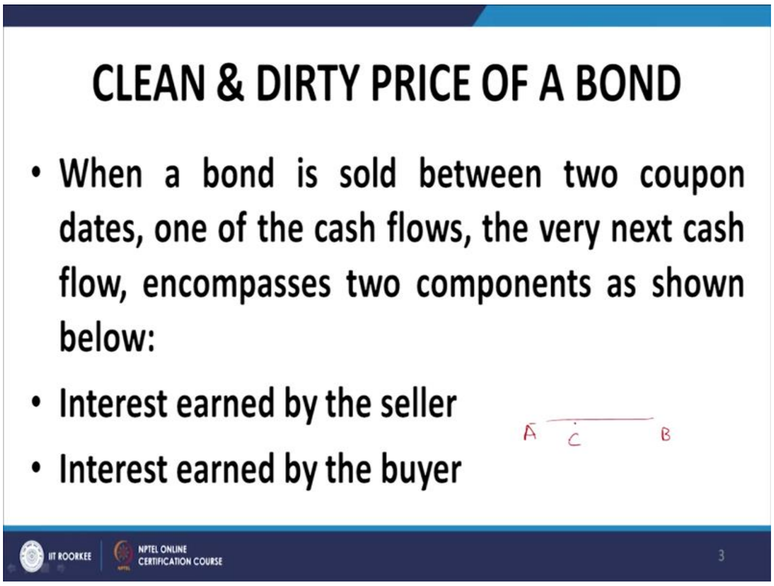

Towards the end of the last lecture, I was discussing the concept of clean price and dirty price in a bond. To understand what exactly we mean by these terms, let us look at a transaction that involves a purchase/ sale of a bond which takes place between two coupon dates. So far we have been focusing our attention to transactions that are taking place on the coupon date and as a result of which we have a whole number of periods to deal with.

Now, let us relax that assumption and let us look at a transaction which takes place in between two coupon dates. Let us say the last coupon payment was at A and the next coupon payment is that B. So, AB would be six months in a six-monthly bond and one year in an annual bond and so on. Let us assume that a transaction takes place, a party X sells the bond to another party Y, at some point C between A and B. Then the issue arises as to how to account for the interest that is attributable to the period AC and the period CB. It is very clear that because the party X who is selling the bond has held the bond for a part of this coupon period AB i.e. AC, he should be entitled to the interest for that particular period (AC).

And as far as the buyer of the bond is concerned (Y), he should be getting the interest for the period pertaining to his holding of the bond i.e. CB. Therefore, we are to allocate the interest for the period AC (which is called the accrued interest) to the seller X. Now, because at the point B, the issuer of the bond shall pay interest for the entire period AB to the party whose name appears in the register of bondholders (Y), the party Y is required to reimburse the interest for the period AC to X. It is the convention that this interest for the period AC (called accrued interest) is addedee on to the quoted price of the naked bond to arrive at the transaction price. Thus, the buyer pays up the interest for the period AC to the sller upfront at the instant of buying the bond and leter on (on the immediately following coupon date) he gets reimbursed by the company (at point B). The transaction price of the bond arrived at by including the accrued interest (for the period AC) to the quoted price is the actual cash price or the actual payment that X would receive from the party Y. It is called the dirty price of the bond.

Let me clarify once again. Because X has held upon for a part of the coupon period (AC) and Y has held the bond for the remaining part of the coupon period (CB), the interest for the period AB (which will be received by B in full from the issuer) needs to be allocated between these two holding periods AC & BC. Obviously, when Y buys the bond, he will approach the company and get the bond transferred in his name. As a result, when the next coupon payment arrives (B) the entire coupon (AB)would be paid to the party Y by the issuer. The issuer company will not split up the interest and give part of the interest to X and part of the interest to Y. It will make the entire coupon payment to the party whose name appears in the register of bondholders and that would be party Y because he has bought the bond and got the bond transferred in his name in the company's records. In other words, the interest for the entire period AB gets split up into two parts, the interest that is earned by the seller X for the period that he holds the bond (AC) and the interest that is earned by the buyer (Y) for the period he holds the bond (CB). Because the entire payment of interest (AB) is going to be paid to the buyer of the bond on the next coupon date, which includes the interest for the period which the bond is held by the seller (AC), the buyer needs to compensate the seller for the interest pertaining to the period that the seller has held the bond (AC). This part is called accrued interest and as a convention it is added to the quoted price to arrive at the fiull transaction price and settled upfront i.e. at the settlement point of purchase of the bond by Y..

So the actual transaction price of the bond will comprise of two parts (i) the quoted price of the naked and (ii) the amount of accrued interest i.e. the amount of interest that has accrued on the bond from the date of the last coupon payment to the point at which the settlement is made of the trade between X and Y by the relevant exchange.

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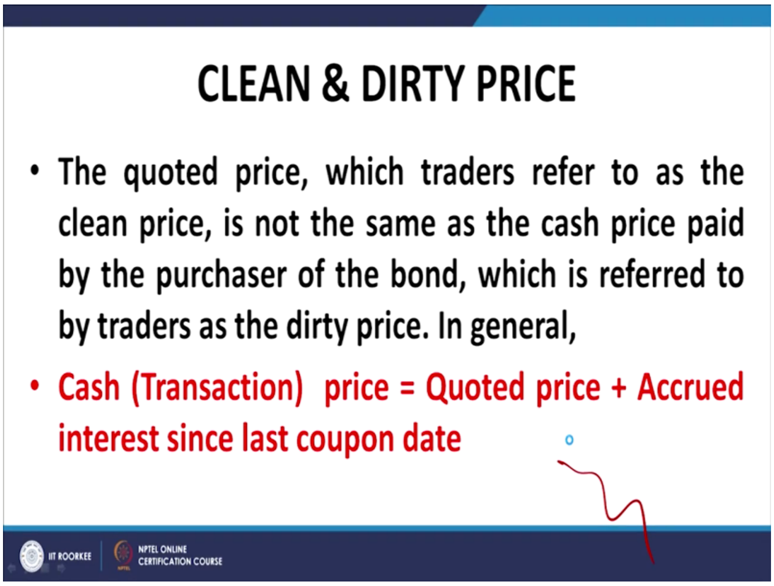

The total price of the bond i.e. the transaction price is called the dirty price of the bond and the price at which the naked bond is quoted is called the clean price.

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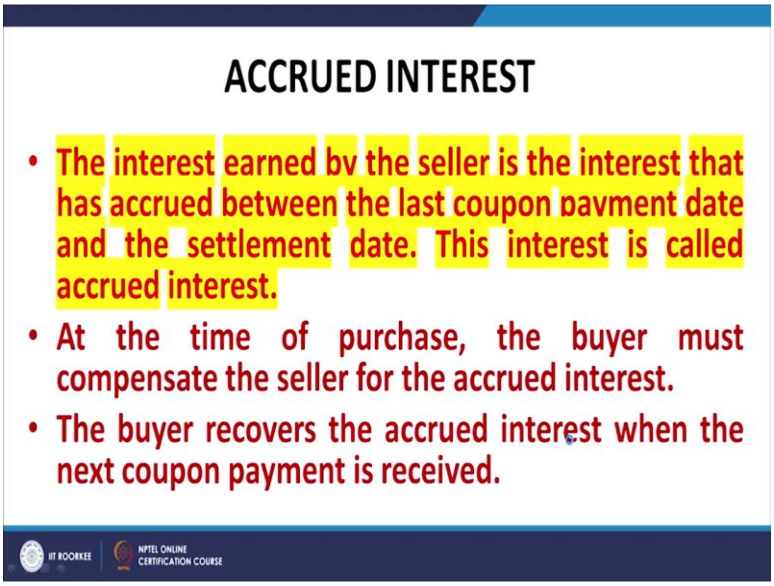

So, let me repeat the definition of accrued interest formally. The amount of interest earned by the seller since the last coupon date to the transaction settlement date i.e. the interest that has accrued to the seller between the last coupon date and the settlement date of the transaction is called accrued interest. Because the buyer is going to recoup this amount of interest from the issuer, (since the issuer will pay the interest for the entire coupon period to the buyer, it will not split up the interest and pay part of it to the seller and part of it to the buyer), he (the buyer) pays up this interest to the seller upfront for the period that the seller has held the bond together with settlement of the transaction. The entire coupon payment will be made to the buyer whose name appears in the register of bondholders. Thus, it is the obligation of the buyer to reimburse the interest for the period of the holding of the bond by the seller (AC) and that is called the accrued interest. At the time of purchase, the buyer must compensate the seller for the accrued interest and the buyer recovers the accrued interest from the company at the next coupon date.

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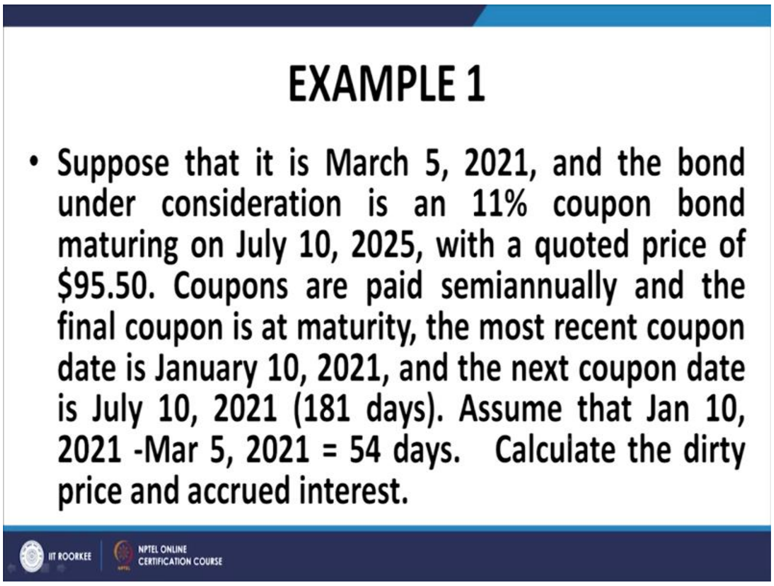

Let us take an example to illustrate what I have been saying. Suppose that it is March 5, 2021, and the bond under consideration is a 11% semi-annual coupon bond maturing on July 10, 2025, with a quoted price of 95.50. This is a clean price of the bond. Coupons are paid semi-annually and the last coupon and final coupon is at maturity that is on July 10, 2025.

The most recent coupon date is January 10, 2021, and the next coupon date is July 10, 2021. It is also given that the interval between January 10, 2021, and July 10, 2021 is 181 days. And that the interval between January 10, 2021 and March 5, 2021 is 54 days. We are to calculate the dirty price and the accrued interest. The clean price of the bond is given to us which is 95.50.

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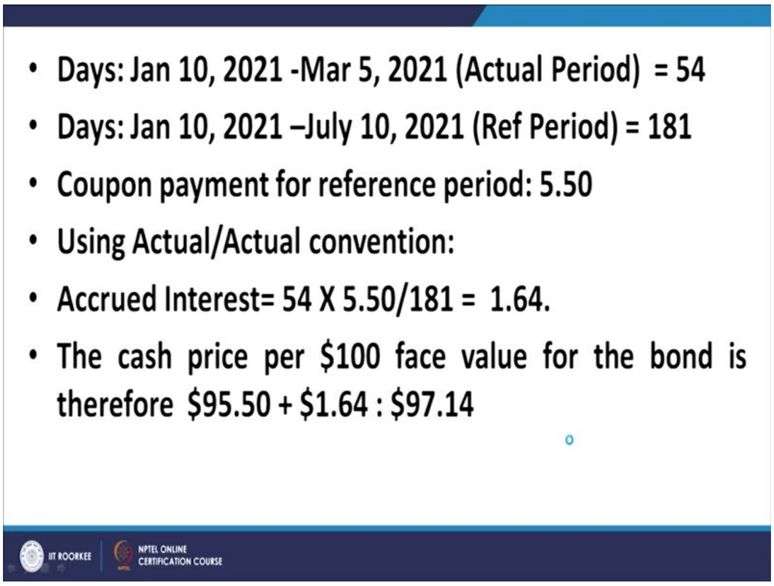

So, the total days between January 10, 2021 and March 5, 2021, which is the part of the coupon period (January 10, 2021 to July 10, 2021) during which the bond is held by the seller is given as 54 days and the total days in the coupon period (January 10, 2021 to July 10, 2021) are given as 181 days. Both these figures are explicitly given in the problem. So, we do not have to work out the number of days in either case. The coupon payment for the semi-annual period of 181 days (that is from January 10, 2021, to July 10, 2021), is half of the coupon rate (which is 11% p.a.). Therefore, the amount of interest for the full coupon period on INR 100 face value of bond is equal to 5.50. Now, using the actual upon actual convention, we allocate the proportion of interest out of this 5.50 to the selling party as 5.50*54/181 = 1.64. This amount will be paid bby the buyer to the seller upfront to gether with the quoted price at the settlement date. Of course it is the buying party that would get the full interest of 5.50 from the issuer on the next coupon date of July 10,2021. The seller will be reimbursed by the buyer on transaction settlement and the issuer company will reimburse the buyer. Therefore, the settlement price of the bond and the cash received by the seller of the bond would comprise of the quoted price which is 95.50 and the accrued interest which is 1.64. So, the dirty price of the bond turns out to be 97.14. It is quite simple, but a bit technical and this is the convention that is usually followed in the bond market.

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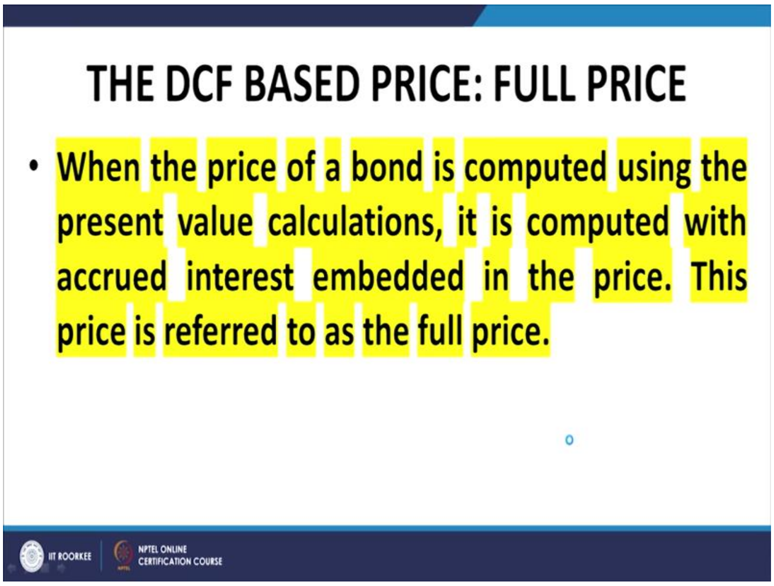

Now a word about the DCF based price which is also termed as the full price. When the price of a bond is calculated using the present value calculations, it is computed with accrued interest embedded in the price. This price is referred to as the full price. Let us try to understand this with an example.

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FACE VALUE	1000
COUPON	10%
REQUIRED RATE OF RETURN	12%
NEXT & LAST COUPON	31 12 2021
01 01 2021	982.1429
01 04 2021	1010.367
01.10.2021	1069.272

Let us consider a bond which has a face value of 1,000. The coupon rate is 10% p.a. payable annually. The required rate of return on this bond is given as 12% p.a. and the next and last coupon payment on this bond is on 31.12.2021. So, the final payment on 31.12.2021 would be the redemption of principal, which is assumed at par of 1,000, and the last coupon payment of 100 (that is 10% p.a. of 1,000) aggregating 1,100. So, the last payment would be 1,100 as on 31.12.2021. Naturally, a bond if valued as on this date, that is 31.12.2021, would be worth 1,100. Let us now look at the valuation at earlier dates. If we value this bond on the 1st of January 2021 using the required rate of return of 12% p.a. and the final cash flow of 1,100 at 31.12.2021, we get a value of 982.1429. If we value the same bond at 1st April of 2021, we get a value of 1010.367. And if we do the same valuation on 1st of October 2021, we get a value of 1069.272.

Now, the important thing that we find is that the value is increasing. Why is the value increasing like this? We discussed sometime back that as the maturity of the bond approaches, the value of the bond must approach its redemption value. Of course, if there is a coupon that coupon also has to be accounted for. Now, in this case, again we are finding this phenomenon. The bond is quoted at a discount and gradually as the maturity of the bond is approaching the value is increasing. But we need to address a different issue here. What we find is that the amount that is being discounted, I repeat, the amount that is being discounted (that is 1,100) is remaining constant irrespective of the point at which the valuation is done. However, as we do the valuation later and later, we come closer to the cash flow that is likely to occur and as a result of which the discounting impact is reduced. The closer we are to the point at which a particular cash flow is likely to occur, the lesser is the impact of discounting. If we discount a cash flow way distant into the future, we will get a very small value. If we discount the same cash flow very close to the date of occurrence of that cash flow, we will get a value, which is very close to that cash flow. This is natural because the discounting period gets reduced, irrespective of the rate of interest, provided it is positive.

So that is the important point, while the coupon payment remains constant, as we approach a coupon payment date, the discounting process results in a multiple which is closer to 1 and as a result that discounting effect is reduced as we approach the coupon payment.

But the important thing here is that the coupon itself is not proportionately reduced. For example, it is not the case that when we were discounting the bond on January 1, 2021, we use the full year's coupon payment and when we discount the bond on April 1,2021, we reduce the coupon payment proportionately for 3 months. We do NOT do this. We are still discounting the entire coupon payment of 100. However, because we have now moved forward in time, we are closer to that coupon payment, the discounting effect is reduced as we can see from the figures that we have obtained.

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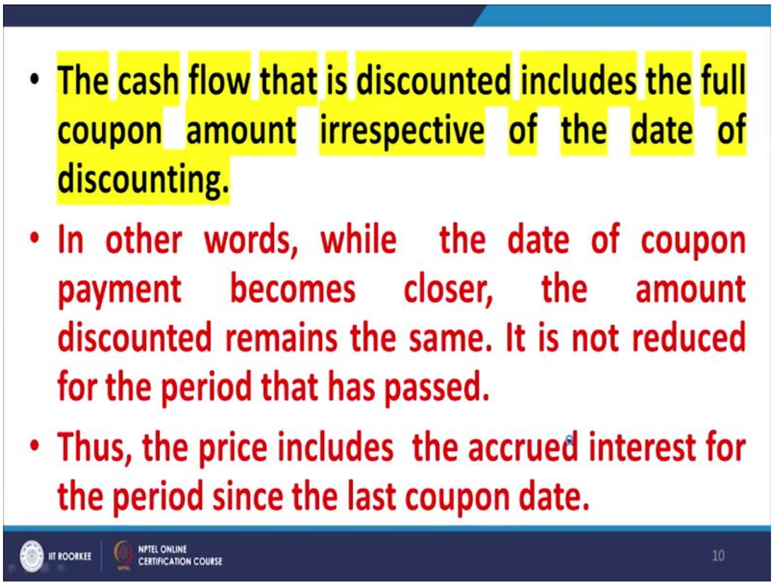

So, that is important. The cash flow that is discounted includes the full coupon amount irrespective of the date of discounting. That is the important part. ***The coupon amount is not proportionately reduced with the reduction in the discounting period.*** The time remaining to the coupon payment goes on reducing, but the amount of the coupon does not get reduced.

In other words, while the date of the coupon payment becomes closer, the amount discounted remains the same, it is not reduced by the period that has elapsed. Thus the price includes the accrued interest for the period since the last coupon date.

Because the coupon payment is not reduced and the coupon is discounted at its full value, the price that we arrive at includes the accrued interest for the period which has elapsed since the last coupon date.

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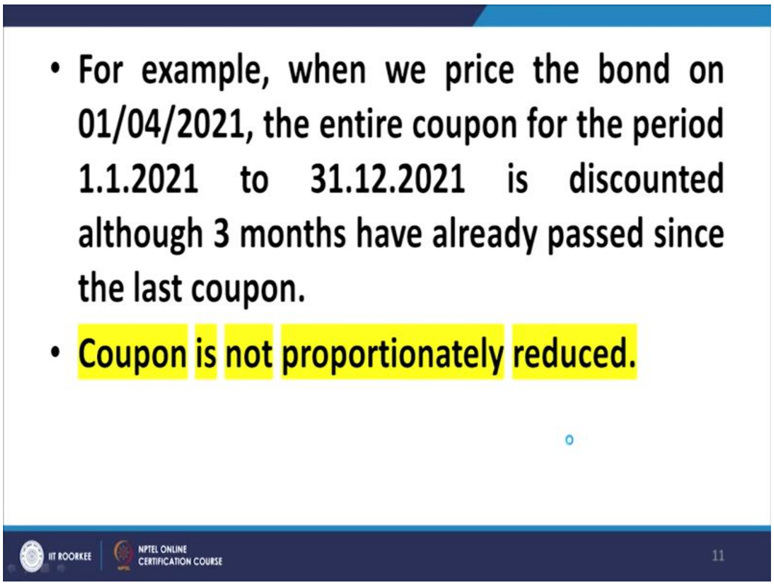

And this is the example that I have alluded to just now, the coupon is not proportionately reduced, let me reiterate the statement.

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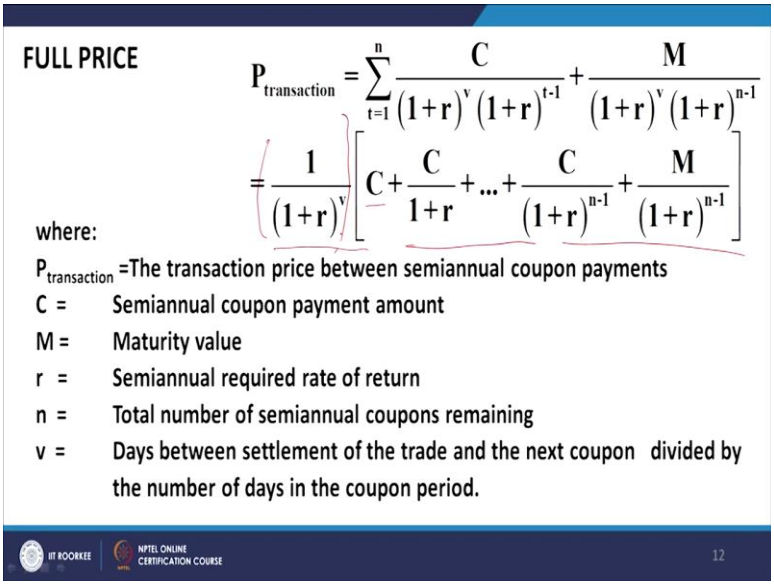

The full price of the bond, as per the cash flow discounting formula, is given by the formula that is here on the above slide. This is the formula that is used to work out the discounted cash flow price or the full price of a bond. Now, please note this point, the expression within the square brackets is the value of the bond as on the next coupon date, immediately before the coupon payment is made. Why that is so? Because of the presence of the coupon C in the calculation of the price.

I repeat the value that is arrived in the expression in the square brackets is equal to the value of the bond immediately before the next coupon payment. This value is then discounted back for the period which is remaining to the next coupon payment by the factor that appears in front of the square brackets, which now I have encircled in the round bracket.

So, this factor that appears as a pre-factor to the expression in the square brackets does the job of discounting the DCF value as on the date of the next coupon payment to the date on which the valuation is done. But again I reiterate, the full coupon payment is being discounted. The C is the full coupon payment or the coupon payment relating to the full period since the last coupon date.

This coupon payment C relates to the full period since the last coupon date, although the discounting is done for the period up to which the transaction is done or to the point at which the value is calculated. So, this formula reiterates the example that we discussed just now, that while the coupon payment is for the full coupon period, the discounting is for a period which is less than the full coupon period if the point of discounting is in-between two coupon periods. As a result of this, the price that we arrive at includes the accrued interest for the period which has elapsed since the last coupon date.

Now, we come to a very interesting and a very intriguing topic, which is usually dealt marginally in books on security analysis and portfolio management. Therefore, I propose to deal with it in some detail to make this course worthwhile. I will talk in detail about the concept of interest rate risk. First of all, I will discuss the concept of interest rate risk, then I will move to measures of interest rate risk and I will end up with the management of interest rate risk. So, let us start.

Now let me get into this concept by virtue of an example. As I discussed in a lot of detail, when I talked about the yield to maturity, one of the fundamental assumptions that goes into calculation of YTM is that the holding period equals the maturity of the bond. The implication of this particular assumption is that the investor recovers the redemption value of the bond (which is known at the time of issue of the bond) on the date of liquidating his investment. As a result of this, there is no impact of the market as far as the value of the bond on the end of the investment horizon is concerned.

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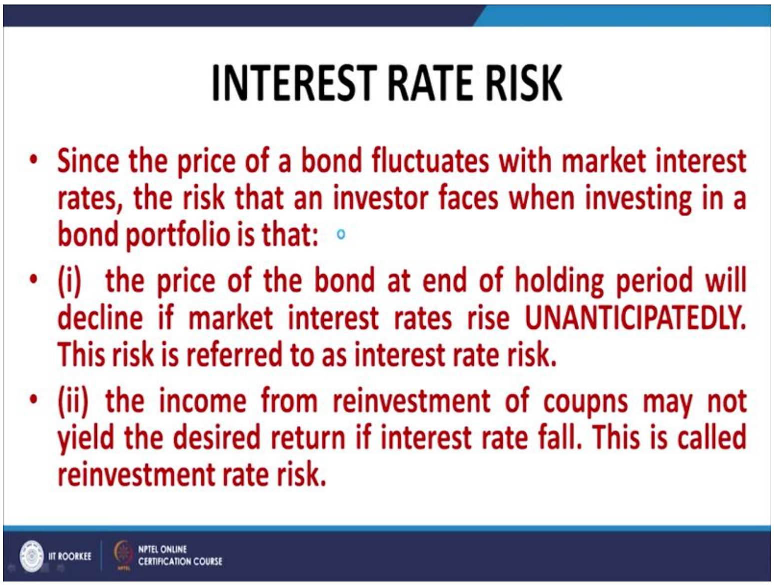

Therefore, when we work out the YTM, we are not bothered about the market value of the bond. Subsequent fluctuations in market value after acquisition of the bond do not impact the YTM at which the investment was entered.

Let us now relax this assumption. Let us say that the holding period of an investor is less than the maturity of the bond. Let us say that the maturity of the bond is 5 years and the holding period of the investor is 2 years. Now, what happens? What will happen is that at the end of 2 years, the investor has to liquidate the investment by selling it in the market to exit the investment. Obviously, this has to be done at the then prevailing market price. Keep this at the back of your mind. Now suppose after I buy a bond in the market, the market interest rates change. Let us say the market interest rates go up. What would be the impact as far as I are concerned, assuming that I have a holding period of 2 years and not 5 years, which is the life of the bond. Now, because the interest rates have increased, I will be able to reinvest the coupon that I get at t=1 i.e. at the end of the first year at a higher rate. So, the re-investment income that I will get at the end of my holding period would be more.

But what about the price? Now, if I liquidate the investment at t=2 years, then the price that the bond would fetch would be such that the buyer of the bond gets a YTM equal to the new market interest rate, which is higher. Let me repeat this very important point. *The price that I would get by liquidating my investment in the market would be the price at which the buyer of* *the bond would get a YTM based on his purchase price equal to the new market interest rate, that is, the increased market interest rates.*

As a result of this what will happen? The market price that I would get on liquidating the bond would be lower. Therefore, the capital gains that I had anticipated (if any) would decrease. Thus, there would be two impacts on the return on holding the investment for a period which is not equal to the date of maturity of the bond.

Firstly, if the interest rates increase, the reinvestment income will increase and if the interest rates decrease, the reinvestment income will decrease. This part is called the reinvestment rate risk. If there is an increase in the market interest rates, reinvestment income increases and if there is a decrease reinvestment income decreases. In other words, the reinvestment income is a function of the market interest rates and would fluctuate depending on the market interest rates that prevail during the holding period. The impact of these fluctuation on the investor's return is termed as the reinvestment rate risk.

The second part is the capital gains. If there is an increase in market interest rates, as I explained just now, the market price of the bond at the point of liquidation would decrease and conversely, if there is a decrease in the market interest rates, the market price at which I liquidate the bond would increase. Thus, the return on account of capital gains that I get at the end of my holding period would be dependent on the market interest rates and would fluctuate in response to the change in the market interest rates.

In other words, any change in market interest rates would impact my returns on two counts, (i) the reinvestment income, this is called the reinvestment rate risk and (ii) the capital gains, this is called the interest rate risk. So reinvestment rate risk and interest rate risk both relate to fluctuations in the market interest rates on account of holding a bond for a period which is less than the maturity of the bond.

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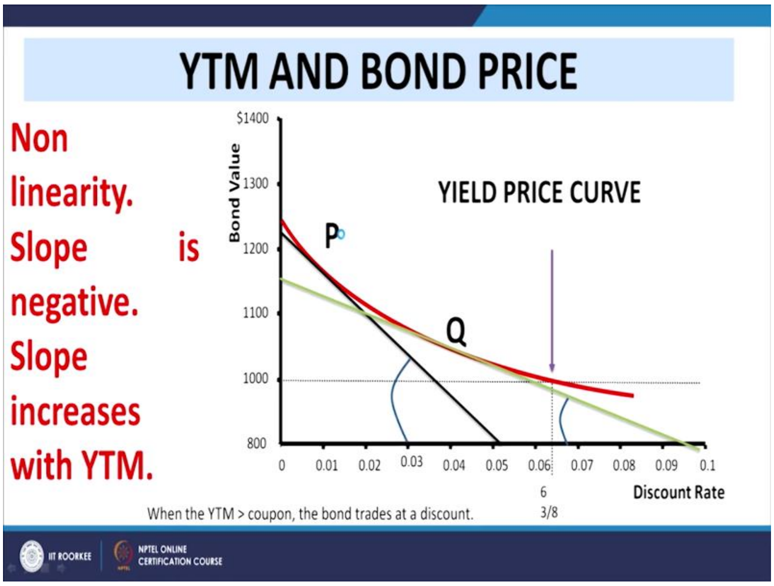

Now, let us look at this diagram. We have seen it earlier. This is a plot which depicts the price of the bond as a function of the YTM. I repeat, it is a yield price curve. In other words, it depicts the price of the bond at various values of YTM. The price of the bond is plotted along the Y axis and the YTM of the bond is plotted along the X axis. One thing is very obvious. It is that the price of the bond decreases as the YTM increases and vice versa. We also know that YTM is a proxy for or an averaging of the market interest rates. So, as the market interest rates increase, the price of the bond decreases and vice versa. This is very much apparent. If you look at the slope of the curve at the point P and Q, both of them are negative, dP/dy is negative at both P & Q. In fact, it is so at all points on the curve. As a result of this, we again infer that price and YTM are inversely related.

Howsoever we may like to infer, the diagram itself is depicting that the YTM and price are inversely related. We will come to a quantification of this in a minute. So, the first inference is that the slope of the yield price curve is negative. Now if we look at the slope of the curve at the point P and compare it with the slope of the curve at point Q, we find that the curve has flattened out. What does it mean? It means that the magnitude of the slope has decreased.

I reiterate, I have used the word magnitude. So, because the slope is negative and the magnitude of the slope has decreased as we move from the point P to the point Q, we can say that as the YTM increases the slope has actually increased. Decrease of the magnitude of a negative quantity means it has actually increased.

So, as we move from the point P to the point Q i.e. as the YTM increases, the slope of the curve is i.e. the first derivative is negative. However, the second derivative is positive. So, we arrive at two conclusions, (i) the slope or the first derivative of the yield price curve is negative throughout showing that price decreases as YTM increases, and (ii) the slope of the slope or the second derivative is positive throughout showing that the slope increases as YTM increases. Let us now move to the next slide.

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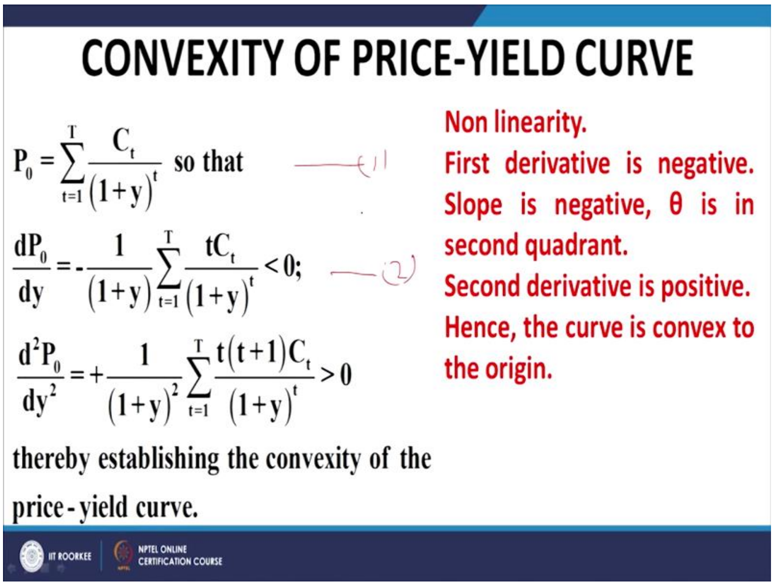

Now, this is the quantification of the results that I have just mentioned. We start with the expression for the market price in terms of the YTM i.e. $\mathbf{P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$. If we differentiate this expression with respect to y, keeping C_t as constant (because it is given in the contract of issue) we get: $\mathbf{\frac{dP_0}{dy} = -\frac{1}{(1+y)} \sum_{t=1}^{T} \frac{tC_t}{(1+y)^t} < 0}$. We find that this is clearly negative for all values of y,t>0. We are assuming as usual that the bond follows a conventional cash flow stream, that is the first cash flow is negative at t=0 which is equal to the price at which the investor enters the investment followed by a stream of positive cash flows.

So, we find that the first derivative is negative as explained on the basis of the figure that we just saw. We also find that the second derivative turns out to be positive. $\mathbf{\frac{d^2P_0}{dy^2} = +\frac{1}{(1+y)^2} \sum_{t=1}^{T} \frac{t(t+1)C_t}{(1+y)^t} > 0}$. Again, this is a straightforward differentiation with respect to y and we find that this second derivative turns out to be positive for all values of positive y, t given the conventional set of cash flows.

So the results that we arrived at on the basis of a perusal of the figure are vindicated by explicit mathematical calculations. I repeat, the slope is negative but the second derivative is positive. The slope is increasing, because it is negative and it is decreasing in magnitude.

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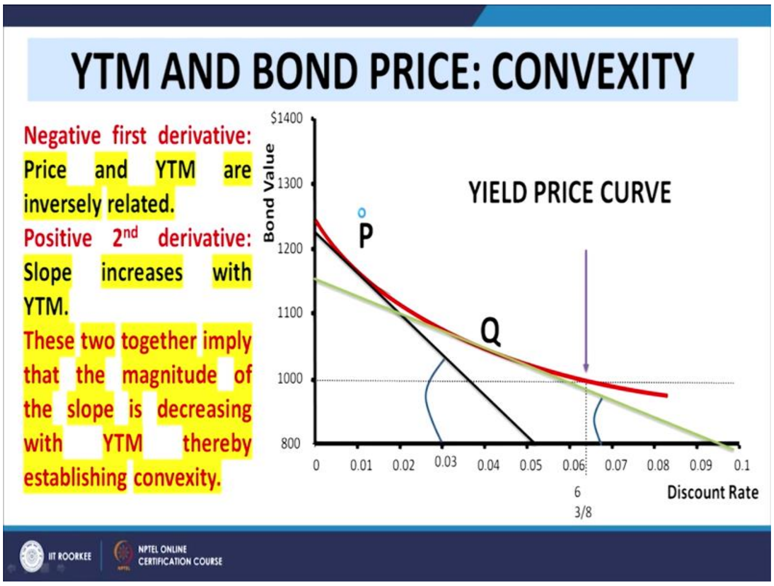

And so, these are the conclusions. I have reiterated the conclusions in this slide which I have explained in the last few minutes.

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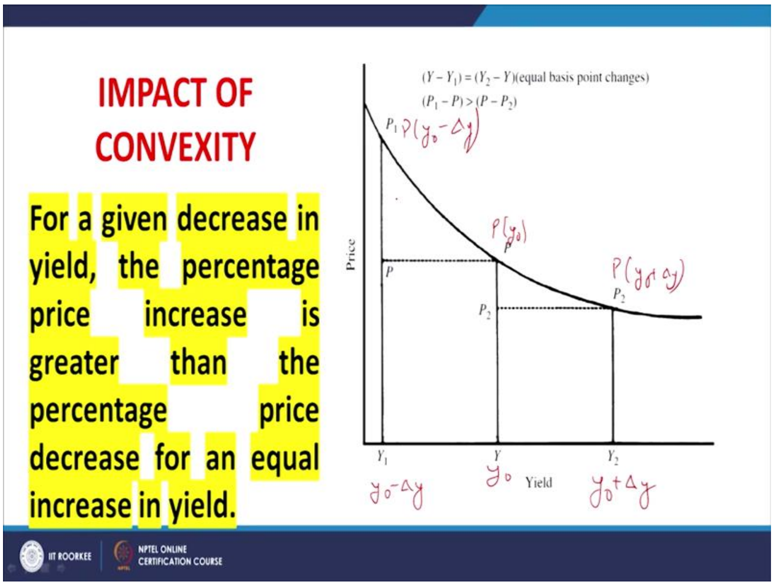

What is the impact of convexity. Now that is a very interesting. Look at the figure that is on the right hand side of your slide. What we find is that if the value of YTM (i) increases by a small amount Δy from a point y_0 with a corresponding change in price from $P(y_0)$ to $P(y_0+\Delta y)$ and (ii) decreases by a small amount Δy from a point y_0 with a corresponding change in price from $P(y_0)$ to $P(y_0-\Delta y)$ then (because of the convexity of the yield-price curve) we must have: $P(y_0-\Delta y)- P(y_0)> P(y_0)- P(y_0+\Delta y)$.

So, what do we conclude: For a given decrease in yield, the percentage price increase is greater than the percentage price decrease for an equal increase in yield. In other words, $P(y_0-\Delta y)- P(y_0)> P(y_0)- P(y_0+\Delta y)$.

This is attributed to the convexity of the yield price curve towards the origin, which is explained by the fact that the first derivative is negative and the second derivative is positive.

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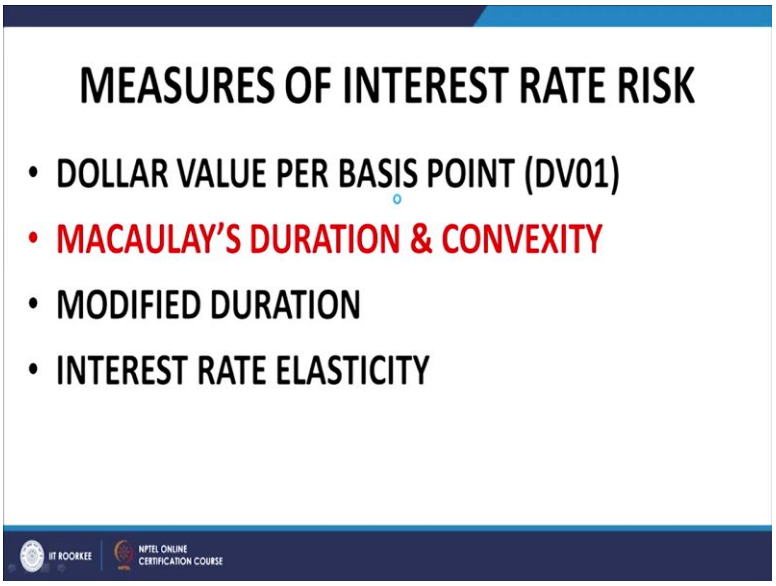

Now, we come to the measures of interest rate risk. There are four common measures of interest rate risk and they are (i) dollar value per basis point (DV01) (ii) Macaulay's duration and convexity (iii) modified duration and (iv) interest rate elasticity. I shall take each of them in detail.

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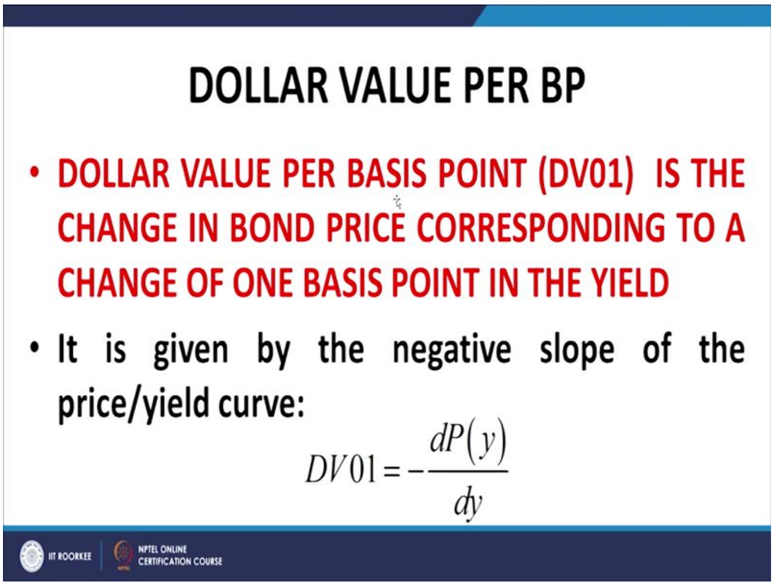

As far as dollar per basis point is concerned, it is quite simple. Dollar value per basis point (DV01) is the change in the bond price corresponding to a change of 1 basis point in the yield. I repeat, dollar value per basis point (DV01) is the change in the bond price corresponding to a change of 1 basis point in the yield. It is given by the negative slope of the yield price curve.

Now, please note an important point. We use, by convention, a negative sign in front of the derivative. The reason for this is that we want to return a positive number. The relationship between the yield and the price of a bond is invariably inverse and as a result of this the derivative dP/dy is always negative. But for convenience, we prefer a positive number. As such we introduce a negative sign in the very definition itself. So, DV01 will be returning a positive number. Nevertheless, we must understand that there would be a decrease in price corresponding to an increase in YTM for a level coupon bonds with conventional cash flows. Now, we come to the concept of duration. This is the most important concept here. I will explain this in detail after the break. Thank you.