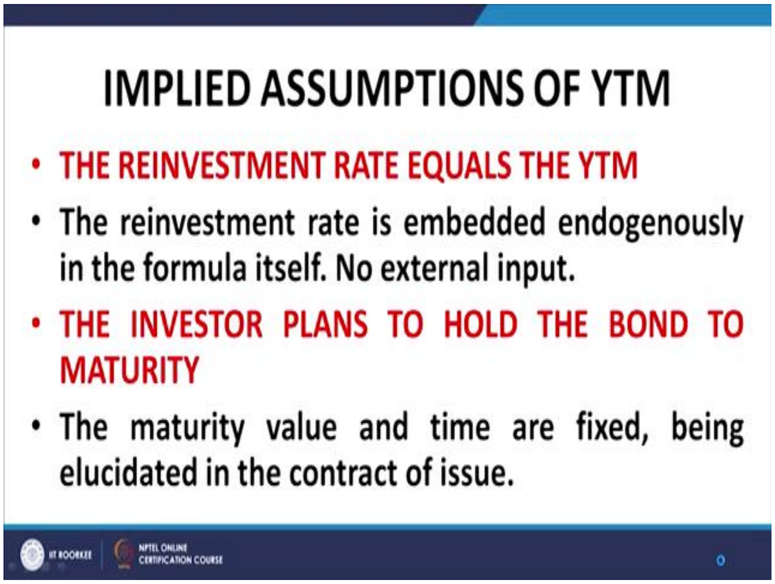
Security Analysis & Portfolio Management
Professor J P Singh
Department of Management Studies
Indian Institute of Technology, Roorkee
Lecture 15
Yield To Maturity V

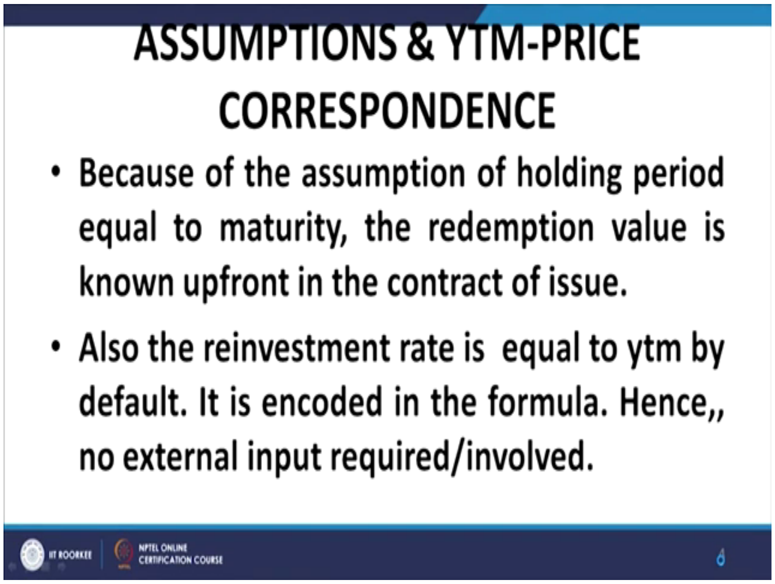
Welcome back. So let us continue from where we left off. But before we do that, a recap of the important issues relating to YTM.

(Refer Slide Time: 00:40)



The assumptions of YTM. The first assumption that is incorporated into the YTM formula is that all intermediate cash flows are deemed reinvested at the YTM rate. This assumption is there by default. There is no design in it. The formula for YTM is based on this premise. This is implicit in the formula itself. Secondly, because we sum the cash flows up to maturity, there is the assumption that the bond would be held up to maturity. All the cash flows arising from the bond up to the maturity of the bond are discounted when we work out the YTM.

(Refer Slide Time: 01:13)



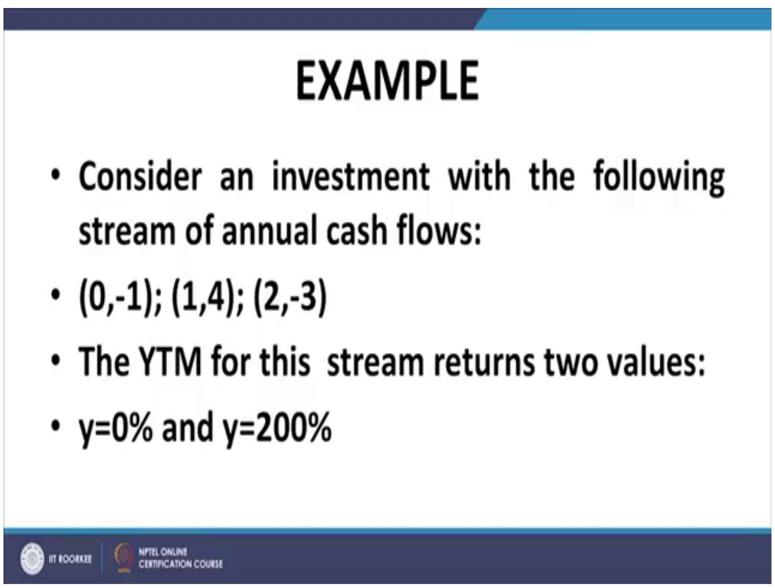
Because of these two assumptions, we isolate ourselves from the environment. What I explicitly mean is that, because the reinvestment rate is equal to the YTM by default, there is no external input involved there. At the same time because the bond is deemed to be held to maturity, the redemption proceeds equal the maturity value of the bond and is known upfront in the terms of issue or the contract of issue. Therefore there are no external inputs, except for the market price, at the point in time at which we are calculating the YTM. Even that is explicitly known, because the price at which the security is traded is known at the point at which we are working out the YTM. Because the price constitutes the only external input into the formula, there is a 1-1 correspondence between the price & YTM of a given bond. A given cash flow pattern will return a unique YTM corresponding to a given price. The relationship between the price and the YTM is that of one to one correspondence for a given cash flow pattern. In other words, the YTM is uniquely determined by the price of the instrument.

However, this is subject to a caveat. The caveat is that in explaining this particular point that there is a one to one correspondence between the YTM and the price of a security, we have implicitly assumed that the cash flow pattern arising from the instrument is one of conventional nature. What exactly we mean by conventional nature? A conventional cash flow pattern relating to a security, is one in which we pay the price upfront and then we get the returns from the security, Thus, there is a single cash outflow (on account of the price payment) followed by a sequence of cash inflows (representing coupons & redemption value) over the remaining period or the remaining life of the instrument. In other words, there is one cash outflow at the point we take up the investment position, and thereafter the investment rewards you with cash inflows over the life of the instrument. This is a conventional cash flow pattern, which we assume when we talk about YTM and other measures of yield.

However, if there are multiple sign changes in the cash flow pattern i.e. if there is a cash outflow to start with and then a sequence of cash inflows and then again a cash outflow (for example, payment of call money or such similar payments) to the issuer of the security and subsequently some more cash inflows and then again a cash outflow etc, then we may not get a unique YTM.

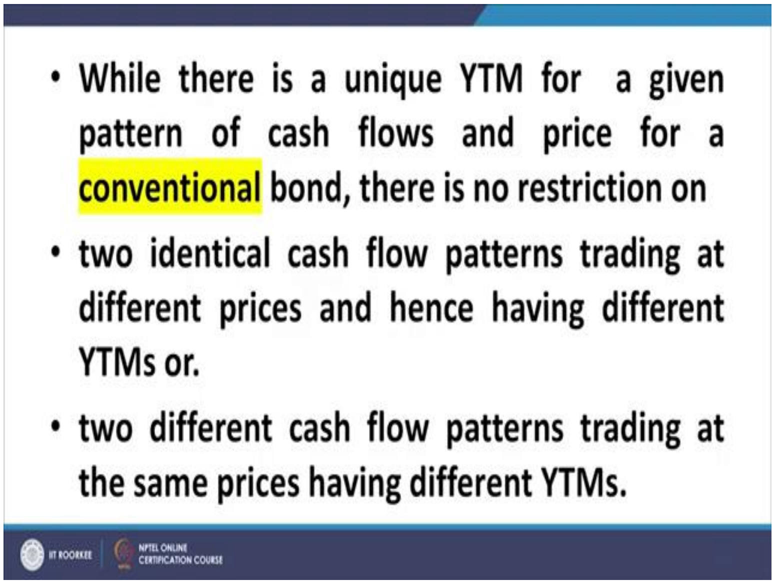
In other words, if the cash flow pattern undergoes multiple sign changes, then the YTM that may be returned for a given price may not be unique.

(Refer Slide Time: 04:30)



To illustrate my point, let us consider the above stream of cash flows. The YTM of this stream returns y=0% p.a. or 200% p.a. So, we end up with having a non-unique YTM for this particular cash flow.

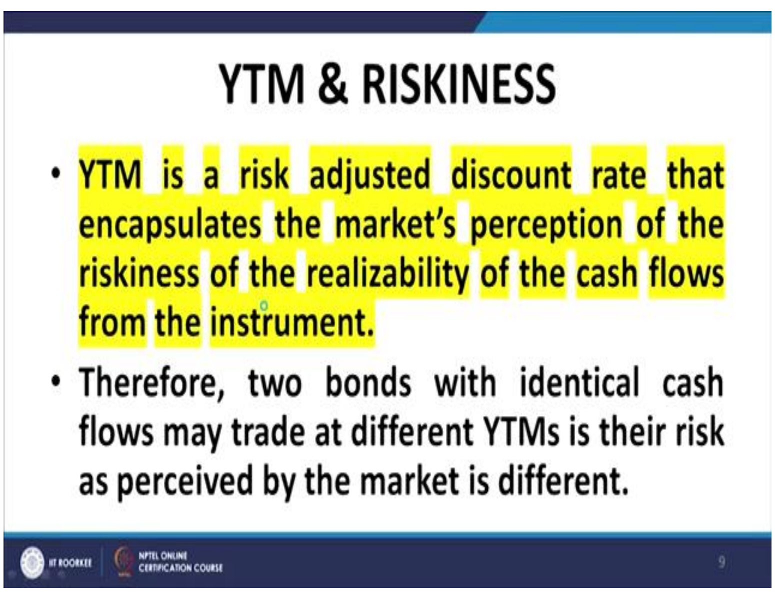
(Refer Slide Time: 05:03)



Therefore, it is necessary to assume that the cash flow pattern relating to the security has only one sign change i.e. it is a cash outflow followed by a sequence of cash inflows.

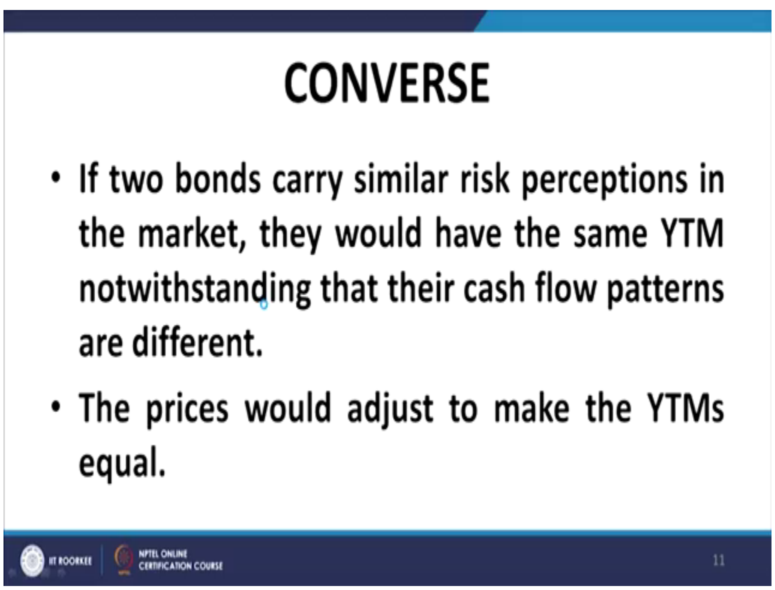
Now, while there is a unique YTM for a given pattern of cash flows and price for a conventional bond, there is no restriction with respect to (i) two identical cash flows trading at different prices and therefore returning differ YTMs, and (ii) two different cash flow patterns having the same price and thereby again returning different YTMs. We come to a very important aspect.

(Refer Slide Time: 05:46)



The important aspect is that YTM is essentially a discount rate. Now, one of the methods for accounting for the riskiness in the realizability of cash flows emanating from an investment is to incorporate the riskiness through a risk premium while arriving at the appropriate discount rate. In other words, the appropriate isk adjusted discount rate that is used for discounting acash flow stream is arrived at by adding a risk premium to the risk free rate. The risk premium is supposed to reflect the riskiness in the realizability of the expected cash flows. ***So, in that sense YTM is also a risk adjusted discount rate.*** It is the discount rate which encapsulates within itself, the risk profile of the investment as perceived by the market because we are working out the YTM on the basis of the market price of the security. So the perception of riskiness that is encoded in the YTM is that of the market or the collective wisdom of the market. Therefore, two bonds which have identical cash flows may very well trade at different prices and return different YTMs if they have differing risk profiles or if the market perceives the risk in realizability of the cash flows to be different. The bond that has a higher riskiness with respect to the realizability of its cash flows would obviously trade at a lower price and return a higher YTM. This justifies the contention that the YTM is a risk adjusted discount rate. Higher the riskiness, higher is the risk premium added to the risk free rate when we work out the YTM of a particular security.

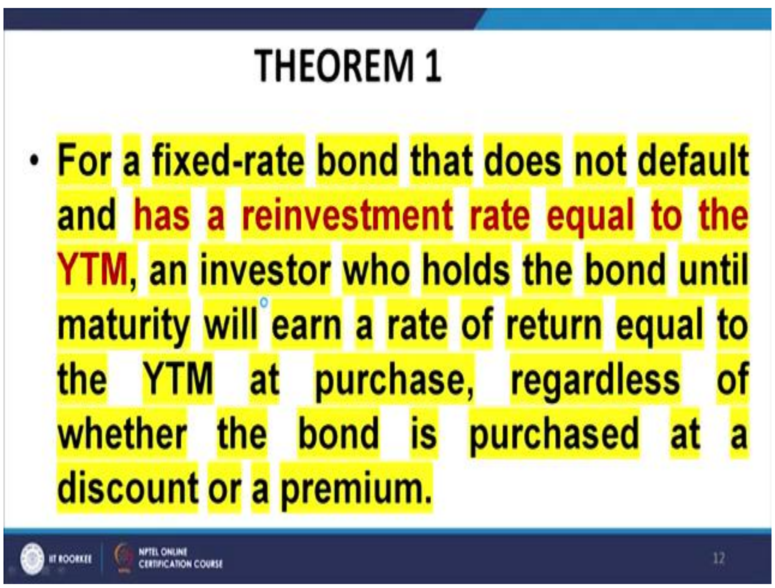
(Refer Slide Time: 07:33)



Conversely, if two bonds carry similar risk perceptions in the market they would be trading at the same YTMs, irrespective of the pattern of cash flows encoded therein. What does that mean? How would this happen? Well this would happen by the price realigning itself in line with the cash flow pattern. Let me give an example. Suppose there is a bond that is quoting at INR 900 and that bond has a coupon rate of 10% p.a. with redemption at the face value of 1,000. Let there be another bond which has a coupon rate of say 20% p.a. with redemption at face value of INR 2,000, and the bonds have similar risk profiles as to the realizability of cash flows. Then the two bonds would be quoting at the same YTM, notwithstanding the fact that the cash flows from the second bond would be double those of the cash flows from the first bond. How would that happen? Well that would happen by the price changing itself to align itself with the cash flow pattern of the second bond compared to the first bond. They would obviously not quote at the same price. The price would align itself in such a way that the YTM on the two bonds remains the same provided the riskiness on the two bonds remains the same.

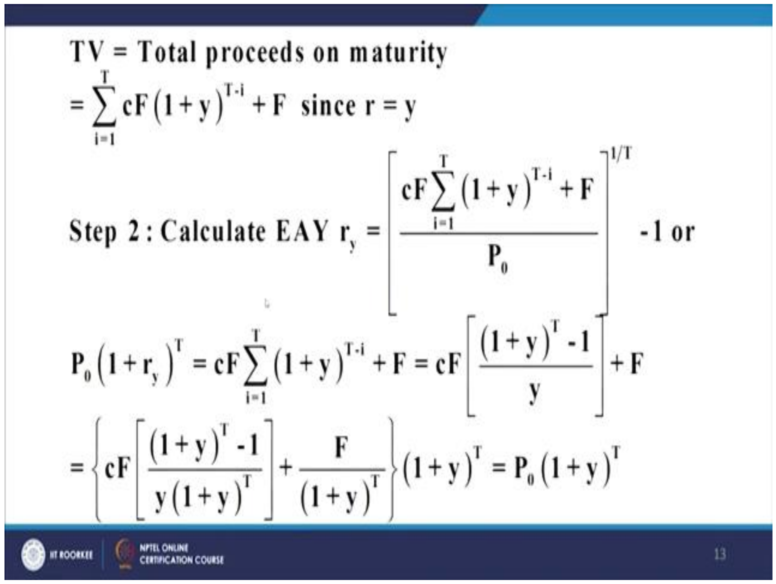
Now we quickly recap the theorems that we did in the last lecture and then move on to some fresh results on the YTM.

(Refer Slide Time: 08:51)



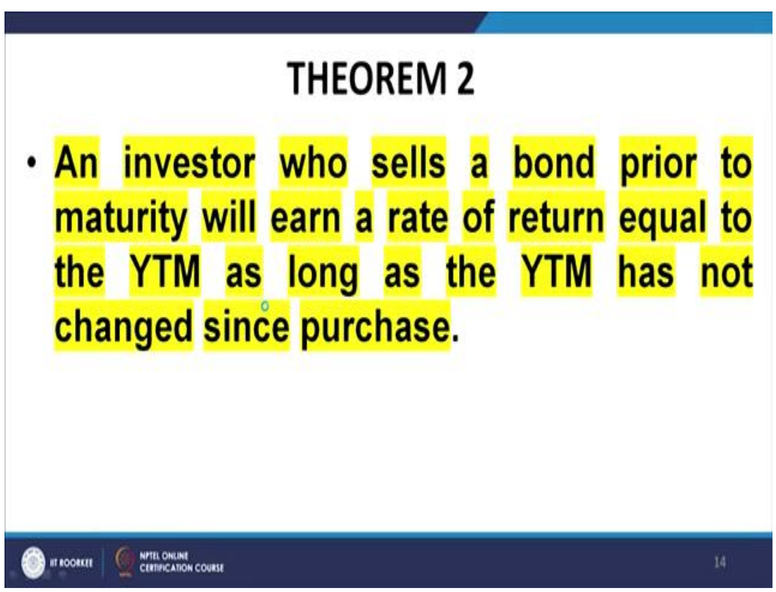
For a fixed rate bond that does not default and has a reinvestment rate equal to the YTM, an investor who holds the bond until maturity will earn the rate of return equal to YTM at purchase regardless of whether the bond is purchased at a discount of a premium.

(Refer Slide Time: 09:20)



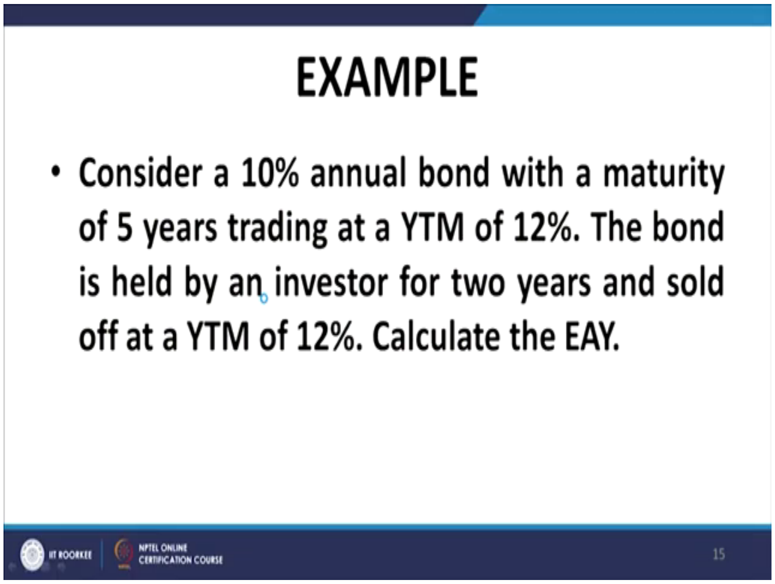
The proof of this is given in this slide. It is quite simple, it involves a little bit of algebra and we find that the effective annual yield on the instrument is equal to the YTM, if the reinvestment rate is equal to the YTM. Obviously this holds for a level coupon bond redeemable at par, as I mentioned in the last lecture also. That being the case, we do infer that if the reinvestment rate is equal to the YTM then the bond's rate of return equals the YTM irrespective of the price of purchase. Please note the entire calculation is independent of whether the bond is quoted at a discount or premium or par, which establishes the theorem.

(Refer Slide Time: 10:11)



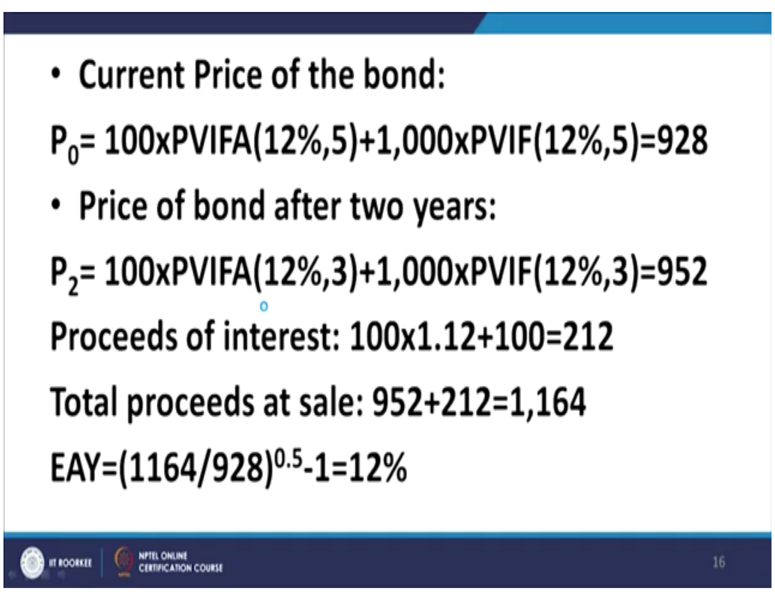
The second theorem, an investor who sells a bond prior to maturity will earn a rate of return equal to its YTM as long as the YTM has not changed since purchase.

(Refer Slide Time: 10:27)



I illustrated this theorem with the above example. Consider a 10% p.a. annual bond with a maturity of five years, trading at a YTM of 12% p.a.. The bond is held by an investor for two years and sold thereafter at a YTM of 12% p.a. That means there is no change in the YTM, no change in the market interest rates during this period of holding of two years.

(Refer Slide Time: 10:53)



We calculate the rate of return to the investor on the basis of his holding period of two years. And after these calculations, we find that the rate of return earned by the investor is also 12% p.a. again establishing the theorem, at least by way of illustration.

(Refer Slide Time: 11:05)

$$P_0\left(1+r_y\right)^H = cF\sum_{i=1}^{H}\left(1+y\right)^{H-i} + P_H$$

$$= cF\left[\frac{\left(1+y\right)^H - 1}{y}\right] + P_H$$

$$P_H = cF\left[\frac{\left(1+y\right)^{T-H} - 1}{y\left(1+y\right)^{T-H}}\right] + \frac{F}{\left(1+y\right)^{T-H}}$$

$$P_0\left(1+r_y\right)^H = cF\left[\frac{\left(1+y\right)^H-1}{y}\right] + cF\left[\frac{\left(1+y\right)^{T-H}-1}{y\left(1+y\right)^{T-H}}\right] + \frac{F}{\left(1+y\right)^{T-H}}$$

$$= \left\{cF\left[\frac{\left(1+y\right)^H-1}{y\left(1+y\right)^H} + \frac{\left(1+y\right)^{T-H}-1}{y\left(1+y\right)^T}\right] + \frac{F}{\left(1+y\right)^T}\right\}\left(1+y\right)^H$$

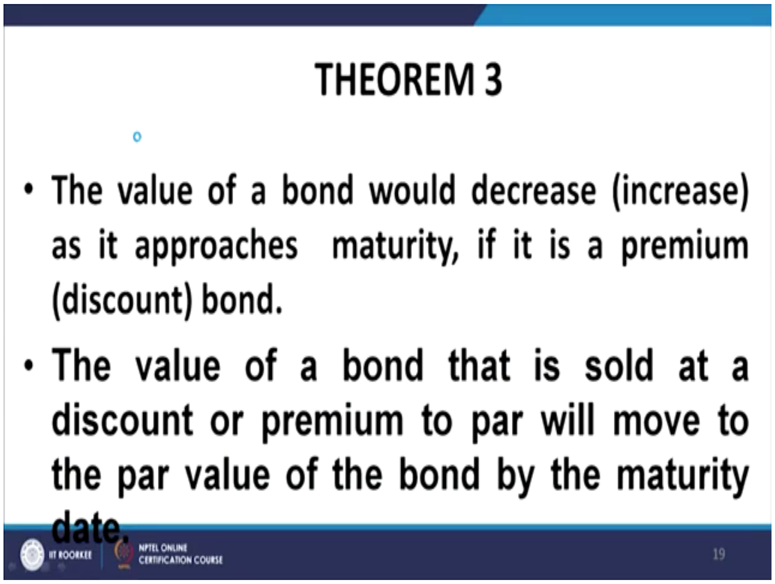
$$= \left\{\underbrace{cF\left[\frac{\left(1+y\right)^T-\left(1+y\right)^{T-H}}{y\left(1+y\right)^T} + \frac{\left(1+y\right)^{T-H}-1}{y\left(1+y\right)^T}\right] + \frac{F}{\left(1+y\right)^T}}\right\}\left(1+y\right)^H$$

$$= P_0\left(1+y\right)^H$$

This is the explicit proof of the theorem. Again it involves a little bit of algebra and explicit expressions for the PVIFA factor at the rate of YTM.. Other than that, there is one issue worth explaining and that is that the term that lies within the curly brackets in the second last equation, is nothing but the price of the bond because y is the YTM of the bond.

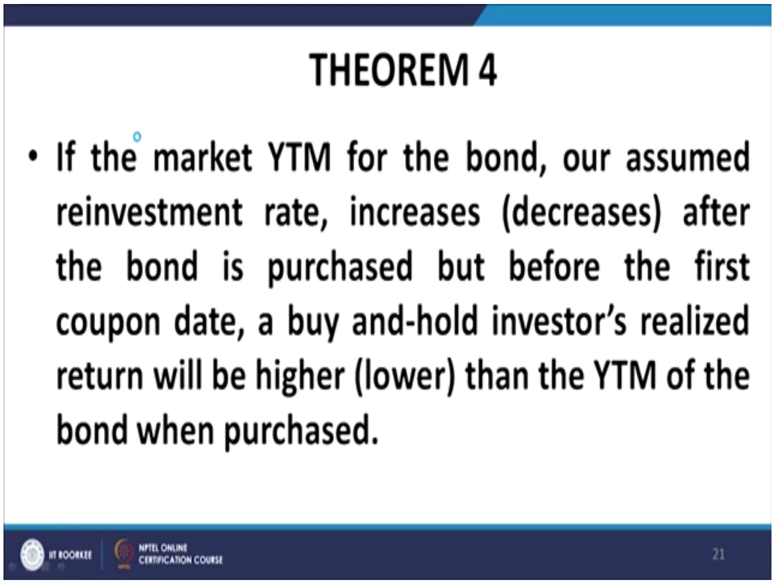
Therefore, by the definition of YTM the expression within the curly brackets gives us P_0 and hence we find that the effective annual yield r_y is equal to the YTM y.

(Refer Slide Time: 12:14)



Then we came to the third theorem. This theorem states that the value of a bond would decrease, (increase) as it approaches maturity if it is a premium (discount) bond. As I emphasized in the last lecture, the value of the bond on the date of its maturity must necessarily converge to its par value if it is redeemable at par. In fact, whatever be its redemption value, it must converge to that redemption value on its date of maturity to ensure no arbitrage, which is usually the underlying principle for financial pricing. Therefore if a bond is quoted at a premium at an earlier date, it must return to a par value (if it is redeemable at par) on the date of maturity. Now, if the amortization of this difference between the price and redemption value (premium) is to be continuous, then at any earlier point in time between $t=t$ and $t=T$ the bond will gradually be amortizing the amount of premium. Therefore the value of the bond would gradually decrease. The premium will decrease and the market price of the bond will gradually decrease correspondingly as the bond approaches maturity. On the date of maturity the bond would be trading very close to its par value if it is redeemable at par. The converse is the case if the bond is quoting at a discount. Thus, we have that the value of a bond that is sold at a discount or premium to par before maturity will move to the par value by maturity date. I have already explained this as part of the previous statement.

(Refer Slide Time: 14:03)

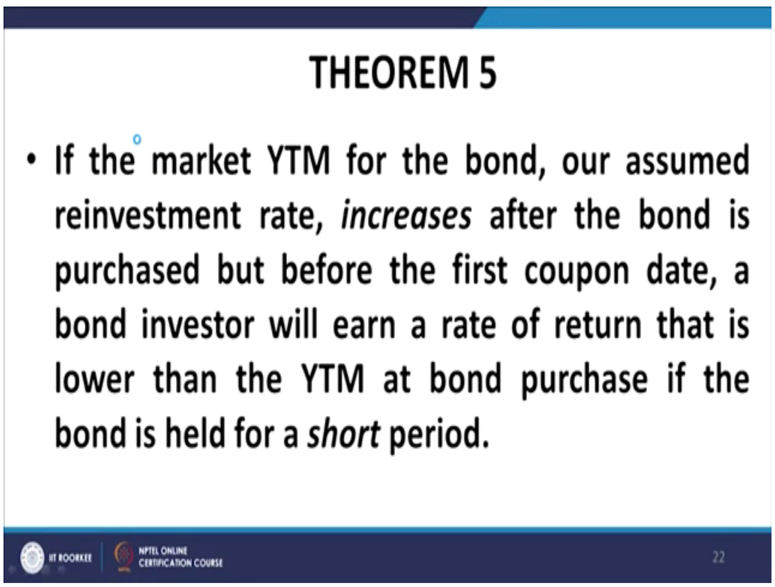


Then we come to the next theorem. If the YTM of for a bond, our assumed reinvestment rate increases (decreases) after the bond is purchased but before the first coupon date, a buy and hold investor's realized return would be higher (lower) than the YTM of the bond, when purchased.

Let me illustrate this theorem with an example. Let us say today we are at t=0 and we buy a bond, say it is a 10% p.a. coupon bond with a maturity of 5 years at 928 which corresponds to the current YTM of 12% p.a.. Very soon thereafter, say at t=0.25 years, before the first coupon payment which is to occur at t=1 year, the YTM increases.

Now if the YTM increases, say to 15% p.a., what will happen? Note that the investor is a buy and hold investor. In other words, he will continue to hold on to the bond irrespective of the increase in interest rates over the life of the bond. Now, because the increase has taken place before the first coupon payment, he can now reinvest all the coupons that he receives from the bond (excluding, of course the last one) at the new rates i.e. 15% p.a .which are higher than the rate at which he had taken the investment i.e. 12% p.a. ***Now please note this important feature that we are talking about an investor who is not going to sell the bond in the market, who is a buy and hold investor. In other words he will continue to hold on to the investment up to the date of maturity of the instrument.*** So clearly the reinvestment income that he gets, the income that he gets on the reinvestment of coupons up to the date of maturity of the bond would be higher than at the earlier YTM of 12% p.a.. Further, the redemption proceeds would not change, because he is holding the bond to maturity. Therefore the redemption value would be the value specified in the issue document. It will not be affected by changes in interest rates. I reiterate, this is because the investor holds the bond to maturity. He is not selling the bond in the market. Thus, the total cash flow that he is going to get on the date of maturity of the bond would be higher at the new YTM of 15% p.a. than the amount that he would have got as per the original YTM of 12% p.a. Therefore, the realized yield over the entire remaining life of the investment, that is, from t=0 to t=5 years for the investor would be higher than what it was when the YTM was 12% p.a.

(Refer Slide Time: 17:33)



Now we come to the next theorem. This theorem says that if the market YTM for the bond, our assumed reinvestment rate, increases after the bond is purchased but before the first coupon date, a bond investor will earn a rate of return that is lower than the YTM at bond purchase, if the bond is held for a short period.

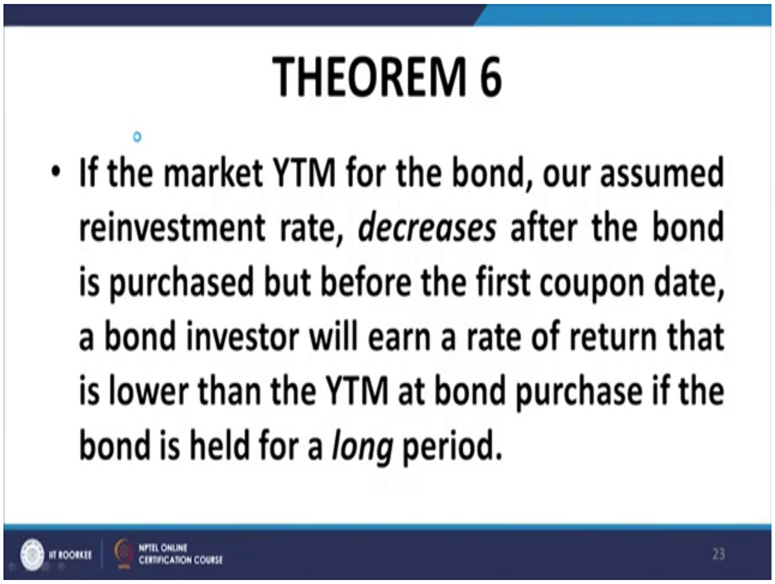
Now let us look at what happens here. Let us continue with the example in context of the previous theorem. We are again considering a bond with a maturity of five years, a coupon rate of 10% p.a. with the current market price of 928 corresponding to a YTM of 12% p.a. Now let us assume that t=0.25, that is, before the first coupon which is to occur at t=1 year, the market interest rates have increased from 12% p.a. to 15% p.a.. ***Now the assumption is that the investor is going to hold the bond for a short period of time.*** Let us say, he holds the bond up to say t=

0.50 years. He disposes off the bond and exits the investment at t=0.50 by selling the bond in the market. Now what happens when the interest rates change from 12% p.a. to 15% p.a. at t=0.25 years? (i) Obviously, this would not affect the reinvestment income of our investor since he holds the bond only upto t=0.50 years. He receives no coupon, so there is no question of any reinvestment of coupons or of reinvestment income. (ii) However, the market price of this bond will fall to the extent that any investor buying the bond at t=-0.50 years (after the YTM has increased to 15% p.a.) will get a rate of return of 15% p.a. upto maturity. Thus, the rate of return earned by our investor holding the bond from t=0 to t=0.50 years would be less than the original YTM of 12% p.a.

It may be noted here that because the interest rates are increasing, the bond has to give a higher rate of return to the new investor. However, if we assume that the new investor holds the bond to maturity, then the redemption value is fixed. The coupon rate is also fixed. So the only thing that can change here is the price of the bond. The price of the bond would decrease so that the new investor who enters the investment at this new price, would get a YTM of 15% p.a. over the remaining life of the bond upto maturity.

Let us review the position of our original investor. He bought the bond at t=0when the interest rates were 12% p.a. and disposed off the bond at t0.50 year, after the bond price has realigned itself. Obviously the price have gone down. Therefore, the return that would be earned by the original investor would be lower than the YTM at which he entered the investment. In other words, the rate of return earned by the investor who had made the investment at t=0 and who exits the investment at t=0.50 would not be 12% p.a. It would be lower. This fall in his return would compensate for the increase in YTM for the investor who enters the investment after the change in interest rates to 15% p.a. So that is how the dynamics would operate. If the investor holds a bond for a short period of time then the decline in prices corresponding to an increase in market interest rates would operate to his disadvantage. If there is an upward shift in YTM then the return that an investor realizes on his short holding period would be lower than the YTM at which he entered the investment.

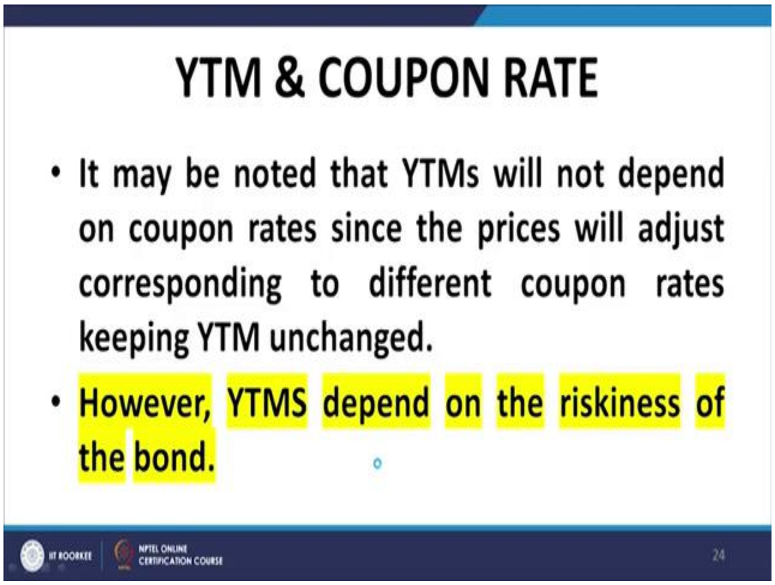
(Refer Slide Time: 22:19)



Next theorem. If the market YTM of the bond, our assumed reinvestment rate, decreases after the bond is purchased but before the first coupon date, a bond investor will earn a rate of return that is lower than the YTM, if he holds the bond for a long period. Continuing with the above example ofa 5-year 10% p.a. bond purchased at t=0 by an investor at 928 corresponding to a YTM of 12% p.a. The investor plans to hold the bond to maturity. Let the market interest rates decline from 12% p.a. at t=0 to 9.00% p.a. at t=0.25. Then, (i) all coupons will be rinvested at the lower rate of 9% p.a. resulting in decline of reinvestment income (ii) the redemption value will be as per the contract of issue and independent of changes in interest rates since the investor holds the bond to maturity. Note here that as far as the benefit of the price escalation due to decrease in YTM is concerned, that benefit also will accrue marginally to the investor. Why? Because he holds the bond up to, or very close to its maturity. Because he holds the bond upto or very close to its maturity, the number of coupons that need to be discounted at the lower discount rate of 9% p.a. would be very few. There will be very few coupons left in the life of the bond which will be necessary for discounting when the investor works out the market price at which he would be exiting the investment. Thus, the total proceeds realized by the investor at maturity would fall due to a fall in YTM and so would the return earned by the investor.

So the net result is that on the one hand the reinvested income declines while on the other the incremental capital gains are marginal. So if that is the case, the overall rate of return that he gets on the investment is lesser than the YTM at which he entered the investment. So we are now clear about the theorems that relate to the YTM.

(Refer Slide Time: 26:21)



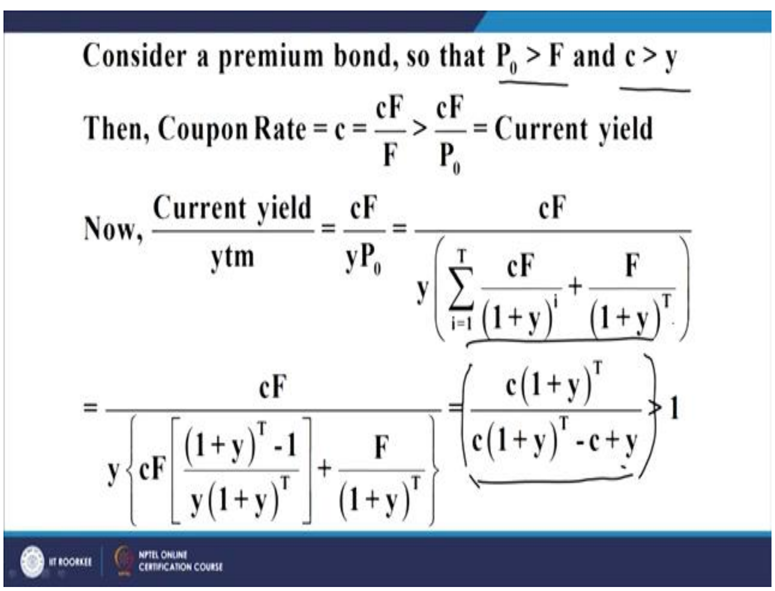
Now YTM and coupon rate. ***It may be noted that the YTMs do not depend on the coupon rates. YTMs depend on the riskiness of the coupons rather than the rates of the coupon. If two bonds carry identical risks in the realizability of their cash flows, they will return the same YTM, even though they have different coupon rates.***

The difference in the coupon rates manifests itself by realigning the prices of the bonds. The prices of the bonds will realign such that if the cash flows of the bond B are higher then the cash flows of bond A and they have the same riskiness then the price of bond B would be higher than the price of bond A to the extent as to make the YTM that they return be the same. So that is an important point. YTM is a discount rate and it is a risk adjusted discount rate. It is a risk adjusted discount rate which is worked out on the market's perception of the riskiness of the underlying cash flows emanating from the bond.

(Refer Slide Time: 27:34)

Now we come to certain relationships which hold for level coupon bonds redeemable at par. (i) For a bond which is quoting at par, the coupon rate= current yield=YTM, (ii) for a premium the coupon rate> current yield>YTM, (iii) for a bond selling at a discount, coupon rate< current yield<YTM.

(Refer Slide Time: 28:00)

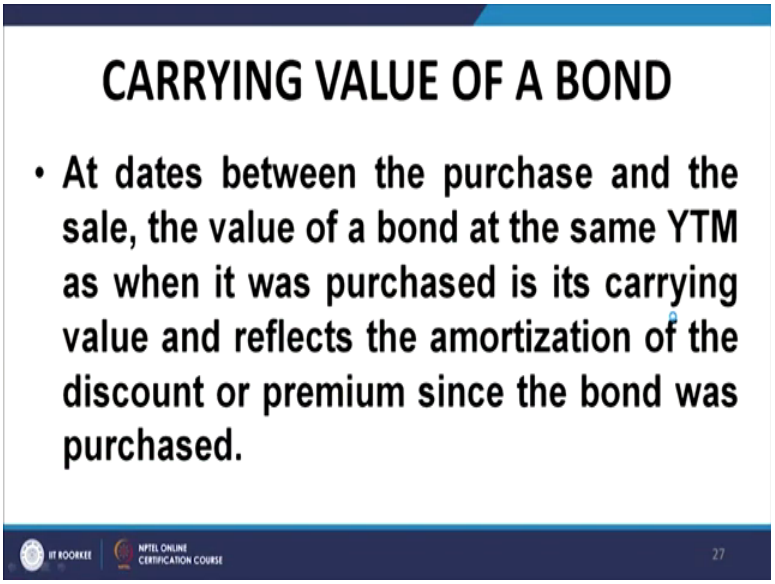


The proof is here on this slide. It is quite straightforward. The only thing that needs explanation is the first step. If we consider a premium bond, the price of the bond is greater than its face value (P_0>F, c>y). We are again talking about the level coupon bond, redeemable at par.

So at least for the premium bond, we have established these results. Absolutely analogous calculations hold for the discount bond and of course for a par bond the calculations are even more easy. All these three measures will return the same value for the par bond. The coupon rate and the current yield and the YTM, all converge in the case of a par bond.

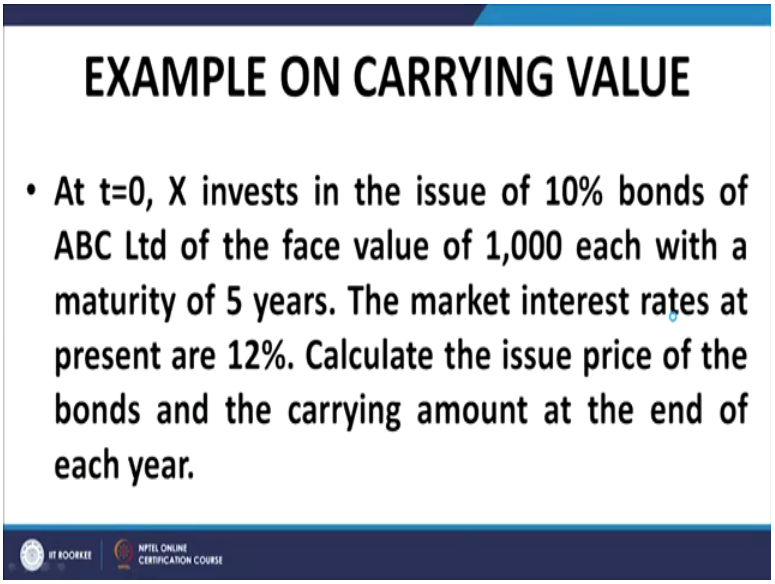
So now we come to the carrying value of a bond. This is again a very important topic. Although it is more related to accounting, we need to be acquainted with this topic, in the context of the accounting for bonds.

(Refer Slide Time: 30:35)



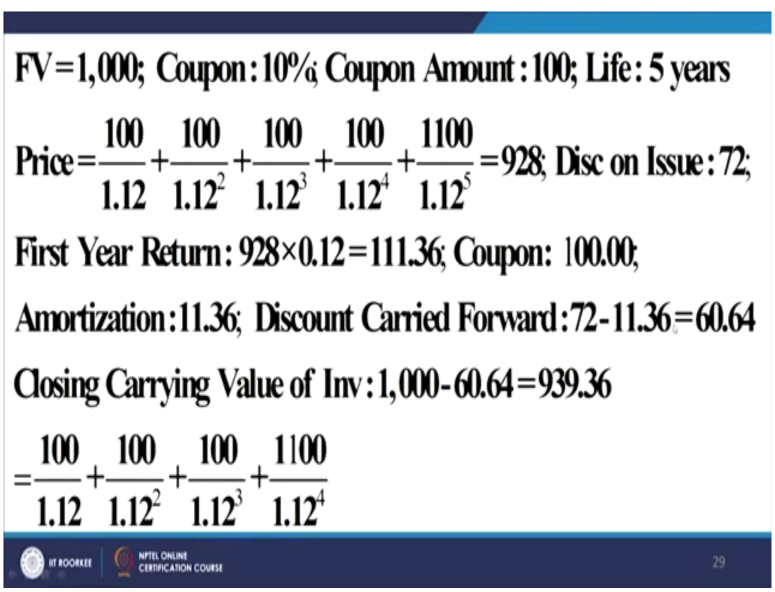
The carrying value of a bond, at dates between the purchase and the sale of a bond, is the amortized value of the bond worked out at the same YTM, at which it was purchased. Let me repeat, at dates between the purchase of the bond and the sale of the bond, the amortized value of the bond at the same YTM, at which it was purchased constitutes the its carrying value. This is the value at which the bond is recommended to be maintained in the books as per the International Financial Reporting Standards. This carrying value reflects the amortization of the discount or premium since the bond was purchased. Now we take up an example on carrying value.

(Refer Slide Time: 31:24)



At t=0, an investor invests in the issue of 10% p.a. bonds of ABC Limited of the face value of 1,000 each with a maturity of 5 years. Let us assume that the current relevant market interest rates are 12% p.a. We are supposed to (i) calculate the price at which the issue is to be made and (ii) calculate the carrying value at the end of each year to be recorded in the books of accounts of the investor.

(Refer Slide Time: 32:05)



Let us now look at the solution. The face value is 1,000, the coupon rate is 10% p.a., the maturity is 5 years and the market rates are 12% p.a. Thus, the current price is obtained as the discounted cash flow price and is 928. We discount all future coupons at the csurrent interest rate.

Please note we are not to discount the cash flows at the coupon rate, but at the relevant market interest rate. This relevant market interest rate represents the required rate of return as perceived by the market, corresponding to the risk class to which the investment belongs. So, we work out the price on the basis of the market interest rate, which works out to 928 as I mentioned earlier.

So the amount of discount at which the bond is issued is 1,000-928=72. The price at which the issue would be marketable is 928. It would not be marketable at 1,000, because people demand a return of 12% p.a. and are not happy with the coupon rate of 10% p.a. that is contracted on this bond. Thus, the investors would not be willing to accept a return of 10% p.a. (i.e. the coupon rate) on this bond, by taking up the bond at 1,000. They would take the bond only at a discounted price, because they want a higher return on this bond since the current prevailing market rates on similar risky investments is higher at 12% p.a. So that being the case the discounting of the cash flow needs to be done at 12% p.a. and we return a price of 928.

We shall continue from here after the break, thank you.