Security Analysis & Portfolio Management

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Lecture 14

Yield to Maturity IV

Welcome back. So before the break I was talking about the assumptions that underlie the calculation of YTM.

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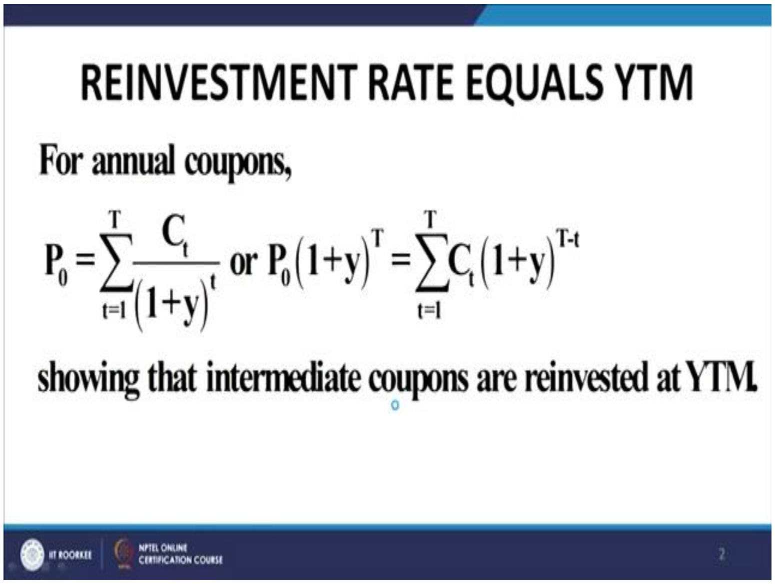

They are (i) reinvestment rates equals the YTM of the instrument, and (ii) the investor's holding period of the instrument equals the maturity of the instrument. Both are encoded in the formula that defines the YTM $\mathbf{P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$ **or** $\mathbf{P_0 (1+y)^T = \sum_{t=1}^{T} C_t (1+y)^{T-t}}$. As you can see from this formula, the reinvestment rate is clearly equal to the YTM. This is evident from the right hand side equation. Further, in the left hand side equation, the summation extends from 1 to T where T is the maturity of the instrument. In other words, we are discounting all the cash flows from the instrument up to the date of maturity of the instrument including, of course, the redemption value. The impact of these assumptions is that the market uncertainty does not percolate down to the YTM calculation. The only market input is the current price, which is known unambiguously.

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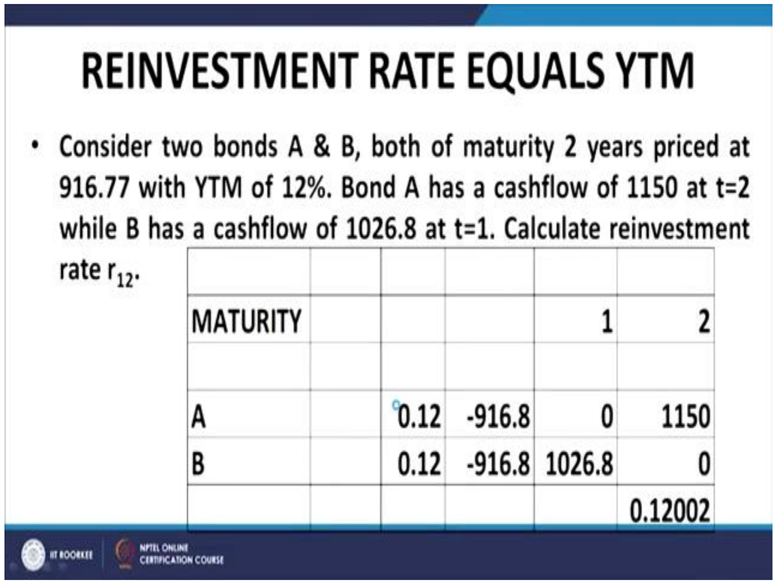

REINVESTMENT RATE EQUALS YTM

- Consider two bonds A & B, both of maturity 2 years priced at 916.77 with YTM of 12%. Bond A has a cashflow of 1150 at t=2 while B has a cashflow of 1026.8 at t=1. Calculate reinvestment rate r_{12}.

MATURITY				1	2
A		0.12	-916.8	0	1150
B		0.12	-916.8	1026.8	0
					0.12002

The fact that reinvestment rate equals the YTM is illustrated by this example. We have two bonds A & B eadh of face value 1,000. Bond A pays 0, 1,150 at the end of year 1 and year 2 trespectively while B pays 1026.80 and 0. Both are quoting at a price of 916.80 corresponding to a YTM of 12% p.a. We construct a portfolio P of A long and B short. The cash flow at t=0 is 0, because the inflow that we receive on account of the short bond B, will exactly match the outflow, on account of buying the bond A. The cash flows of the portfolio P will be -1026.80 at the end of year 1 and 1,150 at the end of year 2. Further, since 1026.80(1+0.12)=1150, this portfolio clearly corresponds to a reinvestment rate of 12% p.a.. By invoking the principle of no arbitrage, the future value of 1026.80 must be equal to 1150. If you work out the implied rate of interest, it turns out to be 12% p.a. In other words, the cash flow at t=1 is invested precisely at the YTM to arrive at the cash flow at t=2. So this example clearly illustrates the assumption that when we calculate the YTM we impliedly assume that all the cash flows, during the life of the instrument are reinvested at a rate equal to the YTM of the instrument. This vindicates our contention that the YTM computation assumes reinvestment at the YTM rate.

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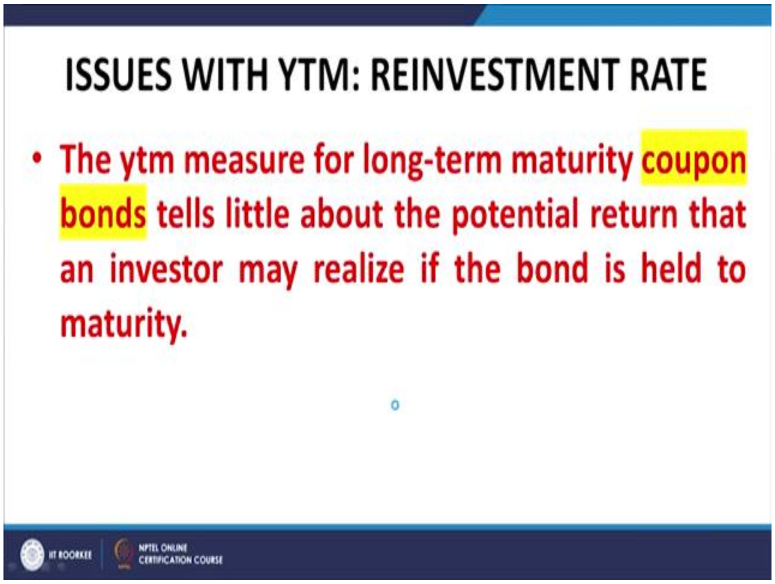

What are the issues with YTM? We have so far discussed the anatomy of YTM, but what are the practical limitations of YTM? Let us now come to this particular point. ***The YTM measure for long term maturity coupon bonds tells us little about the potential return that an investor may realize if the bond is held to maturity.*** Let me repeat, the YTM measure for long term maturity coupon bonds tells us little about the potential return that an investor may realize if the bond is held to maturity. Please note the ***coupon bonds*** word is highlighted here. So this particular issue may not apply when we are talking about zero coupons bonds.

For the zero coupon bonds, if they are held up to maturity, then obviously there is no reinvestment risk, because no coupons are being paid on the instrument and therefore there is no issue of reinvestment. However, in the case of coupon bonds, particularly of long maturity, YTM, turns out to be not a very precise measure of yield. Let us see it with an example.

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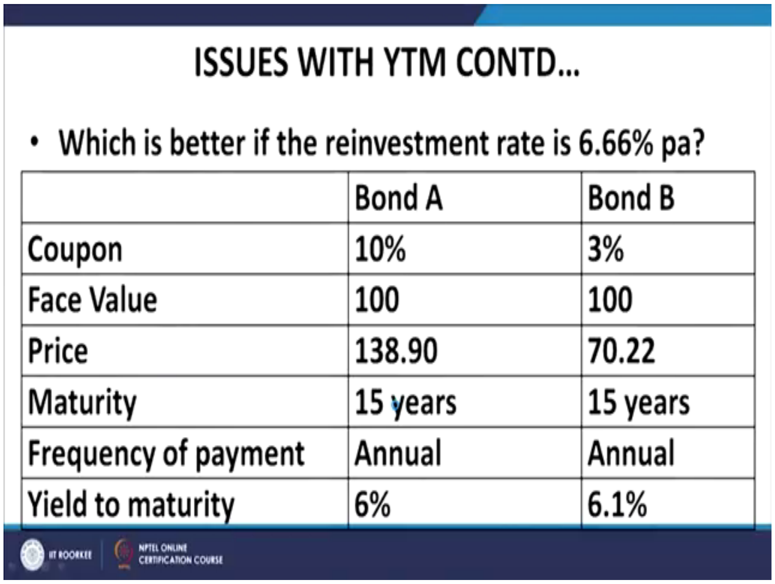

ISSUES WITH YTM CONTD…

- Which is better if the reinvestment rate is 6.66% pa?

	Bond A	Bond B
Coupon	10%	3%
Face Value	100	100
Price	138.90	70.22
Maturity	15 years	15 years
Frequency of payment	Annual	Annual
Yield to maturity	6%	6.1%

In this example we have two bonds A & B. The reinvestment rate is given as 6.66 percent. The coupon rate of bond A is 10% p.a. and of B is 3% p.a. The face value of both the bonds is 100. The current market price of the bond A is 138.90 and that of B is 70.22. The maturity of both the bonds is 15 years. Coupon payments are annual. The YTM of the bonds turns out to be 6% p.a. for A and 6.1% p.a. for B. If we were to assume that YTM is the correct measure of yield then, obviously, bond B would be preferred.

However, there is a very important assumption underlying the concept of YTM and that is that all intermediate payments are deemed reinvested at the YTM rate. So, while calculating YTM, it is impliedly assumed that the reinvestment rate of bond A coupons is 6% p.a. while that of B's coupoms is 6.10% p.a. But it is given in the problem that the actual rate of reinvestment is 6.66% p.a. Let us see the results.

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		A	B
REINVESTMENT RATE		6.66%	6.66%
REINVESTED COUPON		243.6823	73.1047
REDEMPTION VALUE		100.12223	100.321
TOTAL		343.80453	173.426
PRICE		138.9	70.22
EAY		1.062284	1.06213

These are the results. The reinvestment rate is 6.66% p.a. The total value of reinvested coupons at the date of maturity i.e. at the end of 15 years turns out to be 243.6823 for bond A and 73.1047 for bond B. Please note, this is the total value of reinvested coupons calculated at the reinvestment rate of 6.66% p.a. I have worked out the redemption value backwards and it turns out to be 100.12 & 100.32 for A & B respectively. The minor difference from face value is apparently due to rounding off of figures. It seems that actual redemption is at face value of 100. However, the difference is insignificant and unimportant in the given context. Thus, the total cash flows at maturity for A (that includes reinvested coupon proceeds & redemption value) is 343.80 and 173.43 respectively corresponding to a price of 138.90 for A and 70.22 for B. These figures imply an effective annual yield of 6.2284% p.a. for A & 6.2113% p.a. for B.

Now let us recall that the YTM of bond A was 6% p.a., the YTM of bond B was 6.1% p.a. So, if we had accepted YTM as the correct measure of yield and accordingly invested in bond B, we would have actually fared worse off in terms of the effective annual yield. Why is this so? Well there are two reasons (i) the YTM implicitly presumes that all coupons are reinvested at the YTM rate. Thus, the coupons of A are deemed to be reinvested at 6% p.a. while those of B are deemed to be reinvested at 6.1% p.a. But the actual reinvestment rate in both cases is 6.66% p.a. (ii) The coupons of A (10% of 100=10) are much higher than those of B (3% of 100=3). Thus, the impact of the higher reinvestment rate (6.66%) manifests as greater incremental yield in the case of A than for B.

In working out the YTM, we assume that the coupons of A (10 p.a.) are reinvested at 6% p.a but in actual fact these coupons of A are reinvested at 6.66% p.a. B's coupon rate is is less, it is 3% p.a. So in the case of B, the impact of the higher actual reinvestment rate is less because the reinvestments are less. The coupons are smaller so the reinvestments are less. Therefore, we end up with the EAY of A being higher than that of B. So the inference is that if the actual/projected reinvestment rates differ from the YTM, then the YTM may give a distorted picture. This impact gets magnified when the reinvestment income turns out to be a significant contributor to the overall yield on the instrument. This point needs to be kept track of, and it is very well illustrated by this example. The reinvestment rate has a strong say in the overall yield of an instrument, particularly if the coupon rate is high. YTM assumes that these coupons are reinvested at the YTM by default. There is no choice, there is no discretion, it is the default rate embedded in the formula. So if the actual reinvestment rate is different, then we should use that different reinvestment rate instead of using the concept of YTM and thereby calculate the EAY.

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PRICE	A	138.9	B	70.22		
REINV RATE	TOTAL CASH FLOW		TCF/PRICE		RETURN	
	A	B	A	B	A	B
0.06	332.88	170.15	2.39654	2.4231	0.06	0.0608
0.061	334.66	170.68	2.40936	2.43065	0.0604	0.061
0.065	341.94	172.87	2.46177	2.46183	0.0619	0.0619
0.066	343.8045	173.4261	2.47519	2.46975	0.0622	0.06212
0.07	351.41	175.71	2.52995	2.50228	0.0638	0.0631

The above table illustrates the EAY corresponding to different reinvestment rates. We find that so long as the reinvestment rate is below 6.66% p.a. the EAY for bond B is higher than that of A, but for reinvestment rates aat or above 6.66% the situation reverses and the effective annual yield for A becomes more than effective annual yield for B.

Why? Let me repeat, because A's coupons are more and therefore the impact of the higher reinvestment rate compared to the YTM manifests more prominently in the case of A and we get a higher yield if we use the actual reinvestment rate compared to that for B, where the coupon size is very small. In fact if we have a zero coupon bond the reinvestment rate will have no impact, because there are no coupons to reinvest. So in the case of a zero coupon bond YTM will turn out to be the correct measure of yield, whatever the reinvestment rate may be.

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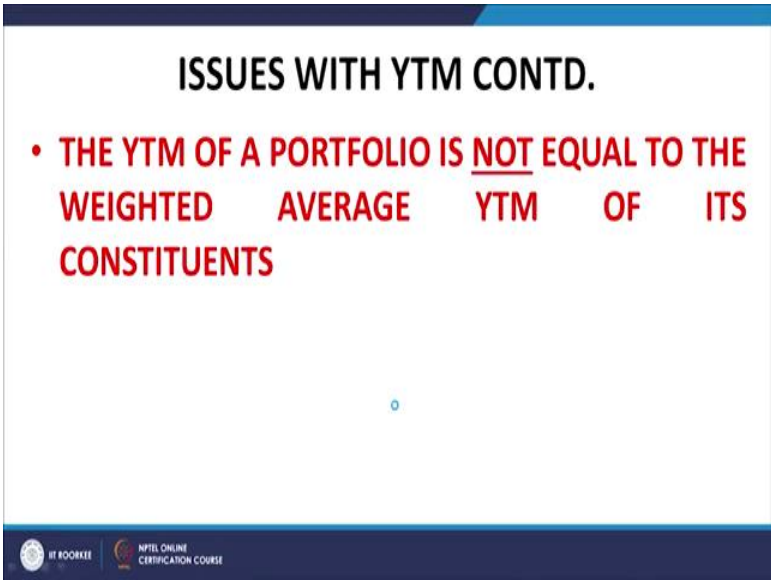

The second issue with YTM is that the YTM of a portfolio is not equal to the weighted average YTM of the constituents. Now we all know that the expected return on a portfolio is equal to the weighted average expected return of the constituents of the portfolio. However, when we talk about the YTM, this formula does not hold. The YTM of a portfolio is not equal to the weighted average YTM of the constituents. Let us see a simple example.

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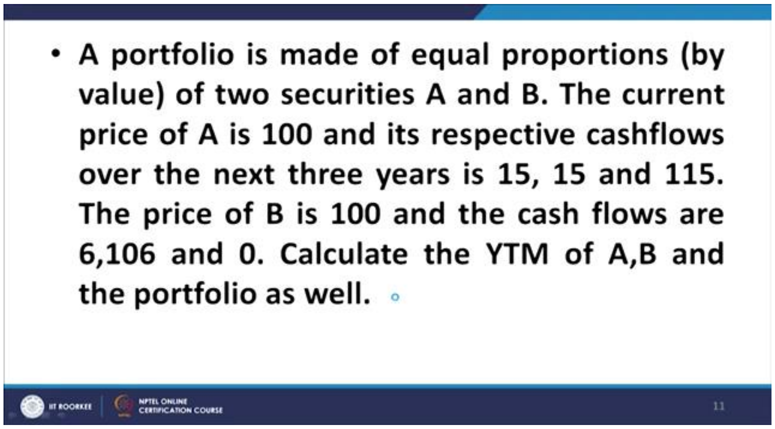

A portfolio is made of equal proportions by value of two securities A & B. The current price of A is 100 (face value) and its respective cash flows over the next three years are 15, 15 and 115. Clearly the YTM of A is 15% p.a. since it has a coupon of 15% p.a. and is quoting at face value. The price of B is also 100 (face value) and the cash flows are 6 and 106 in year 1 and 2 respectively. So, the YTM of bond B (which has a coupon rate of 6% p.a. and is quoting & redeemable at par) is 6% p.a. The weighted average YTM would be (6+15)/2=10.50 % p.a. Let us see what the actual YTM of the portfolio turns out to be. The total cash flows are (-)200 at t=0, 21 at t=1, 121 at t=2 & 115 at t=3. When we work out the YTM of this stream of cash flows using Excel we find that it is 11.29% p.a.

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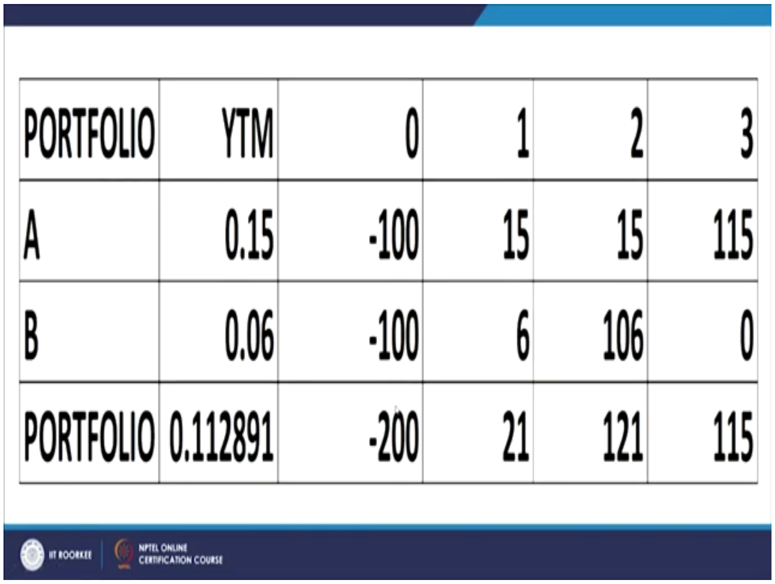

PORTFOLIO	YTM	0	1	2	3
A	0.15	-100	15	15	115
B	0.06	-100	6	106	0
PORTFOLIO	0.112891	-200	21	121	115

Recall that the weighted YTM of this portfolio was 10.50% p.a. Obviously, there is a significant difference between the weighted YTM of the constituents and the actual YTM of the portfolio. So, the linear portfolio combination rule that applies to expected returns does not apply to YTMs.

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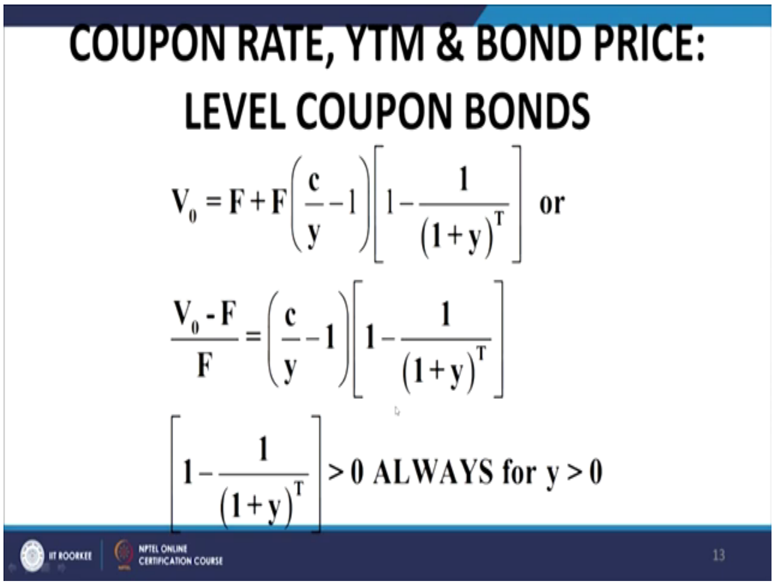

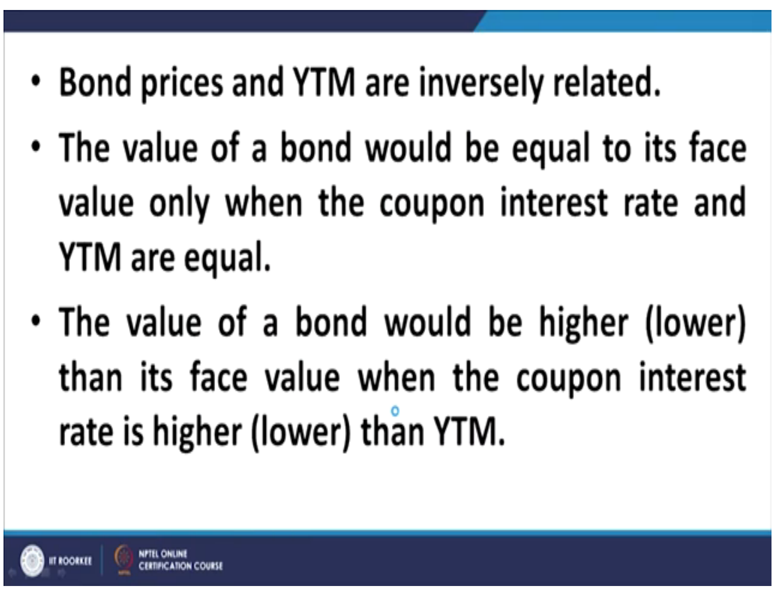

Now just to recall the relationship between coupon rates, YTM and bond price. Bond prices and YTM are inversely related. The value of a bond would be equal to its par value if and only if the coupon rates are equal to YTM and the bond is redeemable at par and it is a level coupon bond. The value of a bond would be more than its face value if the coupon rate is more than the current interest rate. Again, I repeat, this is applicable for a level coupon bond which is redeemable at par value. The inverse is true in the case of discount bonds.

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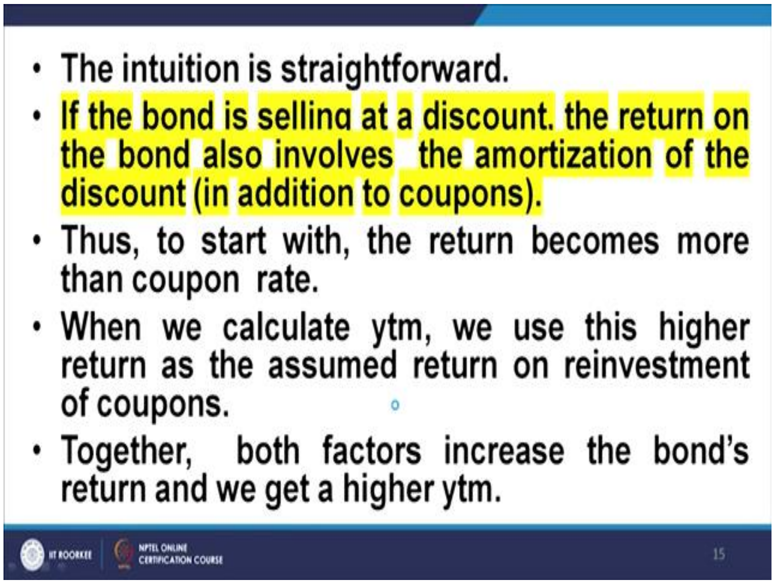

What is the reason for this? Let us talk about a discount bond. We are saying that in the case of a discount bond the YTM of the bond would be higher than the coupon rate provided of course the bond as a level coupon bond redeemable at par. But we know that the YTM of a bond is symbolic of the current market interest rates on similar instruments. Because, the YTM is higher, it means that there are similar instruments that are offering higher rates than our bond. Accordingly, the investors require a higher rate from this instrument than the coupon offered by the bond. This would mean that the demand for this bond will fall with a corresponding fall in price and increase in return until the bond offers the same return as the investor's remand whence equilibrium will set in.

It may be noted that if a bond is quoting at a discount, the return therefrom can be split into two parts viz. (i) the coupons and (ii) the amortization of the discount every year. Suppose the life of the bond is five years, then at the end of the five year period the bond (because it is redeemable at par) shall necessarily carry a value of par on account of no arbitrage considerations. Thus, the difference between the current price (which is below par since the bond is quoting at a discount) and the par value will be amortized over the life of the bond. Thus, every year there will be a certain amount of amortization which will be part of income to the investor in addition to the cash coupon. Hence, the investor will get return on two accounts (i) the coupon cash flow and (ii) this amortization part. So both these parts will add together and as a result the overall return to the investor will be higher than the coupon rate. Thus, the YTM would be higher than the coupon rate. In other words the net result is that the YTM of a discount bond is more than the coupon rate and vice versa. If a bond has a coupon rate which is lower than the YTM it would be quoting at a discount.

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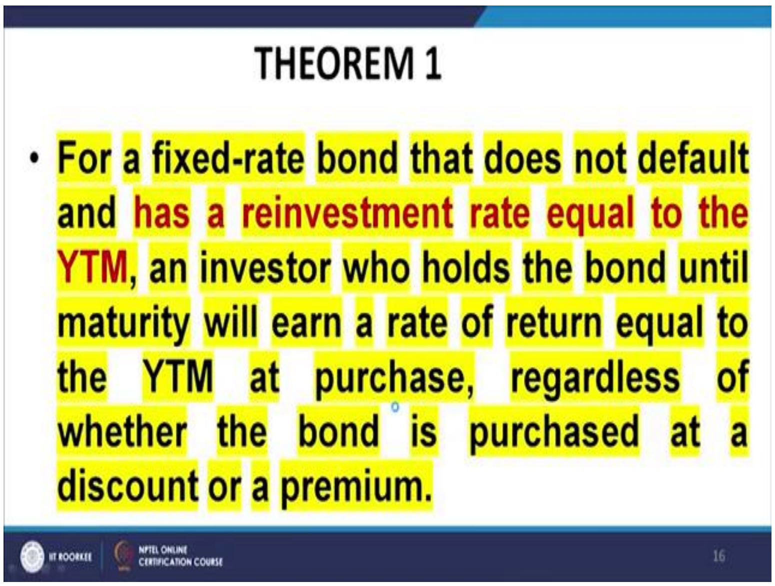

Now let us look at this theorem. For a fixed rate bond that does not default and has a reinvestment rate equal to YTM an investor who holds the bond till maturity will earn a rate of return equal to the YTM at purchase, regardless of whether the bond is purchased at a discount or a premium. It is quite a simple theorem. If the reinvestment rate is equal to the YTM, then obviously the investor will get a return equal to the YTM at the point in purchase. We have already in fact established it earlier

$$\mathbf{TV = Total\ proceeds\ on\ maturity} = \sum_{i=1}^{T} cF(1+y)^{T-i} + F \quad \text{since } r = y$$

$$\mathbf{EAY}\ r_y = \left[\frac{cF\sum_{i=1}^{T}(1+y)^{T-i} + F}{P_0}\right]^{1/T} - 1 \text{ or } P_0(1+r_y)^T = cF\sum_{i=1}^{T}(1+y)^{T-i} + F = cF\left[\frac{(1+y)^T - 1}{y}\right] + F$$

$$= \left\{cF\left[\frac{(1+y)^T - 1}{y(1+y)^T}\right] + \frac{F}{(1+y)^T}\right\}(1+y)^T = P_0(1+y)^T$$

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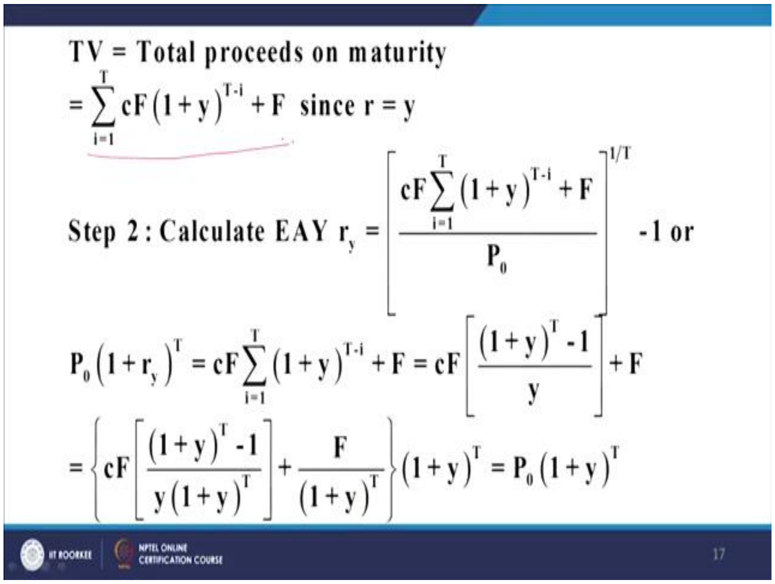

You can see it here again. If reinvestment of the coupons is taking place at the YTM, then the total cash flows at the end of the holding period i.e. at maturity will be given by $\sum_{i=1}^{T} \mathbf{cF}(\mathbf{1+y})^{\mathbf{T-i}} + \mathbf{F}$ **since** $\mathbf{r = y}$. Using this expression, if we work out the effective annual yield over a holding period equal to maturity and simplify we get that the effective annual yield equals the YTM. In other words, the effective annual return r_y is equal to y, provided the reinvestment of the coupons is at the YTM rate and the yield is calculated over the term to maturity. Of course, we are talking about a level coupon bond redeemable at par. So, in the case of a level coupon bond redeemable at par if all coupon reinvestments are at the YTM then the yield earned by the investor over the term to maturity of the bond is also equal to the YTM irrespective of the price of purchase if. Clearly this yield is independent of P_0, whatever be the price of purchase, whether it is below par, above par or at par, the yield that we get is equal to the YTM.

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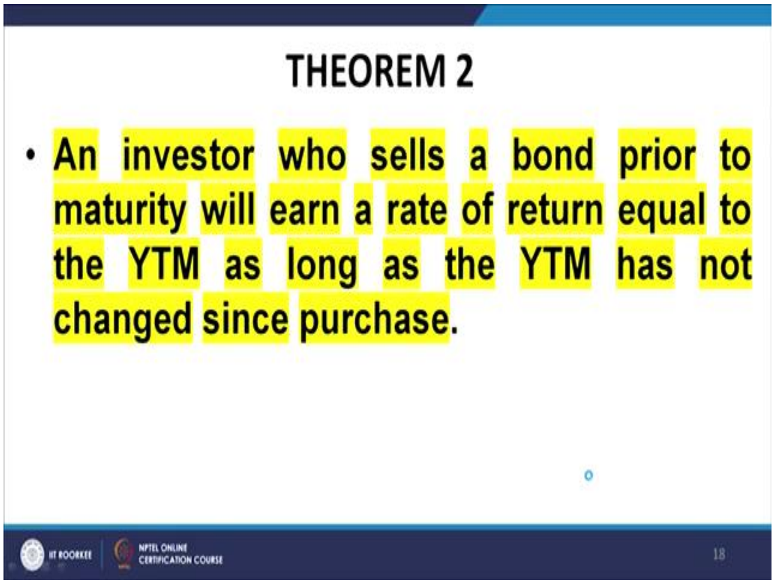

The next theorem. An investor who sells the bond prior to maturity will earn a rate of return equal to the YTM as long as the YTM has not changed since purchase.

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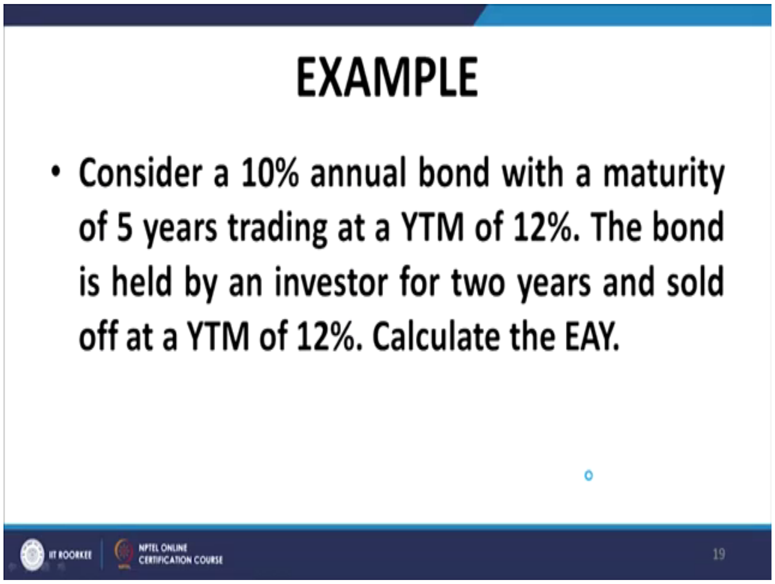

Let us take an example to illustrate this theorem first. Consider a 10% p.a. annual coupon bond, with a maturity of 5 years trading at a YTM of 12% p.a. The bond is acquired by an investor at this YTM at t=0 and held for two years at the end of which it is disposed off at the same YTM. In other words, we assume that the YTM has not changed over the holding period of the investor. We want to calculate the effective annual yield. Now,

Current Price of the bond: P_0= 100xPVIFA(12%,5)+1,000xPVIF(12%,5)=928
Price of bond after two years: P_2= 100xPVIFA(12%,3)+1,000xPVIF(12%,3)=952
Proceeds of interest: 100x1.12+100=212; Total proceeds at sale: 952+212=1,164
EAY=$(1164/928)^{0.5}$-1=12%

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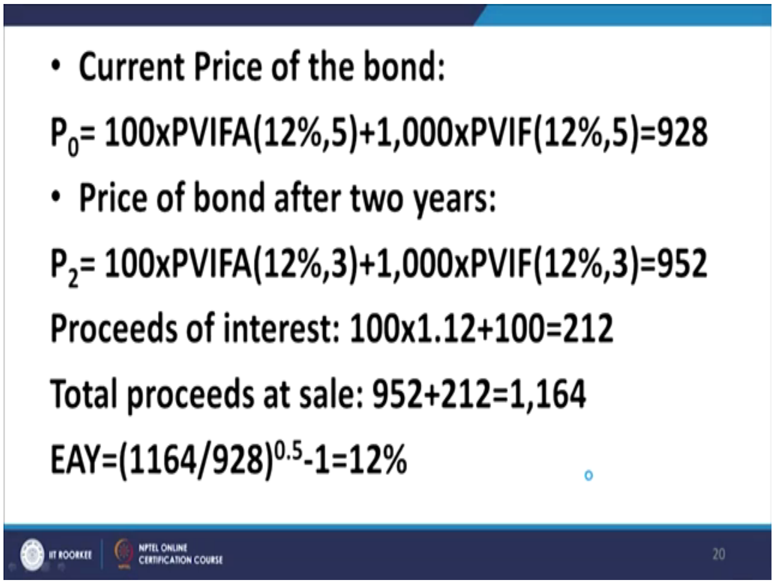

The effective annual yield turns out to be 12% p.a. thereby illustrating the theorem.

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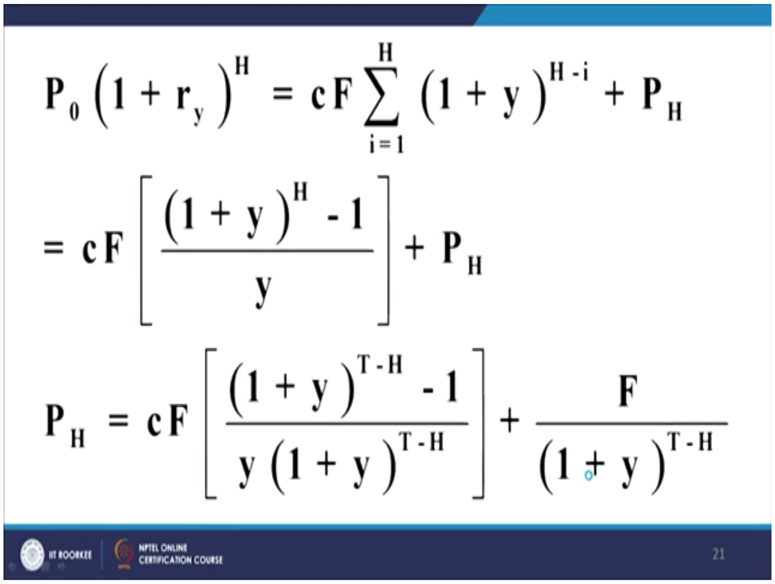

$$P_0\left(1+r_y\right)^H = cF\sum_{i=1}^{H}\left(1+y\right)^{H-i} + P_H = cF\left[\frac{\left(1+y\right)^H - 1}{y}\right] + P_H \text{ where } P_H = cF\left[\frac{\left(1+y\right)^{T-H} - 1}{y\left(1+y\right)^{T-H}}\right] + \frac{F}{\left(1+y\right)^{T-H}}$$

$$\begin{aligned}
P_0\left(1+r_y\right)^H &= cF\left[\frac{\left(1+y\right)^H - 1}{y}\right] + cF\left[\frac{\left(1+y\right)^{T-H} - 1}{y\left(1+y\right)^{T-H}}\right] + \frac{F}{\left(1+y\right)^{T-H}} \\
&= \left\{cF\left[\frac{\left(1+y\right)^H - 1}{y\left(1+y\right)^H} + \frac{\left(1+y\right)^{T-H} - 1}{y\left(1+y\right)^T}\right] + \frac{F}{\left(1+y\right)^T}\right\}\left(1+y\right)^H \\
&= \left\{cF\left[\frac{\left(1+y\right)^T - \left(1+y\right)^{T-H}}{y\left(1+y\right)^T} + \frac{\left(1+y\right)^{T-H} - 1}{y\left(1+y\right)^T}\right] + \frac{F}{\left(1+y\right)^T}\right\}\left(1+y\right)^H \\
&= P_0\left(1+y\right)^H
\end{aligned}$$

This is the explicit proof for the theorem. Again it is simple algebra, where we use the expressions for the PVIFA and simplify.

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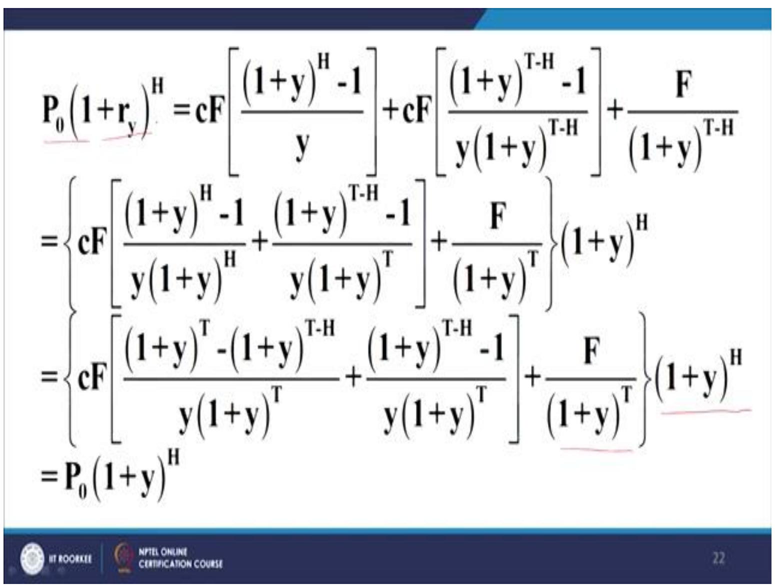

The expression $\left\{cF\left[\frac{\left(1+y\right)^T - \left(1+y\right)^{T-H}}{y\left(1+y\right)^T} + \frac{\left(1+y\right)^{T-H} - 1}{y\left(1+y\right)^T}\right] + \frac{F}{\left(1+y\right)^T}\right\}$ simplifies to P_0 by the definition of YTM. It is the present value of all future coupon payments, plus the present value of the redemption value calculated at the YTM. So the entire expression in the curly brackets must equal P_0. The important condition is that the YTM has not changed during the holding period of the investor and provided we are talking about a level coupon bond a bond that is redeemable at par.

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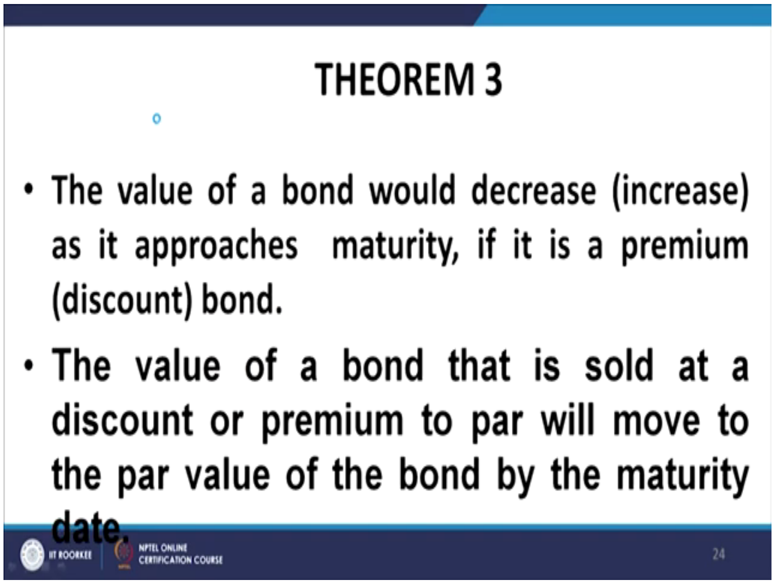

The next theorem. The value of a bond would decrease (increase) as it approaches maturity if it is a premium (discount) bond. I mentioned a few minutes back that the value of a bond on its state of maturity must equal its redemption value. A bond that is to be redeemed at face value, must be quoting at face value on the date of maturity to prevent any arbitrage from taking place. For example, if a bond was to be priced at a value lower than the redemption value at maturity unlimited arbitrage (riskless) profits could be made by buying the bond in the market and delivering it to the company and take the redemption proceeds and vice versa. Therefore it must necessarily be that the price of the bond on the date of redemption must equal its redemption value. So if a bond is quoting at a premium at an earlier date or at a discount at an earlier date then that premium or discount, as the case may be, must necessarily be amortized over the remaining life of the instrument, because, I repeat, at the maturity of the bond the bond must quote at its redemption value.

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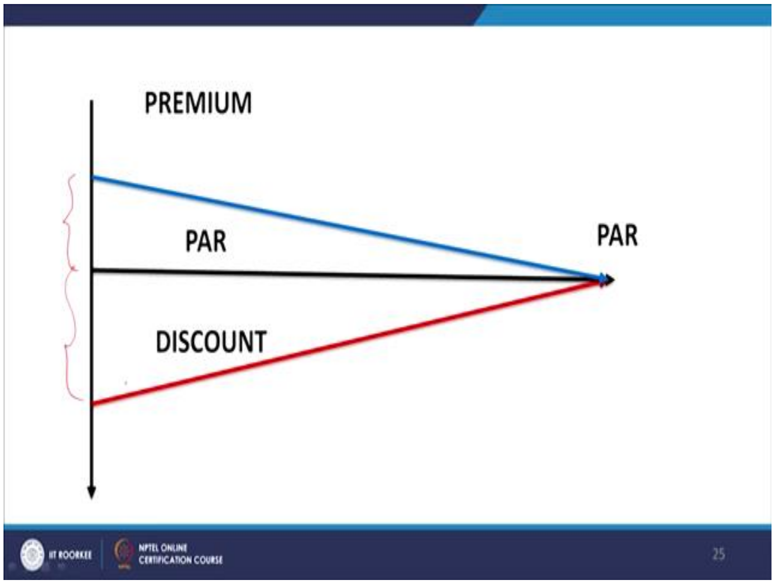

Please note that this amortization may not be linear. In fact, it will not be linear. But for the simplicity of exposition I have represented it via straight lines. In the case of a premium bond, the amount of premium must be amortized over the remaining life of the bond so that the bond comes to par value on the date of maturity. Similarly for a discount bond, the amount of discount must be amortized by the point in time it reaches its maturity so that it may be trading at par value on the date of maturity.

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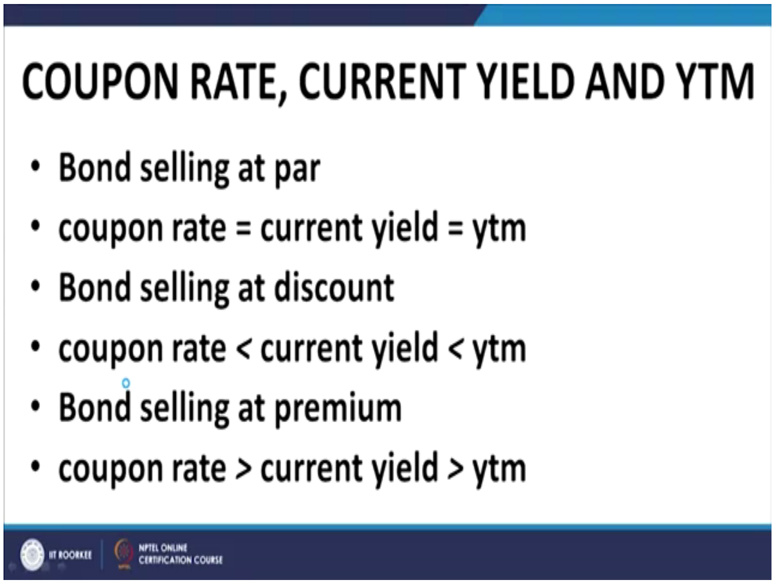

Right, we will continue from here in the next lecture, thank you.