**Security Analysis & Portfolio Management**
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**Lecture 13**
**Yield To Maturity III**

Welcome back. So let us continue from where we left off, but before that a quick recap of what we have done so far. In the last lecture, I introduced the concept of intrinsic value.

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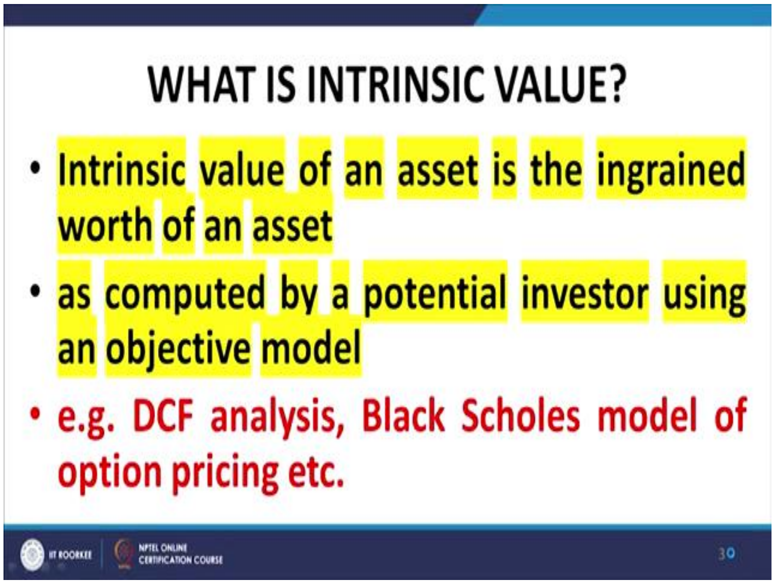


We define intrinsic value as the ingrained worth of an asset. In some sense, it is the internal value of the asset. It is computed by using a model by a potential investor. So, it is computed by a potential investor using a model that he perceives to be appropriate for the valuation and is based on the inputs that are estimated by the investor himself. So to that extent the intrinsic value is investor centric. Of course, it is also security centric.

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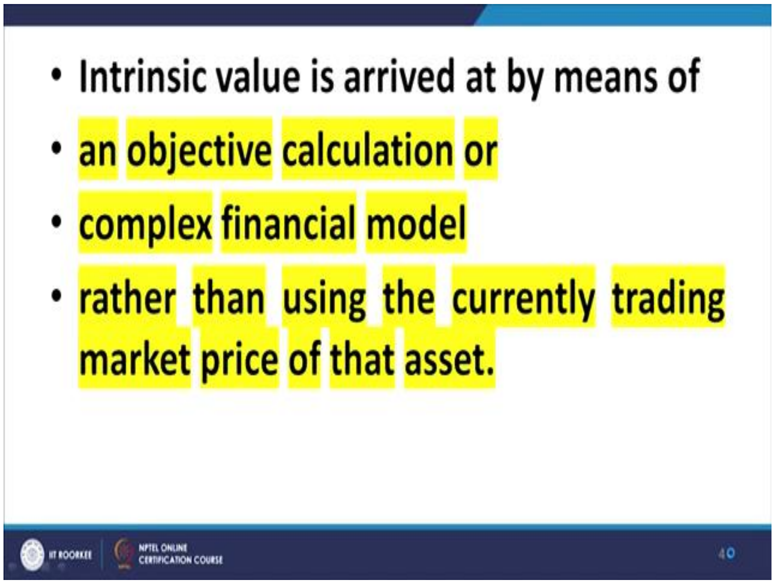


So, to reiterate, intrinsic value is arrived at by means of an objective calculation or a complex financial model, rather than using the currently traded market price of the asset.

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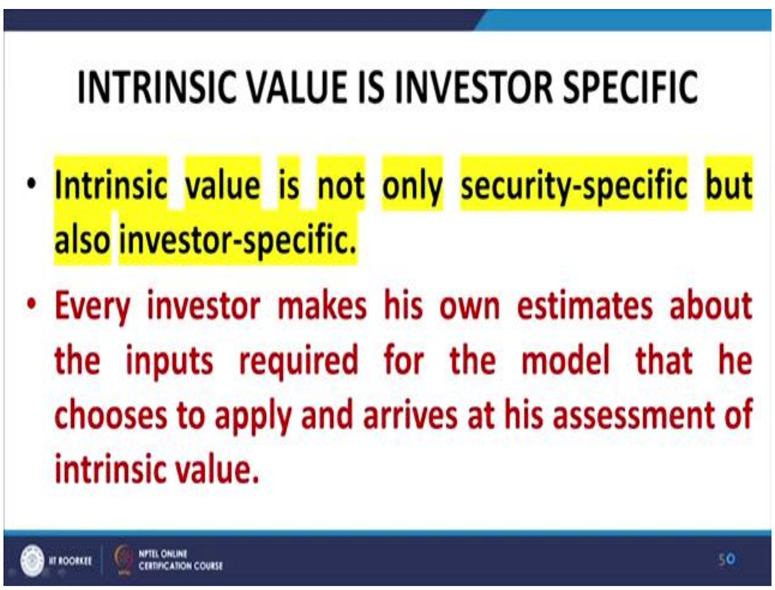


As I mentioned just now, intrinsic value is investor specific. The model is selected by the investor on the basis of which the intrinsic value is calculated. The inputs to that model, for the computation of the value of the asset are also estimated by the investor and imported into the model.

If we are using the DCF model which is the most commonly used model for bond and equities, then we define the intrinsic value as the present value of all future expected cash flows attributable to that particular asset, discounted at the appropriate risk adjusted discount rate.

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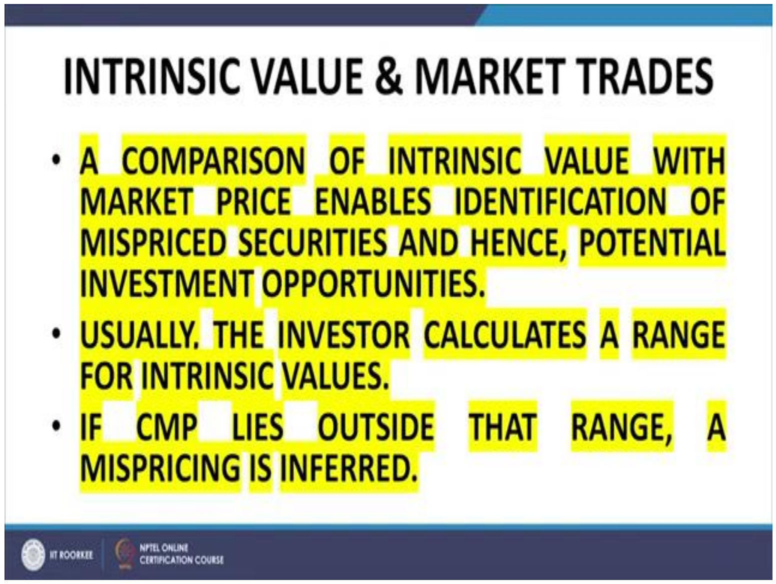


Now the value of calculating the intrinsic value for the investor is that he can compare that intrinsic value with the currently prevailing market price and thereby locate mispriced securities in the market as per his perception, and then take investment decisions accordingly.

Now the important thing is that because the inputs that go into the intrinsic value calculation as well as the selection of the model itself is somewhat subjective. The normal practice, therefore, is that the analyst uses or computes a range for intrinsic value within which, he feels, that the correct value of the security should lie. Then, if the market price is outside that range, he comes to the conclusion that the asset is underpriced or overpriced as the case may be. He can, thereby take the appropriate risk decision.

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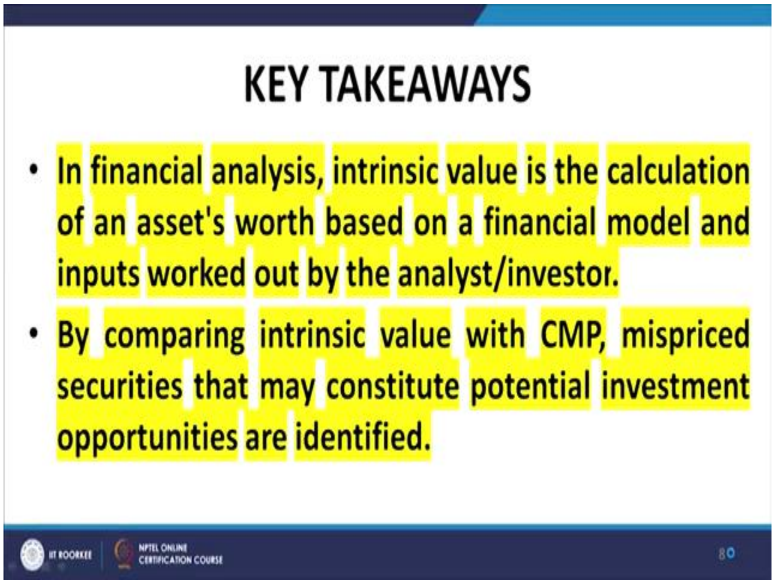


So the key takeaways, just to reiterate, intrinsic value is the calculation of an asset's worth, based on a financial model and inputs worked out by the analyst or the investor. By comparing the intrinsic value with the current market price, mispriced securities that may constitute potential investment opportunities are identified.

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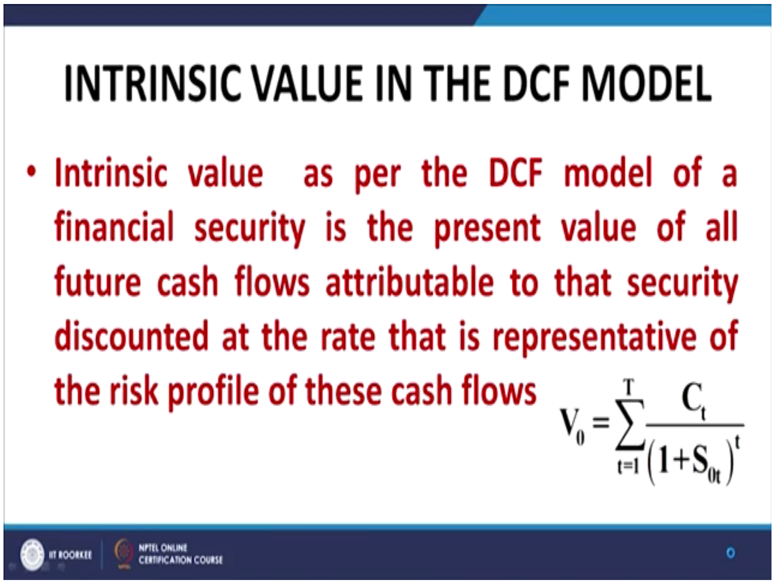


In the DCF model, as is given in the formula right in the bottom right hand corner of your slide, we obtain the intrinsic value of an asset as the present value of all future expected cash flows attributable to that security, discounted at the appropriate risk adjusted discount rate.

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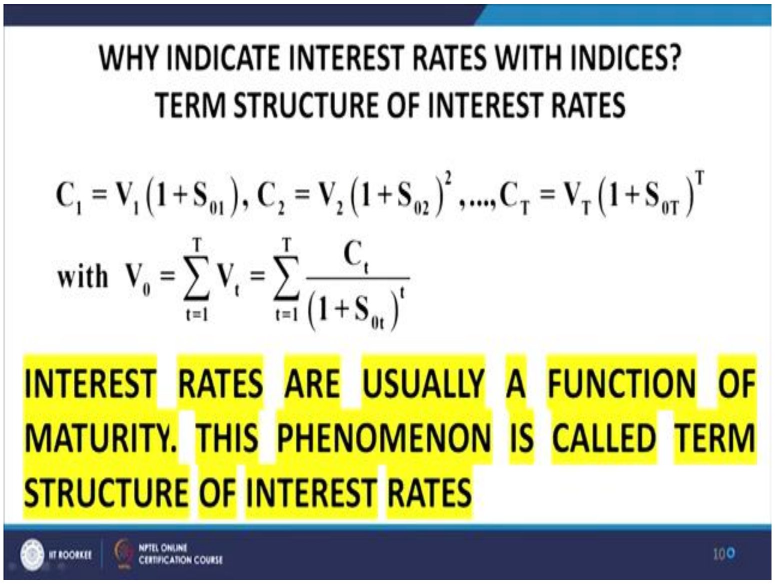


Then I mentioned that we usually represent the interest rates with subscripts to indicate that the given rate refers to the relevant discounting period. In other words, the subscripts indicate that the cash flows are being discounted at the spot rates pertaining to maturity corresponding to the timing o the cash flows. This is necessary, because of the phenomenon of term structure of interest rates which implies that the interest rate is a function of the maturity of the underlying deposit. In other words, if we make a deposit of one year, we get a certain rate. If we make a deposit of two years we get a slightly different rate. For longer maturity, rates may diverge even more with the length of the deposit. So keeping that in mind, we need to specify the rate at which the discounting is to be done because we cannot choose a common rate as such since the cash flows are occurring at different points in time. Therefore, for discounting those cash flows the appropriate rates which are relevant to those points in time need to be considered.

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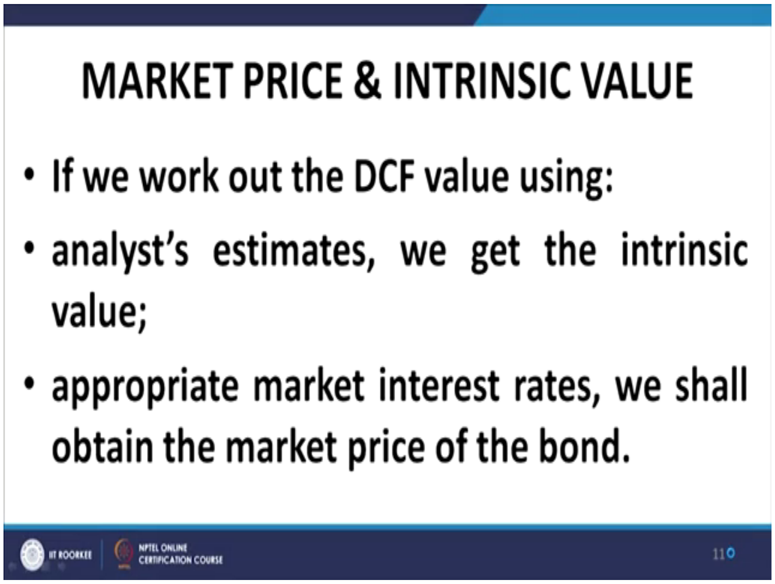


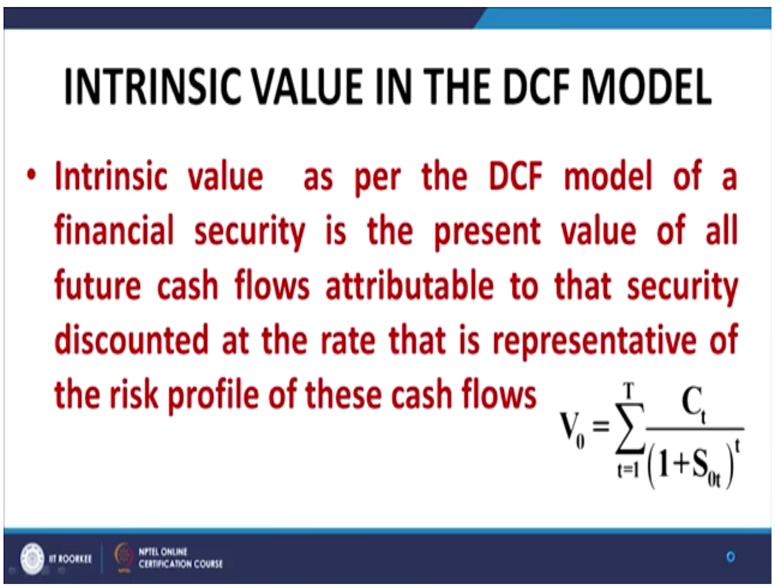


In the DCF model, as I mentioned, if I use the estimates of the interest rates and the cash flows as worked out by the analyst or the investor we get is the investor centric value, or the worth of the asset as perceived by the investor. However, if we replace these inputs by the market inputs, by the perceptions of the market encoded in the relevant interest rates, then what we should get on the left hand side is the market price of the asset.

But I need to mention here that the market price of the asset is the more prominent observable, because it manifests itself straightaway as the trading platform output. Therefore the process that actually takes place in the market is to observe the market price and thereafter work backwards and arrive at the relevant interest rates which are appropriate for discounting the cash flows of a particular maturity.

Now, an important observation. As far as the fixed income securities are concerned, we have the advantage and a very significant advantage that the cash flows that relate to the fixed income security are normally given by the contract of issue or the issue document. Therefore there could little variation in the estimation of cash flows among the investor community and hence, the market. In other words the estimates of the market and of the investor of the numerator in the valuation formula i.e. the cash flows are likely to be converging, likely to be similar amongst investors  because the cash flows essentially arise from the contract of issue which is encoded in the issue document.

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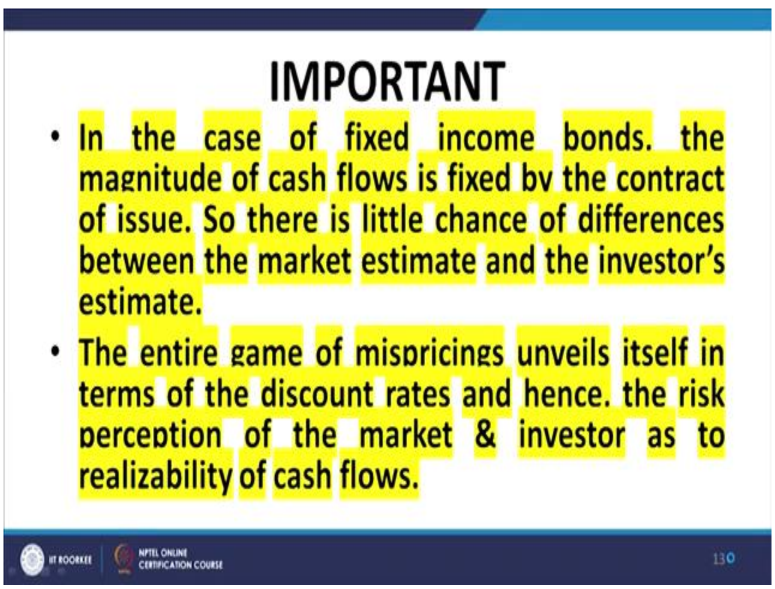


Therefore the entire game of mispricings manifests itself in terms of the discount rate that is used for calculating the present value of the future cash flows arising from the asset for calculating the intrinsic value.

If we use the market estimates of appropriate interest rates, we get the market price on the left hand side and if we use the investor specific interest rates (the interest rates as estimated by the investor) we get the investor based value or the intrinsic value of the asset. Therefore by comparison of the intrinsic value as estimated by the investor and what the market has estimated the investor can arrive at appropriate investment decisions.

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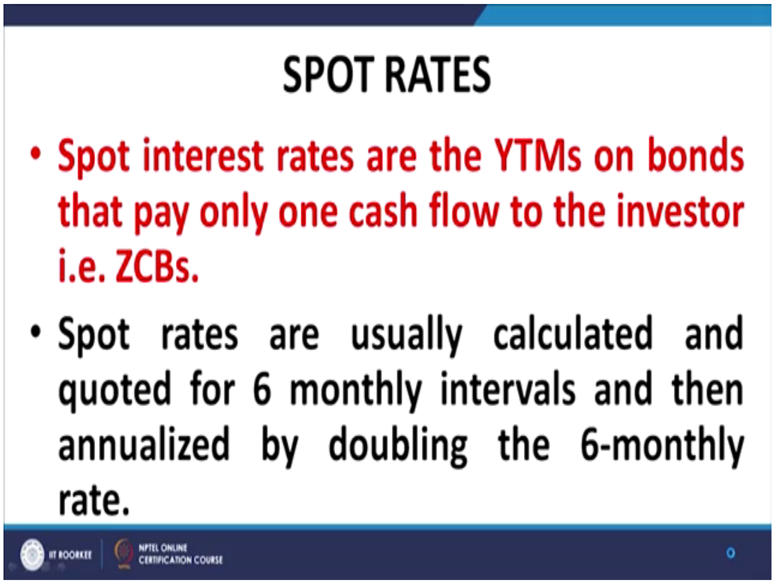


Then we discussed the concept of spot rates. Spot interest rates are the YTMs on bonds that pay only one cash flow to the investor, that is, zero coupon bonds.They are usually calculated for intervals of six months i.e. spot rate for a deposit of six months, one year, one and half year, two years and so on, and then they are doubled to express them on an annualized basis.

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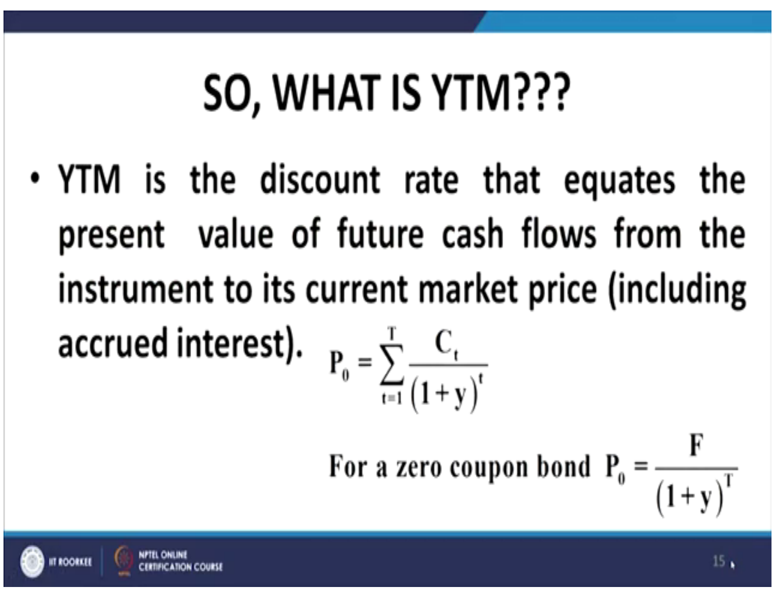


Now, the next issue is, what is YTM? We define the yield to maturity (YTM) of an instrument as that discount rate at which the present value of all future cash flows arising or emanating from the asset equal the current market price of the asset. Let me repeat, it is very important, it is that discount rate which when used for discounting all future cash flows pertaining to a particular asset give us the market price of the asset.

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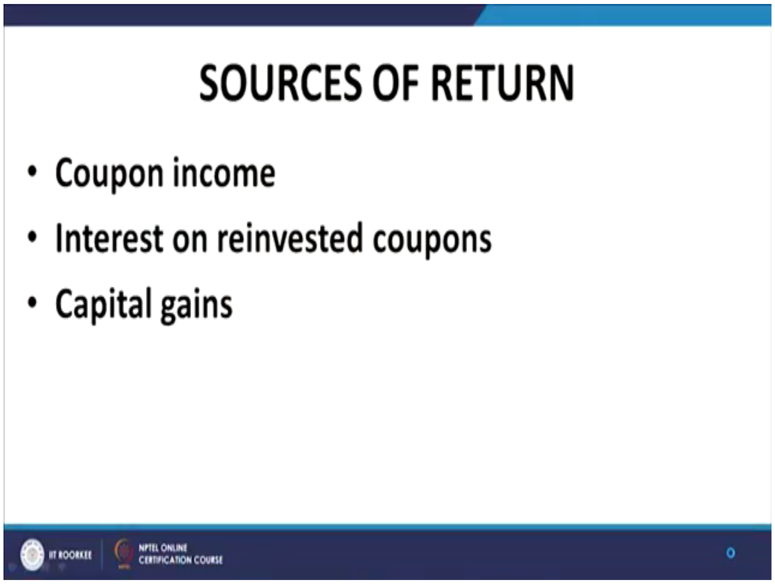


Now I talk about the sources of return on the fixed income security. First of all, we have the coupon payments. Then, if the investor reinvests those coupon payments, he gets interest on reinvested interest i.e. interest on interest. Finally, if the investor decides to liquidate the investment before the date of maturity of the instrument, he could end up with a capital gain or a capital loss. The capital gain or loss is the difference between the market price or the price at which the investment is liquidated by selling in the market, less the carrying value of the investment as represented in the books pf the investor. This carrying value is the amortized value worked out at the YTM at which the bond was purchased by the investor.

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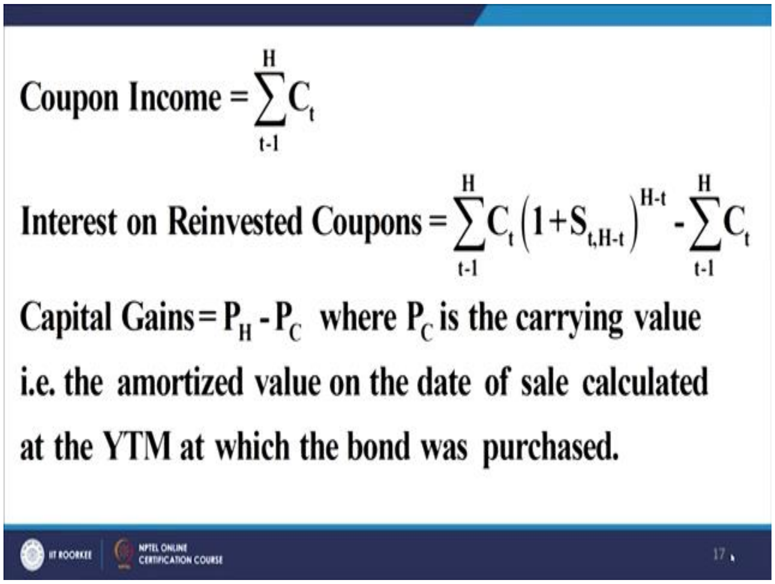


So the coupon income is given by the first expression right at the top of the slide **Coupon Income** $= \sum_{t-1}^{H} C_t$ . The interest of reinvested interest is given by the second expression **Interest on Reinvested Coupons** $= \sum_{t-1}^{H} C_t \left(1 + S_{t,H-t}\right)^{H-t} - \sum_{t-1}^{H} C_t$ . The capital gains are given by the difference between the price of the asset on the date of liquidation and the carrying value of the investment **Capital Gains** $= P_H - P_C$, The carrying value is the amortized value on the date of sale calculated at the YTM at which the bond was purchased. I will elaborate this with an example later on in this lecture.

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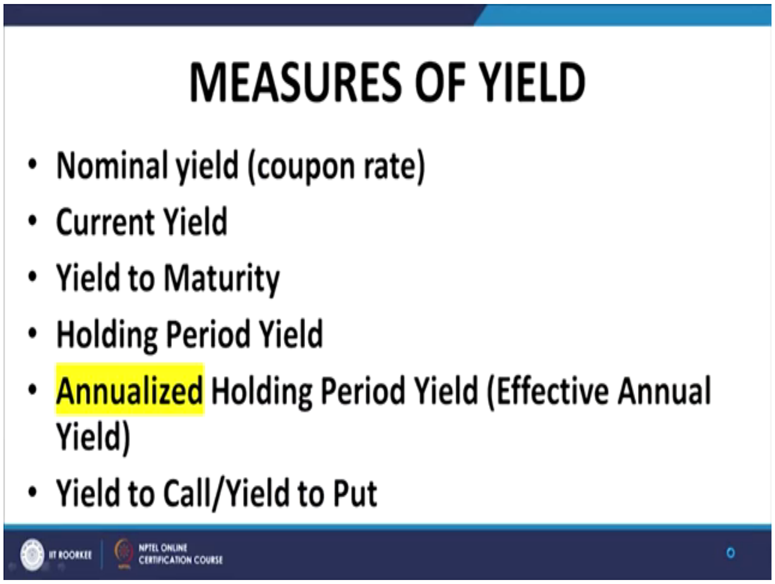


Now, I come to the measures of yield. Nominal yield is the coupon yield or the coupon rate. The current yield is given by the ratio of the annual coupon payments to the current market price of the instrument. Obviously, it does not consider the issue of reinvestment of coupon payments as well as of the capital gain or loss on the liquidation of the investment. Then I talked about yield to maturity. I was discussing about YTM at the end of the last lecture, so let us continue from there.

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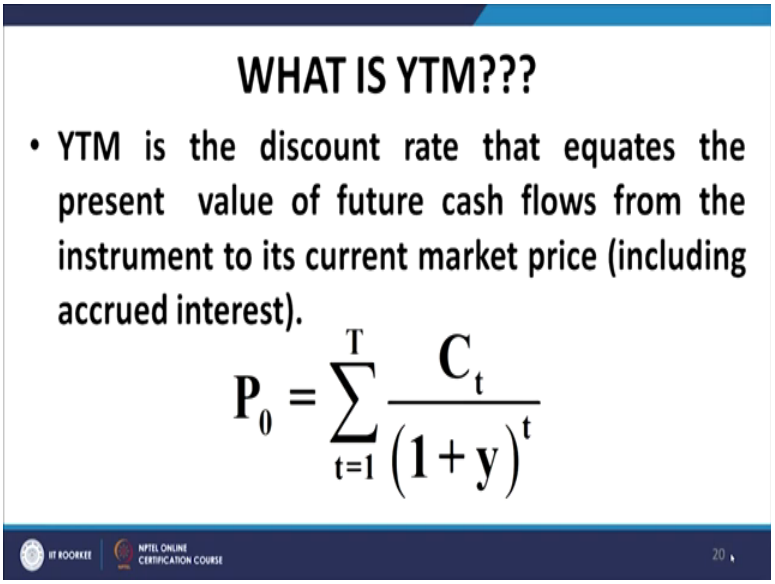

So we define yield of maturity as the discount rate that equates the present value of all future cash flows attributable to the asset to its current market price.

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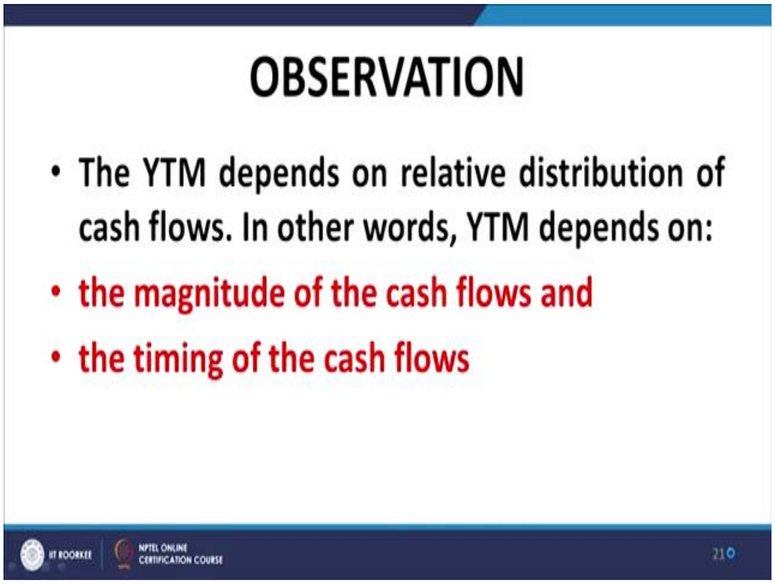


So the important thing here is that the yield to maturity depends on the pattern of cash flows encoded in the investment as specified in the issue document. Pattern of cash flows includes the amount of those cash flows i.e. the magnitude of the cash flows and also the points in time at which these cash flows occur i.e. the timing of the cash flows.

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**YTM DEPENDS ON CASH FLOWS**

| MATURITY | | YTM | PRICE | 1 | 2 |
|---|---|---|---|---|---|
| COUPON | | | | 15% | 15% |
| SPOT RATE | | | | 8% | 12% |
| DISC RATE | | | | 0.9259 | 0.79719 |
| A | | 0.117187 | -1056 | 150 | 1150 |
| B | | 0.104216 | -1060 | 650 | 575 |

This is easily seen by reference to the examples. Here we have two bonds, A & B. Bond A has a stream of cash flows of 150 in year t=1, and 1150 in year t=2. Bond B has a cash flow 650 in year t=1 and 575 in year t=2. The relevant spot rates are $S_{01}$=8% p.a. and $S_{02}$=12% p.a. We find the YTMs to be 11.72% p.a. and 10.42% p.a. respectively for A & B. So clearly the YTM is a function of the magnitudes of the cash flows.

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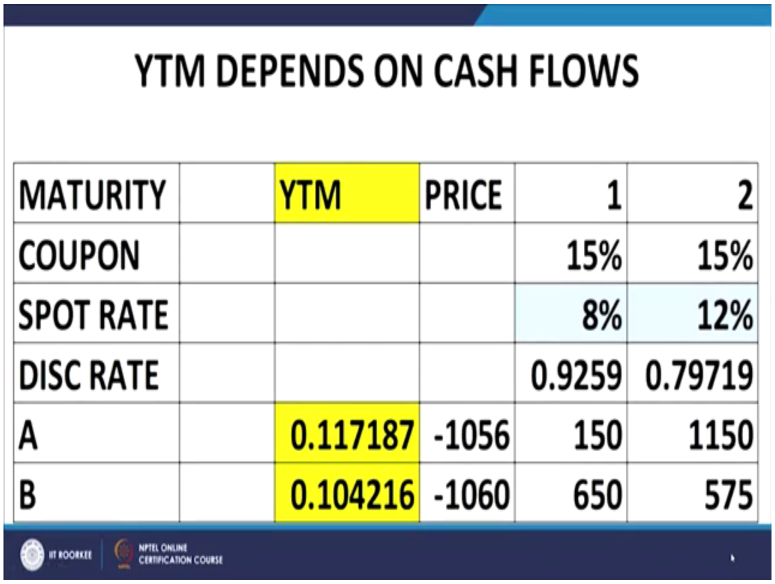
**YTM DEPENDS ON MATURITIES**

| MATURITY | | YTM | PRICE | 1 | 5 |
|---|---|---|---|---|---|
| SPOT RATE | | | | 8% | 12% |
| DISC RATE | | | | 0.9259 | 0.56743 |
| A | | 0.118371 | -791 | 150 | 1150 |
| B | | 0.10937 | -928 | 650 | 575 |



YTM DEPENDS ON CASH FLOWS

| MATURITY | | YTM | PRICE | 1 | 2 |
|---|---|---|---|---|---|
| COUPON | | | | 15% | 15% |
| SPOT RATE | | | | 8% | 12% |
| DISC RATE | | | | 0.9259 | 0.79719 |
| A | | 0.117187 | -1056 | 150 | 1150 |
| B | | 0.104216 | -1060 | 650 | 575 |

Now the YTM is also a function of the timing of the cash flows. We, again, consider two bonds A & B. Bond A pays 150 at the end of year t=1 and 1150 at the end of year t=5 with no intermediate payments. Bond B pays 650 at the end of year t=1 and 575 at the end of year t=5 with no intermediate payments. We again compute the YTMs and find that the YTMs have changed to 11.83% p.a. and 10.93% p.a. respectively compared to the values of 11.72% p.a. and 10.42% p.a. in the earlier case.

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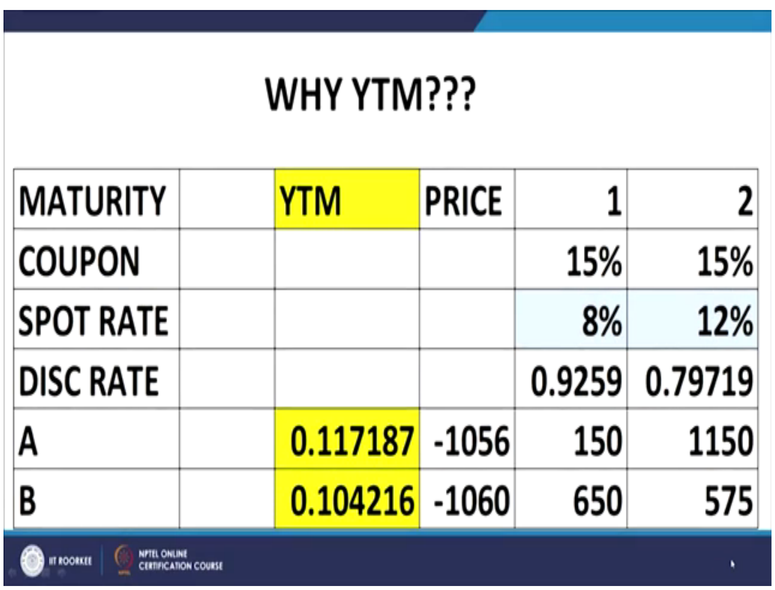

WHY YTM???

| MATURITY | | YTM | PRICE | 1 | 2 |
|---|---|---|---|---|---|
| COUPON | | | | 15% | 15% |
| SPOT RATE | | | | 8% | 12% |
| DISC RATE | | | | 0.9259 | 0.79719 |
| A | | 0.117187 | -1056 | 150 | 1150 |
| B | | 0.104216 | -1060 | 650 | 575 |

So the outcome of these two examples is that the YTM of an instrument depends not only on the magnitude of cash flows but also on the timing of the cash flows.

Why YTM? Well this is also illustrated with this example. We have two 15%% p.a. coupon bonds and the spot rates are $S_{01}$=8% p.a. and $S_{02}$=12% p.a. We want to find out which is the more appropriate investment. Clearly this is an ambiguous issue. We need some consistent measure that we can use as estimate for the return in such situations where the spot rates are different for different years. In our case, there is a significant difference between the interest rates on one year deposits and two year deposits. So we need a consistent method, whereby we can estimate the overall return on these two year bonds. This is what the YTM purports to attempt. By working out the YTM, we find that the YTM is 11.72% p.a. for bond A and 10.42% p.a. for bond B. We, thereby, conclude that in the given scenario, bond A is a superior investment compared to bond B if we are planning to hold the bonds to maturity.

Now why the YTM of bond A is superior is because it has a greater amount invested at the higher spot rate of 12% p.a. In the case of B, 50% of the bond value is redeemed at the end of the first year. As a result of this a lesser amount remains invested at 12% p.a. So naturally it results in a lower YTM for bond B. We will come back to it in more detail very soon.

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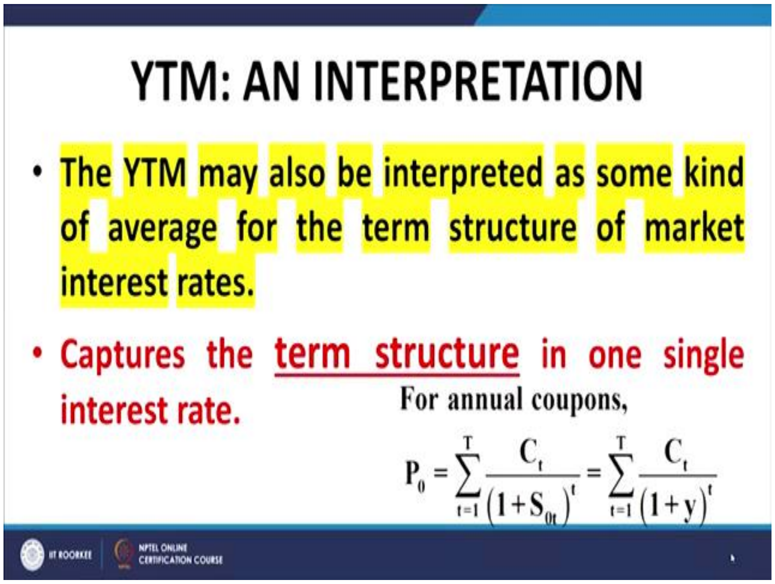


We look at an interpretation of YTM. Consider the equation: **For annual coupons,** $\mathbf{P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t} = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$. When we look at this equation, in the first expression for $P_0$ we have the spectrum of spot rates $S_{01}$, $S_{02}$, $S_{03}$ and so on, which constitute the discount rates for the respective cash flows occurring at t=1,2,3…that is $C_1$, $C_2$, $C_3$ etc. but I the second expression for $P_0$ we find that there is a common single rate which is used for discounting all the cash flows. But on the extreme left side we have the same output $P_0$ of both these calculations and that is the current market price. So the important thing that we infer from this is that the y value that we obtain (which is a unique value, a solitary value used for discounting cash flows of all maturities), must represent some kind of averaging or some kind of representation for the entire spectrum of interest rates, which is relevant to the calculation of the value of the asset. In other words we can say that this y value, that is the YTM, captures the entire term structure which is relevant to the evaluation of the bond in one single number.

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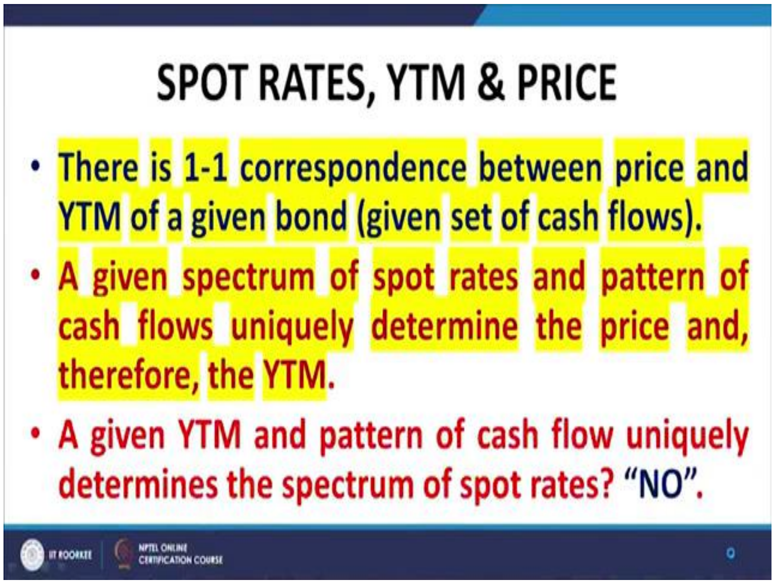

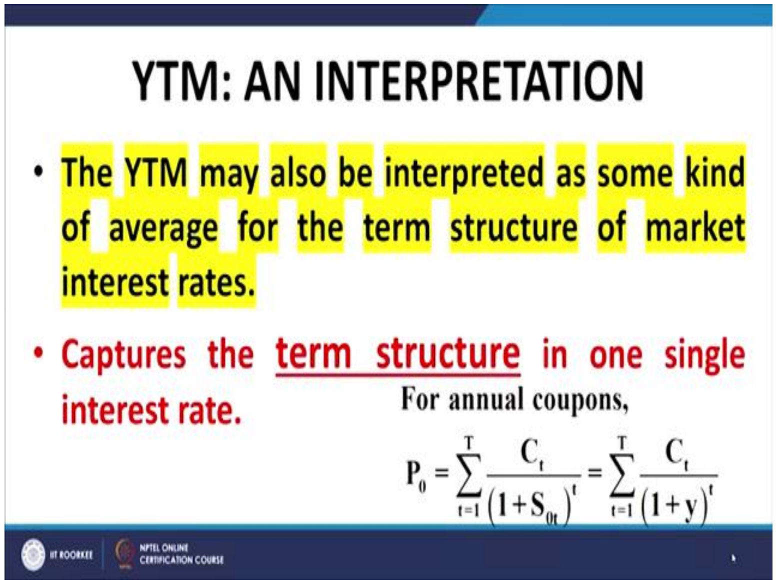


As you can see from this formula **For annual coupons,** $\mathbf{P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$ , there is only one input into this model. As far as the $C_1$, $C_2$, … are concerned i.e. as far as the cash flows are concerned, they are pre determined in the sense that they are contained in the contract of issue. They do not vary for a given instrument. They are constants (subject to the possibility of default). As far as the timing of the cash flows is concerned, it is also given in the contract of issue. So, in a sense, the magnitude and timing of cash flows do not constitute variables for a particular bond.

So the only thing that constitutes a variable for a given bond is the left hand side i.e. the current market price. Therefore there exists a one to one correspondence between the current market price $P_0$ and the YTM of the bond. We have, thus, established that the current market price has one to one correspondence with the YTM of the bond. There is another point, the current market price is also determined with reference to the entire spectrum of spot rates which are relevant for discounting the cash flows at different points in time as seen by the first part of this equation $\mathbf{P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t} = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$ . This particular equation tells us that the current market price is equal to the discounted value of future cash flows discounted at the appropriate risk adjusted spot rates. So that being the case, we can establish a correlation between the middle term $\mathbf{\sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t}}$ and the extreme right hand term $\mathbf{\sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$ and we see that given bond i.e. given a structured set of cash flows and the spectrum of corresponding interest rates, we can uniquely determined the market price and thereby uniquely determined the YTM of the bond.

So we have two inferences: (i) there is a one to one correspondence between the current market price and the YTM of a bond and (ii) given a spectrum of spot rates there is a one to one correspondence between the spectrum of spot rates in relation to a bond and the YTM of the bond.

However, the converse does not hold. If we are given the YTM of a bond, we cannot uniquely determine the spectrum of spot rates which are appropriate for discounting the bond. I mentioned just now that given a bond and a spectrum of spot rates cash flows appropriate to discounting of the bond we can work out its current market price and therefore we can work out its YTM uniquely. However, the converse, that is, given a YTM and the current market price (which is obtainable from the YTM) and the cash flows on the bond, we cannot work out the entire spectrum of interest rates.

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| MATURITY | YTM | | 1 | 2 |
|---|---|---|---|---|
| SPOT RATE | | | 0.08 | 0.12 |
| DISC RATE | | | 0.92593 | 0.79719 |
| BOND PRICE | 0.1042 | -1060 | 650 | 575 |
| NEW SPOT RATE | | -1060 | 0.0829 | 0.11803 |
| | | | | |

This is illustrated by this example. We have a pair of spot rates relating to maturities of one and two years. One pair comprises of 8% p.a. for maturity of one year and 12% p.a. for a maturity of two years. It is shown that there exists another pair of spot rates viz 8.29% p.a. for one year and 11.803% p.a. for two years that yield the same market price and hence the same YTM for a bond paying 650 and 575 in year one and year two respectively of 10.42 percent.

In other words, given a YTM of 10.42% p.a. of a bond paying 650 and 575 at the end of year 1 and year 2 respectively, can admit two different pairs of spot rates viz $S_{01}$= 8% p.a. S02=12% p.a. and also $S'_{01}$=8.29% p.a. and $S'_{02}$=11.80% p.a.

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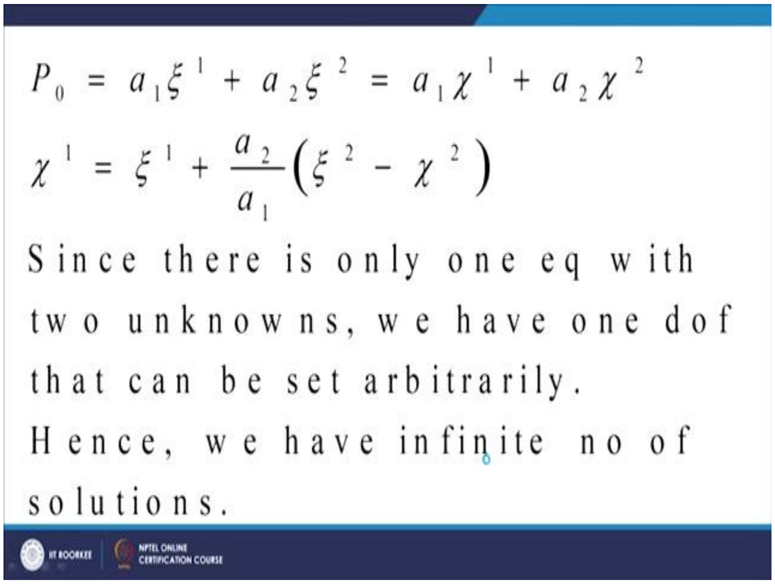


The above slide gives the theoretical explanation of the illustration. We have lesser equations than the degrees of freedom and hence we cannot uniquely determine the spectrum of spot rates, corresponding to a given YTM. I repeat we cannot determine a unique spectrum of spot rates corresponding to a given YTM for a bond.

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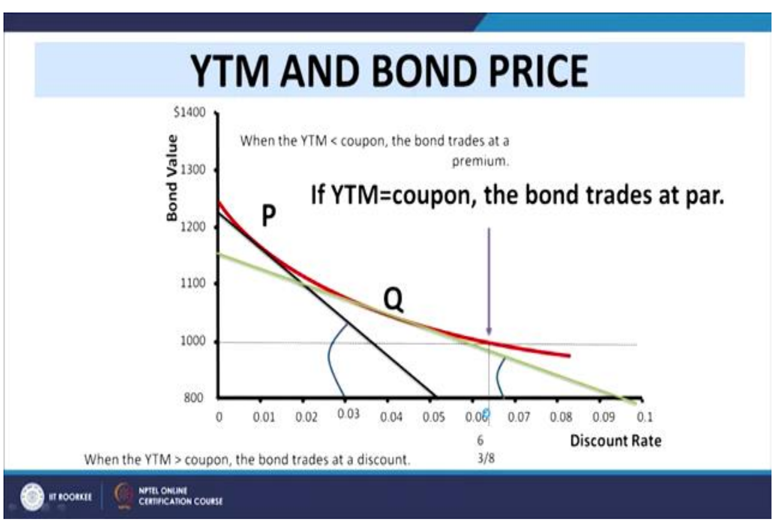

Now, this is a very important figure, a very interesting figure. What does this figure represent? This figure represents the price of the bond at various YTMs. The YTMs of the bond are plotted along the X axis and the price of the bond is plotted along the Y axis. There are some very important observations from this:

(i) The curve is not linear. It is not a straight line, it is curve.

(ii) The slope of this curve is negative.

(iii) The magnitude of the slope goes on decreasing as the YTM increases. The magnitude of the slope goes on decreasing as we move more and more to the right i.e. as  as the YTM increases. (iv) Because the slope is negative and the magnitude decreases, we can say the value of the slope increases as we move more to the right hand side i.e. as we move to higher YTMs. In other words, the slope is negative and it increases as the YTM increases.

Putting it in the language of differential calculus, we can say that (i) the derivative of the price with respect to YTM is negative, and (ii) the second derivative of the price with respect to the YTM is positive. Combining these two expressions, we infer that the yield price curve is a convex curve to the origin i.e. that is it is bulging towards the origin. As you can see from the shape of this curve itself, it is bulging towards the origin and that is convexity. This curve is convex.

So the yield price curve is not a straight line. I repeat, the first derivative of the curve is negative throughout and the second derivative is positive. This shows that the slope is negative to start with and secondly the slope increases as the YTM increases, but because the slope is negative, the magnitude of the slope decreases as we increase the YTM.

Anyway,  I will come back to this in more detail when I  talk about interest rate risk. But for the moment, we need to keep in mind that the curve is not a linear curve and it is a convex curve, towards the origin.

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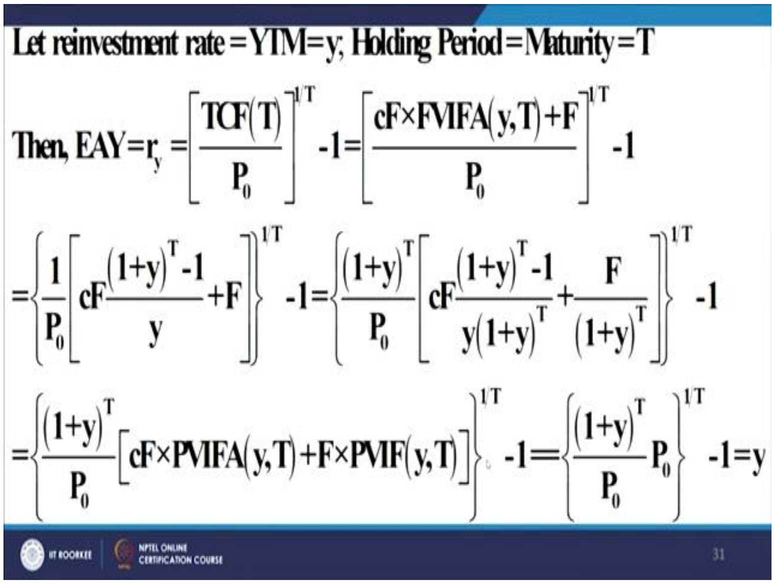


Now I discuss the assumptions of YTM. This is very, very important. Let us start with this particular derivation:

**Let reinvestment rate = YTM = y; Holding Period = Maturity = T**

$$\textbf{Then, } EAY = r_y = \left[\frac{TCF(T)}{P_0}\right]^{1/T} - 1 = \left[\frac{cF \times FVIFA(y,T) + F}{P_0}\right]^{1/T} - 1$$

$$= \left\{\frac{1}{P_0}\left[cF\frac{(1+y)^T - 1}{y} + F\right]\right\}^{1/T} - 1 = \left\{\frac{(1+y)^T}{P_0}\left[cF\frac{(1+y)^T - 1}{y(1+y)^T} + \frac{F}{(1+y)^T}\right]\right\}^{1/T} - 1$$

$$= \left\{\frac{(1+y)^T}{P_0}\left[cF \times PVIFA(y,T) + F \times PVIF(y,T)\right]\right\}^{1/T} - 1 == \left\{\frac{(1+y)^T}{P_0}P_0\right\}^{1/T} - 1 = y$$

Consider an investment in a level coupon bond of face & redemption value F and coupon rate c. Let us assume that the reinvestment rate, that is the rate at which the coupons are reinvested by the investor is equal to the YTM, y. Secondly, let us assume that the holding period of the investor is equal to the maturity of the bond, T. We find that under these two assumptions, the effective annual yield on the investment works out to exactly the YTM figure. The expression within these square brackets $\left[cF \times PVIFA(y,T) + F \times PVIF(y,T)\right]$ is equal to the present value of all coupon payments calculated at the YTM plus the present value of the face value which is also the redemption value, and therefore this must equate, by the definition of YTM itself, the current market price of the instrument $P_0$. What we end up is, on simplification, ***that the effective annual yield equals the YTM under the assumptions that (i) the bond is a level coupon bond (ii) the reinvestmentment rate of coupons equals the YTM and (iii) the investment horizon equals the maturity of the bond.*** So that is the inference of this slightly extended algebraic stuff. I repeat, if we consider a level coupon bond and make two assumption (i) the reinvestment rate is equal to YTM and (ii) the holding period of the investor is equal to the maturity of the instrument, then the effective annual yield derived by the investor is equal to the YTM of the instrument. So if we want that the YTM should reflect the true yield of the instrument then these two assumptions must be implied, must be underlined in the calculation of YTM. This leads us to the next slide.

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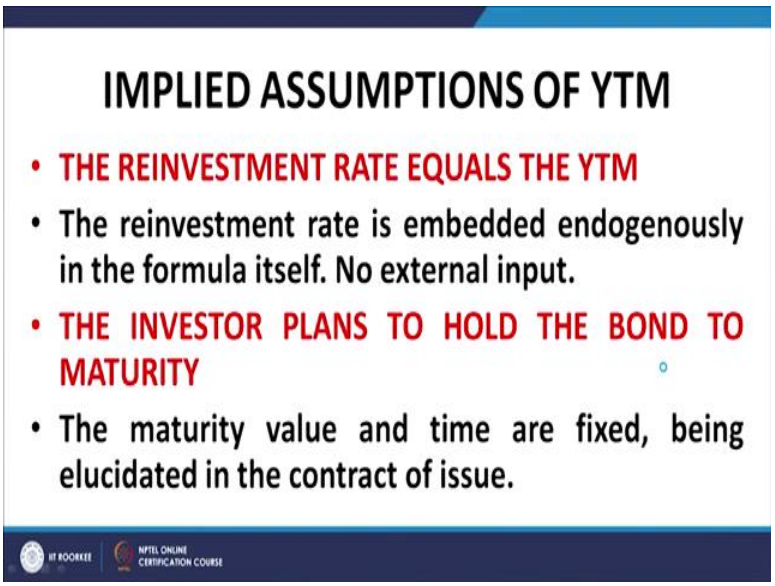


I have used the word "implied" in the context of assumptions undelying YTM. It is here because the formula for calculating the YTM $\mathbf{P_0} = \sum_{t=1}^{T} \frac{\mathbf{C_t}}{(\mathbf{1}+\mathbf{y})^{t}}$ itself encodes the assumptions that we are referring to (i)The reinvestment rate equals the YTM. This is the first assumption encoded in the YTM formula and (ii) The investor plans to hold the bond up to maturity. These two are fundamental assumptions that are encoded in the formula itself for computing YTM. You do not have to impose them from outside. You are not setting any external constraints or whatever. They are part of the mathematical framework of YTM itself.

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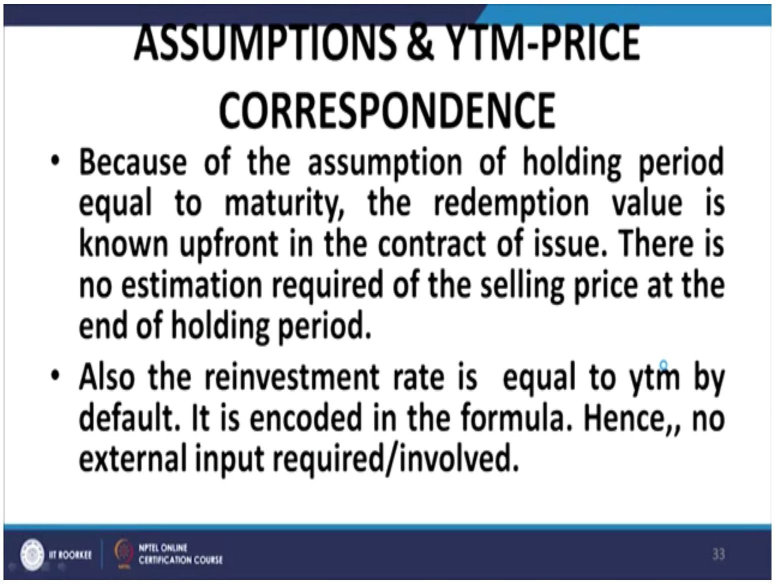


But what is the consequence of these assumptions? The consequence is this that, because the holding period is equal to the maturity of the instrument, there is no uncertainty as to the value of the redemption proceeds i.e. as to the value at which the investor is going to exit the investment. Why it is so? It is so because the holding period of the investment is assumed to be equal to the maturity and the maturity value of the investment is given upfront, it is encoded in the issue document. So it is predetermined and it has nothing to do with the market randomness. It will not fluctuate corresponding to market conditions, because it is there in the issue document or the contract of issue. What does this mean? This means that there is no uncertainty as far as the redemption value of the bond is concerned. It is known with absolute certainty being a part of the issue document. Secondly, as regards the reinvestment rate being equal to the YTM, it is again a part of the formula itself, it is not imposed from outside. Thus, again there is no uncertainty, there is no unknowability, there is no randomness, as far as the reinvestment rate is concerned. The YTM formula itself, as we shall see just now, encodes the assumption that all intermediate coupon payments are reinvested at the YTM rate. These two assumptions play a very significant role in the YTM. I repeat that YTM turns out to be a correct measure of yield only under these two assumptions. Only if these two assumptions hold, then the YTM equals the effective annual yield which is the correct yield on the investment.

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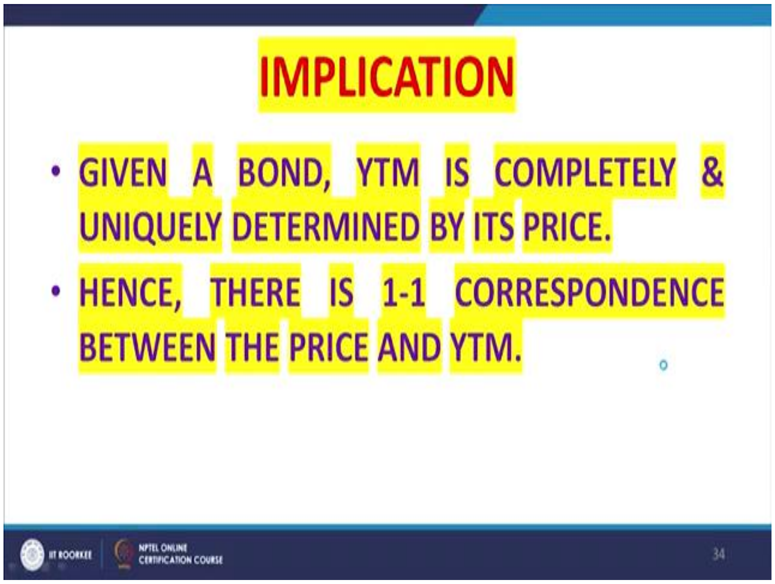


Now what is the implication of this? The implication of this is that if you are given a bond i.e. a spectrum of cash flows, then the YTM is uniquely determined by the current market price. That is what I have been emphasizing. There is a one to one correspondence between the price and the YTM. Now we understand why it is so. It is so because there is no randomness encoded in the calculation of YTM, the only input is the current market price.

As far as the reinvestment rate is concerned, YTM assumes that the reinvestment is at the YTM and as far as the holding period is concerned YTM assumes that the bond is going to be held up to maturity. So, with both of these imputed into the formula for calculating the YTM, we end up with an unambiguous situation where the market has no role to play, except for the price at which the instrument is taken up or invested in.

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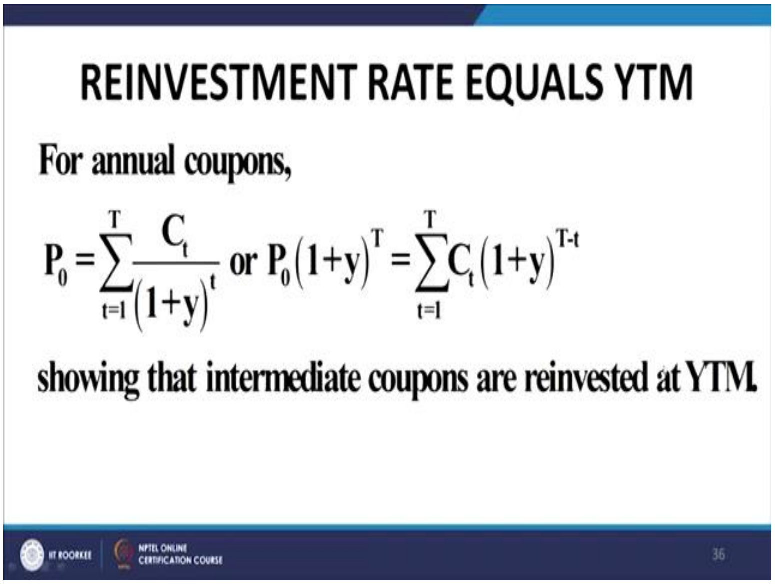


What is reinvestment rate? Reinvestment rate is the rate at which the coupon payments are reinvested by the investor. Now, the YTM formula can be written as: **For annual coupons,** $\mathbf{P_0=\sum_{t=1}^{T}\frac{C_t}{(1+y)^t}}$ **or** $\mathbf{P_0(1+y)^T=\sum_{t=1}^{T}C_t(1+y)^{T-t}}$. Both assumptions are pretty obvious from the latter formula. (i) It is seen from the latter formula explicitly that all coupons are reinvested at the YTM. (ii) Secondly, as seen explicitly from the first formula, the summation is over all cash flows right upto $C_T$ i.e. upto the maturity of the instrument. So we are summing the present value of all cash flows up to the maturity of the instrument. So that is the second assumption, that the investor is holding the instrument up to the date of maturity of the instrument. So both the assumptions are encoded within the formula for the YTM. We will continue after the break. Thank you.