Security Analysis & Portfolio Management
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Lecture 12
Yield to Maturity II

Welcome back, before the break I was talking about the various sources of return that arise from investment in a fixed income security. The first of course is the contractual coupon interest i.e. the coupon payments that the investor gets. The second arises from the reinvestment of the coupons at a certain rate of interest. This gives the investor interest on reinvested coupons. The third source is the capital gains or loss, which is the difference between the market price at which the investor exits the investment and the carrying value of the investment as per his books.

I shall come back to the definition of carrying value and an example at a later point in time. For the moment let us restrict ourselves to what carrying value is.

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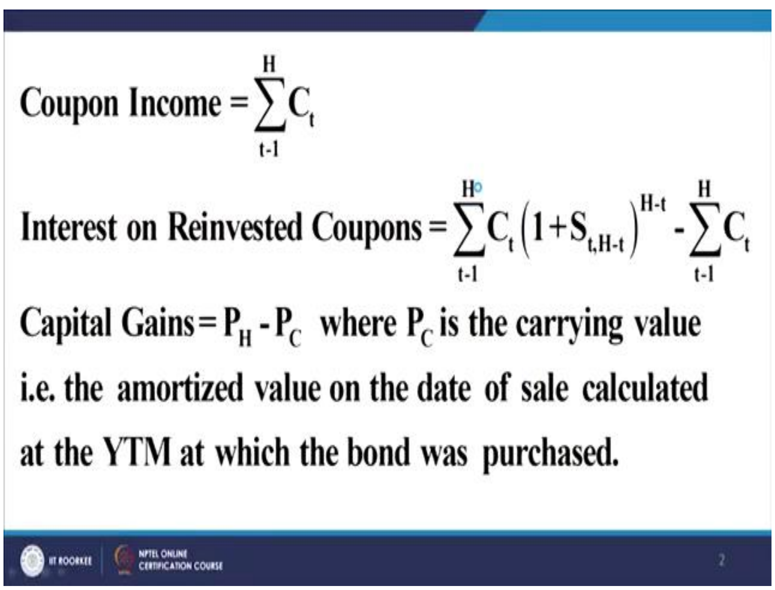

Carrying value on a given date is the amortized value of the security on that date as it appears in the books, calculated at the YTM at which the security was purchased.

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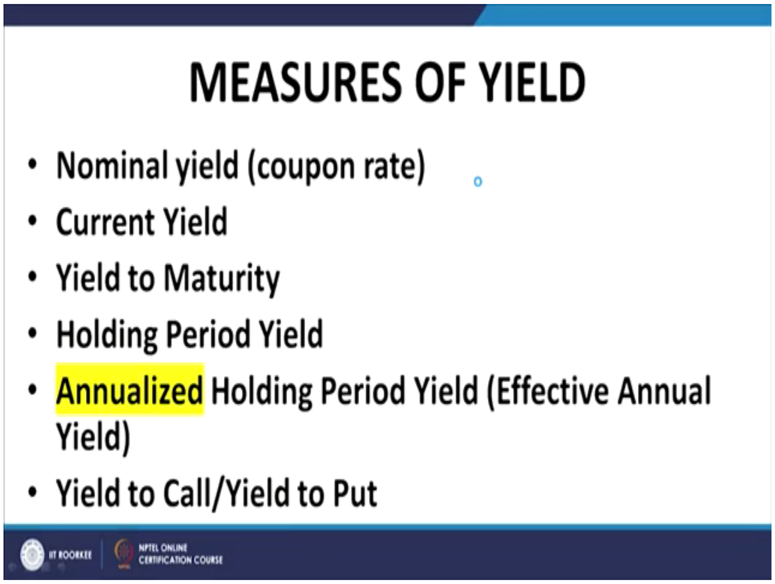

Then I talked about measures of yield (i) nominal yield, which is the coupon rate or the contracted rate that the investor gets on the investment in the bond; (ii) current yield, (iii) yield to maturity, (iv) holding period yield, (v) annualized holding period yield, (vi) yield to call and yield to put, if the bond has these additional option features.

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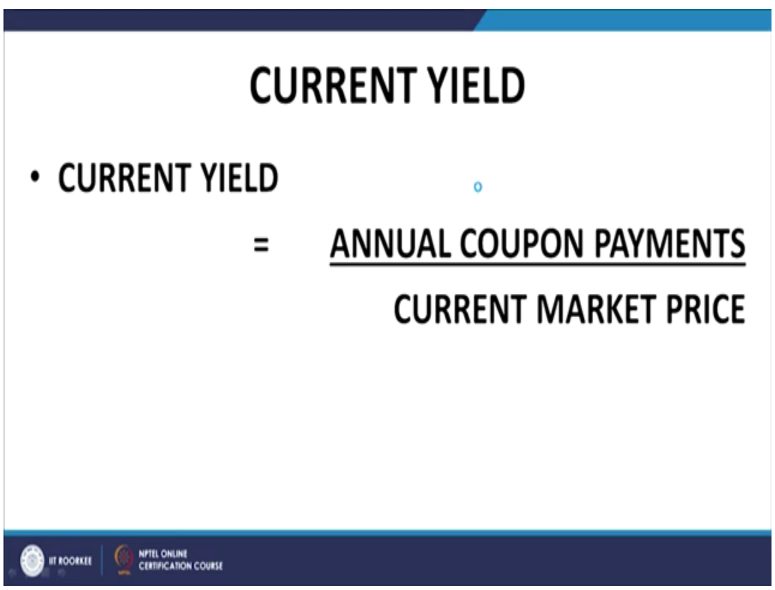

Now, current yield is the aggregate of the coupon payments for the year divided by the current market price. This is not a very useful or a very precise measure of yield in the sense that it does not take any account of the reinvested income, as well as of the capital gains or losses that may arise from the investment. Hence it is seldomly used in practice. It is a layman's measure of yield. It has serious pitfalls which restrict its accuracy and applicability. So current yield is just not the right measure of yield.

Then we come to yield to maturity. I have already elucidated the definition of YTM at an earlier point in time. Yield to maturity is the discount rate that equates the present value of future cash flows attributable to the security to its current market price $\mathbf{P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$.

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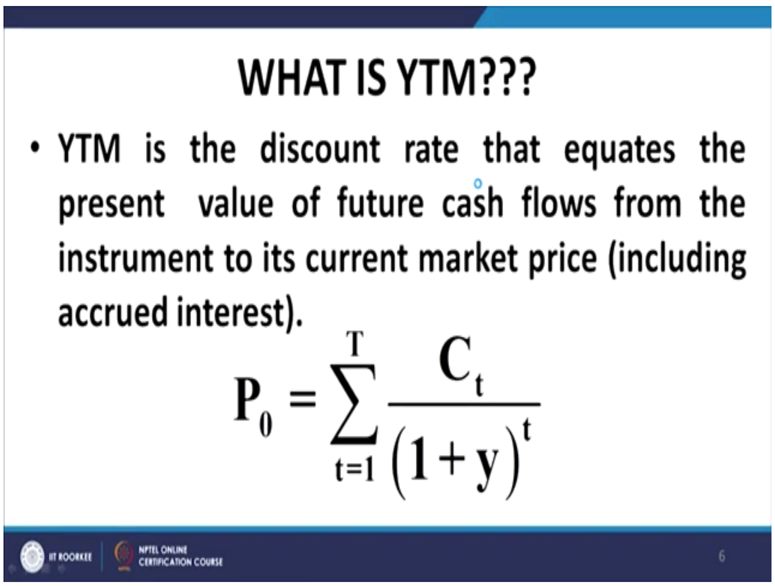

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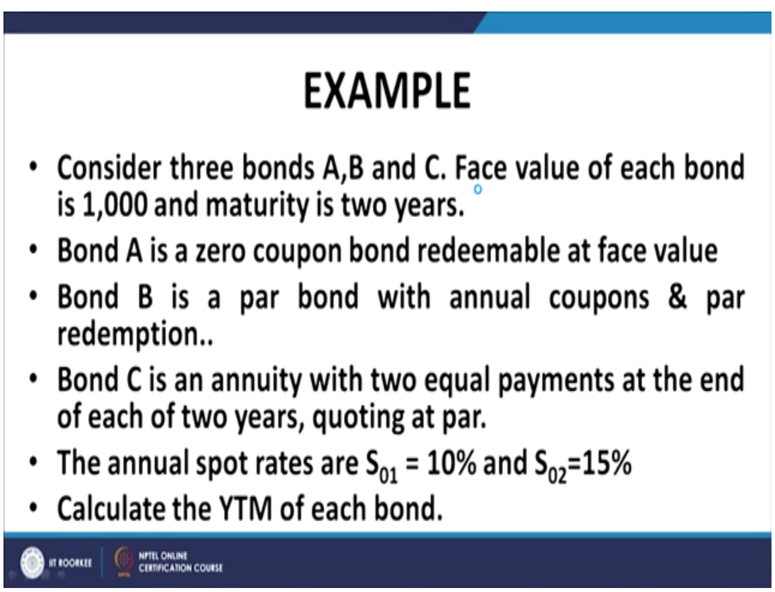

Let us do an example to be more clear about what this definition actually means and how YTM is worked out in practice. Consider three bonds, A, B and C. Face value of each bond is 1,000 units of money, whatever they may be, and the maturity of each bond is two years.

Bond A is a zero-coupon bond, redeemable at face value. Bond B is a par bond i.e. it is quoting at par in the market, with annual coupons and par redemption. Redemption is at face value and its present quote is also at face value. The annual coupons are paid. Please note the coupon rate is not given here.

Bond C is an annuity bond with two equal payments at the end of the first and the second year, equal total payment, total cash flows. An investment in bond C would yield equal cash flows in the first year and in the second year. In other words, both payments (at the end of the first year and second year) would comprise of coupon interest as well as a certain amount of principal redemption.

The annual spot rates are given as, S_{01}=10% p.a. S_{02}=15% p.a. We need to calculate the YTM of the three bonds.

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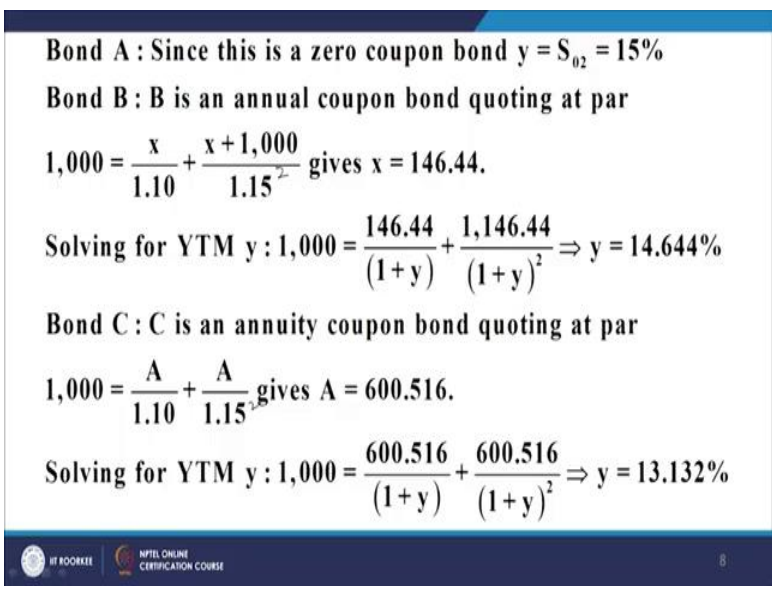

As far as the first bond is concerned, the calculation is quite straightforward because it is a zero coupon bond. Hence, the spot interest rate for the relevant maturity will be the YTM of the bond.

The spot interest rate for a two year bond is given as 15% p.a. Hence, the YTM of bond A is also 15% p.a. So this follows directly from the definition of YTM, because it is a zero coupon bond.

Now we come to bond B. Bond B is a par bond. This means the bond is quoting at par i.e. P_0=F. It is redeemable at par. Since this is a level coupon bond quoting at par, its YTM is equal to its coupon rate. Hence, we have:

$$P_0 = \frac{cF}{(1+S_{01})} + \frac{cF+F}{(1+S_{02})^2} \ or \ F = \frac{yF}{(1+0.10)} + \frac{yF+F}{(1+0.15)^2} \Rightarrow y = 14.644\% \ .$$

The bond C is an annuity bond. This means that it will make equal cash payments at the end of the first year and at the end of the second year. It is quoting at par. Hence,

$$P_0 = \frac{A}{(1+S_{01})} + \frac{A}{(1+S_{02})^2} . \ Also \ P_0 = \frac{A}{(1+y)} + \frac{A}{(1+y)^2}$$

$$or \ \frac{A}{(1+0.10)} + \frac{A}{(1+0.15)^2} = \frac{A}{(1+y)} + \frac{A}{(1+y)^2} \Rightarrow y = 13.132\%$$

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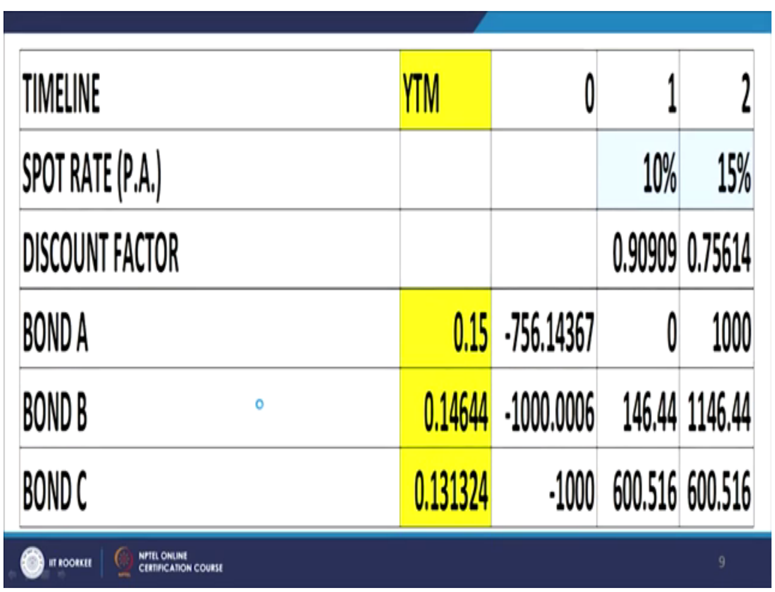

TIMELINE	YTM	0	1	2
SPOT RATE (P.A.)			10%	15%
DISCOUNT FACTOR			0.90909	0.75614
BOND A	0.15	-756.14367	0	1000
BOND B	0.14644	-1000.0006	146.44	1146.44
BOND C	0.131324	-1000	600.516	600.516

This is the excel sheet working of the same example. The YTM of the bonds are yellow highlighted figures. YTM of bond A is 15% p.a., B is 14.644% p.a. C is 13.132% p.a.

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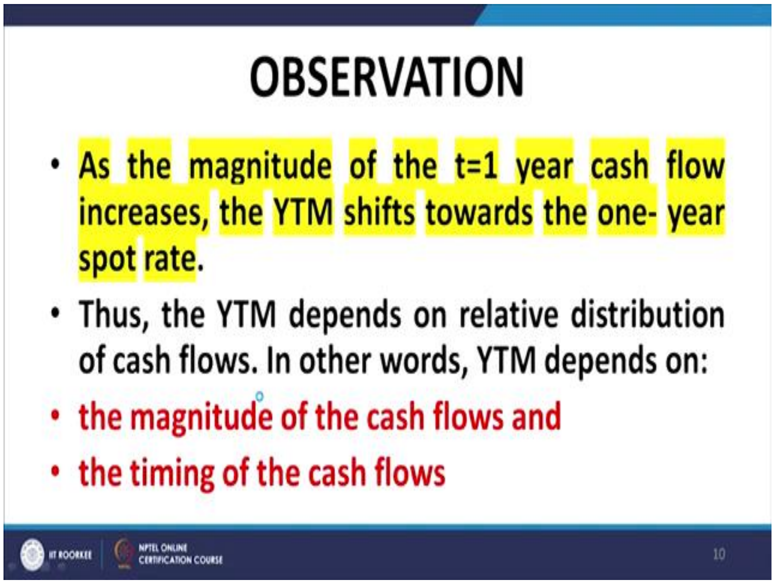

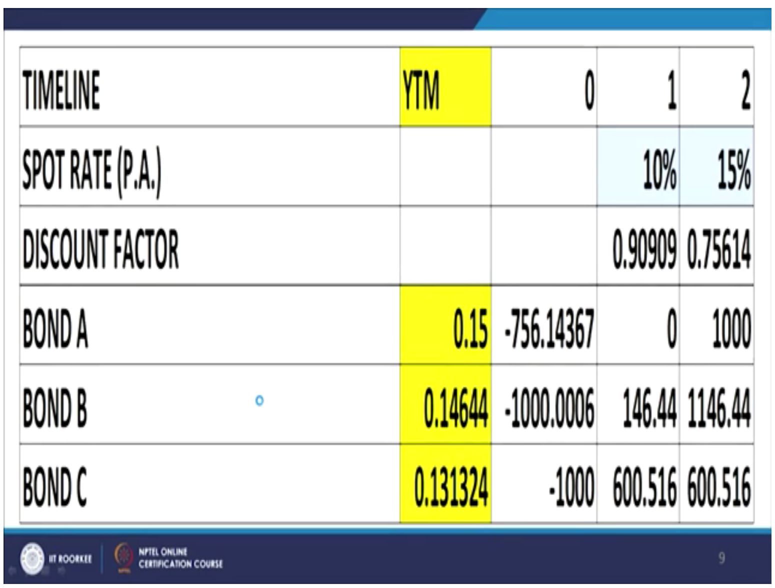

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Now, some important observations are in order.

As the magnitude of the cash flow at t=1 increases, the YTM shifts towards the one year spot rate. Let us look at the relative distribution of cash flows across the two years. In the case of bond A it is 0 in the first year and 1,000 in the second year. In B it is 146 and 1,146 and in C it is 600 and 600. Now, we look at the YTM figures, bond A has 15% (which is the spot rate for two years), bond B has 14.64%, which is lesser than 15% and relatively closer to 10% (which is the one year spot rate). In other words, some movement towards the spot rate for 1 year is evidenced as the first year cash flow increases relatively. C has YTM of 13.13% showing that the shift towards the first year rate gets more pronounced as the relative proportion of first year cash flow increases. Greater is the proportion of cash flows in the first year, greater is the tilt towards the first year spot rate of the YTM figure. Thus the YTM depends on the relative distribution of cash flows. In other words, YTM depends not only on the magnitude of the cash flows but also the timing of the cash flows. We will see that through explicit examples.

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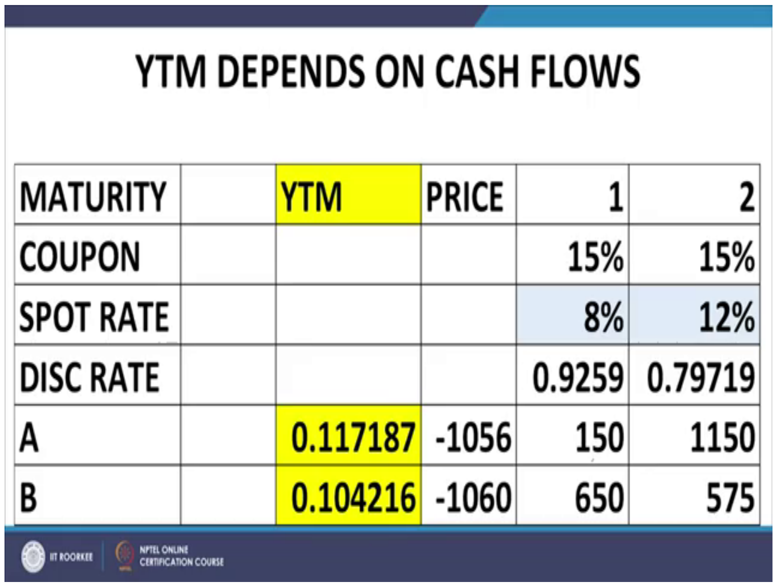

MATURITY		YTM	PRICE	1	2
COUPON				15%	15%
SPOT RATE				8%	12%
DISC RATE				0.9259	0.79719
A		0.117187	-1056	150	1150
B		0.104216	-1060	650	575

We have two annual coupon bonds here A & B. The spectrum of spot rates is S_{01}=8% p.a. & S_{02}=15% p.a. The coupon rate is 15% p.a. for both. Let A be redeemed at face value at the end of 2 years and B be redeemed at face value in two equal instalments at the end of the first and second years. Thus, cash flows from A are: 150 at t=1 and 150+1,000=1,150 at t=2. (It gives you 150 as coupon at the end of the first year and 150 on account of coupon and 1,000 as redemption value aggregating 1,150 at the end of the second year). Bond B gives 150 as coupon plus 500 as redemption value aggregating 650 at the end of the first year (t=1) and 75 on account of coupon plus 500 as remaining redemption value aggregating 575 at the end of second year (t=2). The total cash flow at the end of the first year is 650, comprising of 500 on account of principal redemption and 150 on account of interest and at the end of second year it is 500 of principal redemption and only 75 on account of interest, because coupon will be paid only on the outstanding principal, so interest would be paid on this 500 only.

So the net cash flows for the bond A are 150 and 1,150 respectively for first year and second year and for B they are 650 and 575 respectively. When we work out the YTMs, we find the YTMs to be 11.72 percent for the bond A and 10.42 percent for the bond B. So it is quite clear that the pattern of cash flow determines the YTM of the instrument i.e. the YTM is dependent upon the pattern of cash flows from the instrument. And interestingly higher is the relative proportion of cash flows in the second year, higher is the tilt of the YTM towards the second year spot rate and vice versa.

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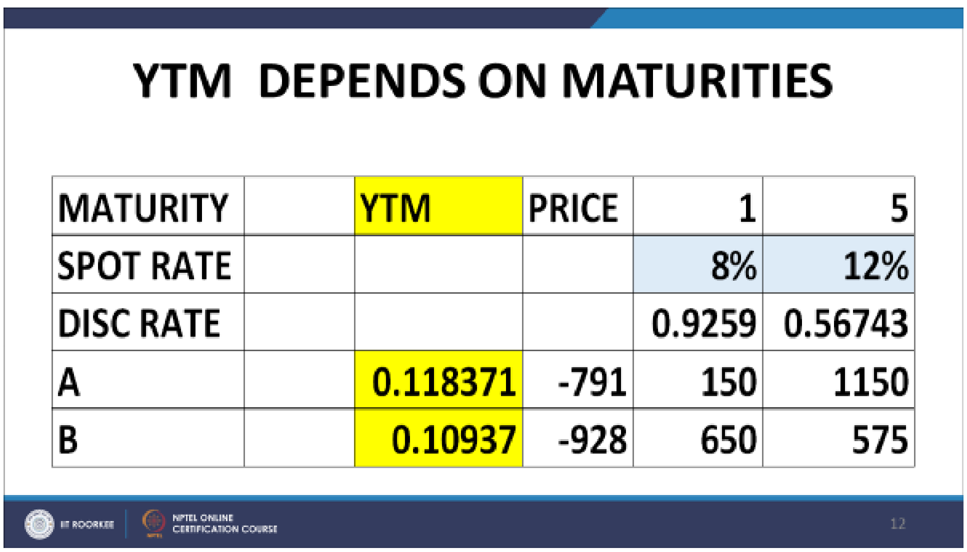

MATURITY		YTM	PRICE	1	5
SPOT RATE				8%	12%
DISC RATE				0.9259	0.56743
A		0.118371	-791	150	1150
B		0.10937	-928	650	575

Now, YTM also depends on maturities. I have talked about YTM depending on the relative magnitudes of cash flows, YTM also depends on the maturities. Now, consider two bonds A & B both of which have a maturity of 5. The spot rates are now S_{01}=8% p.a. and S_{05}=12% p.a. The cash flows replicate the pattern as in the previous example i.e. we have 150 at t=1 year and 1,150 at t=5 years for A and 650 & 575 at t=1 year and t=5 years for B. No intermediate cash flows arise in either case. When we look at the YTM figures, we find that the YTM has changed from the previous figures. For bond A, YTM has changed from 11.72% p.a. to 11.84% p.a. B's YTM has changed from 10.42% p.a. to 10.93% p.a.

So the message that I want to convey by reference to these two examples is that the YTM depends not only on the magnitude of cash flows but it also depends on the timings of the cash flows. In other words, the YTM depends on both the magnitude and timing i.e. the pattern of the cash flows from the bond.

Now, why do we need YTM? I try to address this issue by reference to this particular example.

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WHY YTM???

MATURITY		YTM	PRICE	1	2
COUPON				15%	15%
SPOT RATE				8%	12%
DISC RATE				0.9259	0.79719
A		0.117187	-1056	150	1150
B		0.104216	-1060	650	575

Suppose there are two bonds A & B each of maturity 2 years and we propose to hold the bonds to maturity. We want to select, which one of the two bonds A or B is better for us. I reiterate, our investment horizon is upto the maturity of the two bonds i.e. 2 years. Both bonds pay a coupon of 15% p.a. Bond A has redemption in one installment at face value at the end of the second year. Bond B has redemption in two equal installments at face value at the end of the first year and the second year i.e.500 at the end of the first year and 500 at the end of the second year. The spot rates are S_{01}=8% p.a. and S_{02}=12% p.a. So now our problem is to ascertain which of the two bonds is better for us. Now based on only the above information, it becomes very difficult to take a call on which bond is better. They are paying the same coupon rate and they have the same maturity.

We do note that the pattern of cash flow is different because of a different redemption pattern. Nevertheless, we find that even in the second case, because the bond is repaying back 500 at the end of first year, so the investment is reduced and therefore the lower interest for the second year is justified and hence, B is also a reasonable choice. But we need a measure of yield, which gives us an unambiguous answer, subject of course to the assumptions that we make.

There are reasons for qualifying the above statement. I will come back to that, but for the moment we need to identify some kind of a measure with which we could arrive at a decision on which instrument is better. And that is where the role of YTM comes into play. If we check the YTM of the two bonds, we find is the YTM of bond A is 11.72% p.a., the YTM of bond B is only 10.42% p.a. So on the basis of YTM, we feel that the bond A is better for investment than bond B.

So there is a message that the YTM can convey to us. But as a finance student it is more interesting to examine the reason why YTM of bond A is higher than the YTM of bond B. The reason lies in the spectrum of spot rates. The one year spot rate is only 8% p.a., the two year spot rate is 12% p.a. Now, we also note that the price of bond A is P_A=1,056 and that of B is P_B= 1,060. In the case of A, the component of this price (which is the initial investment) at is invested at the one year spot rate of 8% p.a. is 150/1.08=138.89 and that invested at the two year rate of 12% p.a. is 916.77. The corresponding figures for B are respectively 601.85 and 458.39. Clearly, there is a greater investment in the case of A at the higher rate of interest. Hence, A yields a higher YTM.

Another way of explaining the above differential is to constitute a portfolio consisting of (i) a long position in bond A and (ii) a short position in B. Clearly, the cash flows at t=0 almost cancel each other. The cash flows at t=1 year will be 150-650=(-)500. The cash flows at t=2 years will be 1150-575=(+)575. Hence, the rate of return on this portfolio is 15% p.a. which is lower than the no arbitrage forward rate of 16.14% showing that B is overpriced.

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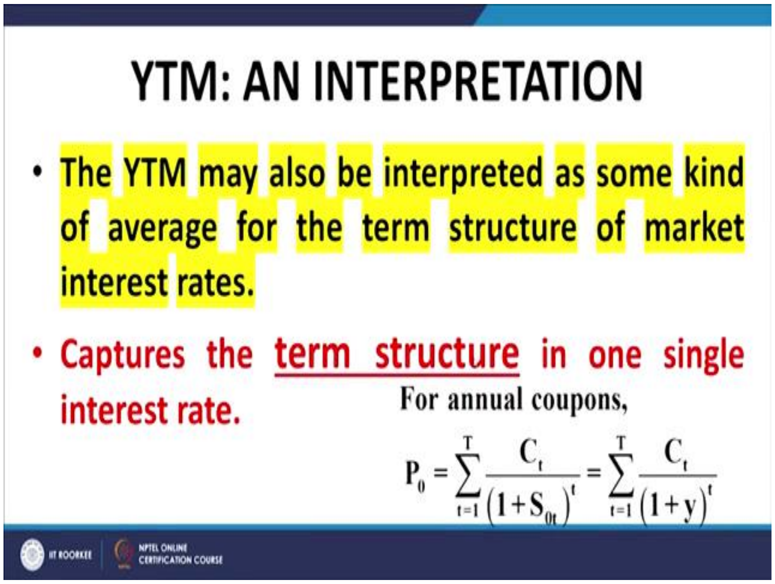

Let us now look at an interpretation of YTM. Let us look at this formula right at the bottom of this slide **For annual coupons,** $\mathbf{P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t} = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$. On the one side, P_0 is equal to summation of C_t divided $(1+S_{0t})$ to the power t. I mentioned a number of times and reiterate once again that there is a functional relationship between the maturity date of the deposit and the rate of interest which is represented by the subscripts in S_{0t}. The cash flow at time t, C_t, is discounted with respect to the rate, S_{0t}, which is relevant to that maturity, and which has to be specified. because we cannot use the same rate for the discounting of all cash flows since these cash flows occur at different points in time and rates vary with maturity.

It is empirically seen that the actual market gives you different rates for different maturities, and therefore I need to add a subscript to each rate (indicating to what maturity it pertains), which is being used for discounting the corresponding cash flow. C_1 has to be discounted with respect to S_{01}. C_2 with respect to S_{02}…., C_T with respect to S_{0T} etc.

But when we look at the YTM formula $\mathbf{P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$, there is nothing which is relating to the term structure of interest rates. In other words, when we work out the YTM, the output returned is only a single number. Let us revisit the definition of YTM. It is that ***single*** discount rate, please note that, at which the present value of all future cash flows, equate its current market price. ***So it is that one discount rate, which when applied over the entire life of the instrument will give you the current market price.***

So, what I am trying to get at is that, in the equation: $\mathbf{\sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t} = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$, on the one side we have a spectrum of rates, S_{01}, S_{02}, S_{03}, S_{04} etc and on the other side we have a single rate. That means that the single rate is, in some sense, representative of the entire spectrum of spot rates. In other words, it does, in some sense, capture the entire spectrum of spot rates which is relevant to discounting the cash flow pattern of a bond, in one single number. That is what the YTM is. It may not be the conventional average, though. That is the interpretation of YTM. So YTM may be interpreted as some kind of average of the term structure of interest rate. It is that one single figure, which captures the entire spectrum of spot rates.

There is another interesting observation. It is that the YTM will always lie between $S_{minimum}$ that is the lowest spot rate and the $S_{maximum}$, the highest spot rate. The YTM of any instrument will always lie between the lowest spot rate and the highest spot rate. Again this property vindicates my contention that the YTM is in some sense the averaging of the term structure of spot rates, relevant to that particular instrument.

$\mathbf{P_0 = \frac{F}{(1+S_{0T})^T} = \frac{F}{(1+y)^T}}$ **OR** $\mathbf{S_{0T} = y}$. This is the YTM of a zero coupon bond, we have already discussed that. So I will not devote time to this.

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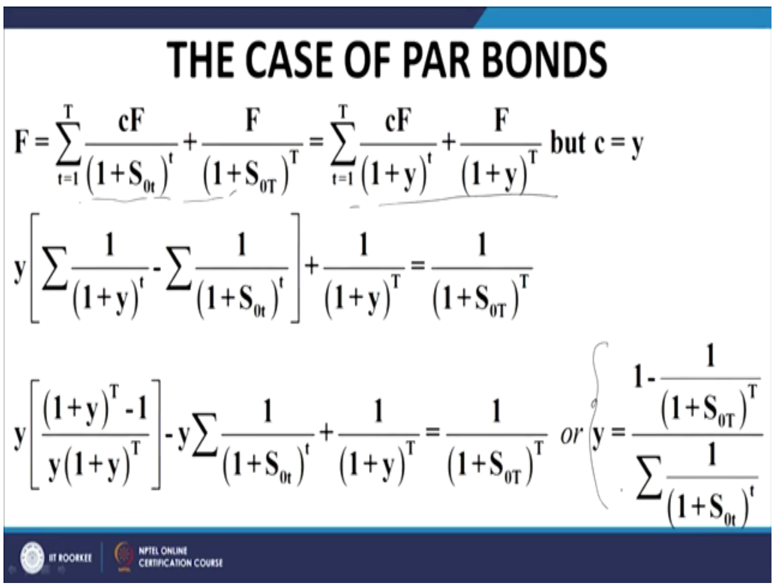

For bonds quoting at par, the formula for YTM simplifies, because in the case of a level coupon bond quoting at par, we have (i) P_0=F, and (ii) c=y. Hence,

$$F = \sum_{t=1}^{T} \frac{cF}{(1+S_{0t})^t} + \frac{F}{(1+S_{0T})^T} = \sum_{t=1}^{T} \frac{cF}{(1+y)^t} + \frac{F}{(1+y)^T} \text{ but } c = y$$

$$y\left[\sum \frac{1}{(1+y)^t} - \sum \frac{1}{(1+S_{0t})^t}\right] + \frac{1}{(1+y)^T} = \frac{1}{(1+S_{0T})^T}$$

$$y\left[\frac{(1+y)^T - 1}{y(1+y)^T}\right] - y\sum \frac{1}{(1+S_{0t})^t} + \frac{1}{(1+y)^T} = \frac{1}{(1+S_{0T})^T} \quad or \quad y = \frac{1 - \frac{1}{(1+S_{0T})^T}}{\sum \frac{1}{(1+S_{0t})^t}}$$

So this is a simplified expression that we get for the case of YTM of par bonds i.e. level coupon bonds quoting and redeemable at par. We can do some simplification in the case of these bonds.

In the normal course of events YTM is not easy to calculate. If we have T cash flows, we get a T^{th} order equation, which will have T roots. It is not necessary that all the roots may be real roots, but then the exercise of solving these equations is quite challenging. We have to use numerical analysis for that purpose. There is no exact formula as of now. For par bonds, we can simplify the situation radically.

Now we come to a very important property of YTM. If we look at the formula for YTM, $\mathbf{P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$ we find that the inputs needed for the YTM are the pattern of cash flows i.e. the magnitude and timing of the cash flows and the market price. The pattern of cash flows is encapsulated in the contract of issue and is singular to the given bond. The cash flow pattern is insulated from the market. It has nothing to do with the market. Market has no influence on the pattern of cash flows, that is, the timing and the magnitude of the cash flows, because they are in the contract of issue. The only other input is the market price, which is again a figure that is known at t=0. So there is no randomness embedded in either the price or the cash flow pattern (ignoring the default possibility, for the moment) and that means that the value of YTM also carries no randomness and is singular to the instrument.

Furthermore, because the numerator, the C_t figure, the cash flows are fixed by contract implying that there is a one to one relationship between the current market price and the YTM of the instrument. I repeat, ***there is a one to one correspondence between the YTM and the market price of the instrument.*** Indeed, in practice, instead of quoting bonds by their market prices, it is very often the case that a bond is quoted by its YTM, because knowing the YTM of a bond, and the pattern of its cash flows, one can easily work out the market price and arrive at it unambiguously. There is no ambiguity about it, because there is a one to one relationship between the market prices and the bond cash flows.

I have so far eluded to the one to one correspondence between the market price and the YTM. From the equation, $\mathbf{P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t} = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$ we see that the market price can also be worked out on the basis of the spectrum of spot rates applied to the cash flow pattern. Thus, if we are given the cash flow pattern and the corresponding spot rate spectrum, we can work out the unique YTM & s market price.

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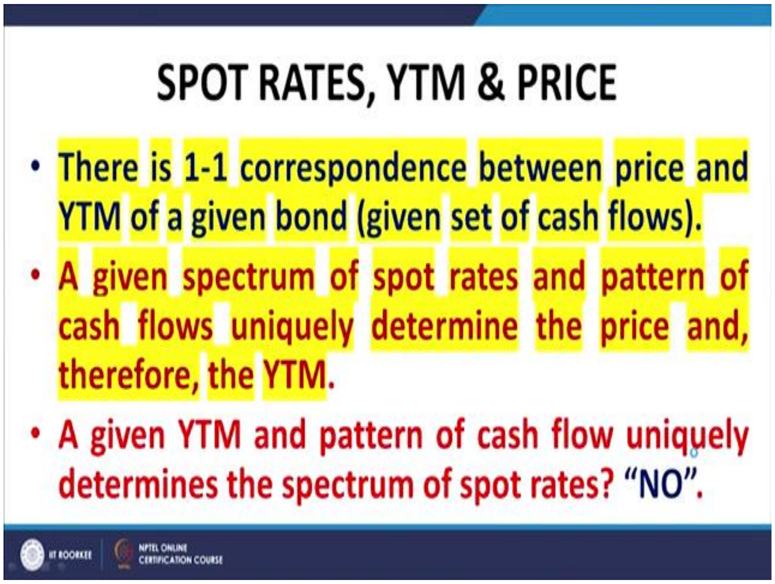

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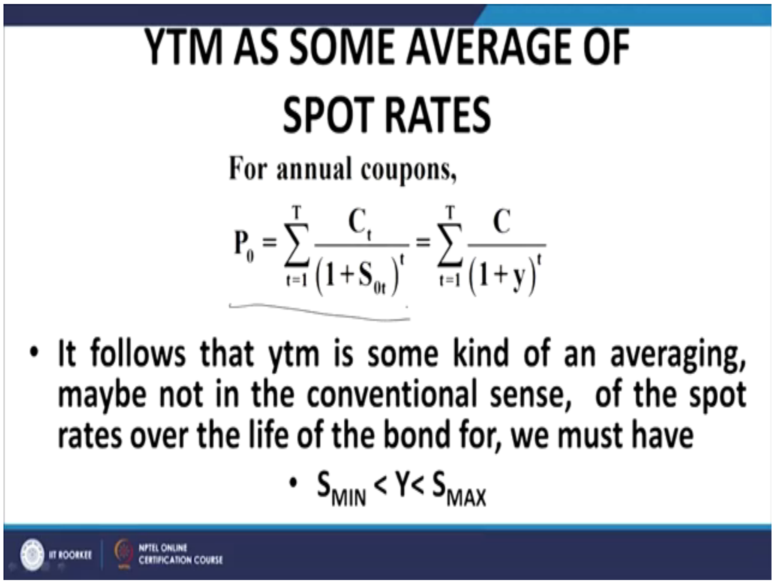

Let us go back to that formula: $\mathbf{P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t} = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$. If we look at the first part of this formula, we can work out the price on the basis of the spectrum of market spot rates and the pattern of cash flows. Thus, because the pattern of cash flows is unambiguously defined in terms of the contract, using the appropriate spectrum of interest rates prevailing in the market, we can work out the market price, again, unambiguously. If we know the market price, we can work out the YTM unambiguously. So that means given a spectrum of cash flows, given a spectrum of spot rates, we can unambiguously arrive at a value of YTM. This is another important feature.

So price and YTM have a one to one correspondence. Further, given a spectrum of spot rates, we can also arrived at an unambiguous value of YTM for a given pattern of contractual cash flows.

But the converse is not true. In other words, if we are given the YTM of an instrument and we are given the pattern of cash flows, we cannot work out the spectrum of spot rates, unambiguously. So given the YTM we cannot work out the spectrum of interest rates, but given the spectrum of market rates we can work out the YTM of a cash flow pattern. We will continue from here. Thank you.