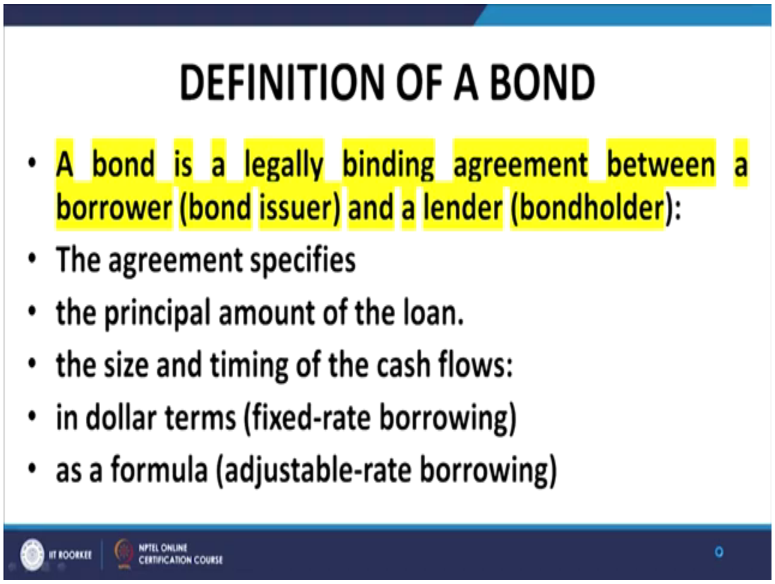
Security Analysis & Portfolio Management
Professor J P Singh
Department of Management Studies
Indian Institute of Technology, Roorkee
Lecture 11
Yield To Maturity I

Welcome back, so before we continue, a quick recap of where we were last time, with respect to bond analysis.

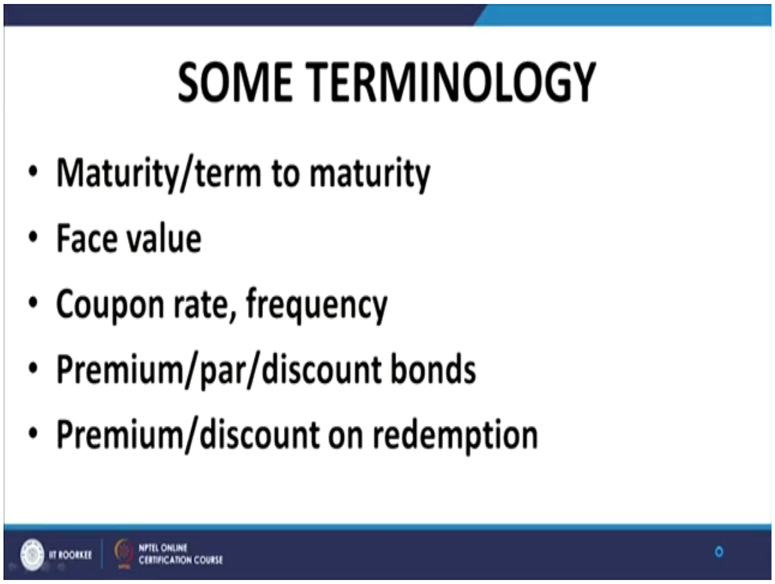
(Refer Slide Time: 00:36)



We defined a bond as a legally binding agreement between the borrower, who is the bond issuer and a lender who is the bond holder. This agreement specifies the principal amount of the loan and the size and timing of the cash flows that are to arise on account of interest and the repayment of principle.

This cash flow interest payments may be defined at a fixed rate or with reference to another variable like LIBOR or MIBOR etc. (in which case the bond is called a floating rate bond). If the interest rate is fixed then the bond is called a fixed rate bond.

(Refer Slide Time: 01:29)



Now, some terminology, maturity is the point in time at which the bond matures for the repayment of principal. The time remaining from today up to the maturity date of the bond is called its term to maturity".

The face value is a very important concept. It is the notional value or nominal value of the bond. It is the value of the bond with respect to which coupon payments are made. In other words, to arrive at the cash flow on account of the coupon payments, we multiply the coupon rate, which is given in terms of the percentages, with the face value.

Similarly, the redemption value is also usually expressed in terms of the face value. Either redemption may be at face value or it may be with reference to the face value at a premium or a discount. Coupon rate is the contracted rate of interest. Please note this very important difference. Coupon rate is not the market rate of interest. It is the rate of interest which is given in the issue document. It is the contracted rate and in the case of a fixed rate security, the coupon rate will remain fixed over the life of the bond.

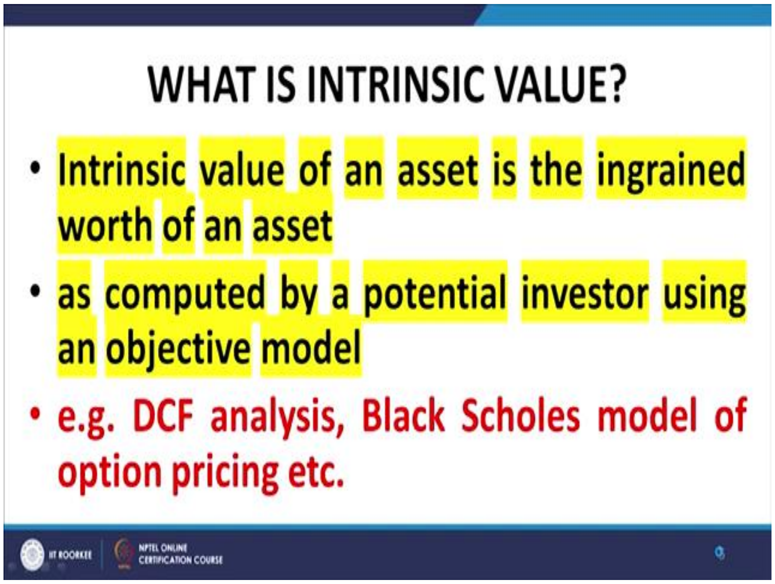
The frequency of coupon payments is also specified in the issue document and is singular to the instrument as is the coupon rate itself. In fact the coupon rate and the frequency of coupon need to be specified together to arrive at the cash flow pattern arising from the bond.

At a particular point in time, bonds may be selling at a premium (that is above face value) or at a discount (that is below face value) or they may be selling at par (that is at face value), depending on the relationship between the coupon rate and the then prevailing market rates of interest.

Bonds may be redeemed at face value, which is usually the case, but not necessarily so. Bonds can be redeemed at a premium or discount to face value. There are no legal restrictions mandating the redemption of bonds at face value. Redemption below or above the face valueis legally admissible. But it is important here to emphasize that whatever the case may be, whether the bonds are to be redeemed at a premium or a discount, the amount or percentage thereof in relation to face value needs to be specified in the issue document at the time of issue of the instruments. In other words, the redemption proceeds need to be unambiguously specified in the issue document. You cannot have a situation where the redemption value is fixed at a later day during the life of the bond. It has to be pre-specified in the contract of issue.

Then, we came to a very important concept, that is the concept of intrinsic value. I emphasized that intrinsic value of security is not only security specific, but it is also investor specific in the sense that the intrinsic value of an instrument is arrived at by an investor on the basis of a model chosen by him, (which he deems appropriate for the evaluation of the security being analyzed), and also on the basis of the inputs that go in to the model as estimated by the analyst or the investor, as the case may be.

(Refer Slide Time: 04:44)

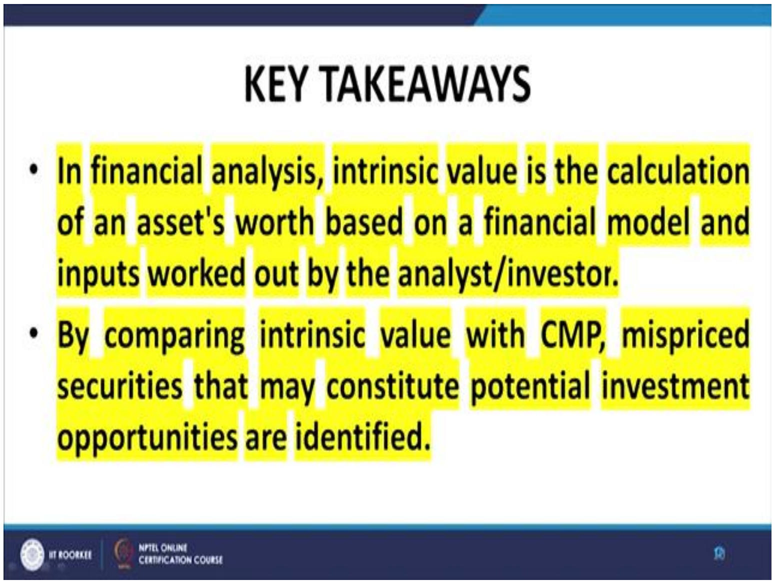


So the important thing is that intrinsic value is not a market based attribute of a security, it is not a market based value. It is a value which is computed by reference to a model which the investor deems to be appropriate and the inputs to this model are also estimated by the investor. So to that extent intrinsic value is singular to the investor and gives the worth of the security as perceived by the investor. Of course, then the investor can compare his perception of the value of a security as encoded in its intrinsic value, with the current market price and take investment decisions accordingly.

(Refer Slide Time: 05:27)

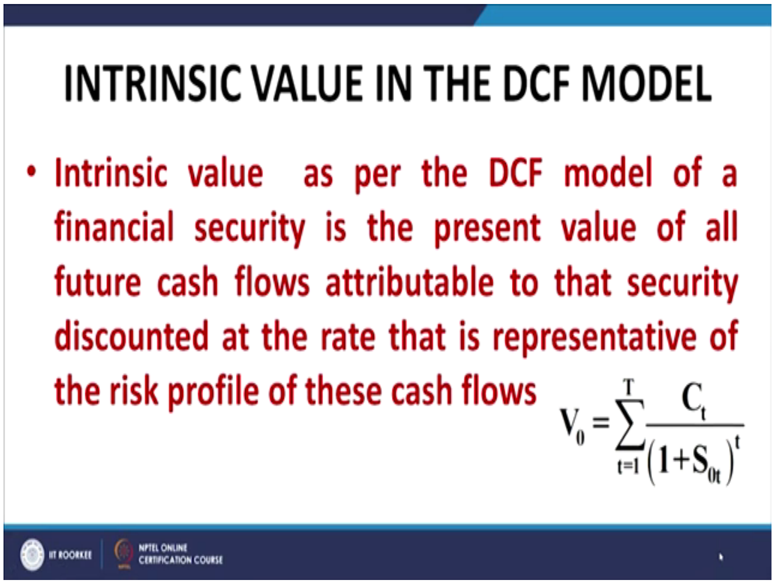
Then we talked about how the market trades emanate. As I mentioned just now, we have a comparison between the intrinsic value that the investor obtains and the current market price and on that basis, the investor identifies mispriced securities and investment decisions follow thereafter.

(Refer Slide Time: 05:51)



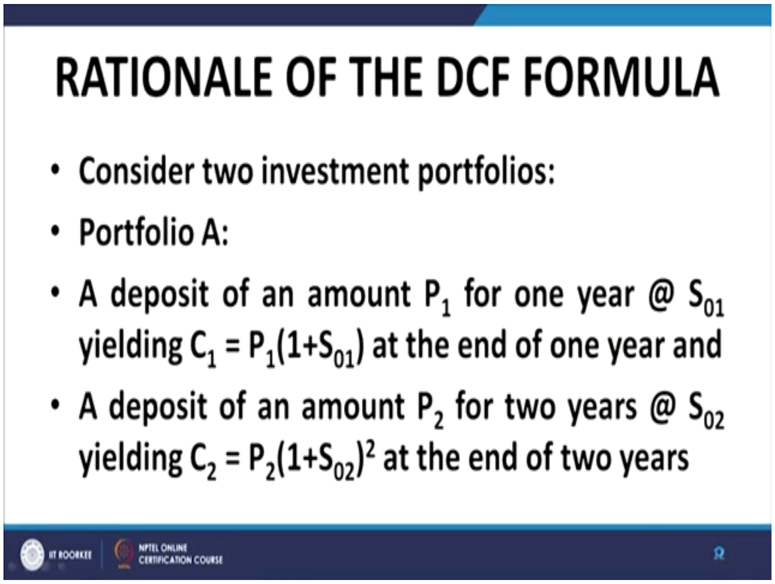
So the key takeaways are, let me read them out. In financial analysis, intrinsic value is the calculation of an asset's worth based on a financial model, with inputs that are worked out by the analyst or the investor. By comparing the intrinsic value with the current marker price, mispriced securities that may constitute potential investment opportunities are identified.

(Refer Slide Time: 06:19)



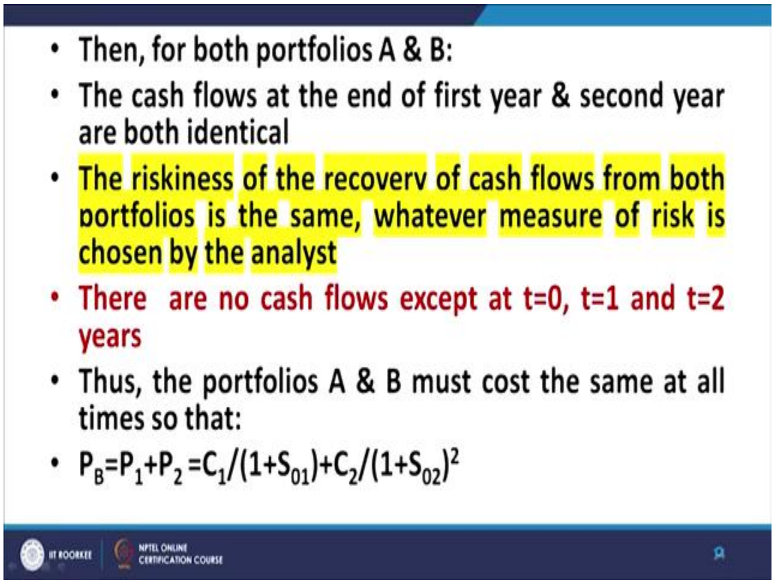
Then, I talked about the intrinsic value as computed by the discounted cash flow model. As per the discounted cash flow model, the intrinsic value of a security is the present value of all future cash flows that maybe attributed to that particular security, discounted at a rate, which is appropriate to riskiness rather of realizability of the cash flows. So, on the basis of the assessment or the riskiness of the cash flows, we arrive at the intrinsic value of the securities by discounting those cash flows at the risk adjusted discount rate. The formula thereof is given right at the bottom of the slide in the right hand corner.

(Refer Slide Time: 07:17)

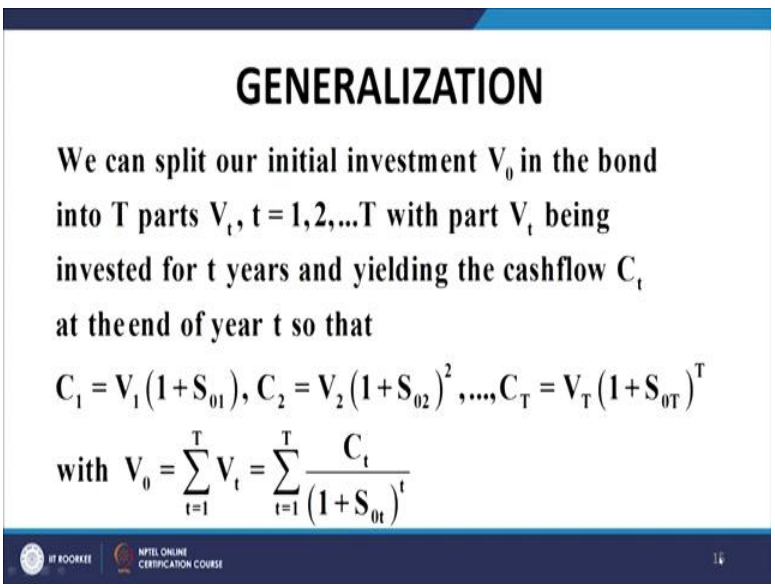


Then I discussed in detail the rationale of the DCF model by constructing two portfolios and invoking the law of one price & arbitrage procedures. Portfolio A comprised of two deposits for valuation of a two year bond, the first with a maturity of one year and the second with a maturity of two years. The cash flows from this portfolio were compared with a portfolio that involved a bond which has a lifespan of two years and which gave cash flow C_1 and C_2 at the end of year 1 and year 2 respectively. I will not go through it again. You can refer to the previous lecture in which this was discussed in a lot of detail.

(Refer Slide Time: 07:45)

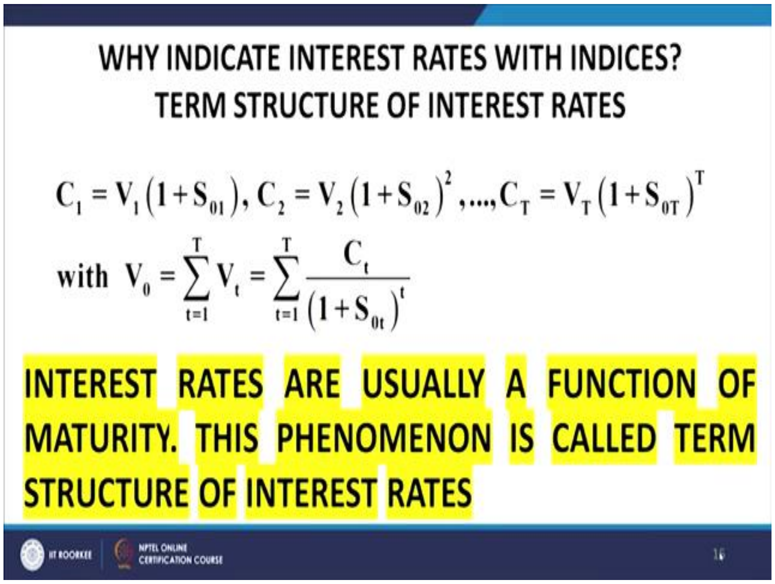


(Refer Slide Time: 07:53)



Then the above was generalized to the entire life of a T-period security, or a security which generates cash flows over T periods, and the formula there of is given right in the bottom line of your slide.

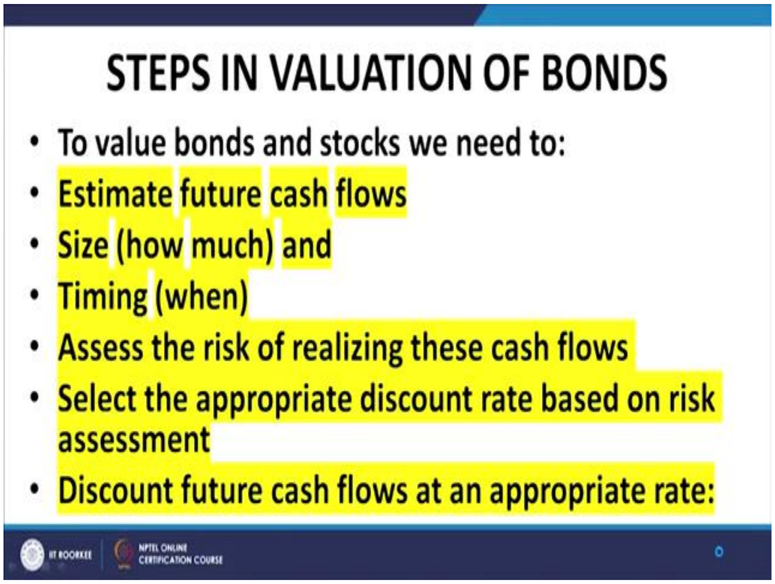
(Refer Slide Time: 08:10)



Then I came back to one particular point and that was the reference to the use of subscripts to describe interest rates. Why I had used subscripts to describe interest rates? This is because of the phenomenon which prevails in the interest rate market,and which is termed as the term structure of interest rates. What this means is that there is functional relationship between the interest rates and the maturity of the underlying deposits. In other words, if you make a deposit for six months you will get a particular rate, which may be different from the rates you get on a deposit for one year or three years.

In other words, the maturity of the deposit has a relationship with the interest rates that we get on the deposit. Therefore, it is necessary to specify the interest rates that are relevant to a particular cash flow discounting because the cash flows arise at different points in time. So they need to be discounted at the appropriate deposit rates for the particular time interval or the time points at which the cash flows are taking place.

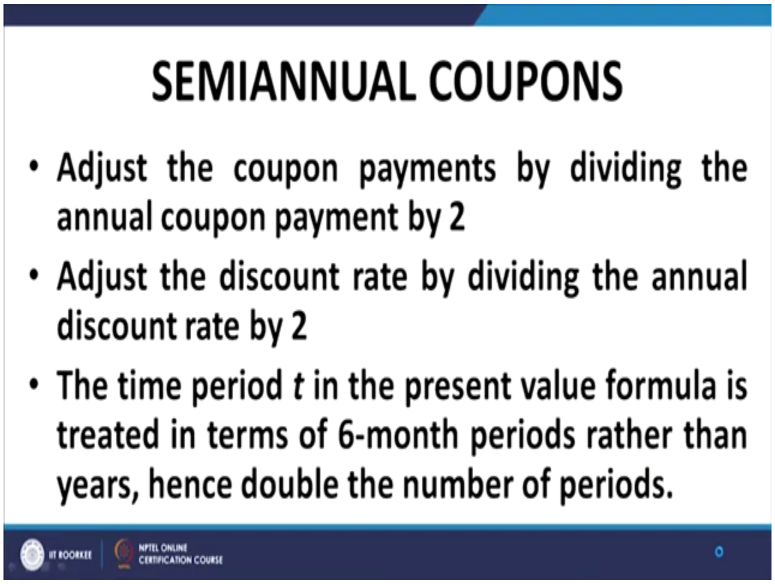
(Refer Slide Time: 09:28)



Let us recap the steps in the evaluation of bonds. (i) Estimate future cash flows. I emphasized that so far as the estimation or cash flows of a bond security is concerned, it is a relatively easy exercise because all the information required therefor is encoded in the issue document and we do not have to look far to arrive at the cash flows that are relevant to a particular fixed income security. The cash flows are pretty much contractual and are contained in the issue document. The size of the cash flows as well as the timing of the cash flows are detailed in the document. The real challenge in the evaluation of bonds lies in the estimation of the risk adjusted discount rate, because this rate needs to encapsulate the risk profile of the cash flows that are going to arise out of the contractual obligation arising from the issue of the bond. In other words, it is here that the possibility of default in the realizability of the cash flows needs to be considered and an appropriate premium added to the risk free discount rate to arrive at the appropriate rate at which these cash flows are to be discounted to arrive at the intrinsic value.

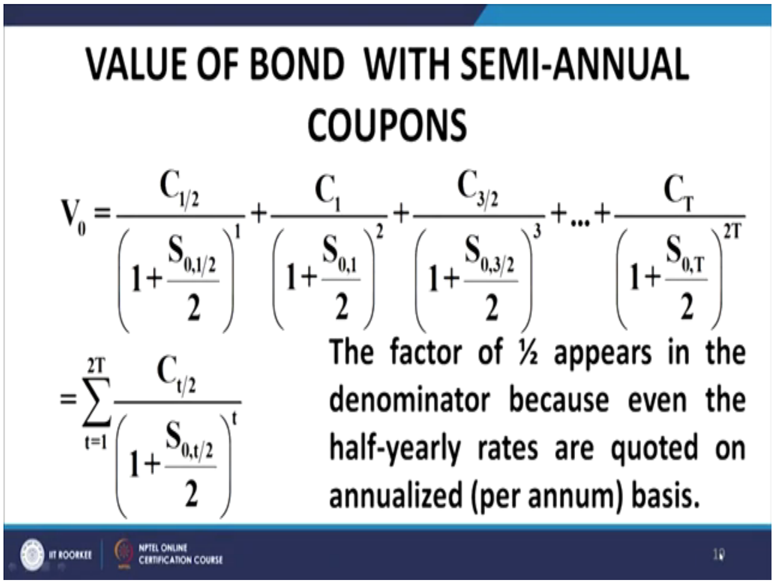
I also emphasized at this point that if we use, instead of the inputs determined by the analyst, the market risk adjusted interest rates for discounting the cash flows arising from the security, we will get the market price of the security on the left hand side. So if we put in risk adjusted market rates, we get the market price, and if we put in the risk adjusted investor ascertained rates i.e. rates based on the investor's risk return perception, we get the investor perceived value or the intrinsic value.

(Refer Slide Time: 11:41)



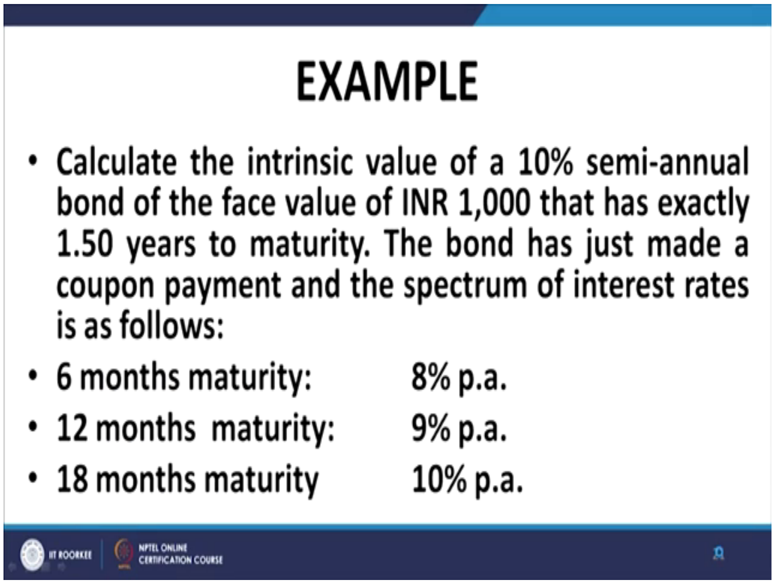
Then I discussed the issue of semi-annual coupons. It is quite straightforward. We double the number of discounting periods, the nominal interest rate or the coupon rate, as you may call it and we discounted for double the number of periods. We also half the discount rate because that is the convention that prevails in the financial bond markets.

(Refer Slide Time: 11:57)



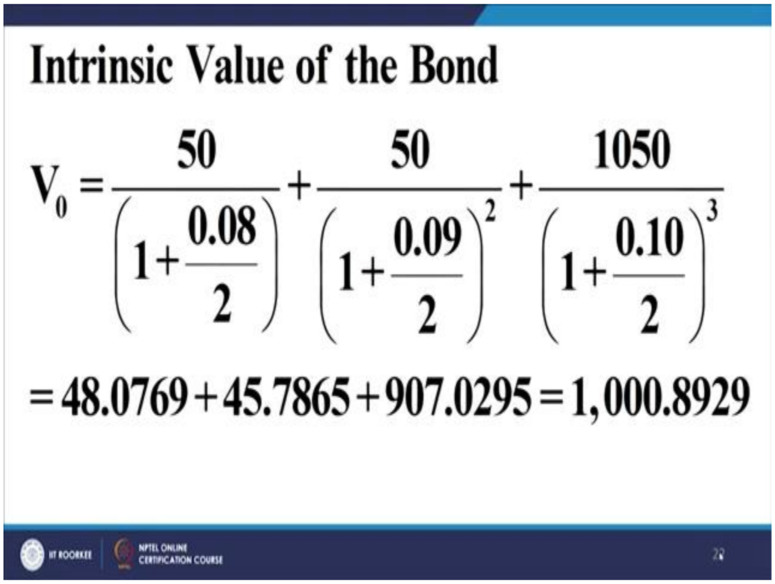
The above is an illustration of how the semi-annual coupons need to be treated. As you can see here, we are having two coupons payments per year because semi-annual payments are made. The discount rate is halved and the number of discounting periods is doubled. In essence, we consider six month period as the unit of time.

(Refer Slide Time: 12:23)

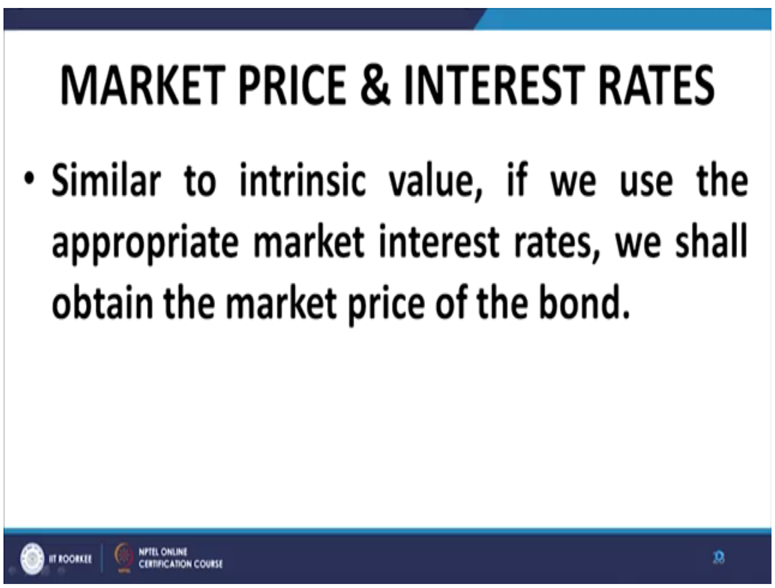


Then, I took up the above example. It is a straightforward example. So I will not repeat the working. The solution is given on the slide.

(Refer Slide Time: 12:29)

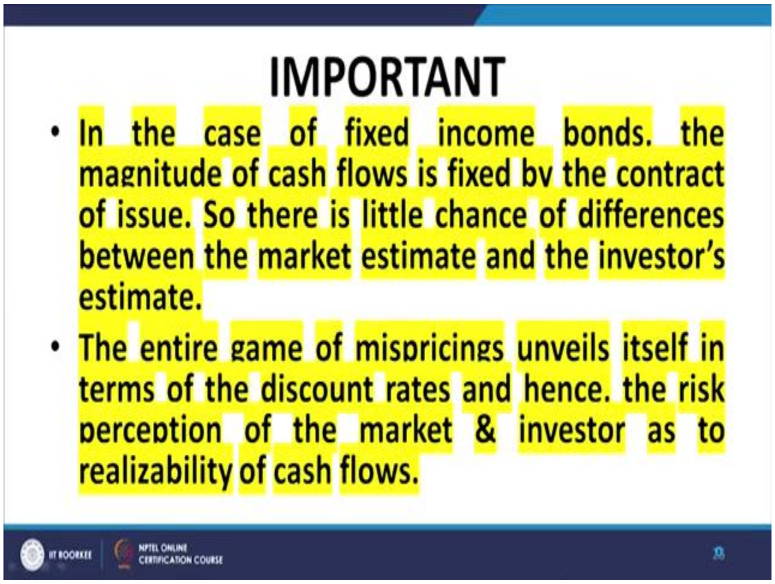


(Refer Slide Time: 12:53)



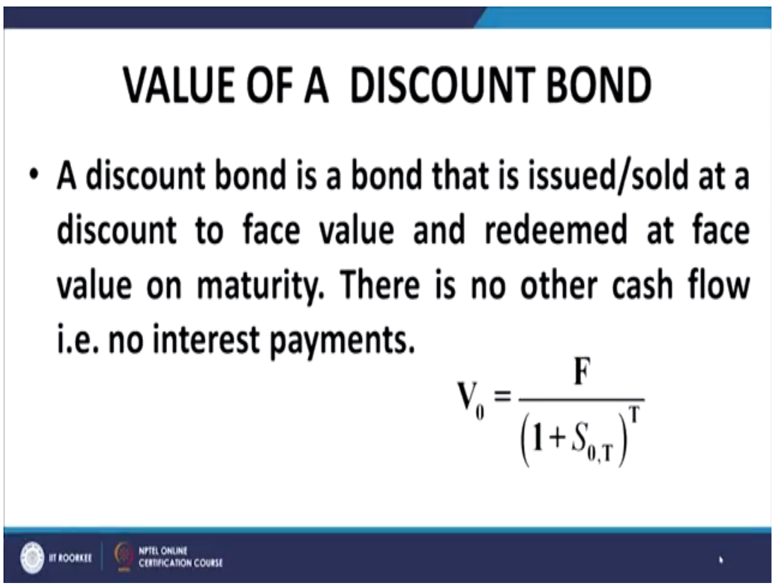
Now, I reiterate what I mentioned just a few minutes back. Similar to intrinsic value if we use the appropriate market rates, what will arrive at the market price of the bond. However, at the macro level, at the level of empirical observation, it is difficult to observe the interest rates which relate to different maturities in the market. It is more convenient to observe the market prices of securities. Hence, the principle that I have just elucidated operates backwards. In other words, on the basis of the market prices which are very easily and very precisely observable quantities, we work backwards and arrive at the relevant interest rates for various periods of discounting. For example, by observing the price of the six-month bond i.e. a bond which has a remaining tenure of six months to maturity, as of today, we can get an estimate of the six month spot interest rate, prevailing the market. Similarly, by observing the current market price of a bond which has a maturity of one year remaining, we can arrive at the one year spot interest rate that is prevailing in the market. So in practice, the principle that is given on the slide operates in the reverse direction, we first ascertain the price as the market input or as the quantity/ parameter determined by the market and on that basis we workout the implied market interest rates.

(Refer Slide Time: 14:14)



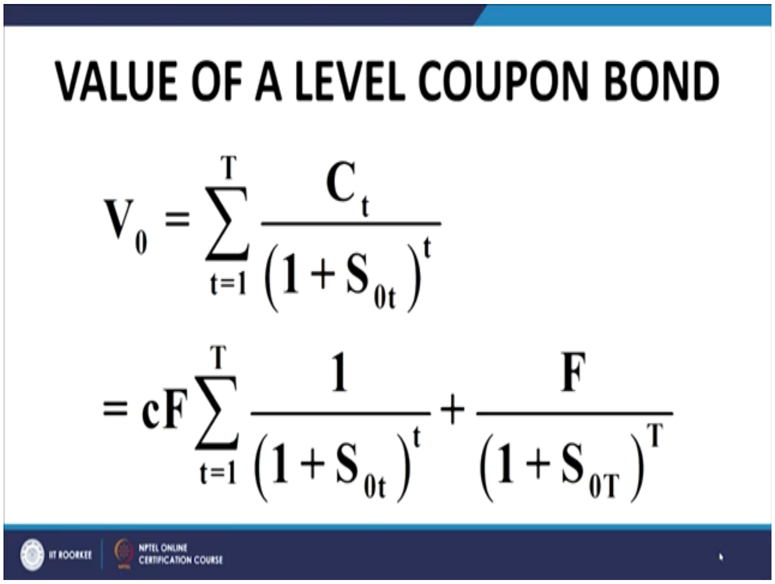
Let us recap one more thing that I have mentioned just a few minutes earlier. I reiterate this because this is important. In the case of fixed income bonds, the magnitude of the cash flows is fixed by the contract of issue. So there is little chance of difference between the market estimate and the analyst's estimate or the investor's estimate. Because the magnitude and timing of cash flows is given in the contract of issue, so whether you do the analysis or I do the analysis or somebody else does the analysis, he is pretty much going to use the information that is contained in the contract of issue. Hence, the entire game of mispricings or perceived mispricings by an investor, rests in the estimation of the discount rate i.e. calculation of the risk adjusted discount rate. Again the risk free rate is pretty much a universally accepted number. Thus, this analysis boils down to the fact that it is the perception of risk or the assessment of risk and the subsequent embedding of that risk in the risk adjusted discount rate i.e. the risk premium that injects subjectivity into the analysis. We need to add on to the risk free rate, an amount based on our risk perception to arrive at the risk adjusted rate for discounting the cash flows from the instrument. Thus, it is the variation of risk premium amongst investors that manifests itself as perceived mispricings in the market. In other words, if my perception of risk of a particular security is different or if I ascribed a different risk premium to the riskiness of the realizability of cash flows of a fixed income security compared to what the collective wisdom of the market does, then obviously I will end up with a situation where I perceive this security as mispriced and I may take investment decisions accordingly.

(Refer Slide Time: 16:22)



We, now, work out the value of a discount bond. In the case of a discount bond i.e. a bond that is issued at a discount and redeemed at a par with no intermediate cash flows, because there is only one cash flow and that is the face value at the date of redemption of the bond (t=T), the current market price is the discounted value of that final cash flow at the appropriate risk adjusted market interest rate. It will be the market rate, if we want to work out the market price or the investor estimated rate if we want the intrinsic value. Conversely if we know the market price, we can work out the prevailing market spot interest rate corresponding to the riskiness and the tenure of the bond. Otherwise, we can also plug in our estimate of the discount rate and arrive at the intrinsic value of the bond $\mathbf{V_0} = \frac{\mathbf{F}}{(\mathbf{1}+S_{\mathbf{0,T}})^{\mathbf{T}}}$.

(Refer Slide Time: 17:05)

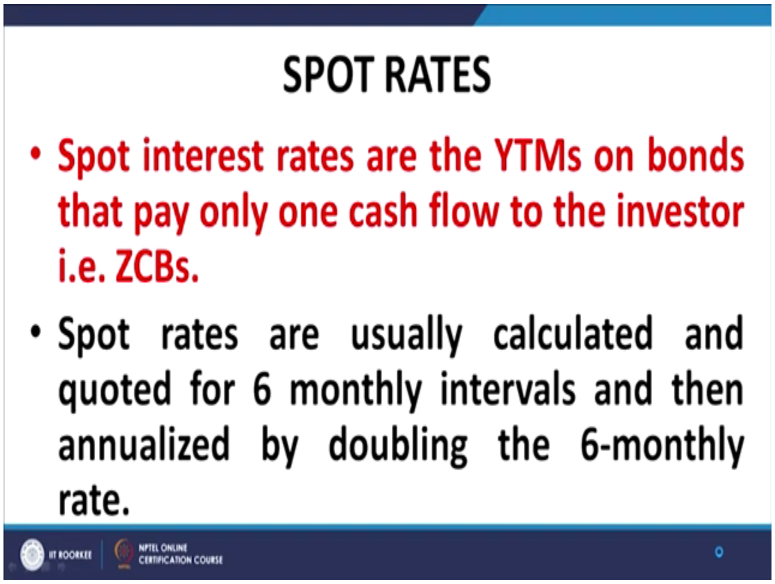


Now, let us examine a level coupon bond. A level coupon bond is a bond which has a fixed coupon rate over the life of the bond and redemption usually at face value. In fact most of the analysis that we are going to do in this particular segment of the course is on the level coupon bond, unless explicitly specified. So, in the absence of any specification or explicit information, we will assume that the analysis we are doing is of a level coupon bond. The coupon rate is fixed, and the redemption is at face value. We also call this bond a plain vanilla bond. Here

$$V_0 = \sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t} = cF\sum_{t=1}^{T} \frac{1}{(1+S_{0t})^t} + \frac{F}{(1+S_{0T})^T}$$

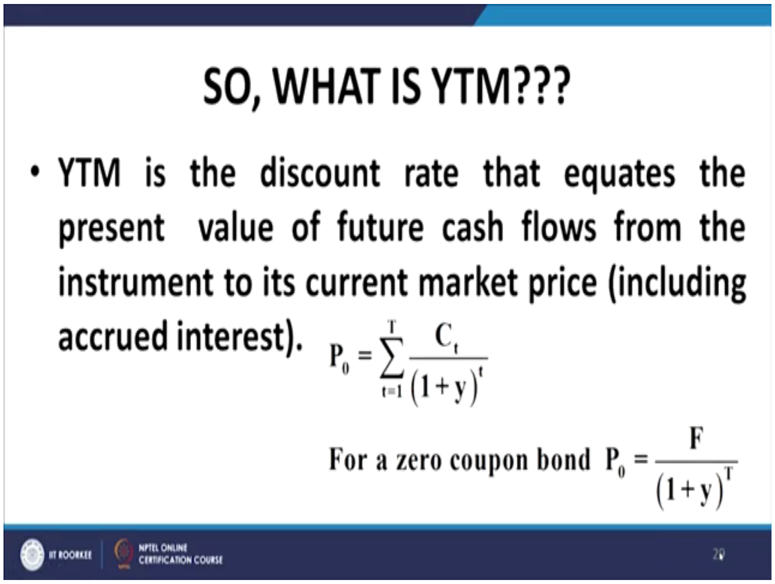
The coupon rate is c, the cash flow at time t is C_t, t is an arbitrary point in time and the summation index, t=T is the bond's maturity. Because the bond is a level coupon bond with coupon rate c, each coupon over its life will be $C_t = cF$. Because the coupon rate is fixed over the life of the bond, we take it outside the summation. At maturity the bond will be redeemed at face value F which accounts for the second term. So c applied on the face value F will give the coupon payment in money terms, which are the cash flow at the coupon payment points, over the life of the bond. Then, of course, we also have to account for the final payment i.e. the repayment of principal or redemption value that is assumed at the face value of the bond, which is given by the second term.

(Refer Slide Time: 18:48)



Now, I will talk about spot rates. It is a very important concept. Spot rates naturally lead us to the concept of yield to maturity (YTM). What are spot interest rates? Spot interest rates are the YTMs (yield to maturity) on bonds that pay only one cash flow to the investor. Such bonds are called zero coupon bonds. So we can say the spot interest rates are the YTMs on zero coupon bonds i.e. bonds which do not have any coupon payments. Bonds that are either issued at discount and redeemed at face value or issued at face value and redeemed at a premium. Spot interest rates are usually calculated and quoted for six monthly intervals, and then annualized by doubling the six month rate, as we saw in the example that we did a few minutes back.

(Refer Slide Time: 19:51)



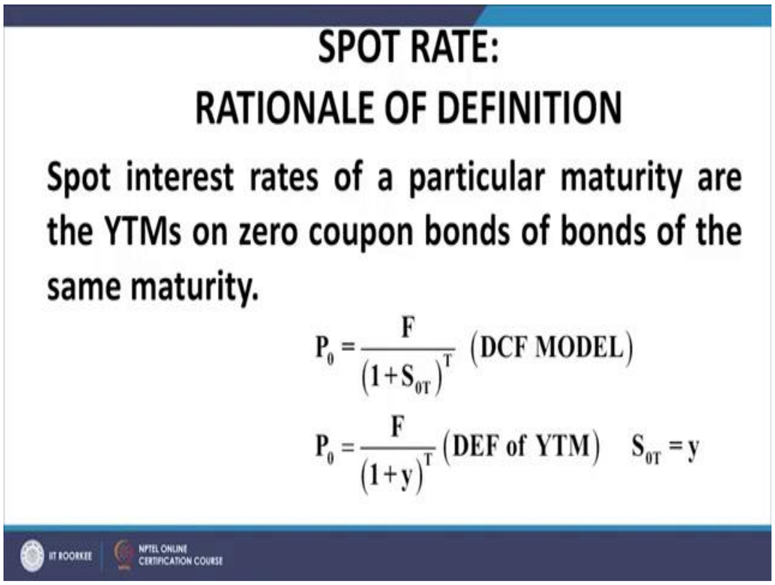
We used the expression YTM while defining spot rates. So what is YTM? YTM is the most important concept that we have in the case of fixed income securities evaluation. ***Yield to maturity or YTM is the discount rate that equates the present value of future cash flows from the instrument to its current market price, including accrued interest.***

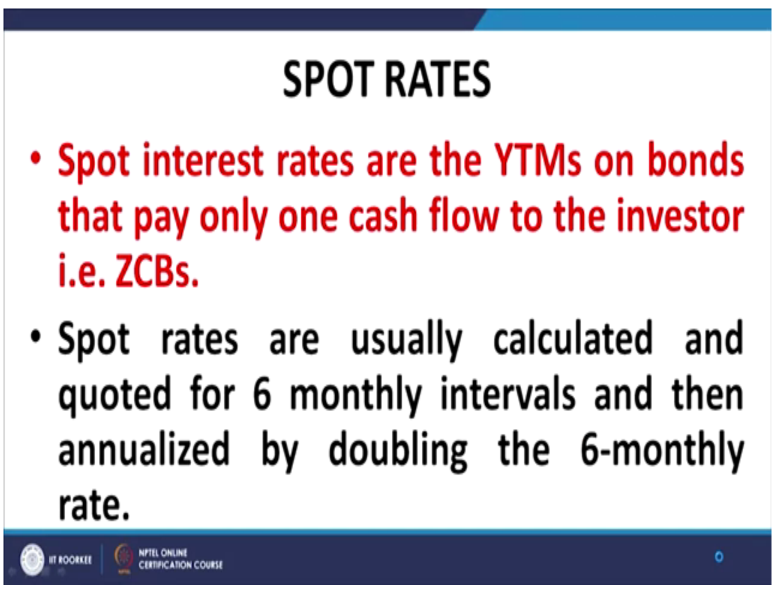
For the moment let us keep the issue of accrued interest separate to keep exposition tractable, simple. We shall return to it at a later point in this segment of the course. So, let us assume that the bond is being valued at one of the coupon payment dates, just after one of the coupon payments has been made. ***Hence, in this situation, the yield to maturity is that discount rate which equates the present value of all future cash flows attributable to that instrument to its current market pric.*** $\mathbf{P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}}$. P_0 is the current market price of the bond, C_t is the cash flow arising from holding the bond at time t and y is the YTM of the bond. Clearly in the case of zero coupon bond or a pure discount bond, because there is only one cash flow and which is at maturity, we will $\mathbf{P_0 = \frac{F}{(1+y)^T}}$, where t=T is the maturity of the bond and y is the yield to maturity of the zero coupon bond or the discount bond.

The rationale of the definition of spot rates in terms of the YTM of zero coupon bonds follows easily from definition of YTM e.g

$$\mathbf{P_0 = \frac{F}{(1+S_{0T})^T}\ \ (DCF\ MODEL);\ P_0 = \frac{F}{(1+y)^T}\ (DEF\ of\ YTM) \Rightarrow S_{0T} = y}.$$

(Refer Slide Time: 22:04)



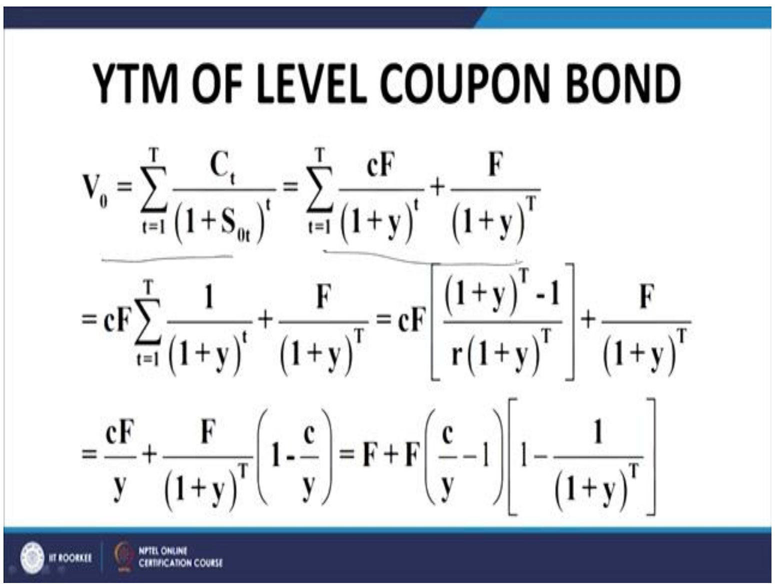


Let us assume that S_{0T} is the current market interest rate relating to a maturity of t=T years, then we arrive on the left hand side is the current market price P_0. $\mathbf{P_0 = \frac{F}{(1+S_{0T})^T}\ \ (DCF\ MODEL)}$

Also, from the definition of YTM, YTM is that discount rate y, at which the future cash flows (there is only one cash flow in this case which is the maturity cash flow and face value), when discounted to present value yield the current market price P_0, $\mathbf{P_0} = \frac{\mathbf{F}}{(\mathbf{1+y})^{\mathbf{T}}} \; (\mathbf{DEF\ of\ YTM})$.

The second equation arises from the definition of YTM, the first equation arises from the definition of the DCF model. If we compare the two, we immediately get is $S_{0T}= y$> this is what the definition of spot rates says. Let me read it ones again, spot interest rates are the YTMs on zero coupon bonds. So that is precisely what we have proved above, the spot interest rate S_{0T} =y the YTM of this zero coupon bond.

(Refer Slide Time: 23:44)



We, now, address the case of YTM of level coupon bonds.

$$\mathbf{V_0} = \sum_{t=1}^{T} \frac{\mathbf{C_t}}{(\mathbf{1+S_{0t}})^{t}} = \sum_{t=1}^{T} \frac{\mathbf{cF}}{(\mathbf{1+y})^{t}} + \frac{\mathbf{F}}{(\mathbf{1+y})^{T}} = \mathbf{cF}\sum_{t=1}^{T} \frac{\mathbf{1}}{(\mathbf{1+y})^{t}} + \frac{\mathbf{F}}{(\mathbf{1+y})^{T}} = \mathbf{cF}\left[\frac{(\mathbf{1+y})^{T} - \mathbf{1}}{\mathbf{r}(\mathbf{1+y})^{T}}\right] + \frac{\mathbf{F}}{(\mathbf{1+y})^{T}}$$

$$= \frac{\mathbf{cF}}{\mathbf{y}} + \frac{\mathbf{F}}{(\mathbf{1+y})^{T}}\left(\mathbf{1} - \frac{\mathbf{c}}{\mathbf{y}}\right) = \mathbf{F} + \mathbf{F}\left(\frac{\mathbf{c}}{\mathbf{y}} - 1\right)\left[1 - \frac{\mathbf{1}}{(\mathbf{1+y})^{T}}\right]$$

The equation $\mathbf{V_0} = \sum_{t=1}^{T} \frac{\mathbf{C_t}}{(\mathbf{1+S_{0t}})^{t}}$ follows from the DCF formula while $\mathbf{V_0} = \sum_{t=1}^{T} \frac{\mathbf{cF}}{(\mathbf{1+y})^{t}} + \frac{\mathbf{F}}{(\mathbf{1+y})^{T}}$ is obtained from the definition of YTM. The rest is algebraic manipulation using summation of geometric progressions.

So if we equate the two expressions **(DCF & YTM)** for P_0, we can get a relationship between the spot interest rates and the YTM in the case of a level coupon bond.

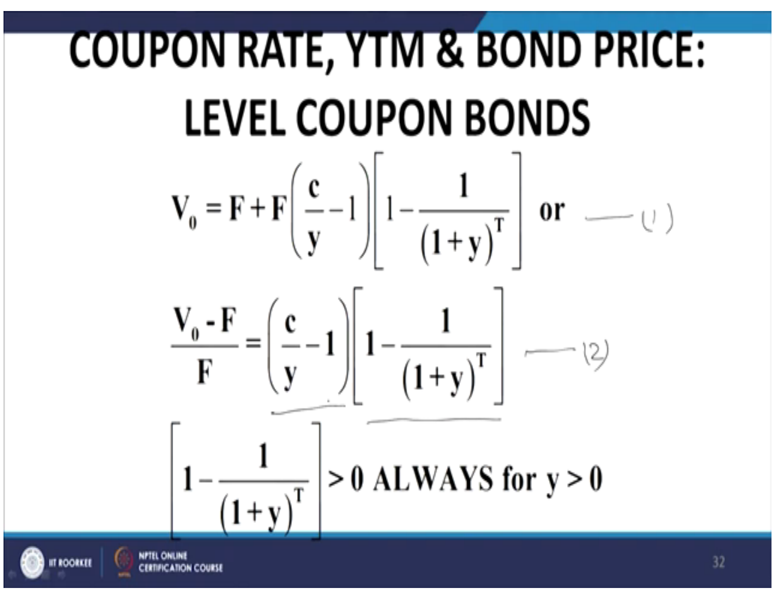
Let us now investigate the relationship between coupon rates and YTM. This is very interesting and can be done on the basis of the formula that we arrived at just now viz

$$\mathbf{V_0 = F + F\left(\frac{c}{y} - 1\right)\left[1 - \frac{1}{(1+y)^T}\right]} \text{ whence } \mathbf{\frac{V_0 - F}{F} = \left(\frac{c}{y} - 1\right)\left[1 - \frac{1}{(1+y)^T}\right]}$$

But. $\mathbf{\left[1 - \frac{1}{(1+y)^T}\right] > 0 \text{ ALWAYS for } y > 0}$.

So now if we look at equation $\mathbf{\frac{V_0 - F}{F} = \left(\frac{c}{y} - 1\right)\left[1 - \frac{1}{(1+y)^T}\right]}$ very carefully, we find that the second factor $\mathbf{\left[1 - \frac{1}{(1+y)^T}\right] > 0 \text{ ALWAYS for } y, T > 0}$, which is normally the case.

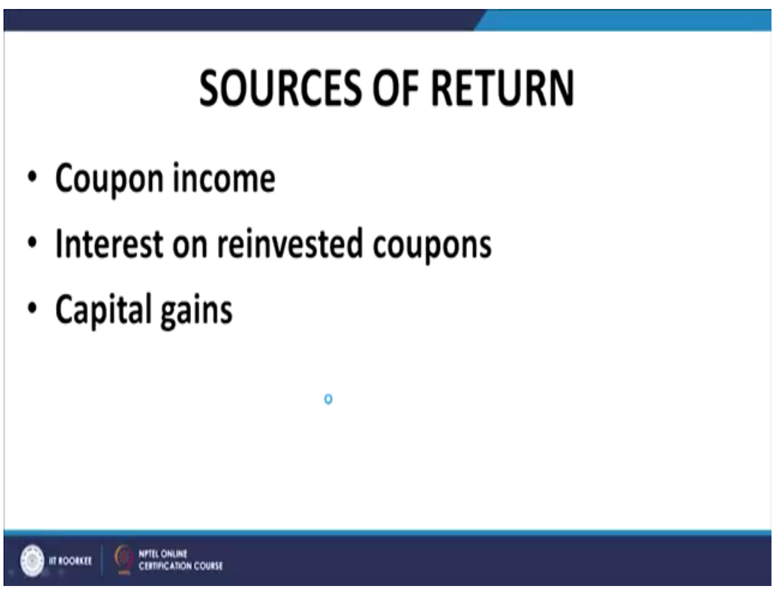
(Refer Slide Time: 24:53)



Because $\left[1-\frac{1}{(1+y)^T}\right] > 0$ **ALWAYS for** $y, T > 0$, the sign of the left hand side, will be determined by whether the first factor that is $\left(\frac{c}{y}-1\right)$ is positive or negative.

If $\left(\frac{c}{y}-1\right)$ ***is positive then left hand side will be positive and if*** $\left(\frac{c}{y}-1\right)$ ***is negative, the left hand side will be negative. What does this mean? It means that if c>y, that is, the coupon rate is greater than the yield to maturity,*** $V_0>F$ ***and bond will trade at a premium, and if c<y then*** $V_0<F$ ***and the bond will quote at a discount. And, of course if c=y then the right hand side becomes 0 and therefore*** $V_0=F$ ***or the bond will be quoting at par.*** So this is a very important information, that depending on the relationship of the coupon rate with the YTM, we can arrive at whether the bond should technically quote at a premium, par or a discount.

(Refer Slide Time: 27:11)



Now we will talk about the sources of return that are generated on a fixed income security.

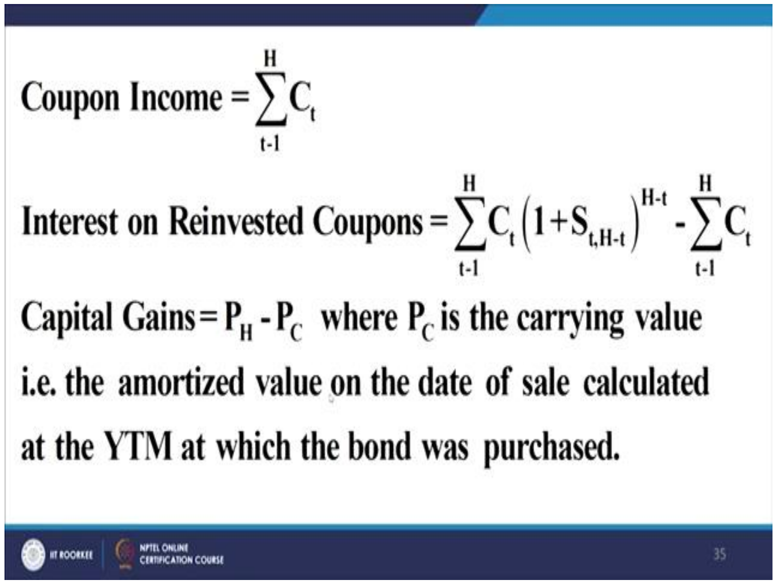
(i) First and the most obvious is the coupon income i.e. the income that arises out of contractual cash flows that are paid on account of interest when a person takes up a long position in the bond, that is, he buys a bond. When an investor invests in a bond, he is entitled to coupon payment, during the life of the bond as per the terms of the issue contract.

(ii) If the coupon payments are reinvested by the investor, he gets interest on interest which constitutes reinvestment income. Suppose you invest in an annual coupon bond at t=0, say with an investment horizon of five years. You will receive the first coupon payment at the end of year t=1. You can reinvest this coupon for the next four years and receive interest on the reinvestment. Similarly, the coupon received at t=2 years may be reinvested for 3 years etc. The interest that aries on account of the reinvested coupons is called reinvested income. You would not like to keep the coupon interest idle. A rational investor would either leave the money in his deposit account or he would invest the money somewhere else. So the return that he would get by reinvesting the coupon payment is called the reinvestment income. This is the interest on interest component, you receive the interest, you reinvest the interest and you receive interest on the reinvested interest. So this is the second component of return that you get.

(iii) Then there is the issue of capital gains. In the event that you do not hold the bond to its date of maturity, you would sell it off in the market to recover the current value of the bond, as on the date that you decide to exit the investment. In that case, of course, it is the market price which will be a factor in determining the overall return, that you are going to get. The market price could be higher or lower than the carrying value of the bond at that time point in time, and correspondingly you would get a capital gain or a capital loss. If you sell the bond a price in the market higher than the carrying value of the bond in your books, you will get a capital gain and vice versa.

What is carrying value? It is defined in this slide.

(Refer Slide Time: 29:53)



The carrying value is defined as the amortized value. I will come back to this again in a later slide and explain how this carrying value is calculated with an example, but for the moment let us stick to the definition. The carrying value is the amortized value on the date of sale, calculated at the YTM at which the bond was purchased. In other words, it is the amortized value which is in your books, provided that amortized value is calculated on the basis of the YTM, at which you had purchased the bond.

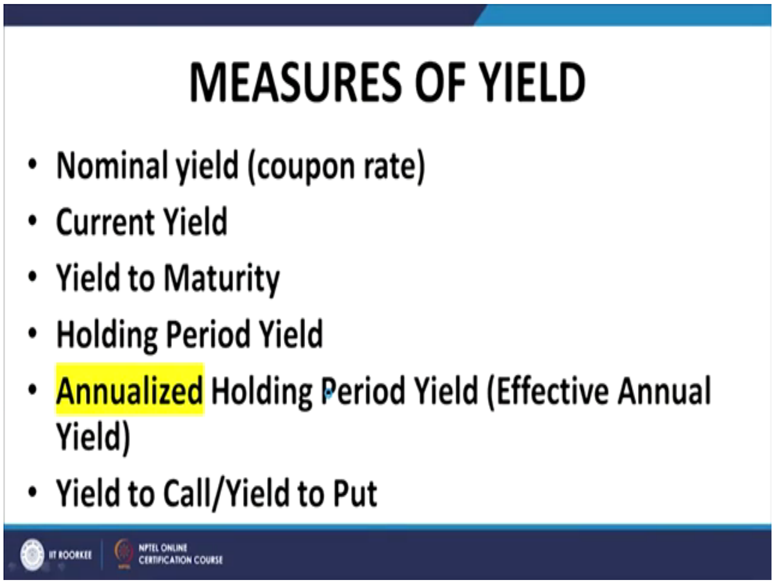
So, if your selling price is higher than the carrying value, you make a capital gain and if your selling price is lower than the carrying value, you make a capital loss.

So there are three sources of income, (i) the coupon income, (ii) the interest on interest or the interest on reinvested coupons and (iii) the capital gains that is the difference between the selling price and carrying value.

If you hold the bond for a period less than its maturity, then the difference between the market price i.e. the price at which you exit the investment, and the amortized value or the carrying value of the bond will constitute the capital gain or capital loss.

Now, I will talk about the measures of yield. I have just talked about the sources of income on the security. I will now talk about the measures of yield.

(Refer Slide Time: 31:28)



(i) First is the nominal yield, that is the coupon rate, so nothing much to say about that.

(ii) The current yield measure is defined as the ratio of the aggregate of coupons over the year (the total coupons that are paid during the year) divided by the current market price of the bond.

(iii) We shall talk about yield to maturity again in a few minutes,

(iv) Holding period yield, I introduced in the context of money market instruments in a previous lecture. I shall be talking more about this.

(v) Annualized holding period yield. Then we have annualized holding period yield which is the same as the effective annual yield EAY

(vi) Then we can also have the yield to call, yield to put if the bond has the properties of having a callability option or a puttability option.

We shall continue after the break, thank you.