Econometric Modelling

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Lecture No: 26

AR, MA & ARMA Processes-I

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Hello, and welcome back to the course on Econometric Modelling. This is Module 26. Module 26 is part of univariate time series modelling, where we are first going to deal with AR, MA & ARMA processes. So, AR stands for autoregressive processes, MA stands for moving average processes and ARMA stands for autoregressive moving average processes.

There are two modules, which would be devoted to this topic, but here in this module, I will be primarily dealing with MA processes that are moving average processes followed by an introduction to autoregressive processes as well.

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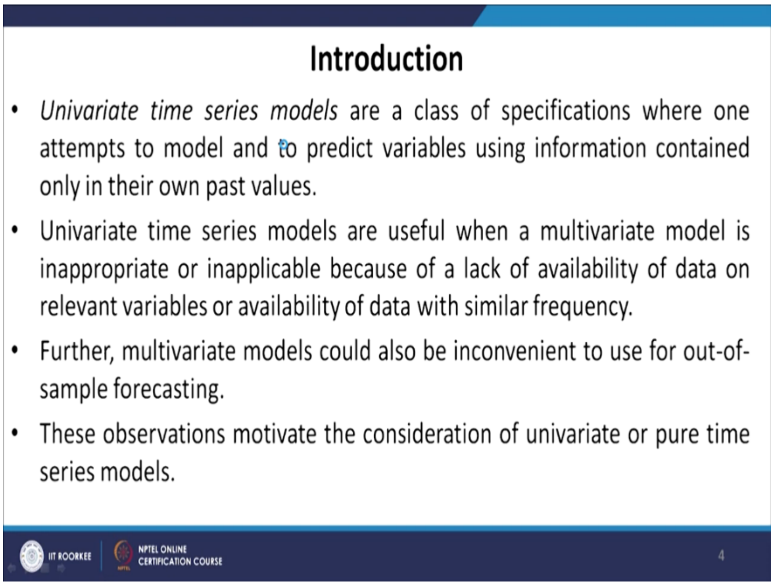

So, univariate time series models are a class of specifications where one attempts to model and to predict variables using information contained only in their own past values. So, we do not consider any other variable that is why it is called univariate modelling. And, of course, univariate modelling is applicable to my understanding only to time series data, because there we are trying to find out whether this series itself contains information that the information which could be extracted from its past values or past observations.

Univariate time series models are useful when a multivariate model is inappropriate or inapplicable because of a lack of availability of data on relevant variables or availability of data with similar frequency. Because for instance, if we are going for a large-scale macroeconomic aggregate model, then it is quite possible that sometimes, relevant data on some of the variables are not available.

The second point is that at times the frequency does not match. For example, if we specifically plan to work with very high-frequency data, for instance, daily stock prices, then there are very few economic or financial data available, which are available at such high frequency. So, as a result of which sometimes it is useful to consider a univariate series only.

So, that the series itself tries to look back at its previous values and we try to find out whether it contains useful information or not. Further, multivariate models could also be inconvenient to use for out-of-sample forecasting. Out of sample forecast basically depending on the setup, you would need information on all the variables for a certain period, which might not be possible, and in that case, this would also be inapplicable.

And finally, these observations motivate the consideration of univariate or pure time series models. Since they do not have any cross-section element in them that is why they could be called pure time series models.

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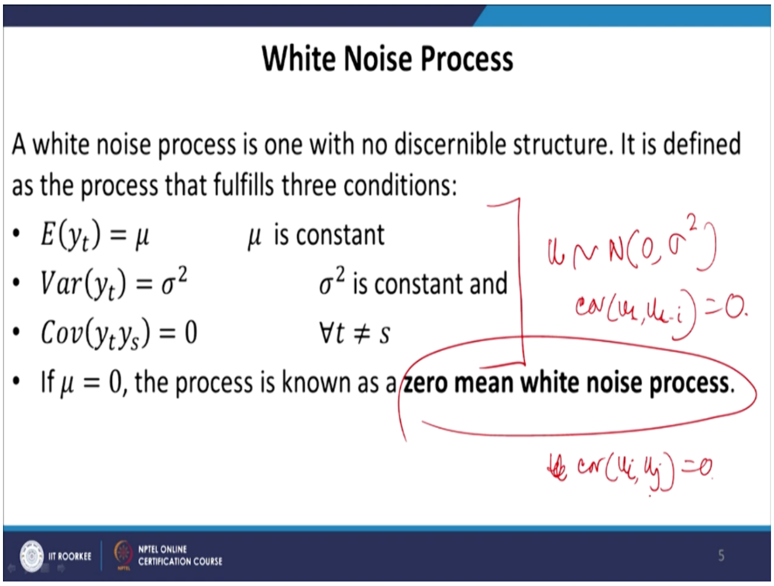

A White Noise Process is one, with no discernible structure. So, before I start talking about the univariate time series model it is important to define the white noise process. We are already now familiar with white noise processes, but the only difference is that probably we have not used that terminology.

So, it is a process with no discernible structure. It is defined as the process that fulfills three conditions. The first one is that it has a constant mean. The mean could be μ, the mean could be 0, if the mean is 0, then we call it a 0 mean white noise process. The variance is a constant term a finite and constant variance it must have that is (refer slide time: 4:26).

So, this is a white noise process. Given this definition, you could understand that so far, the way we have defined the population error terms, which is normally distributed with mean 0 and a constant variance σ^2 and we also assume that (refer slide time: 5:00), then this implies that the error term is a white noise process. That is why I say that we are already familiar with white noise processes, but probably we have not used the term white noise process.

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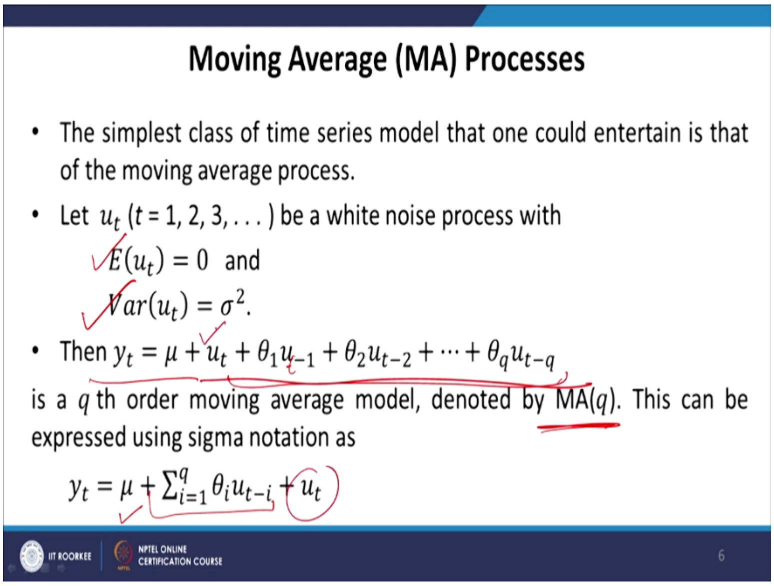

Now defining moving average processes: The simplest class of time series model that one could entertain is that of the moving average process. (Refer slide time: 5:40).

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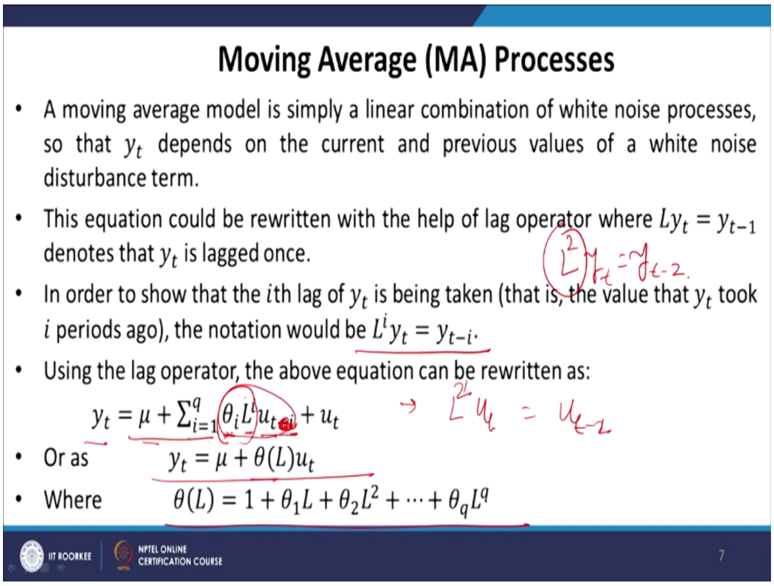

A moving average model is simply a linear combination of white noise processes so that y_t depends on the current and previous values of a white noise disturbance term. This equation could be rewritten with the help of the lab operator where (refer slide time: 6:42). So, this L is called the lag operator and the power of L basically refers to how many lags are there.

In order to show that the (refer slide time: 7:06). So, this is the usual thing that we have.

Ideally, we should have (refer slide time: 7:51).

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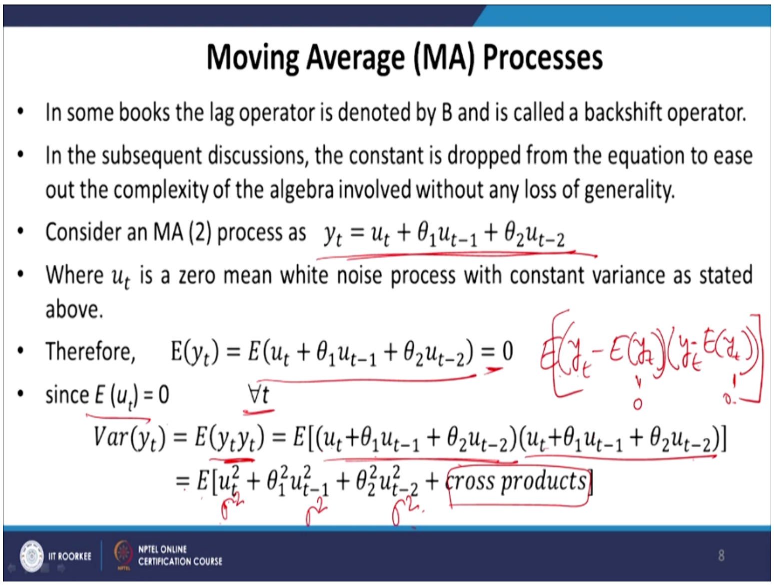

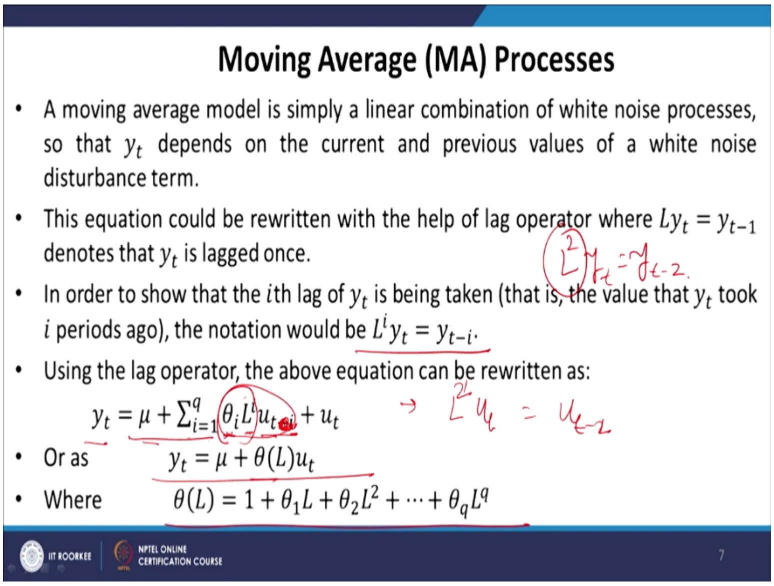

In some books, the lag operator is denoted by B and this is also called the backshift operator. In the subsequent discussions, the constant is dropped from the equation this constant μ is dropped from the equation to ease out the complexity of the algebra involved without any loss of generality. So, now we consider first of all an MA (2) process. So, MA (2) process is written as (refer slide time: 8:54- 9:50).

Since this is equal to 0, this is equal to 0, we are left with only the expected value of y_t multiplied by the expected value of y_t. Then we substitute y_t with this expression. What are we left with? (Refer slide time: 10:04). So, all the cross products are here, which I have not mentioned separately.

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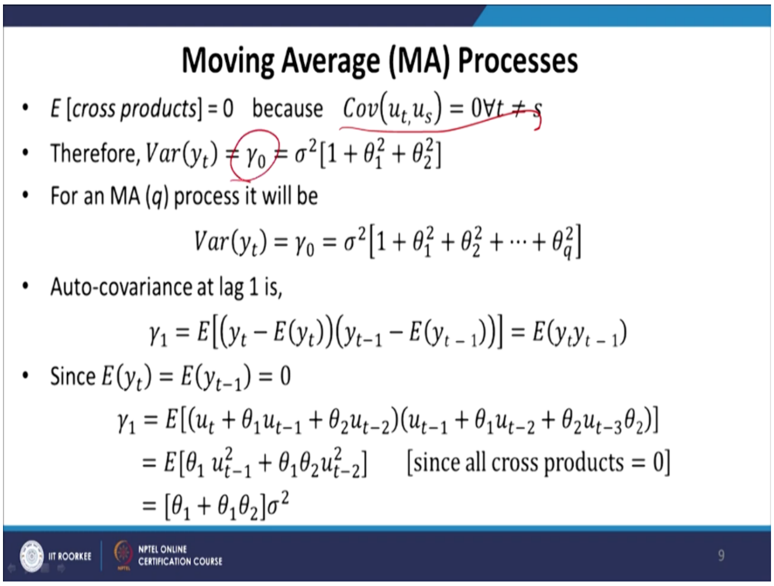

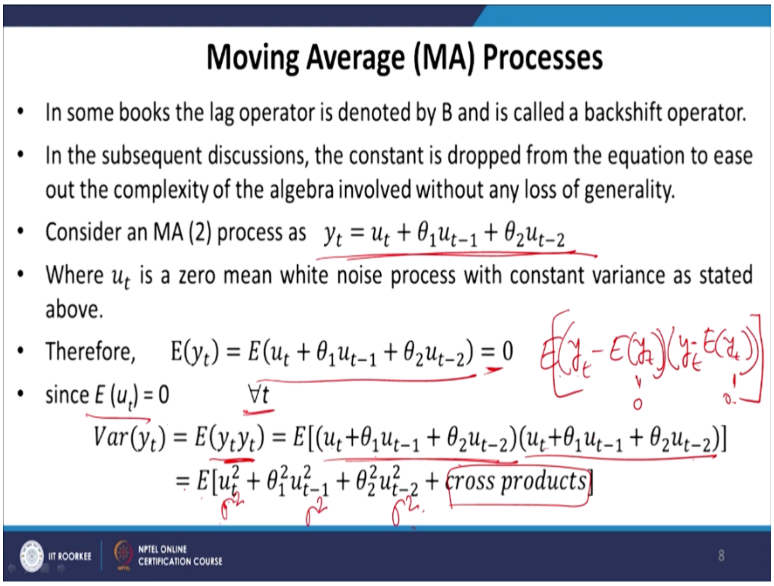

And the reason is that the expected value of all cross products is equal to 0 because u_t is a white noise process, so that is why the current and the previous observations are not correlated with each other. So, (refer slide time: 10:40).

As a result of this (refer slide time: 10:49- 11:30).

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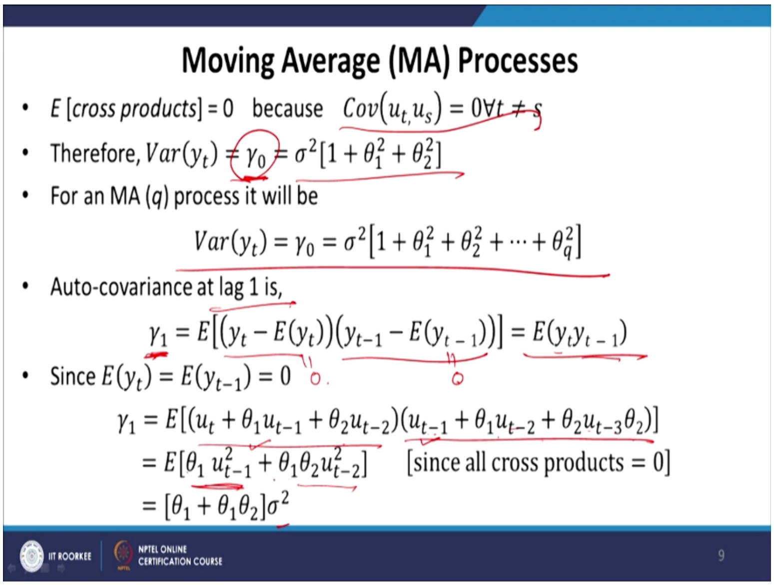

Now, we consider, for an MA (q) process, this is the variance. Now, we consider autocovariance. Auto covariance between autocovariance at lag 1 of the series y_t and not u_t. So, the autocovariance at lag 1 is denoted by γ_1. So, you can see that γ_0 refers to basically, autocovariance at 0 lag, which is equivalent to the variance.

Now, autocovariance at lag 1 is γ_1, which is again, this expression. This is equal to 0, this is equal to 0, so what I am left with is (refer slide time: 12:12). And what we have is, first of all, again, I am going to drop out all the cross products.

So, (refer slide time: 12:31).

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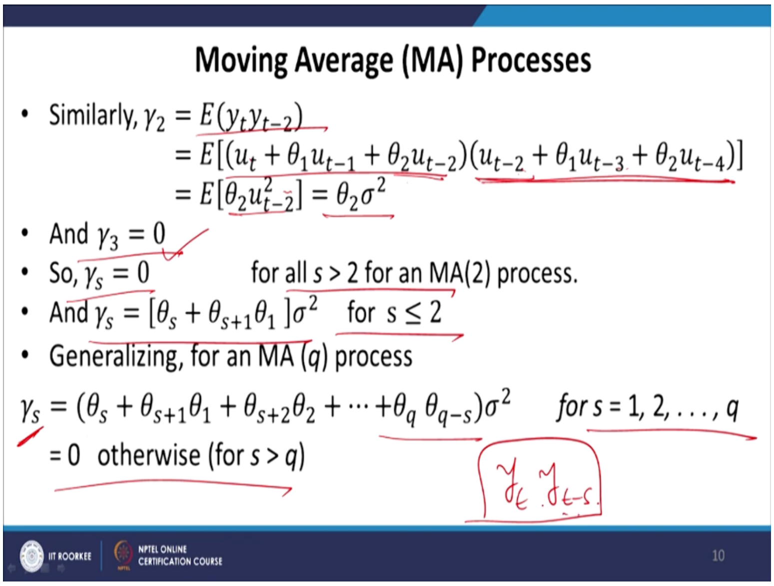

Similarly, γ_2 we have started with an MA (2) process. So, similarly, γ_2 (refer slide time: 13:07- 14:58).

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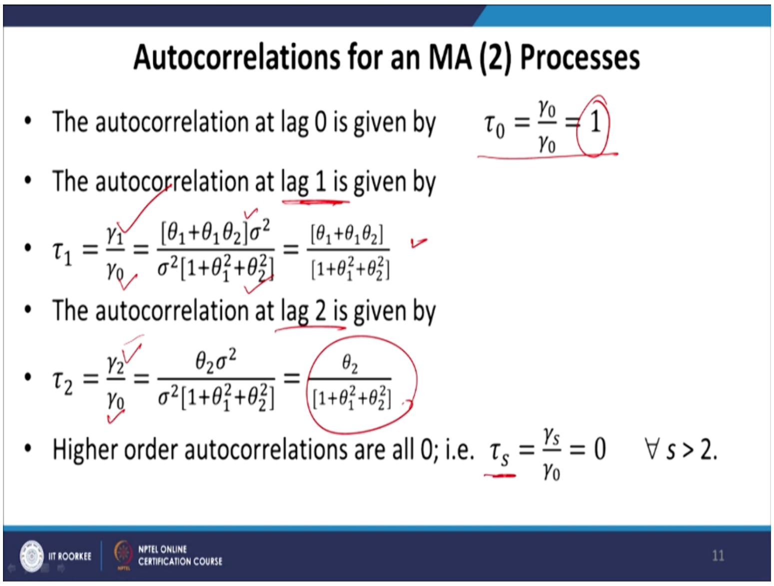

Now, we consider the autocorrelations for the MA (2) process. The autocorrelation at lag 0 is given by 1 because correlation has covariance divided by the variance. So, covariance and variance both are the same at lag 0 and that is why this is equal to 1. The autocorrelation at lag 1 is given by γ_1 that is autocovariance at lag 1 divided by variance.

So, this is autocovariance at lag 1, this is variance, and this is what is my τ_1 which measures the autocorrelation. Similarly, the autocorrelation at lag 2 is given by autocovariance at lag 2 divided by again the variance and this is what we observe. So, for all higher-order autocorrelations, we have this is equal to 0 because γ_s is equal to 0 for S greater than 2 when we are dealing with an MA (2) process.

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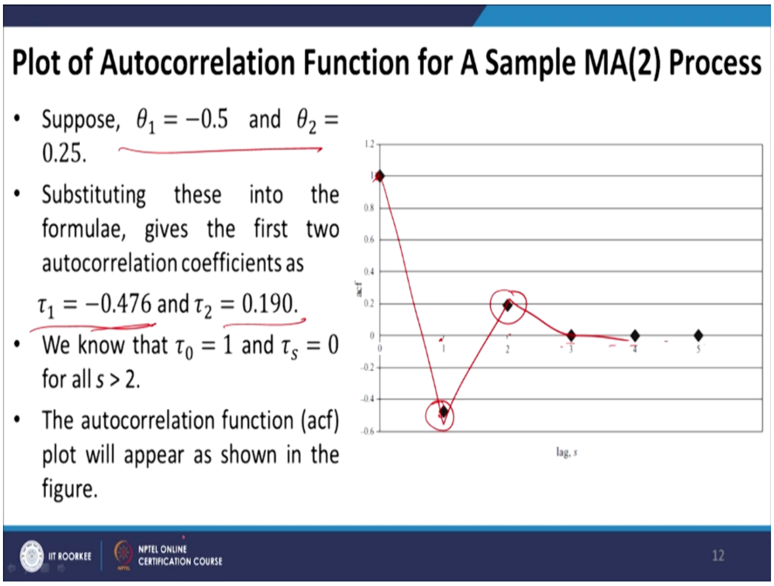

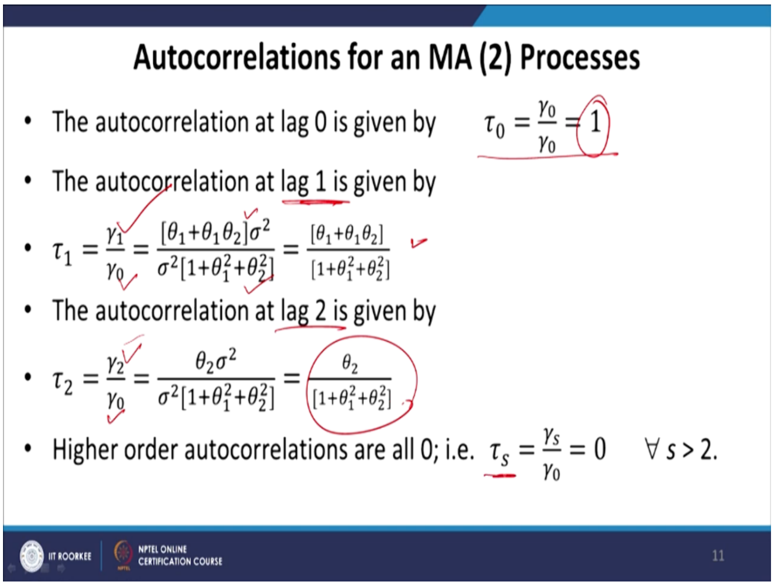

Now, if we plot the autocorrelation functions for a sample MA (2) process. Suppose (refer slide time: 16:06).

So, we are substituting the values of θ_1 and θ_2 in these expressions. And we are getting these values, so we know that autocorrelation at lag 0 equals to 1. Then at lag 1, we have -0.476 this observation, at lag 2 we have 0.190. For any other higher-order lags, this is we are having all 0 autocorrelations.

So, this is a plot of the autocorrelation functions of an MA process with the specific values of θ_1 and θ_2. For different values of θ_1 and θ_2 you would be having different patterns of autocorrelations for an MA (2) process. The autocorrelation function or ACF appears as shown in this figure.

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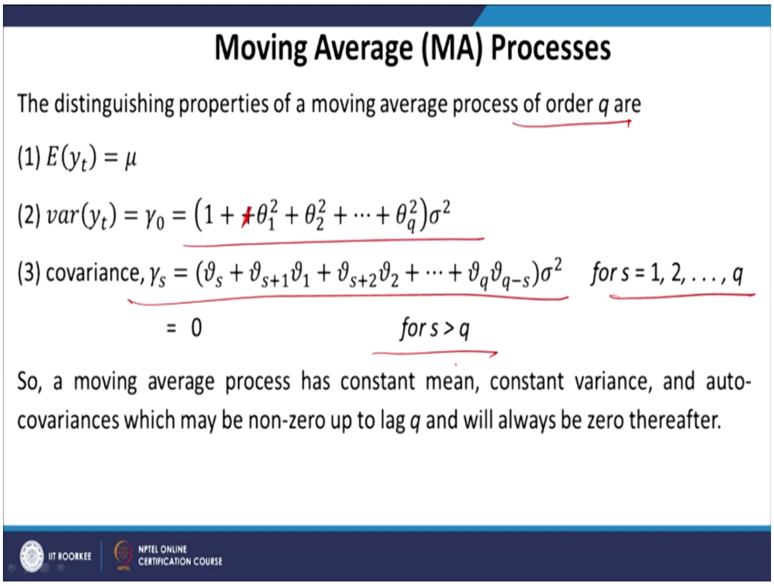

The distinguishing properties of a moving average process of order q are. First of all, (refer slide time: 17:36).

So, a moving average process as constant mean, constant variance, and auto covariances may be non-zero up to lag q and will always be zero thereafter. But one thing that we need to mention or observe here is that a moving average process is actually not estimable in reality, because all your independent variables in a moving average process are the population errors and their lag terms.

Now, how if population errors are not observable then how am I going to utilize them? But, of course, there are statistical packages that are designed to estimate moving average processes. Other than that, moving average processes help us in getting in useful information and insights about other related processes, for example, an autoregressive process. So, next, we are going to first introduce the autoregressive process.

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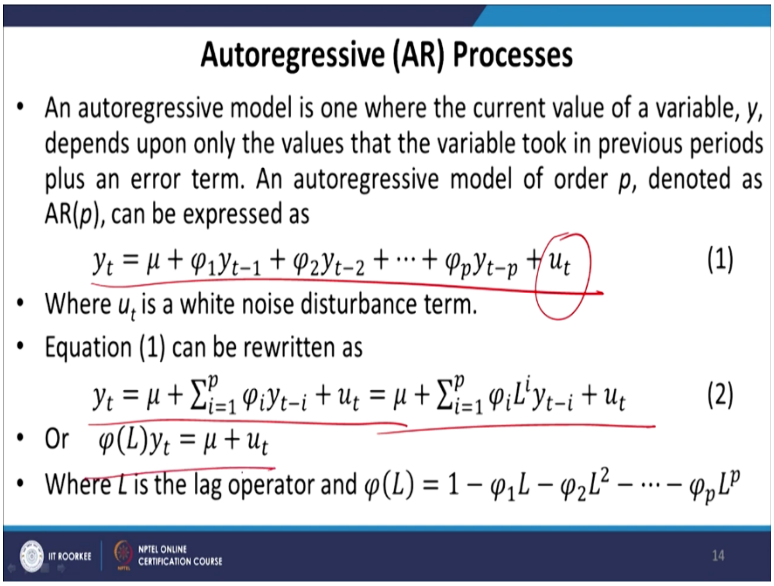

An autoregressive model is one, where the current value of variable y depends upon only the values that the variable took in the previous periods plus an error term. An autoregressive model of order p denoted as AR (p) can be expressed like this. So, this is a structure you are somewhat familiar with as of now, because I have already discussed autocorrelation, and there we had introduced AR (1) models up to ARP models.

Now, here u_t is a white noise disturbance term. So, u_t t has a constant mean, constant variance, u_t most often has a 0 mean constant variance and no autocorrelations. Now, equation 1 can be rewritten again like this, and from there with a lag operator, and finally, something like this, where the lag operator has already been explained and φ (L) the lag operator where this is equal to this expression.

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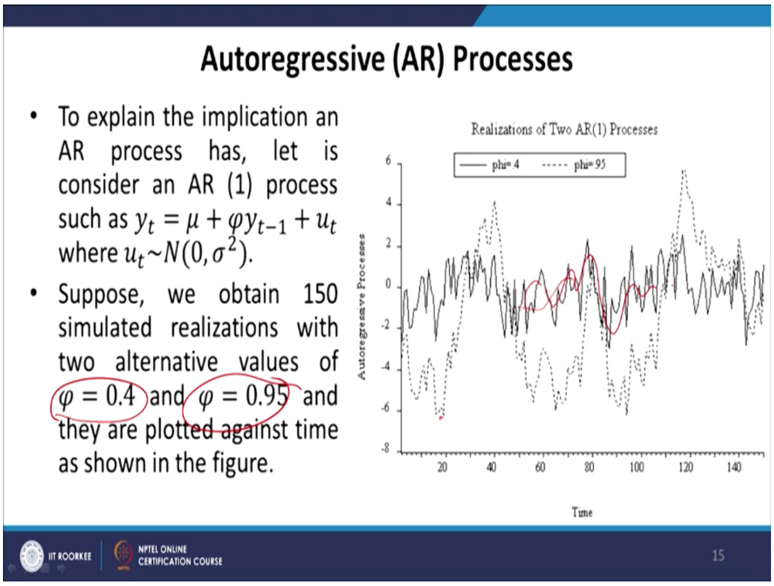

To explain the implication an AR process has, let us consider an AR (1) process such as (refer slide time: 20:12). Suppose we obtain 150 simulated realizations with two alternative values of (refer slide time: 20:26).

Now, what it shows is that the modulations or the fluctuations in the series with $\varphi = 0.95$ is more persistent in the sense when it is going up, it is actually going up for a longer period of time when it is going down also for a longer period of time. So, as a result of which the peaks and the troughs are most often more acute compared to a series with $\varphi = 0.4$.

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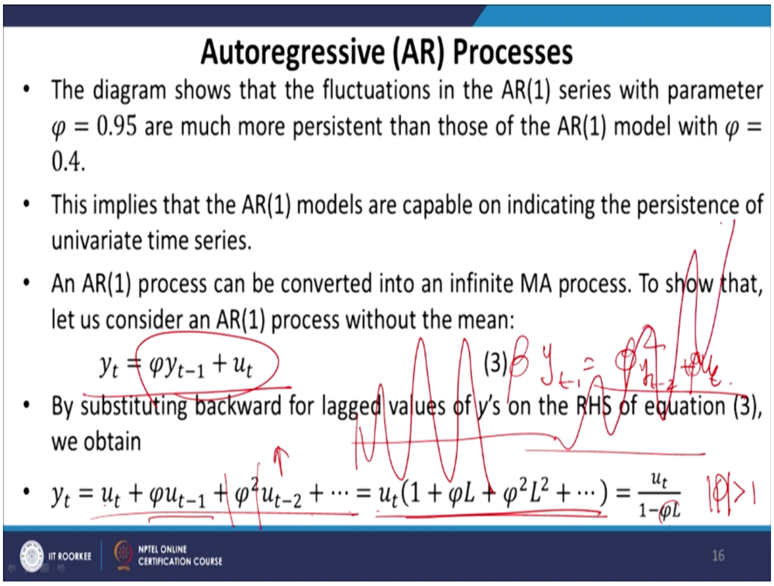

So, the diagram shows that the fluctuations in the AR (1) series with parameter $\varphi = 0.95$ are much more persistent than those of the AR (1) model with $\varphi = 0.4$. This implies that the AR (1) model is capable of indicating their persistence of univariate time series. An AR (1) process can be converted into an infinite MA process.

To show that, let us consider an AR (1) process without the mean. So, again, we are excluding the mean for the sake of simplicity or to avoid the complexity of calculations. Now, by substituting backward for the lagged values of y's on the right-hand side of equation 3, what do you obtain.

First of all (refer slide time: 22:06- 23:04)

This moving average representation for Y is convergent if and only if the mod value φ is less than 1. Because if the mod value of φ is greater than 1, then you can see that first of all, by convergent we mean that if I look at this series not instead of this expression, you can see that if the mod value of i is greater than 1 then as time goes back the value of φ^2 actually keeps on increasing, φ^2, φ^3, φ^4. So, even if φ is a negative number, then, as we go back then the modulations will be like of greater dimensions around the axis. If φ is a positive number, then also we will be having φ probably going up or the series is going upward. So, as a result of which, this is actually in order to have the series convergent, we must have a mod value of φ less than 1.

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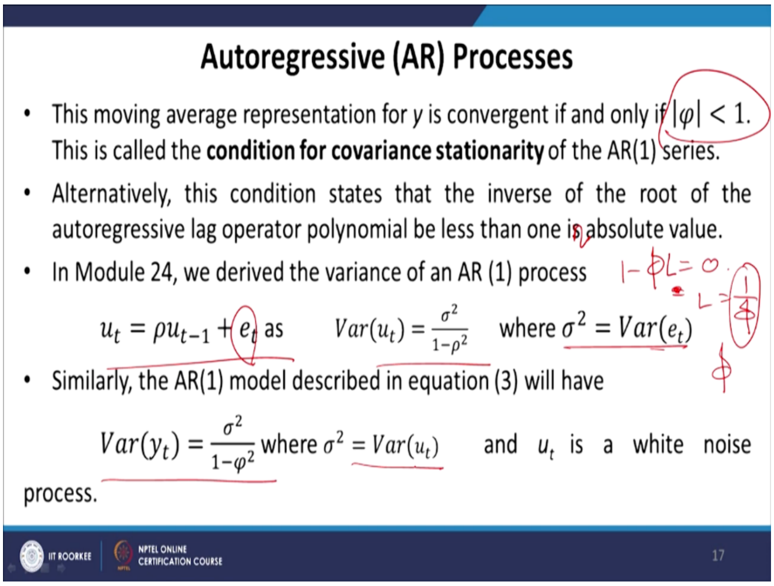

This is called the condition for covariance stationery of the AR (1) series. Alternatively, this condition states that the inverse of the root of the autoregressive lag operator polynomial be less than one in absolute value. So, (refer slide time: 24:32).

Similarly, the AR (1) model described in equation 3, will have (refer slide time: 25:15).

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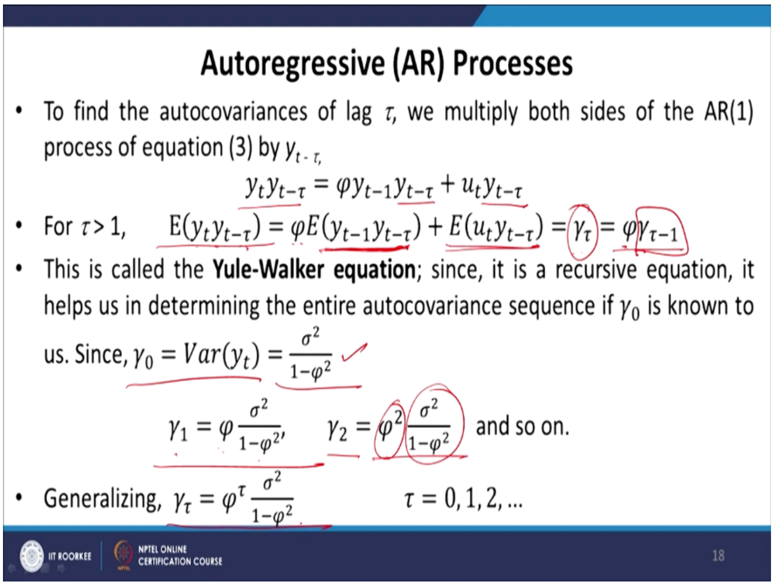

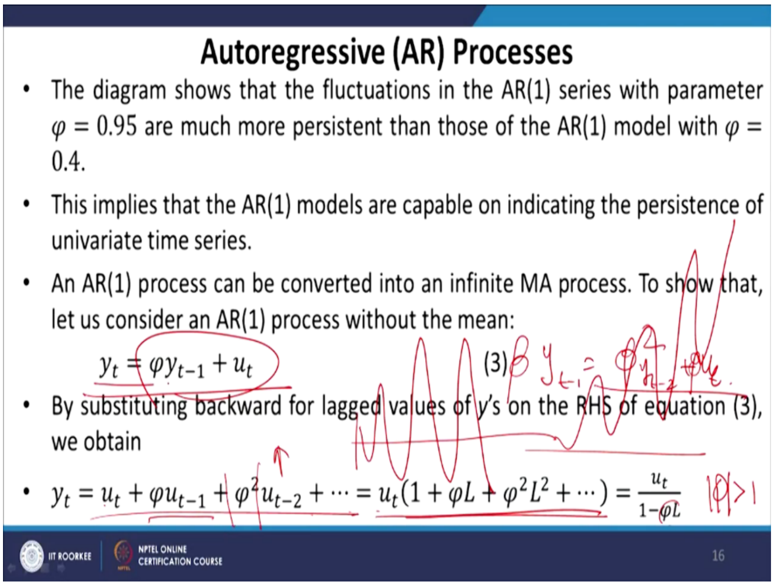

To find the autocovariance of lag τ, we multiply both sides of the AR (1) process of equation 3. So, this was my equation 3, which is simply an AR (1) process. So, we multiply both sides of it by (refer slide time: 25:47- 26:37). This is called the Yule-Walker equation; since it is a recursive equation, it helps us to determine the entire autocovariance sequence if γ_0 is known to us.

Now, we know that (refer slide time: 26:50- 27:39).

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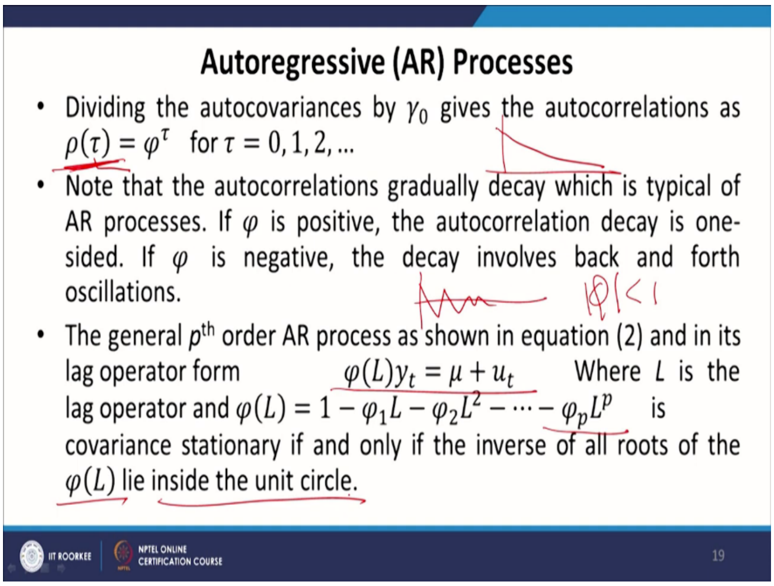

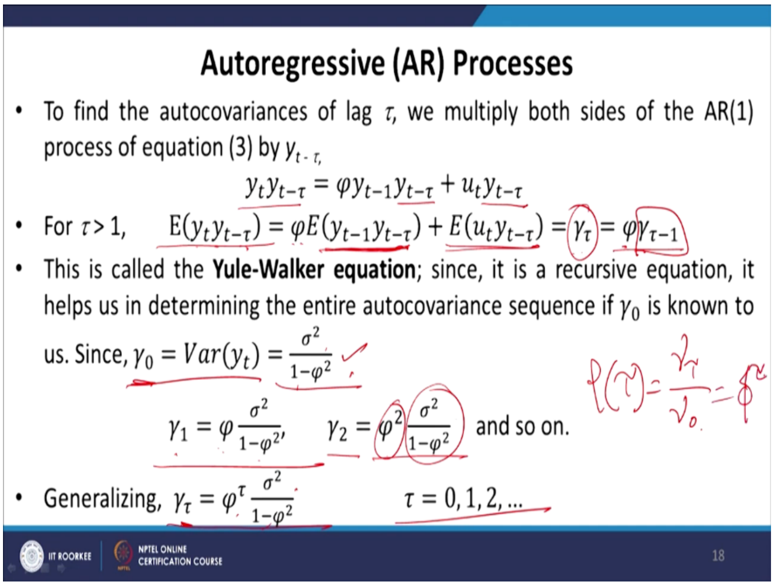

Dividing the auto covariances by γ_0 gives us the autocorrelation as $\rho(\tau)$. We are denoting the autocorrelation here by $\rho(\tau)$ where (τ) refers to the lag between the two observations. And this is equal φ^τ to for all the (τ) equals 0, 1, 2 and so on, you can simply see that this is my denominator always, variance, that is γ_0.

(Refer slide time: 28:10). Note that the autocorrelations gradually decay, which is typical of an AR (1) process. If φ is positive, the autocorrelation decay is one-sided.

So, it decays like this. The autocorrelation function looks like this. If φ is negative the decay involves back and forth oscillation. So, if φ is negative then the decay would be something like this. But, of course, what we need is that φ to be less than 1 because if φ is greater than 1, then we will not have decay, and we will not have convergent series.

The general p^{th} order AR process, as shown in equation 2, and in its lag operator form this is like this, (refer slide time: 29:24).

So, so far, we are dealing with AR (1) model then we gave some examples with AR (2) model. Now, talking about the ARP model which has already been introduced. And we are just now trying to talk about what makes a p^{th} order AR process convergent or covariance stationery. So, the requirement is that the inverse of all the roots of these polynomial lag operators should lie inside the unit circle. Alternatively, all the roots of this polynomial lag operator should lie outside the unit circle.

So, that was about ARP process, just an introduction. We will continue more with the ARP process the derivation of its further characteristics and then with the ARMA process that is autoregressive moving average processes in the next module.

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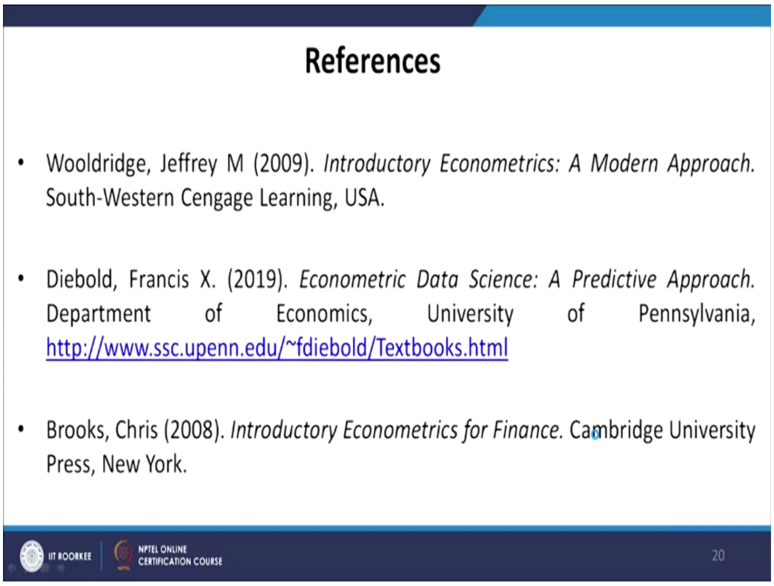

These are the references I have consulted. Thank you.