Path Integral Methods in Physics & Finance
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Lecture – 54
Financial Derivatives

Welcome back. So, before we proceed further, let us take up some examples. In fact, I have been guilty of not taking too many examples so far in the course.

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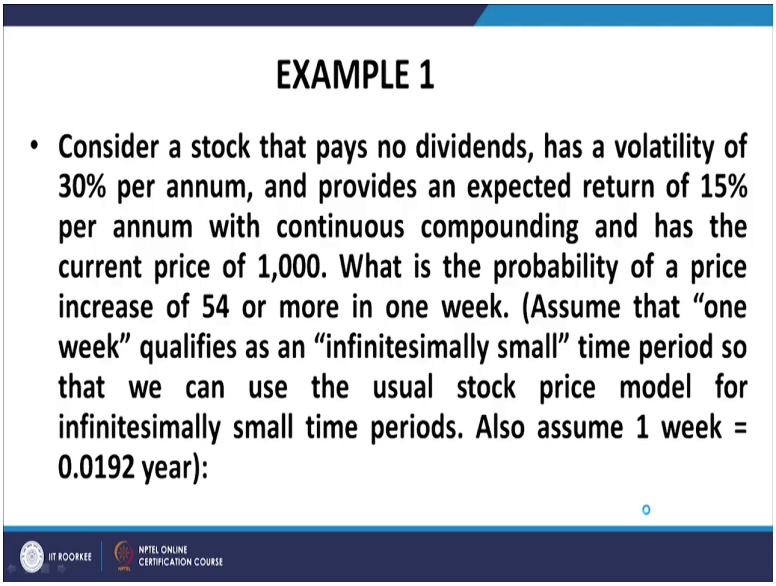

So, let us take up an example. Let us read through this example. Consider a stock that pays no dividends and has a volatility of 30 percent; this is sigma, volatility of 30 percent per annum and provides expected return of 15 percent per annum. This is mu which continuous

compounding and has the current price of 1000; this is the current price this is S naught. What is the probability of price increase of 54 or more in one week?

We want to work out the probability of the price increase of this particular stock which is or share which is presently quoting at 1000 by an amount of 54 or more in one week from now. And, he also clarifies in the problem that assumes that one week qualifies for the infinitesimal time period that is dt e so that we can use the usual stock price model for infinitesimally small time period that is the stochastic differential equation that Langevin equation and he also says that assume that one week is equal to 0.0192 years. So, let us see how this problem is to be done.

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VOLATILITY	0.3	
EXPECTED RETURN	0.15	
CURRENT PRICE	1000	
TIME	0.0192	YEARS
IN THE INFINITESIMAL MODEL		
CHANGE IN STOCK PRICE (dS) IS NORMALLY DISTRIBUTED		
WITH MEAN μSdt	2.88	
$\sigma S\sqrt{(dt)}$	41.56921938	
DESIRED PRICE INCREASE	54	
CORRESPONDING Z VALUE	1.229756073	
REQUIRED PROB	0.1093	

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Recall that what is the model that operates at the level of infinitesimally time periods it is d S is equal to mu S dt plus sigma S d w, a that implies this particular model implies that d S is

normally distributed with a mean of mu S dt and with the variance of sigma square S square dt. So, because d w has a variance of dt. So, the variance is contributed by the stochastic term the diffusion term and that is sigma square S square dt and the mean or the expected return or the expected price change is contributed by the drift term which is mu S dt.

In other words, I repeat d S is normally distributed with the mean of mu S dt and with the standard deviation of sigma S root dt a variance of sigma square S square dt. So, when we workout this relevant expressions substituting mu is equal to 0.15, S is equal to 1000, dt is equal to 0.0192 we get mu as the mean of the mean of d S as 2.88 and we get the standard deviation of d S as 41.5692. The desired price increase is 54.

Therefore, the corresponding Z value that we get is 1.2297 that is obtained by if you are taking 54 minus 2.88 divided by 41.5692. So, that turns out to be 0 point that turns out to be I am sorry, 1.2297. This is the Z value and corresponding to this you can read out from the table the probability of the stock of the value of Z or being greater than this particular Z value is equal to 0.10933. So, this is how this problem is to be attempted.

The important thing is that in this case it is explicitly mentioned that the model that is to be used as the stochastic deferential equation model for which this particular form applies that d S is normally distributed with the mean of mu S dt and with the variance of sigma square S square dt, that is the important part here. The rest of would is calculation stuff.

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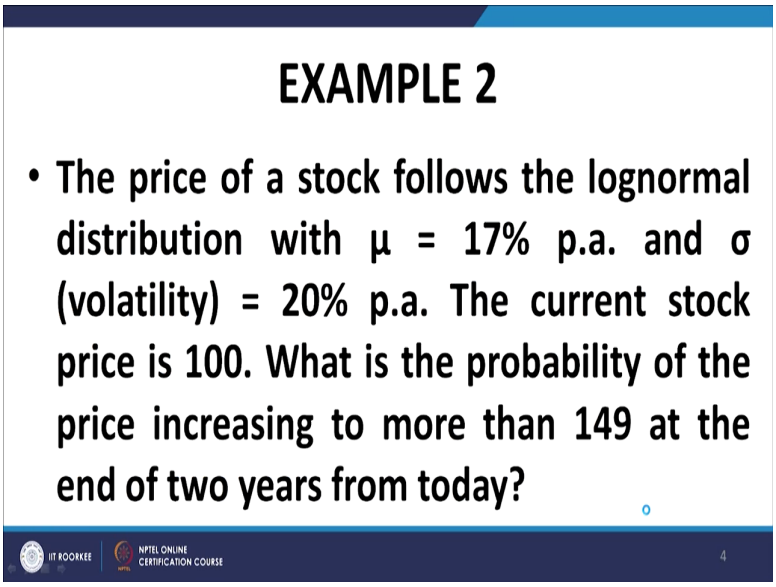

Another problem, slightly different problem. The price of a stock follows the lognormal distribution; now, here it is explicitly mentioned that the price is following the lognormal distribution, mu is equal to 17 percent and sigma is equal to 20 percent. The current stock price is 100, what is the probability of the stock price increasing to more than 149 at the end of two years from now.

Now, please note we are talking about time horizon of two years and more over it is explicitly mentioned that we have to use the lognormal price model. So, we proceed on that basis.

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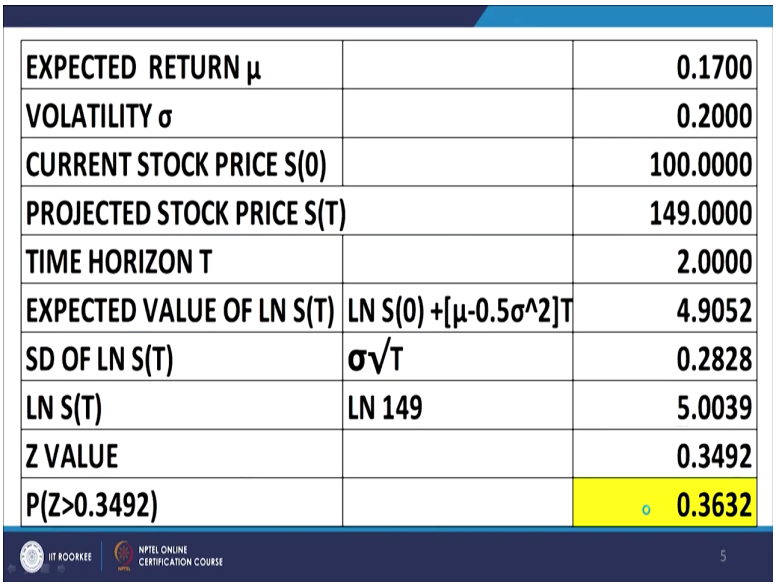

The expected return is given as 17 percent, volatility that is sigma is 20 percent and the current stock price is given as 100, the projected stock price or the desired stock price is 149.00, the time horizon is two years.

Now, remember what is the lognormal model? The lognormal model says that log of S T is normally distributed with the mean of log S 0 plus mu minus 1 by 2 sigma square into T and the variance is given by sigma square T. So, that is the distribution. It is normally distributed with these two parameters log of S T. Log of S T is normally distributed with a mean of this expression, log S 0 plus mu minus 1 by 2 sigma square T and a variance of sigma square T.

So, we workout these two parameters; we find that the means works out to 4.9052 and the standard deviation works out to 0.2828. Now, log 149 works out to 5.0039 and therefore, the corresponding Z value that we have is 5.0039 minus 4.9052 divided by 0.2828, which gives us

0.3492. We want the probability of Z exceeding this value which is equal to 0.3632. Now, this is how this problem is to be attempted.

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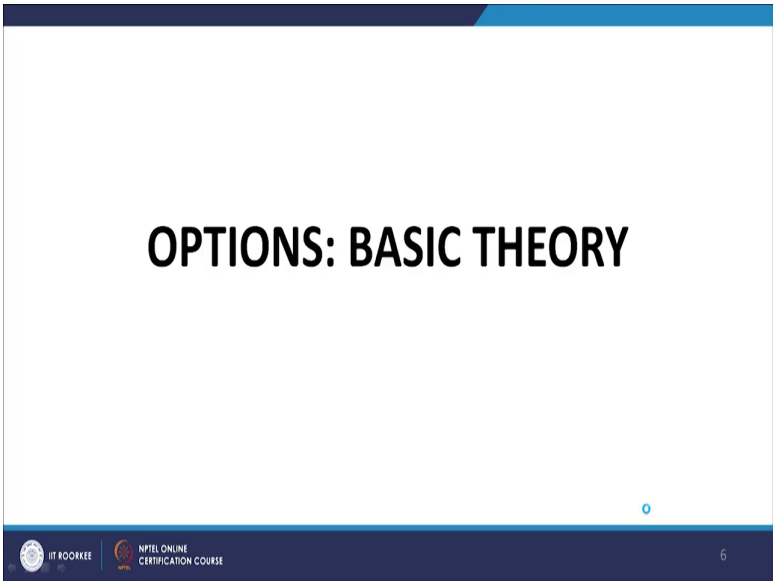

Now, we start with we so far talked about stock price modeling. We have not talked about our ultimate objective of talking of valuation of financial derivative.

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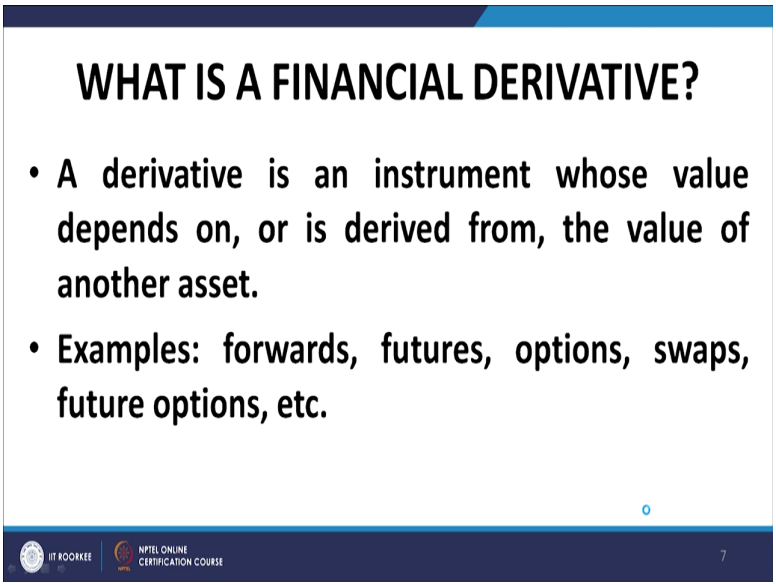

But, before we talk about financial valuation of financial derivatives we need to understand what we mean by financial derivatives. Financial we have talked about stocks financial derivatives are those financial instruments whose price processes depends on the price processes of the stocks such similar financial assets.

In other words, they are financial assets in themselves, but their price processes are dependent on the price processes of some other financial asset like stocks it may be currencies, it may be commodities, it may be interest rates, it may be treasurables or such similar underlying assets. So, financial derivatives I repeat are functions or they are they have a functional dependence on a certain other financial assets certain different financial asset.

And, that different financial asset could be any of these, it could be could be stocks, that is shares that are traded in the market or it could be interest rates, could be stock indices, it could be currencies, could be commodities and so on.

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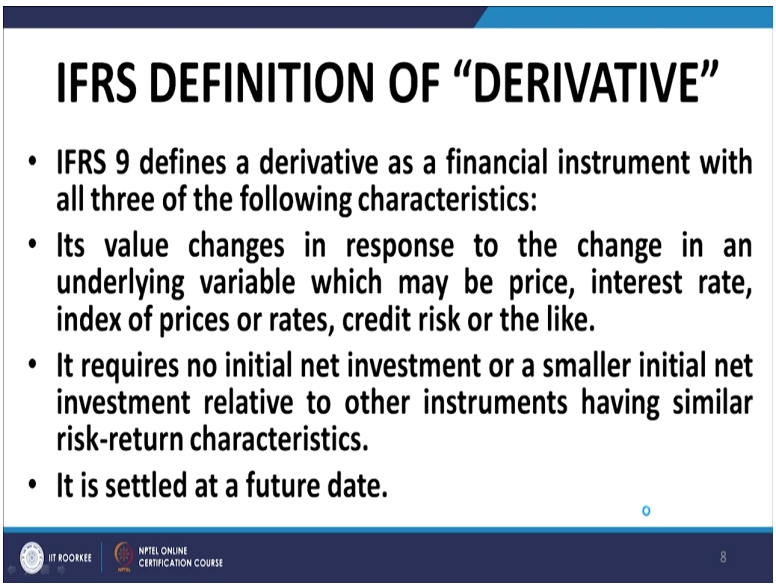

The IFRS definition –IFRS is the International Financial Reporting Standards; the IFRS definition of financial derivatives prescribes three characteristics all of which need to be satisfied in order that an instrument will be classified as the derivative and they are number 1. The value changes in response, the value of the instrument, the value of the financial derivative changes in response to a change the underlying variable which may be the price, interest, or index of prices or rates, credit risk etcetera.

So, there is an underlying asset and there is a derivative asset and this is two tiered structure. The underlying asset is the independent asset and derivative asset has a functional relationship

with the underlying asset such that the price of the derivative asset or the value of the derivative asset depends on the value of the underlying asset.

Number 2: The derivative asset does not require any initial investment or requires a much smaller initial investment compared to other investments that have the similar risk return characteristics. For example, as you will see in the case of a forward contract or a futures contract for that matter options even. They require very little initial investment, but they perform the same job and they provide the investor with the same risk return profile that he would get by investing in the underlying asset and, of course, settled at a future date, as you shall see as we progress.

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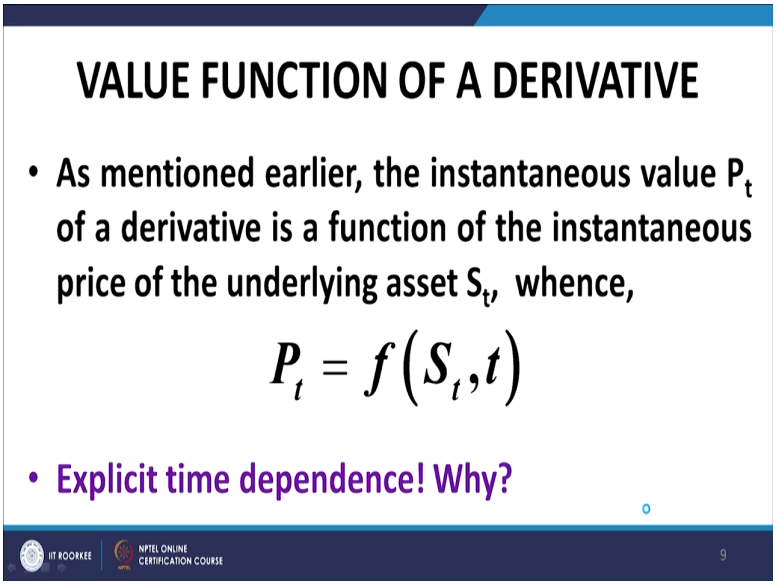

So, this is the value function of a derivative. The value function of a derivative can be represented in this form P t is equal to f S t, S t is the instantaneous value of the underlying

asset or the stock or the whatever that underlying asset is, the independent asset is and of course, there may be explicit dependence time as well.

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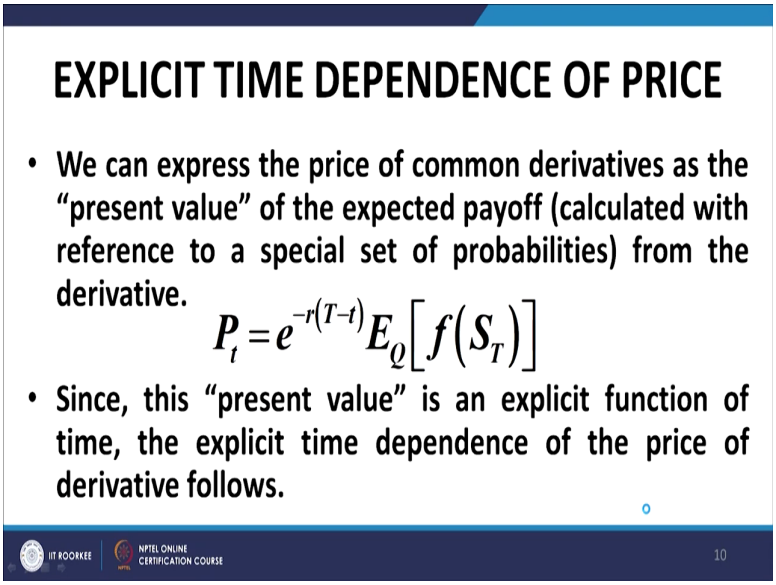

And, now the explicit dependence on time can be explained due to the fact that the value of these derivative instrument are usually obtained in terms of the time value or the present value of the future cash flows that may be attributed to this instruments. Or the expected future cash flows that may be attributed to this instruments, discounted to today's level that expected value being calculated with a set of special probabilities.

But, we shall nevertheless, the important thing is that the value changes as time progresses and therefore, there is an explicit time dependence as well.

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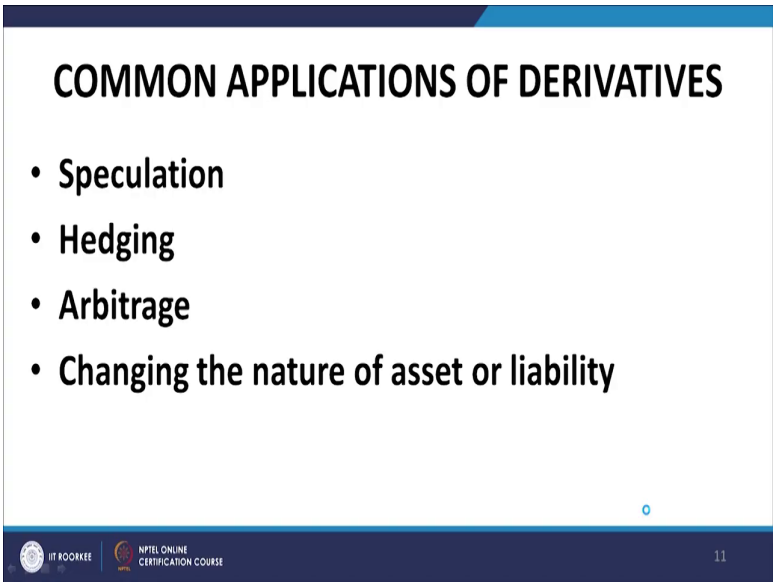

Now, common applications of derivatives, well these derivatives find a huge spectrum of application in modern financial markets, modern business environment, they are used for speculation when people take naked position based on their perception of the future expected market moves and they take positions. For example, if you expect particular stock to increase in value contrary to what the market expects you may take a long forward position or future position.

Then, if you have an exposure at hand you are exposed to currencies you have to make a payment in US dollars, you are based in India you have to make a payment in US dollar then you can cover your exposure. If the payment is to be made at a later date you can protect yourself in against exchange risk arising from the fluctuation of the US dollar Indian rupee

exchange rate during the intervening period by taking up position or taking a forward contract on that.

Then, there is a case of arbitrate if these instruments are trading at different prices in different market and of course, you can take advantage of the differential pricing by buying in the cheaper market and selling in the dearer market. Then swaps can also be used. Swaps is a type of derivative instrument. They can also be used to the nature of an asset or a liability, nature of the currency of the borrowings.

You can also change the interest rate profile of the borrowings from the floating rate to a fixed rate and so on.

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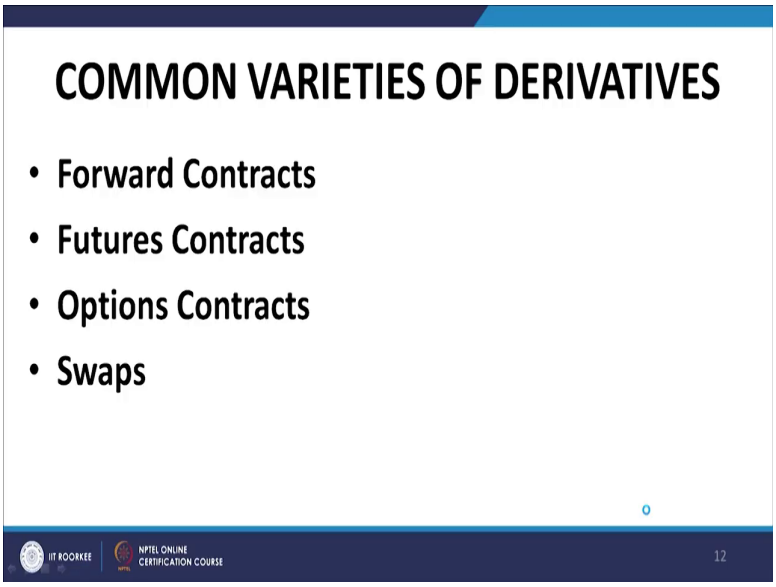

The fundamental derivatives; well, there are four fundamental derivatives that are that are that from the initial strata of derivatives. They are the forward contract, they are the futures contracts and then we have the options contracts then we have swaps.

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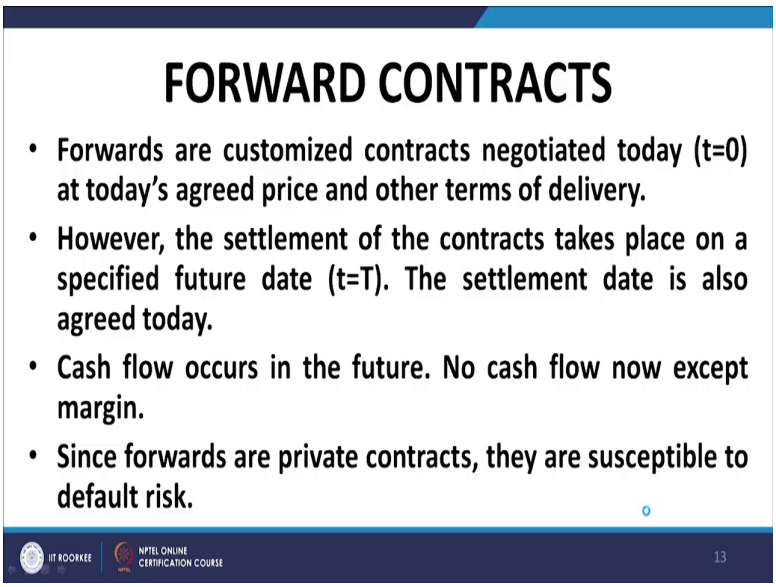

To start with forward contracts, forward contracts are contracts that are not traded on the market that are tailor who made contracts that are negotiated between the two parties that are involved in the contract, that is the first thing.

The characteristics of the forward contract is that the contract is enter into is negotiated at t equal to 0. But, then it entails the delivery of the assets whichever whatever asset follows constitutes the substratum of the contract, it entails the delivery of that asset at future date.

And, of course, the payment of the price is also made at the future date. So, the actual transaction the actual settlement of the contract takes place at a future date the terms of the contract that is the price and other terms of delivery etcetera quality and so, on are all fixed at t equal to 0 that is today.

When the contract is negotiated all the relevant issues that are required to ensure an undisputed settlement of the contract at future date, the date of settlement, the price the quality of the asset, the mode of delivery. And whatever is necessary is relevant for the complete execution of the contract are settled are agreed at t equal to 0.

But, the actual transaction takes place at a future date at a pre-determined future date, date also is determined at t equal to 0, the price is paid and the asset is obtain by the party who is long in the contract and the party who is short in the contract gets the price and delivers the asset. So, that is the nature of the forward contract.

So, the important thing here is that the cash flow, the price takes place at the future date not at not at t equal to 0. Of course, if you go to your bank and request for a forward contract on US dollars to pay to fee to US university say in 3 months time. The bank may require a certain amount of margin to be deposited in the form of an FDR or some other form or a lien on your saving deposit depending on your credit worth in a sense the possibility of your default or withdrawing from the forward contract.

But, other than that the actual transaction price is paid on the date of maturity of the forward contract.

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Futures contract are very similar to forward contract, they are on parallel lines to forward contract, but they have certain distinctive futures. As I mentioned right at the start forward contracts are customized contracts, they are tailor made contracts.

They are contracts which are negotiated between the two parties A and B, there are two parties that enter into the contract. However, futures contracts are tradable. They are traded on recognized exchanges, which has set up for this purposes and which allow trading free trading in such contracts.

Now, obviously, the advantage of tradability is quite straight forward, it adds to the liquidity of the contract. So, a party wants to withdraw from contract at any of time, it can do so without incurring any damage any detriment, without paying any penalty or whatever because

the contracts will be traded in the market and it can obtain the then prevailing market price by taking by taking the opposite position in the contract.

However, there are two fundamental issues that that are required in order that futures contract need to be need to be tradable or to make them tradable. One thing is that they have to be standardized. The contract must necessarily be standardized in order to facilitate liquidity of the contract. If contracts are not standardized it would be difficult to pair up buyers and sellers and thereby the liquidity in the market would not be sustainable.

However, if the contract is standardized just like we have currency notes then the market liquidity can be managed, that is one thing. The second thing is as I mentioned forward contracts are tailor-made contracts or customized contracts and therefore, there is an element of risk in whether one of the parties may default.

But, being private contracts the other party has adequate opportunity to access the risk and take remedial measures in terms of margin and whatever the whatever appropriate restrictions it wants to impose or methodology it wants to impose to cover up the risk factor that is involved in the forward contract.

However, in future this kind of situation is not possible because the futures contract would be changing hands very frequently during the course of the trading just like shares and bonds that trade on the markets. And therefore, there has to be mechanism where these instruments or these futures contracts have to be or are made completely default free in the sense that the buyer of the contract or the party who takes the long position does not have to worry about the performance part of the contract.

And, that is done by the clearing house entering as a mediator in the contract, the clearing house guaranteeing the performance; the clearing house of the exchange where the contracts are traded guaranteeing the performance of both the legs of the contract. And, thereby making them the contracts default free for our practical purposes.

As far as the clearing house is concerned the clearing house protects itself from any event of default by two approaches or two strategies acting and conjunction with each other. One is the margining and the second is the marking to market. Come with the combination of these two strategies of margining and marking to market the clearing house is also able to protect itself against any potential default situation from either party to the futures contract right.

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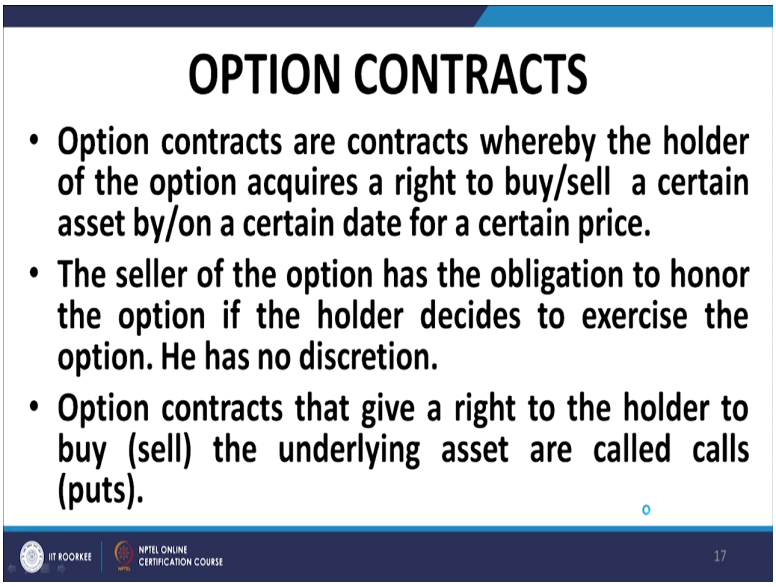

Now, we will talk about options contracts. Options contracts are slightly different to future and forward contracts. The fundamental property of forwards and futures contract is that the two parties that are involved in the contract have both obligations to meet their leg of the contract. For example, if you enter into forward contract with the banker, then it is the; it is your as well as the bankers obligation to meet your respective legs of the contract.

In other words, you will have to pay rupees on the specified date and the banker is obliged to pay US dollars on the said date of settlement of the for future forwards contract. However, options have a slightly different characteristic. In the case of options again as in the case of forwards and futures we have two parties; one party is called the long party the party and the one party who buys the option is called the long party and the party who sells the option is called the short party.

Now, the important thing is that the party that is long in the contract or the party who is long in the option has a right, but not the obligation. Here the party who is long in the option has a right to exercise the option or to let the option lapse. He has the discretion, he has the power and that is why this contract is called an option. He has the option the right the discretion whether he wants to allow the contract to lapse or whether he want to exercise the option as per the terms of the contract.

However, the other party to the contract that is the party who is short was written the option the short party is suppose to is said to have written the option. So, the party who has written the option is obliged is mandated to honor his leg of the contract if the party who is long decides to exercise the contract.

So, the situation is that if the party who was long in the contract that is the party who has brought the option, decides to exercise the contract the party who is short in the option cannot escape the obligation to honor his leg of the contract.

He must honor his leg of the contract. Of course, if the party who is long in the contract allows the contract to lapse, then the party who is short in the option goes scot free. That is natural because the party who is long in the option has not exercised the contract so, the party who is short goes scot free.

So, that is the special feature of options contract. However, because there is nothing called freelance in financial at least in financial markets what happens is that, because the two parties are not on the same platform party who is long is having higher is sitting at a higher pedestal

compared to party who is short party who is long has a right has a discretion, has a power party who is short has no such discretion, no power is simply has an obligation.

And therefore, to get that right to procure that right the party who is long has to pay a price and the price is called the premium of the option and that is in fact, what we shall be talking about in the context of Black-Scholes valuation that is in sense the value of the option. So, it is the premium that the party who is long has to pay in order to acquire the right and the premium is paid to the party who is short who thereby has in a sense gets compensation for the obligation to honor his leg of the contract.

So, that is the fundamental property of the option contract.

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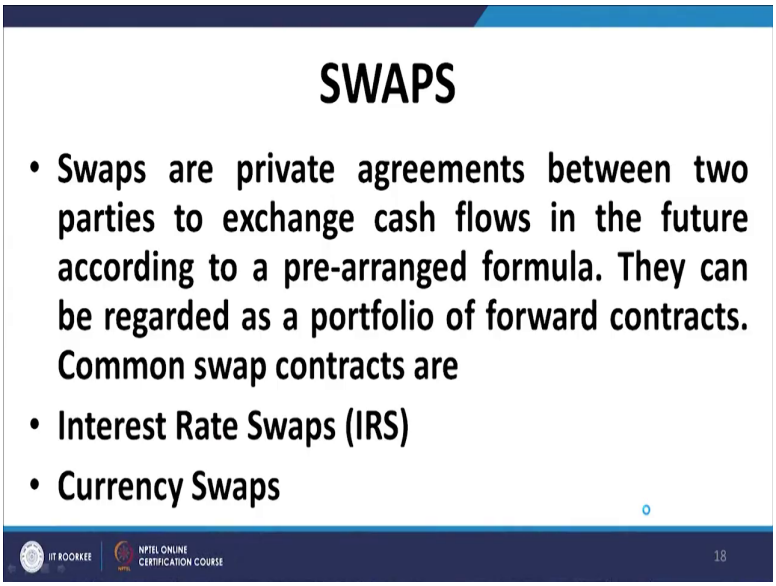

Now, then we talk about swaps. Swaps are contracts which are basically exchange of cash flows. Swap the common terminology meaning of swap is that it is an exchange and that is carried on into the financial literature. Swaps are contracts which entail an exchange of cash flows over future time period. Usually what happens is one of the legs of the cash flows is tied to an underlying variable whereas the other cash flow is fixed at the date of inception of the swap.

Let me repeat. One leg of the swap is fixed at the time of inception of the swap is and that fixing holds for example, if it is an interest rate or currency rate or whatever, but that fixing holds for the entire life of the swap that does not change whatever the market conditions may be.

However, the other leg of the swap is tied to some underlying – it may be an interest rate, floating rate swap. For example, it may be a currency rate and that rate is fixed at periodical intervals and the two parties exchange net payments based on the difference between the fixed rate that was agreed at t equal to 0. That is today when the swap is negotiated and the rate that is fixed on a rolling over bases at every resetting date and in the difference between the two is exchanged and the swaps are usually cash settled.

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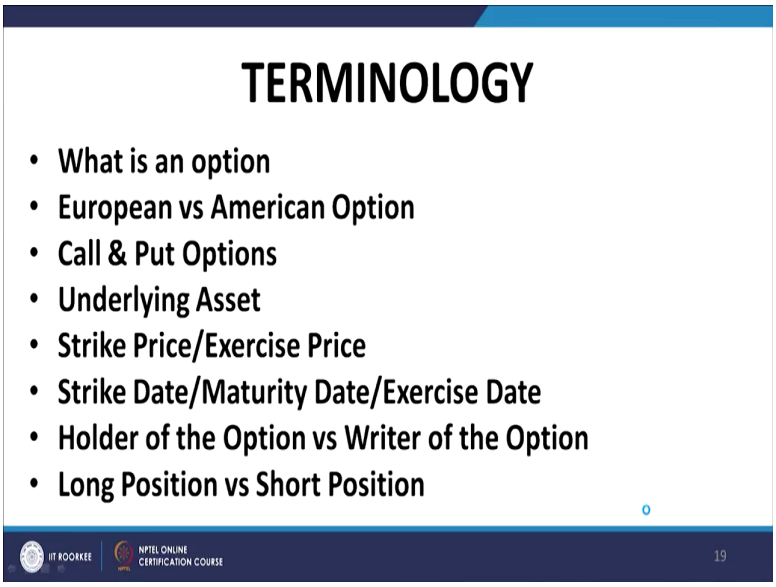

Now, our focus in this particular in the remaining set of lectures shall literally entirely be on options, except for a few article or few issues and the primary approach or the primary issue that we shall be attending to will be Black-Scholes option pricing model.

Before we get into that some more fundamentals we have talked about what an option is. Options can be European, options can be American; European options are options that are exercisable on a particular date, American options are exercisable at any date between the date of inception of the option and the date of maturity of the option.

In other words, American option enjoy a certain amount of flexibility, certain amount of leverage in terms of date of exercise. They may be exercise on any date between the date of

inception of the option and the date of maturity of the option. However, European options are those options that can be exercised sonly on the date of maturity of the option and not earlier.

Call and put options: call options are those options that give the holder of the options, the party who is long in the options are right to buy the asset at a predetermined price from the party who has written the option. Whereas, the put option gives the right to the party who is long in the option, the right to sell the option at the predetermined price at the price that is agreed upon in the option contract to the party who is short in the option.

Let me repeat. Call option is the right to buy at pre-determined price, the price is predetermined fixed in the option contract on a particular date that is called the maturity date for European call options from the party who is short in the option who has written the option. Put options behave similarly except for the fact that they have the they have the right to sell the option to sell the underlying asset.

Underlying asset is the asset that forms the substratum of the option contract. In other word, it is the asset that can be bought or that can be sold under the option contract. These underlying assets as I have been mentioning earlier may be may take any form, may be commodities, may be stock indices, may be stocks, may be currencies and so on. So, these are the assets that can be bought or sold in perseverance of the option contract they form the substratum of the option contract.

The strike price – the strike price which is also sometimes called the excess price. The strike price or the exercise price is the price at which the which is embodied which is incorporated in the option contract. And which the party who is long in the option can either buy in the case of a call option or sell in the case of a put option to the party who is short in the option contracts. So, the price the strike price or the exercise price is the price at which the option contract operates at which the underlying asset can be bought or sold under the option contract.

The strike date or maturity date I am already explained. It is the date on which the options becomes exercisable if it is European option; it is a date up to which the option becomes

exercisable if it is a American option. The holder of the option as I mentioned the option the party who is long of the option, the party who has paid the premium, the party who has bought the option is called the holder of the option; the party who is sold the option is called the writer of the option, is also called short in the option.

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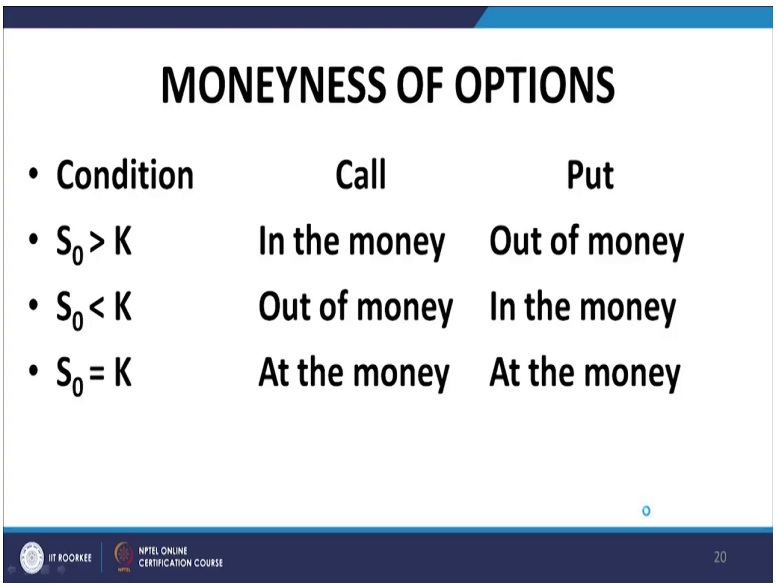

Now, there is another term which is called moneyness of the option. If the current stock price exceeds the excess price the call a call option is said to be in the money and the put option is said to be out of the money. The logic the rational of this terminology is that if the option were to be exercised at a particular point in time let us say t equal to 0 whatever that particular time is.

And at that point in time if S T or S 0 is greater than K that means, and it is a call option then what you could do is, you could buy the buy the underlying asset under the option contract at

the price K and sell it in the market at S 0 and thereby you could make profit of S 0 minus K. And, therefore, the call option is said to be in the money.

Let me repeat if an option is if S 0 is greater than K that is and you have a call option what you can do is, you can buy the asset under the option contract at the strike price K and sell it in the market at S naught. Because S naught is greater than K, you can make a profit out of this equal to S naught minus K and therefore, the option is said to be in the money.

However, you cannot do the same thing if it is a put option and therefore, it is said to be out of the money. Conversely, if S 0 is less then K then you consider case of a put option here. If you have a put option here you can buy the asset from the market at S naught and sell it under the option contract the put option contract to the writer of the option at K and thereby again make a profit of K minus S naught. So, again you have in this case the put options become in the money and the call option becomes out of the money.

So, to reiterate if S naught is greater than K, the call option is in the money, the put option is out of the money and if S naught is less than K the call option is out of the money, the put option is in the money.

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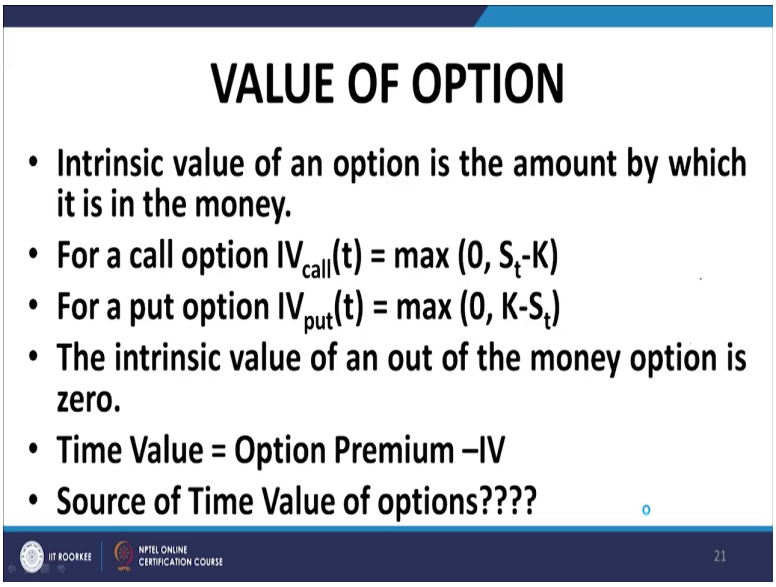

Now, the value of the option. Well, the value of the option means it consist of two parts; it essentially consist of the intrinsic value of the option and the time value of the option. The intrinsic value of the option is in the in a sense related to the moneyness of the option and it is given by this expression here intrinsic value of the call is equal to the maximum of zero obviously, the option cannot be negative the value of the option cannot be negative.

So, the for a call option the intrinsic value becomes maximum of 0 comma S naught or S t minus K whatever the time at whatever time the value is being calculated. And, for a put option the intrinsic value becomes equal to maximum of 0, K minus S t where S t is the t is the time at which the value is being calculated.

Now, please note this is the intrinsic value this is not the total value. The total value of the option is usually is usually greater than the intrinsic value and the differential between the two.

The total value is greater than the intrinsic value of the option in the differential between the two is usually terms as the time value of the option. Now, an issue that is intricated is the origin of the time value of the option.

Now, we need to understand that to understand the concept of time value we also need to understand that the stock price or the price of the underlying asset whatever it may be, usually I shall be using the stock epitomizing all form the underlying assets. So, let us use it use the term stock price this stock price is a random variable. So, the possibility of an option which for example, which has which is has a very low payoff if exercised today or for which S t and K are very closed to each other, they it has very low intrinsic value.

May at the time of its maturity increase, in other words, the possible pay off at maturity may be more then the payoff from the option at the current point in time. This possibility of the option payoff increasing and finite probability existing for this event happening carries a certain value in the market and that value in the market is called time value.

And, therefore, usually when we talk about where options the total value of the option is usually more than the intrinsic value. And the differential between them is called the time value which originates from the uncertainty over the future behavior of the price of the underlying asset and therefore, the payoff from the option, right.

We should continue from here in the next lecture.

Thank you.