Path Integral Methods in Physics & Finance
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Lecture – 43
Euclidean Field Theory [3]

Welcome back. So, before I move to the final topic in the segment which is the quantum field theory on Minkowski space. Let us have an extended recap of what we have done so far after the zero dimensional field theory. So, we then from the zero-dimensional field theory, we moved on to the one-dimensional lattice structure discrete lattice structure.

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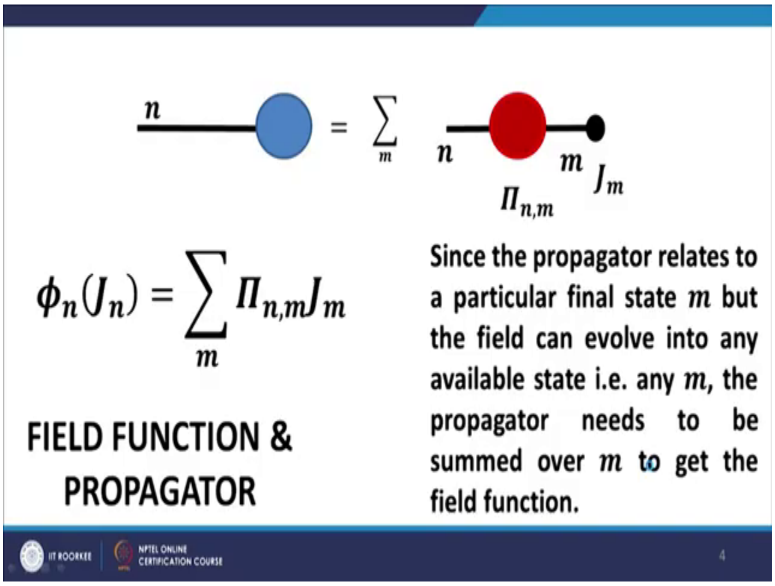

And we introduce the concept of the propagator as representing the moment of the field from once from a state n to another state m. And because the field function could move out of the system in any of these state, so we need to have a summation over the states m when we establish a relation between the field function and the propagators which is given in the equation on the slide.

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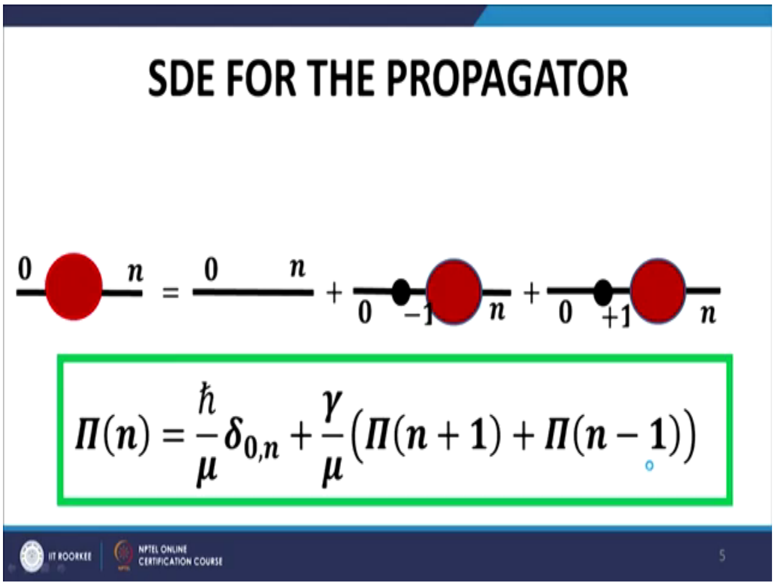

We then had establish these Schwinger-Dyson equation in terms of the propagator, and we based it on the set of Feynman diagrams which are given on the slide. The propagator being represented by a red blob, field function entering the system at point 0, and emit emerging from the system at point n could emerge without interaction if the point 0 and the point n coincide that is represented by the in delta function.

Or it could interact with a two point vertex representing a neighbouring point and then encounter, then move out from the system after propagating to the point n. And this one of them represents the interaction with the left hand field point and the other represents the interaction with the right hand neighbour.

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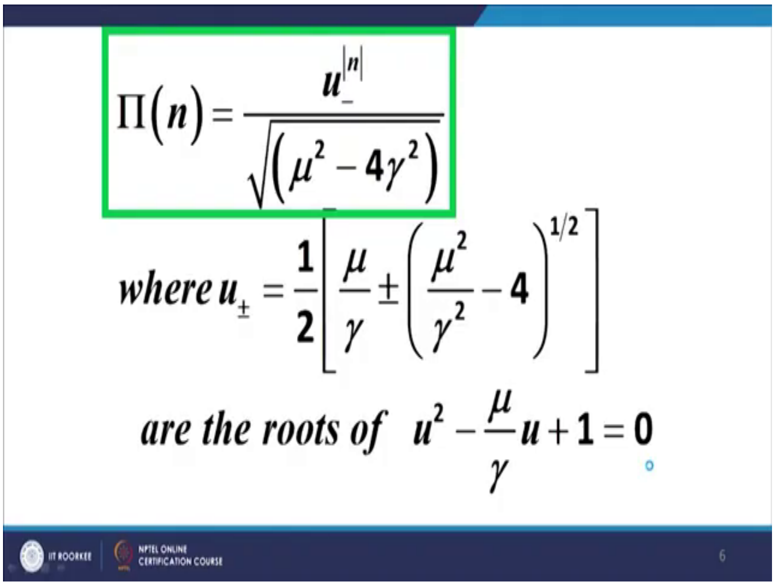

We establish this expression on the basis of the Schwinger-Dyson equation that was given on the previous slide, the green box here. And we got the expression for the propagator as the expression that is given in the top green box here, where u plus minus are defined by this expression here.

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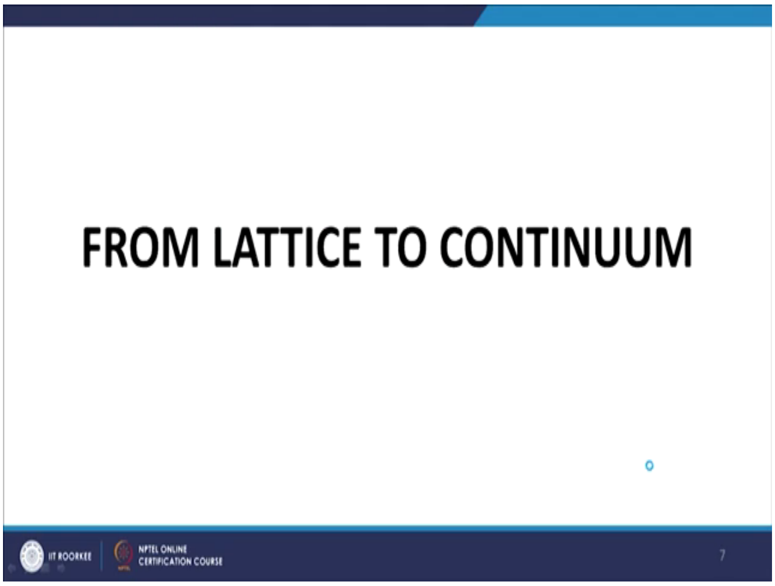

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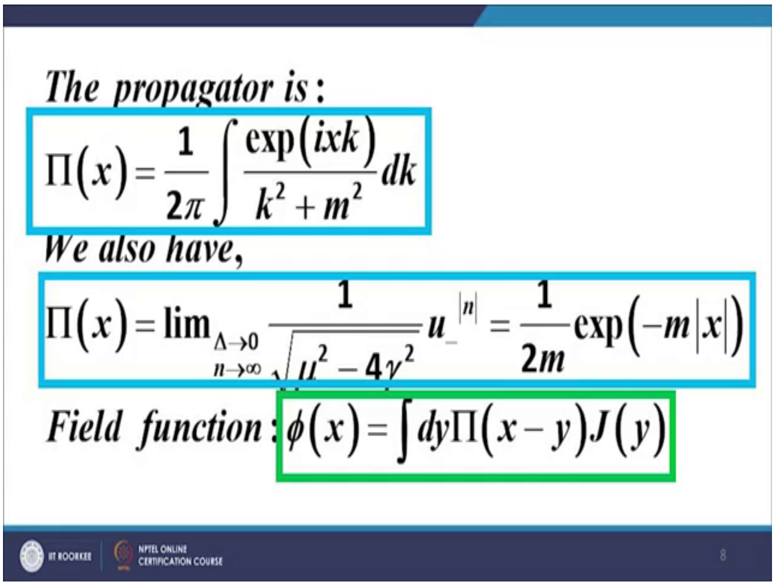

And then we moved over to the continuum in one-dimensional space. And the propagator in this continuum one-dimensional space was shown to be the expression that is given in the upper blue box. The propagator also particularly for large x takes the form of an exponential which is given in the second blue box.

And the field function in terms of the propagator now takes the form of an integral from the summation that was there in the context of a lattice structure. It takes the form of an integral which is given in the green box at the right at bottom of your slide.

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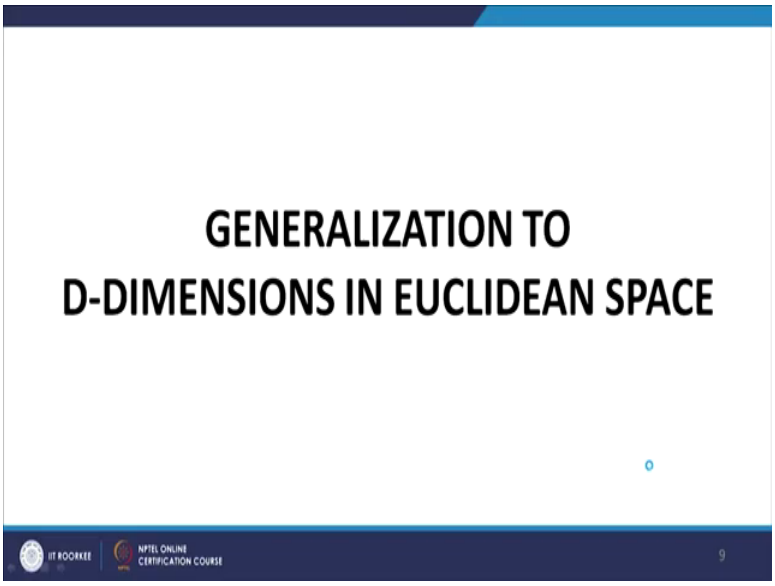

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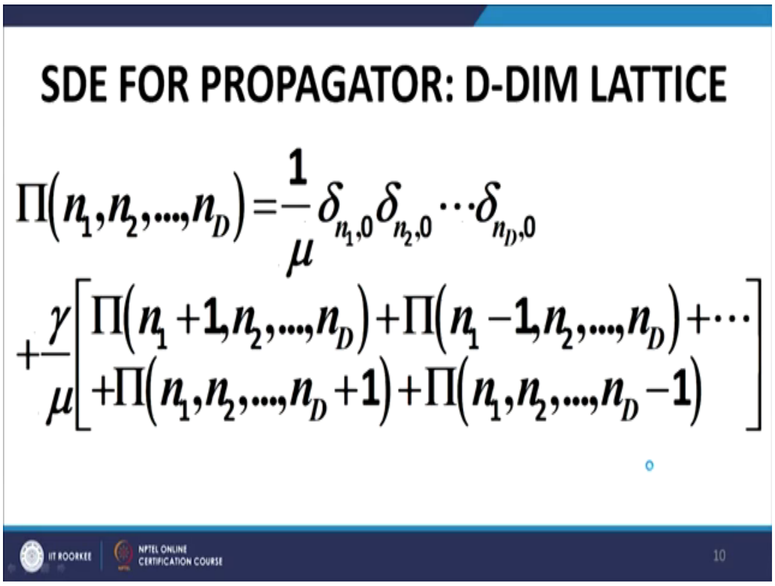

Then we generalize these expression to D-dimension Euclidean space, the Schwinger-Dyson equation for the propagator an D-dimension space is given here. Of course, you have more neighbours in the case of one-dimensional lattice you only have two neighbours. Here you have many more neighbours depending on the value of the dimension of the lattice, and accordingly the Schwinger-Dyson equation for the propagator also becomes more involved.

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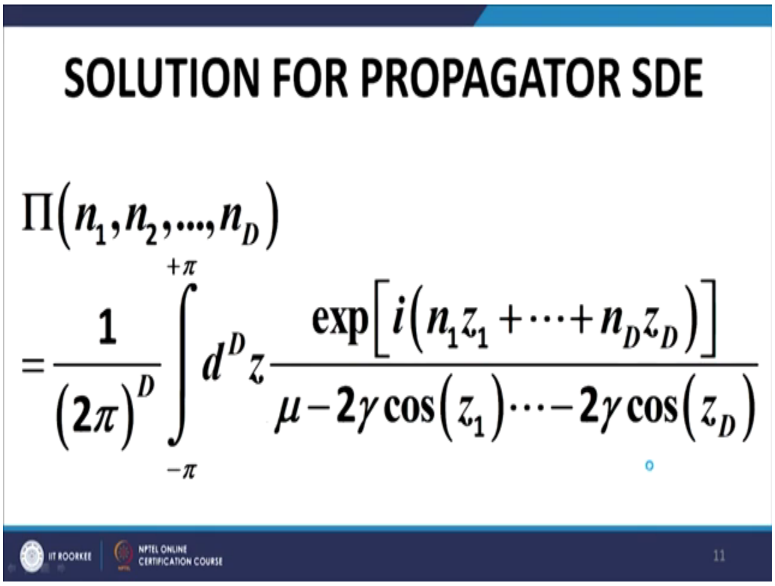

The solution to the Schwinger-Dyson equation representing the expression for the propagator is similar to what we had for the one-dimensional lattice, and takes the form given in the bottom equation of the slide.

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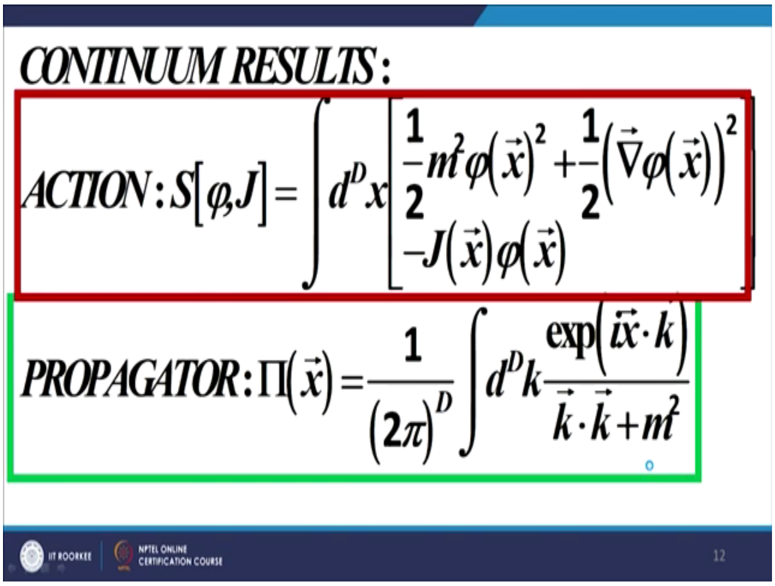

Other continuum results that we are obtained were for the action in the presence of a source, where the source is subsumed within the action. And the propagator now takes the form which is given in the green box at the bottom of your slide.

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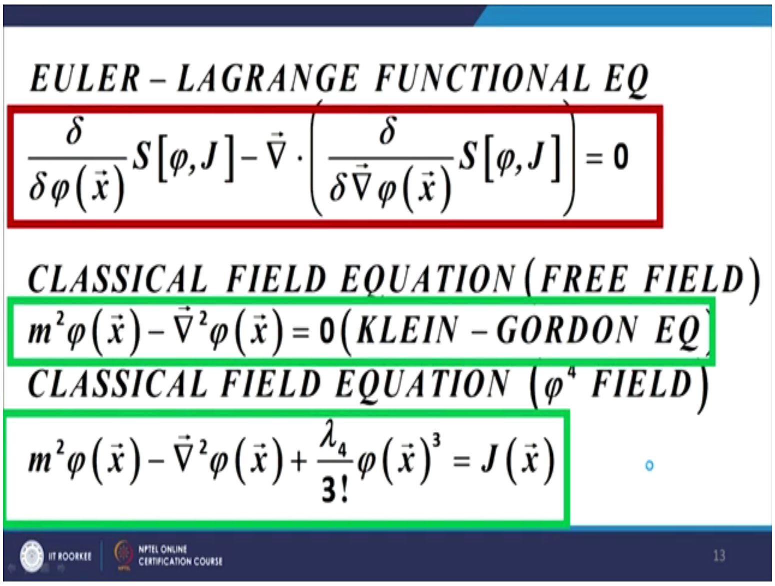

The Euler-Lagrange functional equations takes the form which is given at the in the red box at the top of few slide. The classical field equation, the Klein-Gordon equation takes the form in Euclidean space. Please note this point this is the Klein-Gordon equation in Euclidean space, a free equation a pre field equation, and this is given in the green box upper green box.

And in the case of the phi 4 theory together with the source the equation gets modified to the expression that is given in the lower green box with the introduction of the coupling terms and also the source terms into the free field Klein-Gordon equation.

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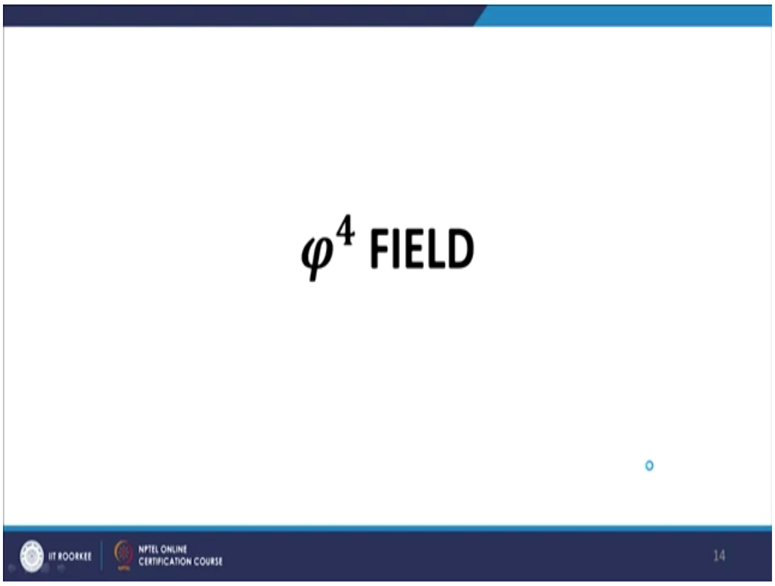

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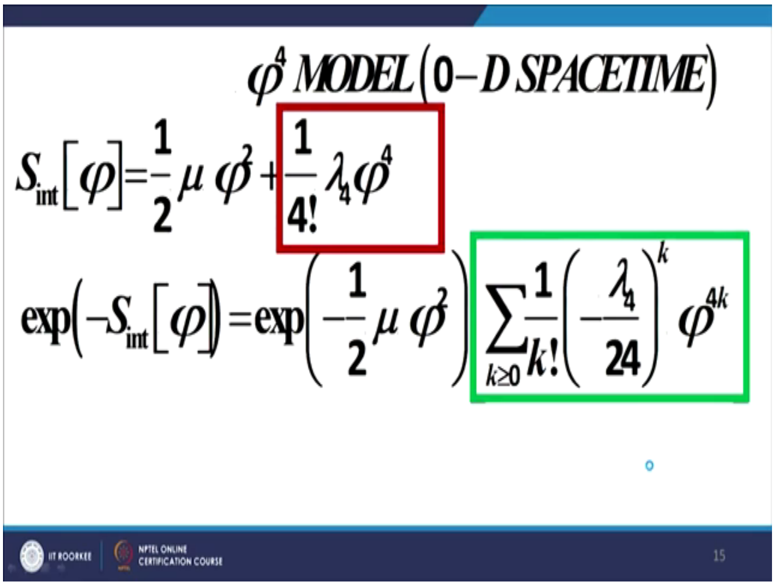

Then we discussed the issue of the phi 4 field. We introduce phi 4 field through an interaction into the action by introducing the term that is given in the red box here, 1 upon 4 factorial lambda 4 phi to the power 4. The exponential of the action was then expanded as a perturbation series in the coupling constant in the interaction constant lambda 4 as a summation series so exponential series.

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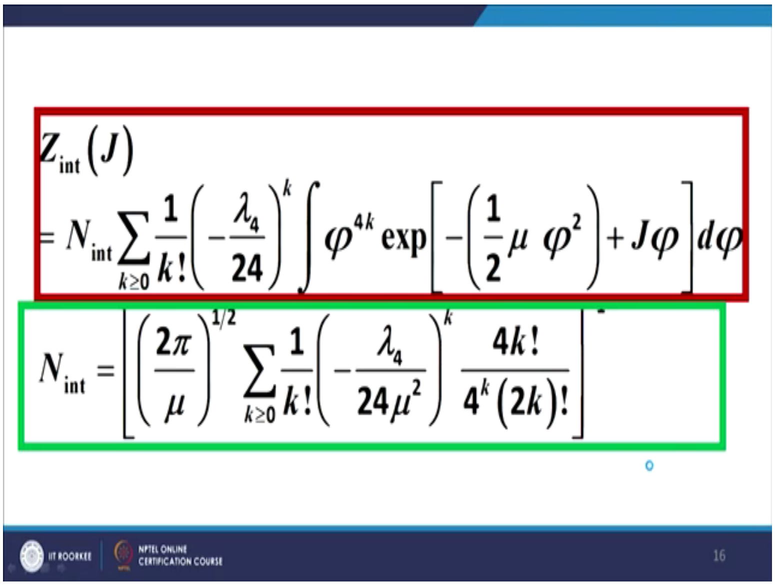

That enabled us to arrive at the interaction generating functional which takes the form given in the red box here, and the normalization of the interaction theory takes the form that is given in the green box at the bottom of your slide.

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$$G_{2n} = N_{\text{int}}\sqrt{\left(\frac{2\pi}{\mu}\right)}\frac{1}{\mu^n}\sum_{k\geq 0}\frac{1}{k!}\left(-\frac{\lambda_4}{24\mu^2}\right)^k\frac{(4k+2n)!}{2^{2k+n}(2k+n)!}$$

$$=\frac{\dfrac{1}{\mu^n}\displaystyle\sum_{k\geq 0}\frac{1}{k!}\left(-\frac{\lambda_4}{24\mu^2}\right)^k\frac{(4k+2n)!}{2^{2k+n}(2k+n)!}}{\displaystyle\sum_{k\geq 0}\frac{1}{k!}\left(-\frac{\lambda_4}{24\mu^2}\right)^k\frac{4k!}{4^k(2k)!}}=\frac{H_{2n}}{H_0}$$

The two point functions, two point Green functions were obtained. And through the perturbation mechanisms, perturbation series and were represented by H 2 n upon H 0. H 0 in a sense capturing the normalization impact, and H 2 n are being related to G 2 n. ah

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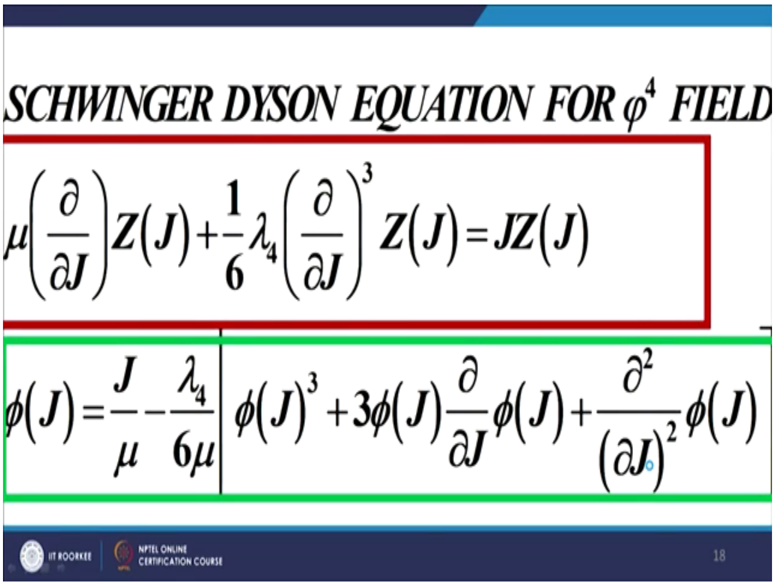

The Schwinger-Dyson equation for the phi 4 field for the generating functional for the phi 4 field takes the form that is given in the red box here. And for the on simplification on substituting the various values, we get the expression for the field function, the Schwinger-Dyson equation for the field function takes the expression that is given in the bottom green box.

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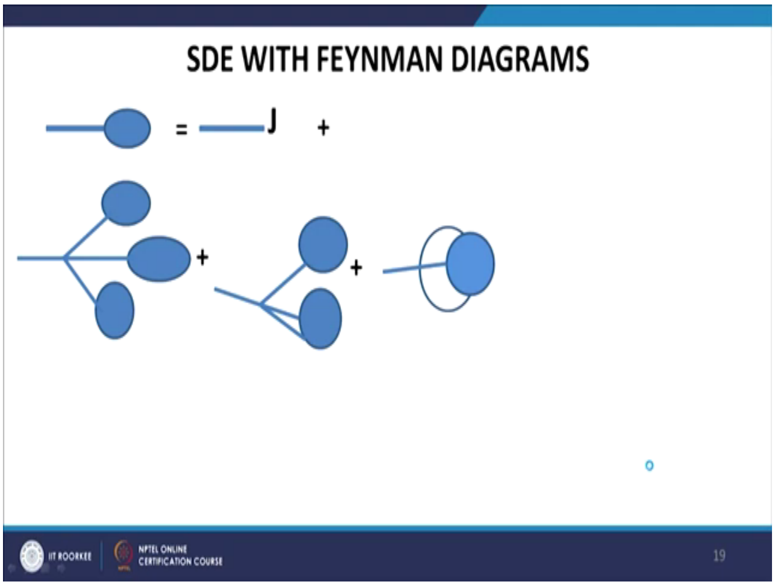

The Schwinger-Dyson equation that is given here in terms of the green box has a direct correspondence to the set of Feynman diagrams which are given here on your slide, which has been explained elaborately in an earlier lecture. So, I would not go through it again, but there is a clear cut correspondence between the terms that are incorporated in the Schwinger-Dyson equation and that are present here.

The first term for example, represents the first term J upon mu the straight line with a source; this represents the term J upon mu. The term the second term with an interaction what is 4 point vertex.

And each of the outgoing legs, external legs ending in a blob is represented by this phi J cube term with the lambda 4 being the vertex. And similarly the other two cases where there is a reunion of two branches emerging from the vertex, and all the three branches from the vertex are represented by the and the second last, and the last terms in the Schwinger-Dyson equation.

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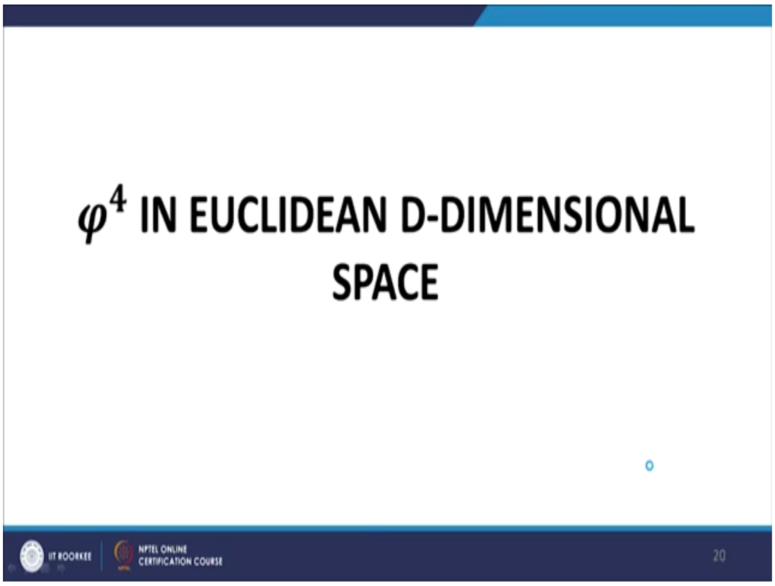

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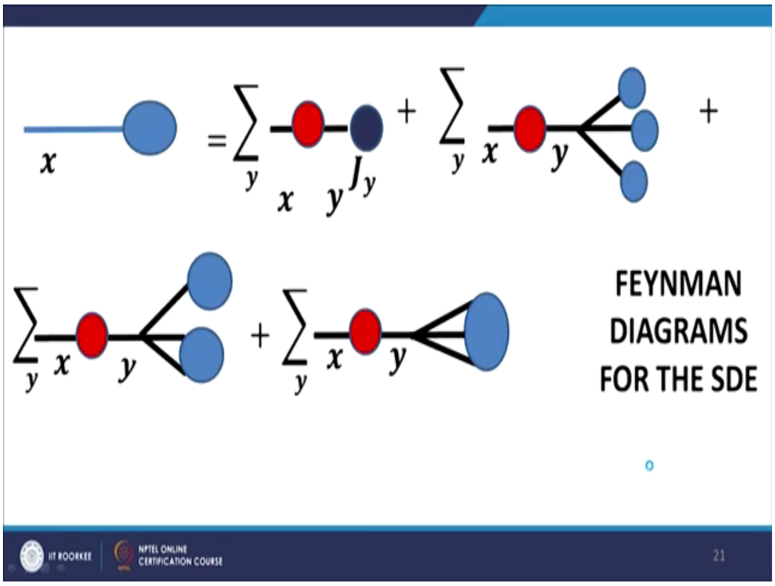

Then continuing with the phi 4 theory in terms of propagators, we have the following set of Schwinger following set of Feynman diagrams corresponding to a field that enters a field function that enters through an external leg at the point x. It could interact, it could first move to the point space time point y, and then interact with a source or it could move to the space time point y, and then encounter a four point vertex.

And the three emerging or the external lines emerging outgoing external lines emerging from the vertex could have separate blobs, each of them or any two blobs could unite or all the three blobs could reunite, and form the same one single blob after emerging from the vortex interaction.

So, these are the various possibilities. Remember in each case we need to sum over the over the fields space time points at which the fields emerged from the through the propagator. And as a result of with summation sign here over y appears. Of course, as we move from this into the continuum arena, the summation will be replaced by the integral and that gives us the Schwinger-Dyson equation which is now shown on your slides.

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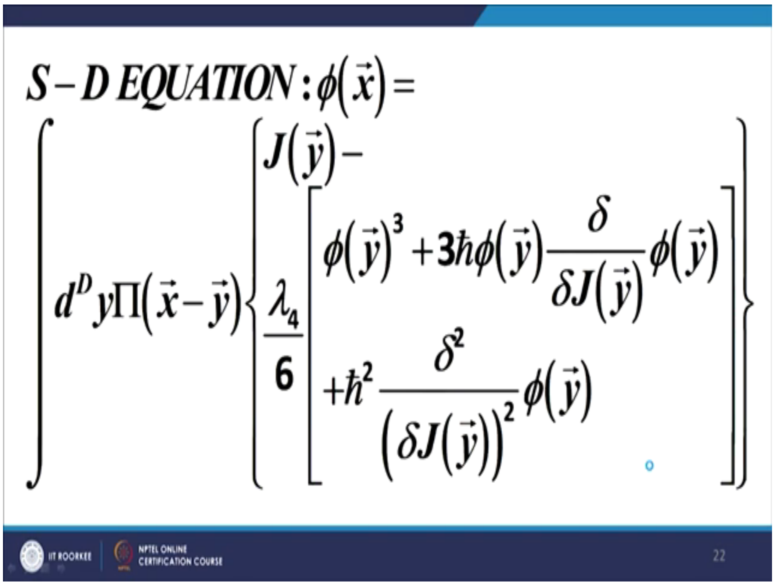

Here I have also incorporated the loop parameter or the h bar parameter representing the number of loops as you can see. In the second last equation, we have one loop; and in the last equation, we have two loops which is represented by h bar in the second last term, and h bar square in the last term.

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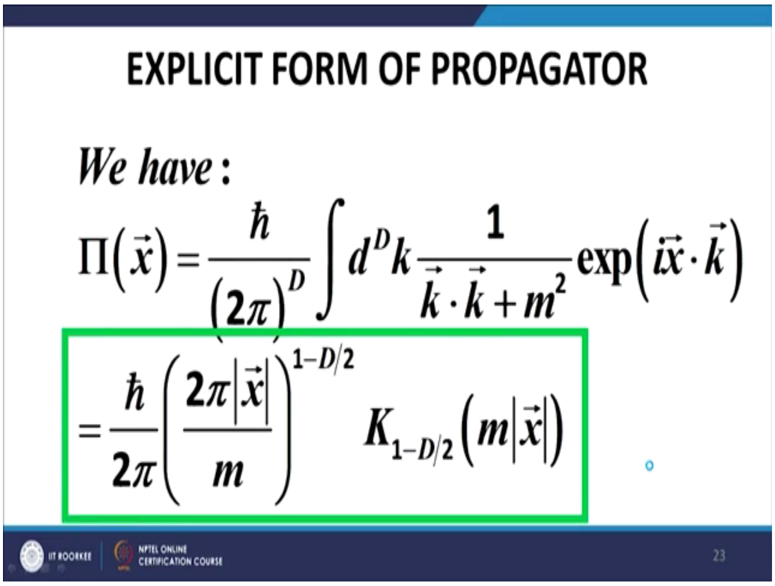

The explicit form of the propagator that we end up with is given on this slide. When you simplify this expression, we get it in terms of the modified Bessel function of the second kind. Remember D is the dimension, and m is the mass of the field, and k is the modified Bessel function of the second kind.

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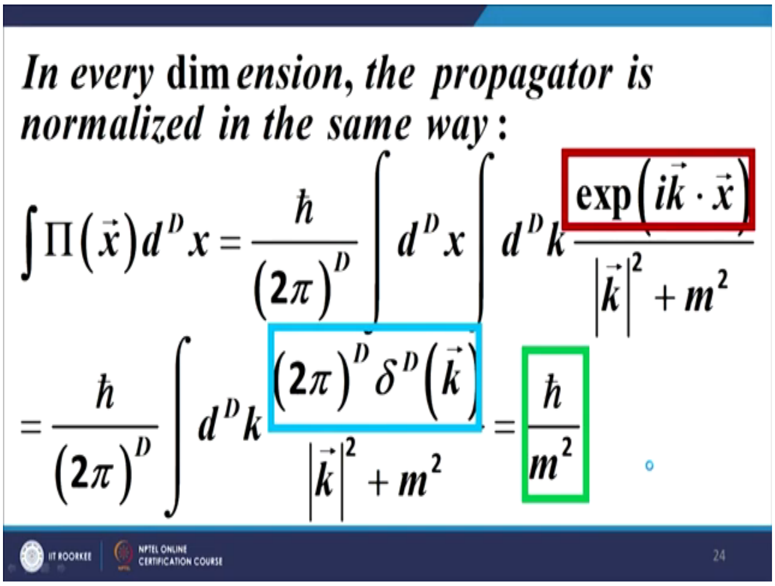

As far as the normalization of the propagator is concerned, the normalization of the propagator is done as per the following scheme which is given here on this slide. And this results in a normalization factor of h bar upon m square for every corresponding to every dimension of the underlying space.

Now, the interesting thing is that for D greater than equal to 3 that is for dimensionality greater than equal to 3, the propagator diverges pi 0 which is the expected value of phi square x diverges x tends to 0.

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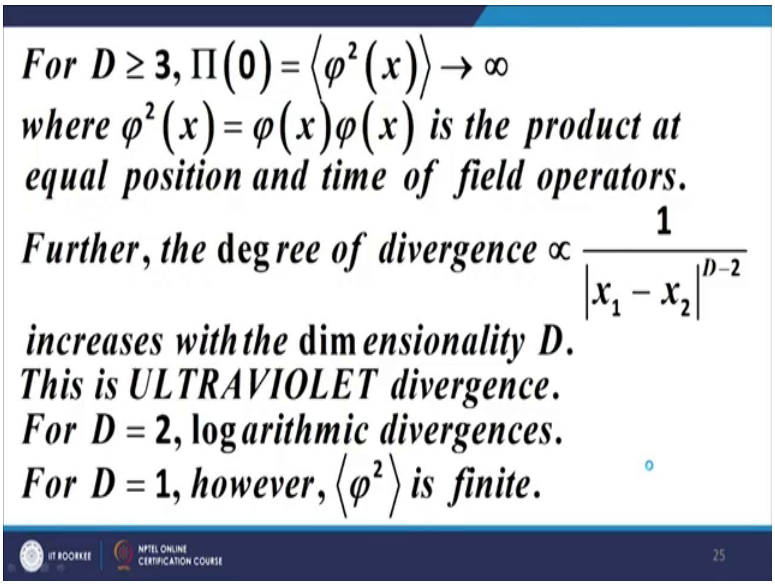

Phi square x is phi x phi x is the product of equal position and time field operators. And this expected value tends to diverge as the two field functions come closer and closer to each other and they become on the equal position and time field operators, then the expected value tends to diverge.

This is ultraviolet divergence. And the degree of divergence tends to increase as the dimensionality increases. For D equal to 2, we encounter infrared divergences which are in the sense of logarithmic divergences as was discussed earlier.

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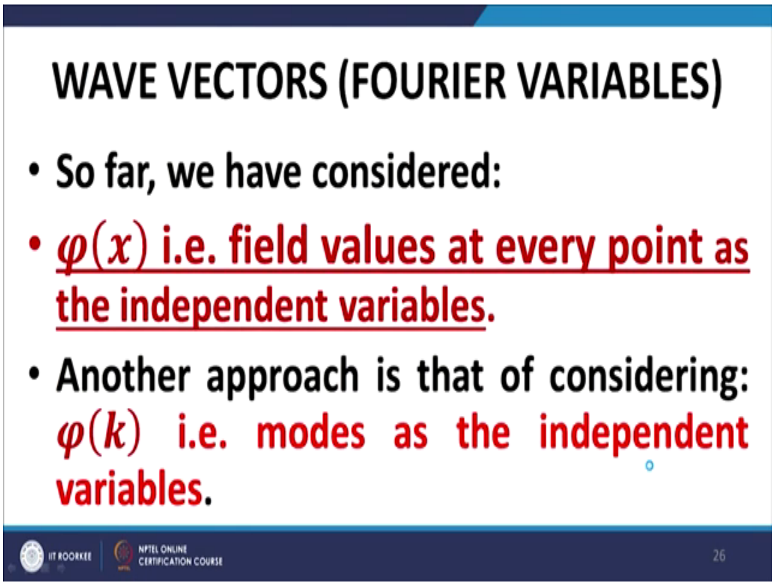

Then we discuss the issue of moving from the position space representation to the wave vector space representation. So, far we had used the field values at every point as the independent variables; now we can we are considering the modes of the fields as the independent variables. This representation has certain advantages.

The most important being the conservation of wave vectors or conservation of momentum as you may say; and moreover in practice the data that is collected the attributes of the system that are actually measured relate to the wave vectors of the momenta rather than positions.

And then we the relationship between the position and the wave vector representation is very well-known; it is straightforward. It is the one is the Fourier transform of the other and therefore, we write phi k is the Fourier transform of phi of x. So, wave vector space is nothing but the Fourier space corresponding to the position space.

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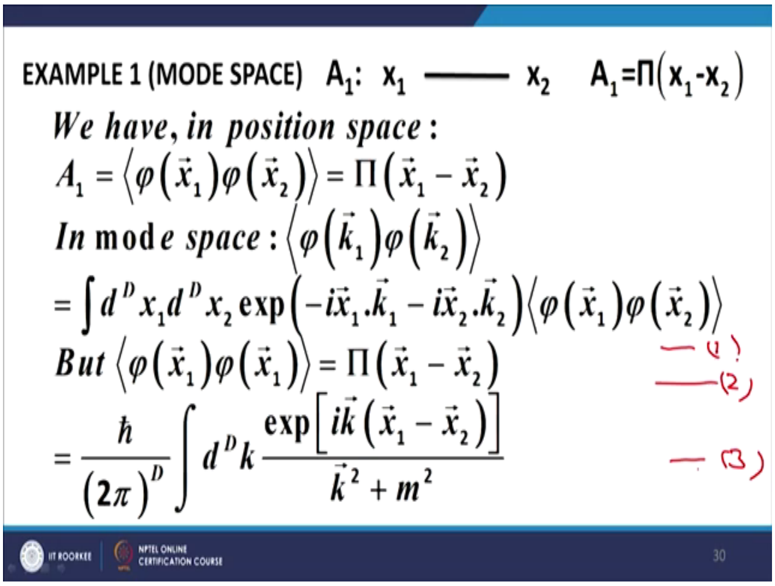

We exemplified computations in the mode space in the context of three examples; one of them is reproduced here. A 1 is the two point function which is represented by the propagator at the points x 1 and x 2. In mode space we have a corresponding function being the expected value of phi k 1 and phi 2 that is a momentum space or in Fourier space.

And when we this expression phi k 1 phi k 2, the expected value thereof works out to a when you convert the variables k 1 and k 2 to the corresponding or when you represent k 1 and k 2 in terms of the corresponding position variables you get the expression which is equation number one here. But phi x 1 phi x 2 as shown above is nothing but the propagator between x 1 and x 2. So, this is equation number 2 here.

And we also know that the propagator between x 1 and x 2 has the explicit expression given in equation number 3. So, we have 3 equations; one equation representing the relationship between phi k 1 phi k 2 expected value and phi x 1 phi x 2 expected value; the second expression representing phi x 1 phi x 2 as the propagator between x 1 and x 2; and the third one representing the expression for the propagator.

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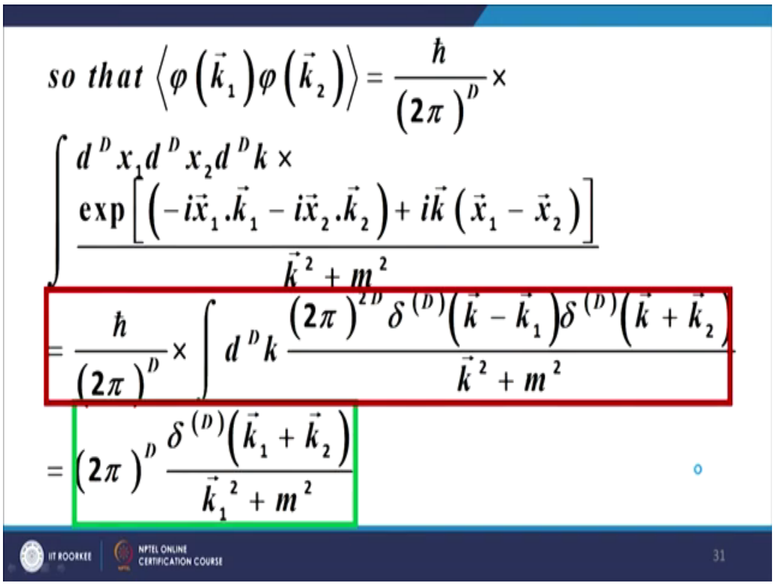

Combining all these three, we can arrive at an expression for phi k 1 phi k 2 expected value in terms of the expression that is given in the green box below. Remember we have a delta function here that delta function is nothing but the conservation of momentum.

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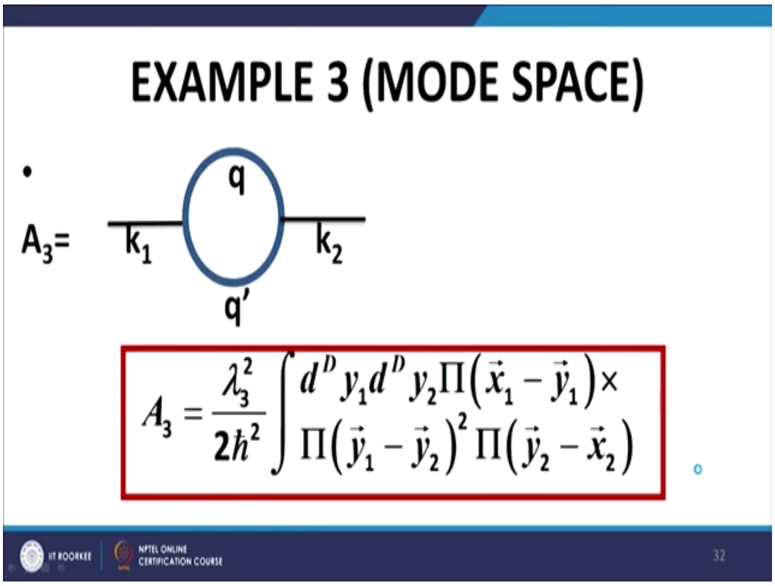

And there is another illustration here which is in the context of a phi 3 theory with two vertices and which as you can see here contains a loop.

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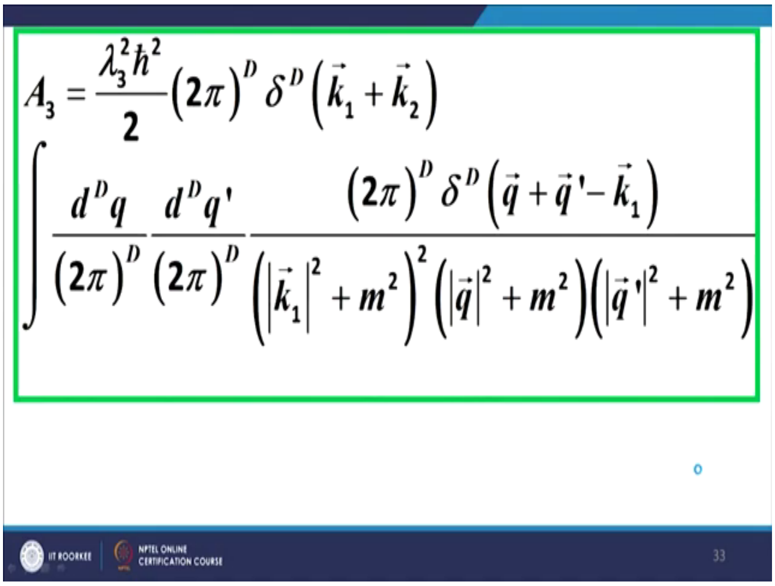

The interesting part as far as the involvement of the loop is that if the expression is solved, you have here two integrals to be done, but we have just the one delta function. You have just one delta function. Now, this one delta function when it is integrated over, it will give me a, it will fix one of the qs, but q still remains a undetermined.

So, in the case of these kind of diagrams which are here on your slide, we have a situation where corresponding to the number of loops, there are a number of internal momenta or internal wave vectors that are not completely determined and that may turn out to be divergent, and that need to be handled in appropriate manner to end up with finite results.

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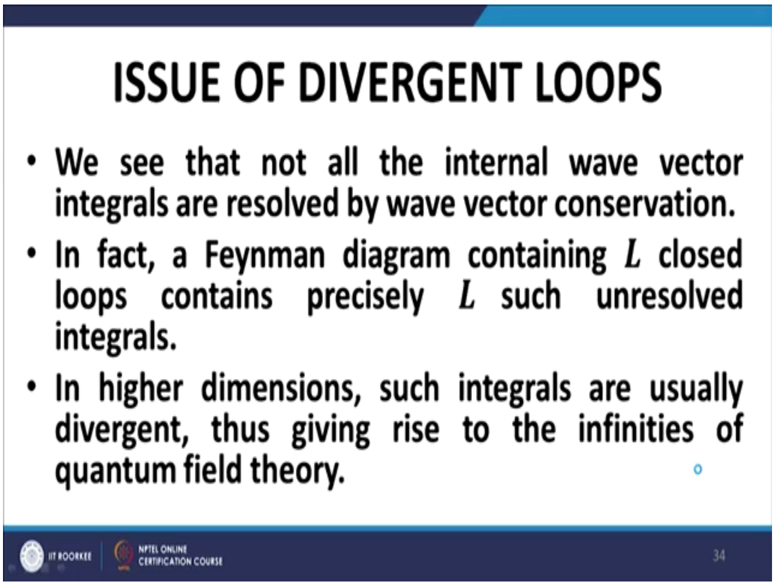

That in fact relates to the issue of renormalization we had discussed in the context of zero dimensional field theory. So, not all the internal wave integrals are resolved by wave vector conservation in diagrams where such loops are present. If there are L loops, L closed loops in a Feynman diagram, then there will be L such unresolved integrals,

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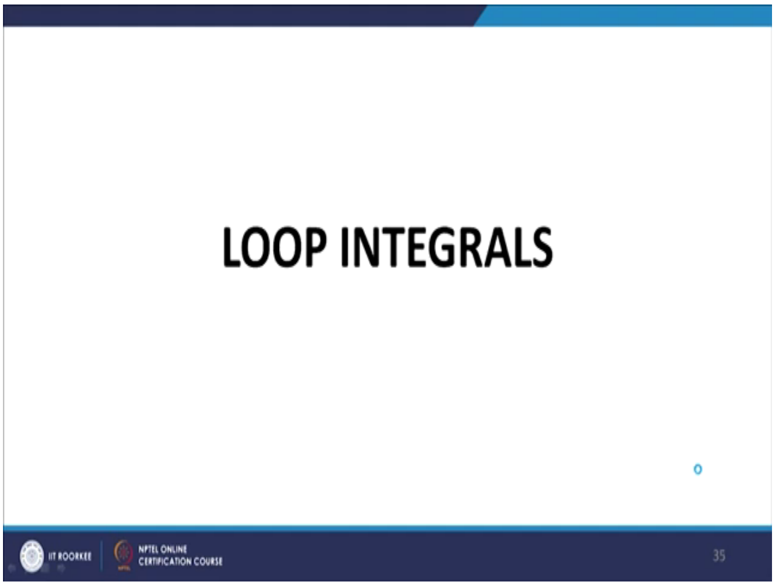

And in higher dimensions these loop integrals tend to be divergent.

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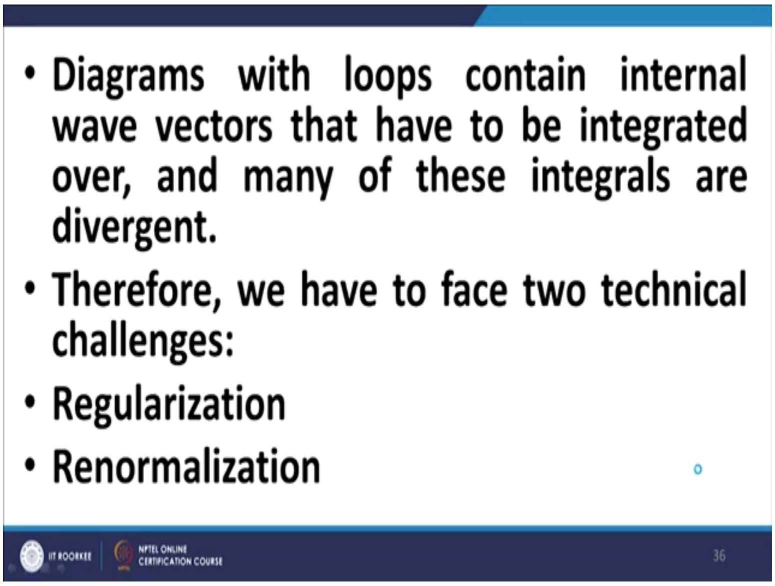

Then we discuss the issue of regularization and renormalization, we discriminated between them.

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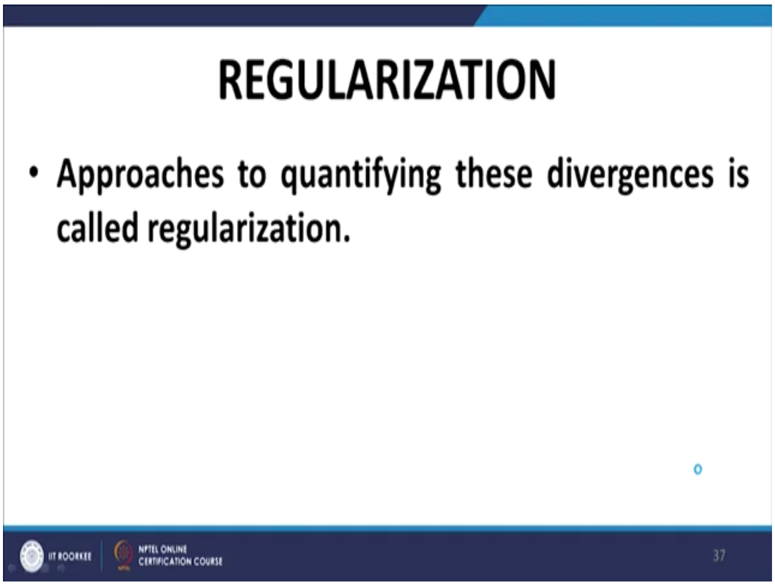

Regularization attempts to quantifying the divergences, so that they can be identified. Renormalization tends to manage these divergences. So, first of all the regularization process I aims to identify these divergences by introducing an external parameter. For example, in the context or dimensional regularization we introduced a dimensional regularization parameter epsilon with which later on goes to 0 of course.

But the point is by using this introducing this parameters, external artificial parameters, we are able to identify these divergences, and therefore identify the existence of situations where renormalization may be required. So, regularization is the quantifying of these dimensions, but it does not make them go away.

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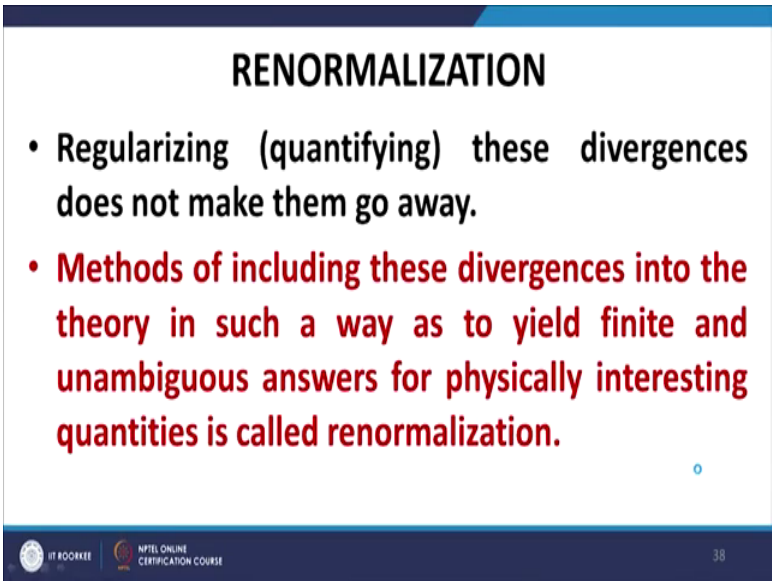

The process of making them go away that is the process of managing these divergences including these divergences in the theory in such a way. So, that we end up with finite results in the context of various n point functions and other quantities of interest in relation to the field theory is called renormalization.

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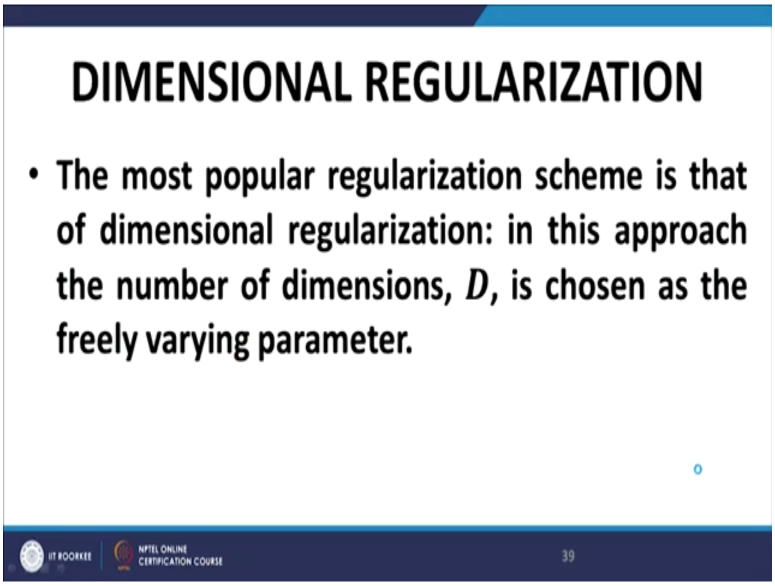

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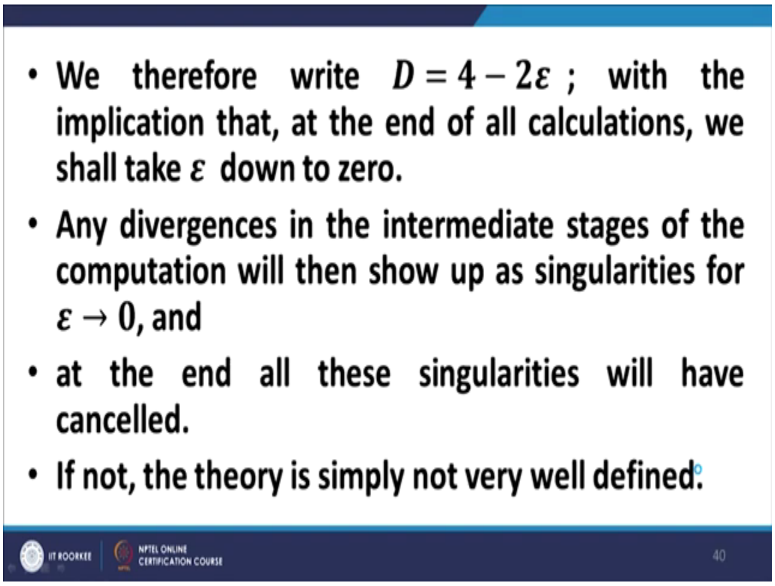

Then we discussed an example of dimensional regularization. We wrote D as 4 minus 2 epsilon with the condition that at the end of all calculation, we shall take epsilon down to 0. And any divergences in the intermediate state will show up as singularities when we take epsilon tends to 0.

But at the end of the day we should be able to cancel those singularities, and emerge with a situation where the final quantities that are of interest to us contain epsilon in such a way that if we take the limit epsilon tending to 0, we still end up with finite results.

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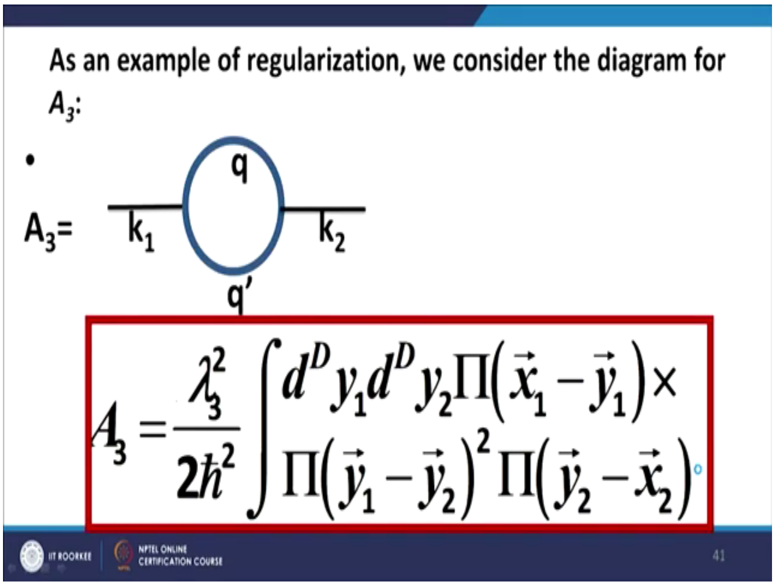

We discuss the same diagram that we had earlier where I mentioned the existence of one loop, and either q dash over q is unresolved has to be integrated over the loop.

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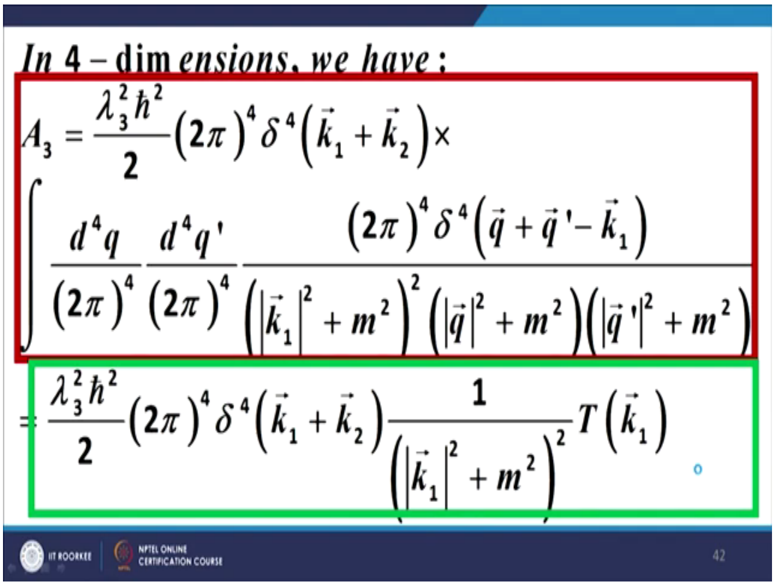

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$$F/A: A_3 = \frac{\lambda_3^2 \hbar^2}{2} (2\pi)^4 \delta^4\left(\vec{k}_1 + \vec{k}_2\right) \frac{1}{\left(\left|\vec{k}_1\right|^2 + m^2\right)^2} T\left(\vec{k}_1\right)$$

$$\textit{where } T\left(\vec{k}\right) = \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 q'}{(2\pi)^4} \frac{(2\pi)^4 \delta^4\left(\vec{q} + \vec{q}\,' - \vec{k}\right)}{\left(|\vec{q}|^2 + m^2\right)\left(|\vec{q}\,'|^2 + m^2\right)}$$

$$= \int \frac{d^4 q}{(2\pi)^4} \frac{1}{\left(|\vec{q}|^2 + m^2\right)\left(\left|\vec{k} - \vec{q}\right|^2 + m^2\right)}$$

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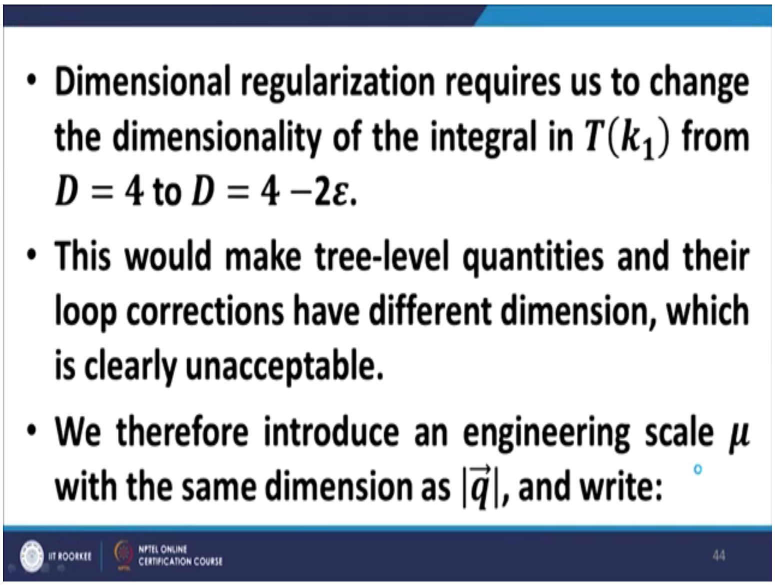

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$$T\left(\vec{k}\right)=\int\frac{d^4q}{\left(2\pi\right)^4}\frac{1}{\left(\left|\vec{q}\right|^2+m^2\right)\left(\left|\vec{k}-\vec{q}\right|^2+m^2\right)}$$

$$\rightarrow T\left(\vec{k}\right)=\mu^{2\varepsilon}\int\frac{d^{4-2\varepsilon}q}{\left(2\pi\right)^{4-2\varepsilon}}\frac{1}{\left(\left|\vec{q}\right|^2+m^2\right)\left(\left|\vec{k}-\vec{q}\right|^2+m^2\right)}$$

We tried to simplify this way. We modified the integral in terms of T k and then we use the Feynman's trick. This is what T k ends up, if I do the; if I do the q dash integral here using this delta function, the one delta function in the blue box, if I do this q dash integral then I get the expression which is here in the green box.

Now, the simplification of this expression which I have now which is now for T k please note this mu 2 epsilon is an is a parameter which is introduced to make the treat quantities compatible with the loop quantities. This is called the engineering dimension. So, we write T k in this form.

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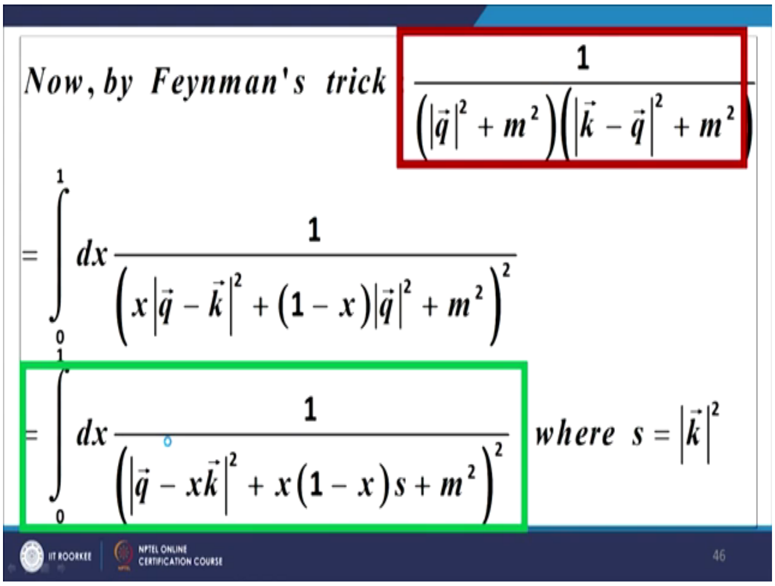

And then we use the Feynman trick. Feynman trick what I will do is I will place it as a part of the presentation, it is slightly extensive may not have the time available to discuss that.

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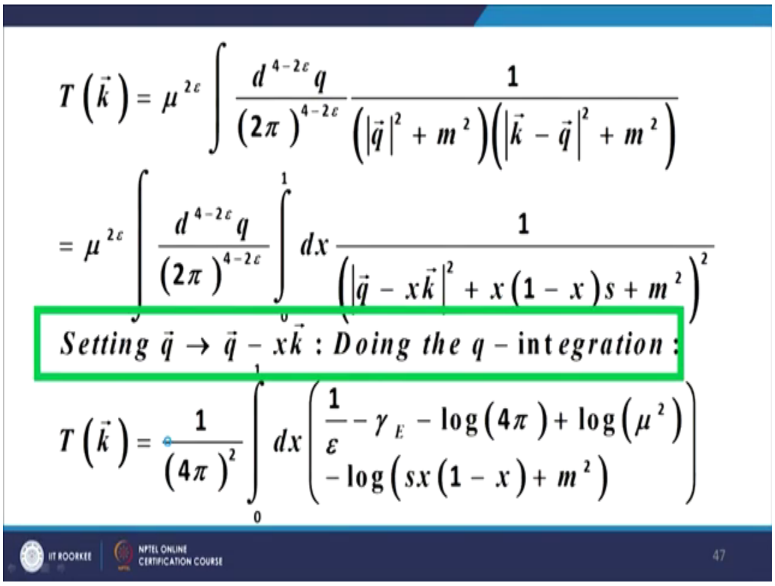

But at the end of the day, it is a mathematical manipulation which enables us to arrive to write T k in the form which is given in this blue box here with F s being given in terms of the green box.

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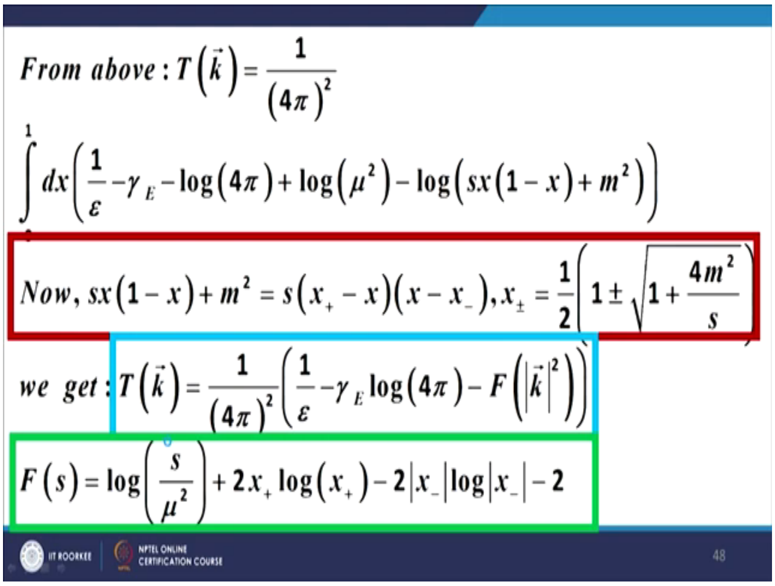

So, through extended mathematical manipulation, we are able to get a manageable expression for the involving the dimensional parameter 1 upon epsilon. And we can thereby manage the divergences that are there in the picture remember gamma is the Euler constant, the rest of it is more or less self explanatory. As I mentioned the details of this computation I shall put in the part of the PPT relating to this lecture.

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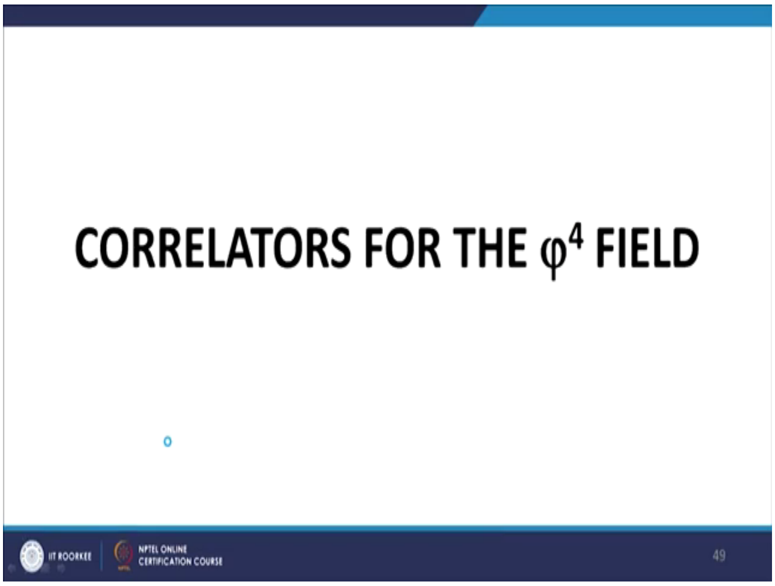

Now, we talk about the correlators of the phi 4 field.

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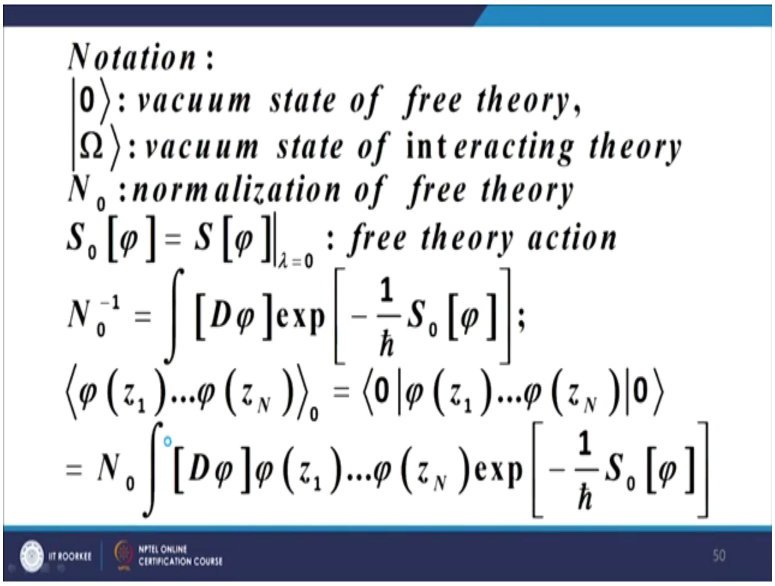

The important thing here is that we have two different states with respect to which expectation values are calculated we have the ground state represented by the vacuum state or the zero state, and we also have the vacuum state of the interacting theory. So, we have two vacuum states one is the vacuum state of the free theory, and the other is the vacuum state of the interacting theory, and normalization could be done with respect to either of the two.

So, N 0 is the normalization with respect to the free vacuum. And N interaction and int is the normalization with respect to the interaction vacuum. So, the similarly the free field action is represented in terms of the lambda equal to 0 action, the interaction parameter is missing.

And the normalization of the free field is given in terms of this expression for N 0 inverse is equal to integral D phi exponential minus 1 by h S 0 of phi. So, this is the normalization of the free field. Similarly, we define the various correlators either in relation to the; in relation to the free field vacuum or in relation to the vacuum of the interacting field.

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$$N_{\text{int}}^{-1} = \int [D\varphi] \exp\left[-\frac{1}{\hbar} S[\varphi] \right]$$

$$N^{-1} = \frac{N_0}{N_{\text{int}}} = \frac{\int [D\varphi] \exp\left[-\frac{1}{\hbar} S[\varphi] \right]}{\int [D\varphi] \exp\left[-\frac{1}{\hbar} S_0[\varphi] \right]}$$

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$\langle \varphi(z_1)\ldots\varphi(z_N)\rangle_0$: $N-$point correlators of free theory in free theory ground state $|0\rangle$

$\langle \varphi(z_1)\ldots\varphi(z_N)\rangle_{\text{int}}$: $N-$point correlators of interacting theory in free theory ground state $|0\rangle$

$\langle \varphi(z_1)\ldots\varphi(z_N)\rangle$: $N-$point correlators of interacting theory in INTERACTING theory ground state $|\Omega\rangle$

(Refer Slide Time: 23:11)

$$Let: \left\langle \varphi(z_1)\ldots\varphi(z_N)\right\rangle_0 = N_0 \int [D\varphi]\,\varphi(z_1)\ldots\varphi(z_N)\exp\left[-\frac{1}{\hbar}S_0[\varphi]\right]$$

$$= \frac{\int [D\varphi]\,\varphi(z_1)\ldots\varphi(z_N)\exp\left[-\frac{1}{\hbar}S_0[\varphi]\right]}{\int [D\varphi]\exp\left[-\frac{1}{\hbar}S_0[\varphi]\right]}$$

So, we need to be a bit careful in terms of which quantity precisely are we talking about. But I shall be explaining them in the this is equal, so this information is this is the expression for the correlators in the free field with respect to the free field vacuum.

Free field correlators, this is the expression that is given on your the slide is the free field correlators. And please note this is the free field action which is appearing in the numerator as well as the denominator. So, this is the free field correlators.

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$$Let: \langle \varphi(z_1)\ldots\varphi(z_N)\rangle_{\text{int}} = N_0 \int [D\varphi]\varphi(z_1)\ldots\varphi(z_N) \exp\left[-\frac{1}{\hbar}S[\varphi]\right]$$

$$= \frac{\int [D\varphi]\varphi(z_1)\ldots\varphi(z_N)\exp\left[-\frac{1}{\hbar}S[\varphi]\right]}{\int [D\varphi]\exp\left[-\frac{1}{\hbar}S_0[\varphi]\right]}$$

The interaction correlators int are the correlators with respect to the field functions are the interacting, now look at the right hand side here, the numerator contains the expression for the interacting action, and the denominator contains the free field action. So, in that sense, these are interacting field points or correlators of the interacting field with respect to the free field vacuum. Correlators of the interacting field with respect to the free field vacuum.

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$$Let : \langle \varphi(z_1)...\varphi(z_N) \rangle = N_{\text{int}} \int [D\varphi]\varphi(z_1)...\varphi(z_N) \exp\left[-\frac{1}{\hbar}S[\varphi]\right]$$

$$= \frac{\int [D\varphi]\varphi(z_1)...\varphi(z_N)\exp\left[-\frac{1}{\hbar}S[\varphi]\right]}{\int [D\varphi]\exp\left[-\frac{1}{\hbar}S[\varphi]\right]}$$

And these are the correlators of the interacting field in the in relation to the interacting vacuum as you can see from the expressions on the right hand side in each case.

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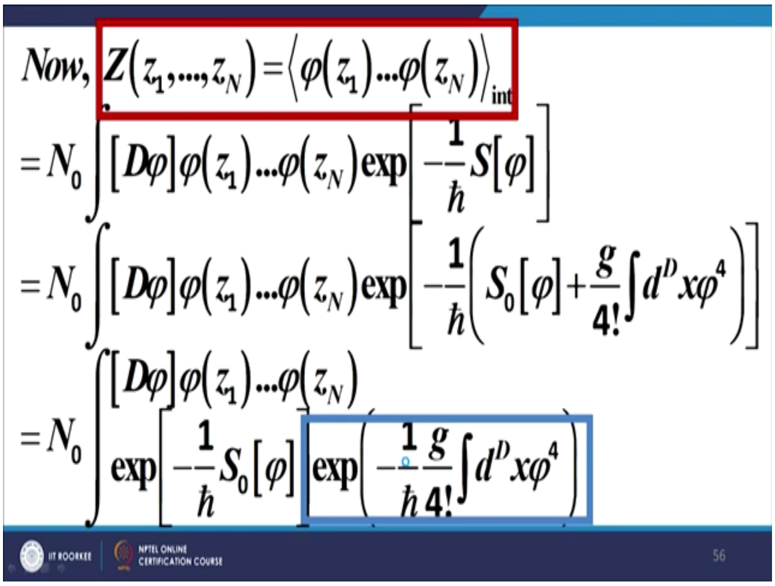

So, now we elaborate on the expressions for each of them. Let us look at the correlators of the interacting field in the context of free field vacuum. We write them in the form which is given in the which is given in the blue box here.

And you find here if you in a process which is simply parallel to what we are done earlier, we write it in the as the interacting action. We expand the interaction term g upon 4 lambda integral d x phi 4 as an exponential series that is this expression is taken separately as the exponential. And then here you see it is expanded as an exponential series given in the red box here.

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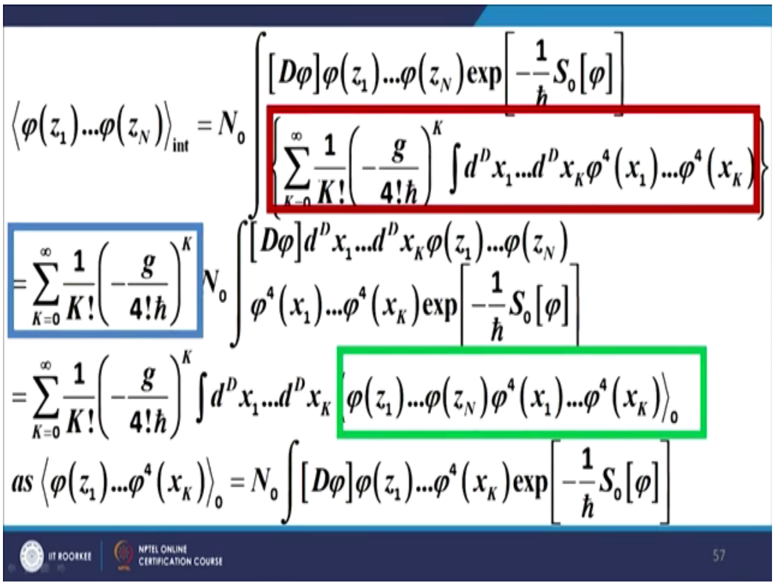

So, this again as in earlier cases, we flip the integral and the summation, and this summation goes outside the integral. Inside the integral what we are left with is the expression that is given in the green box here. And which if you look carefully is nothing but the free field correlators, free field correlators with respect to the free field vacuum that is what is given that is the expression in the green box.

In other words, what we have done is we have been able to express the interacting field correlators interacting field correlator as a power series in the coupling constant with their respective free field correlators. I repeat the interacting field correlators with respect to the free field vacuum have been now been expressed as a power series in the coupling constant with the free field correlators.

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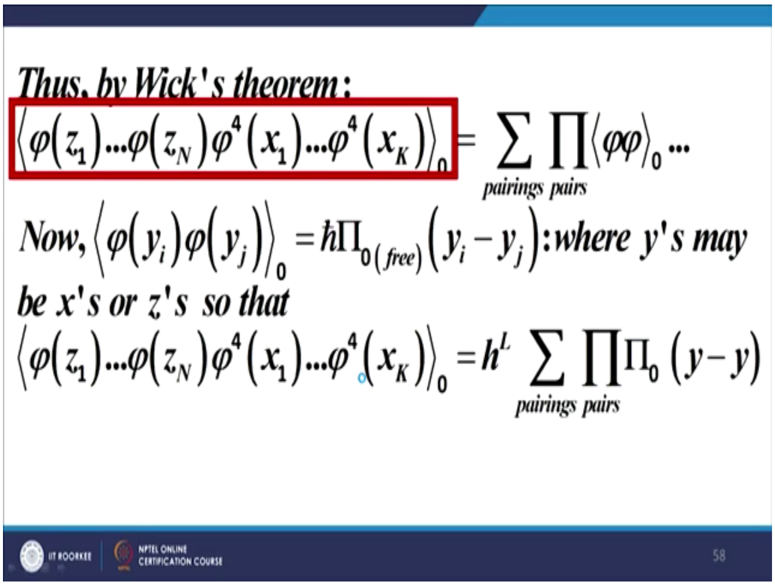

The free field correlators, of course, can be worked out in the usual manner using the Wick's theorem by summing over the all possible pairings.

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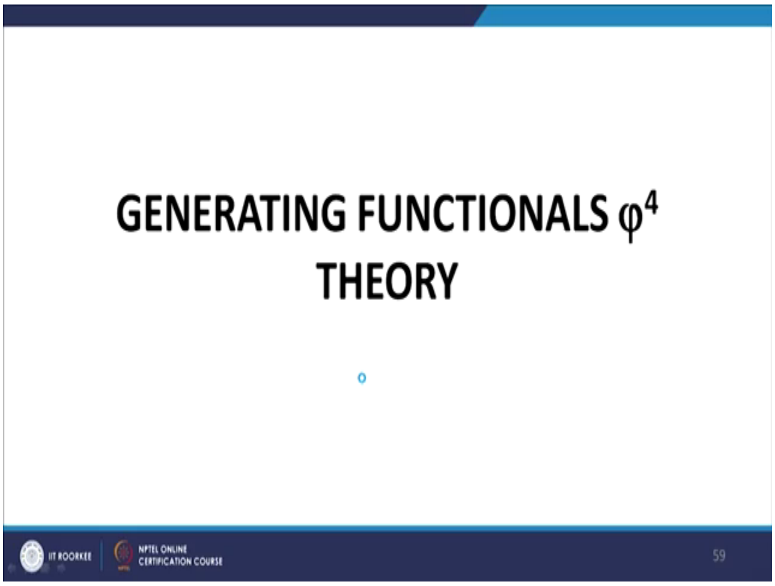

Now, the generating functional for the phi 4 theory.

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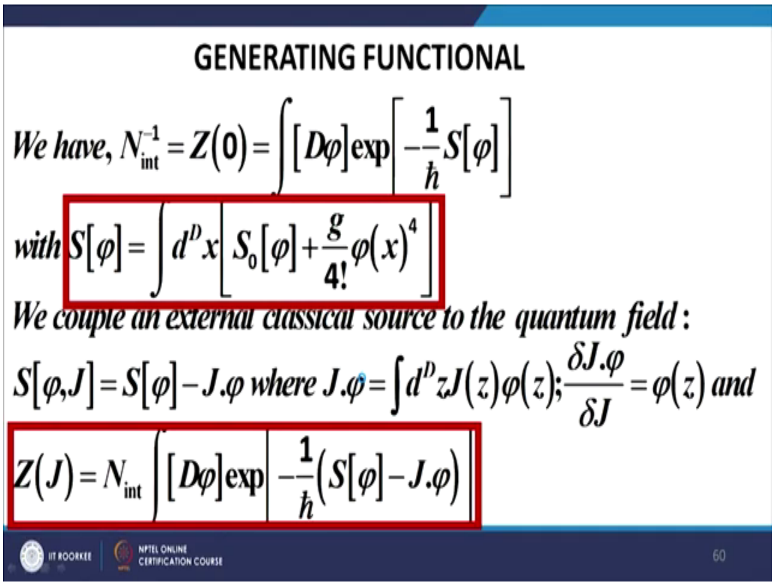

The generating for the function for the phi 4 theory is given by the expression the first expression at the top of the box Z 0 is equal to this. This is the normalization; I am sorry. This is the normalization the interaction field normalization.

The generating function is given at the bottom red box you see the source term being added here and J phi is added as the source term and this gives us the generating functional for the various green functions. And please note these Green functions that we are going to get are the interacting field Green functions with respect to the interacting vacuum.

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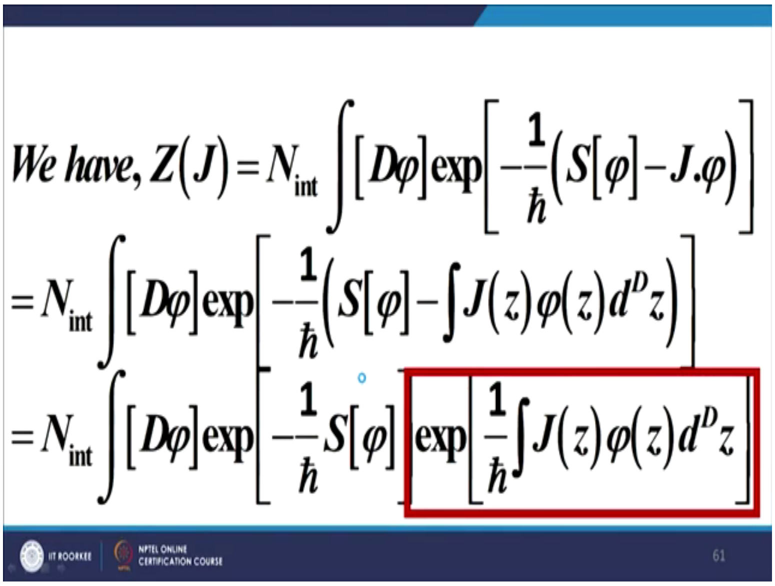

How we get them? Well, what we simply do is very similar to what we are just now we expand the source term integral J z phi z and d D of Z. We expand this term in the exponential as an exponential series as a power series in J.

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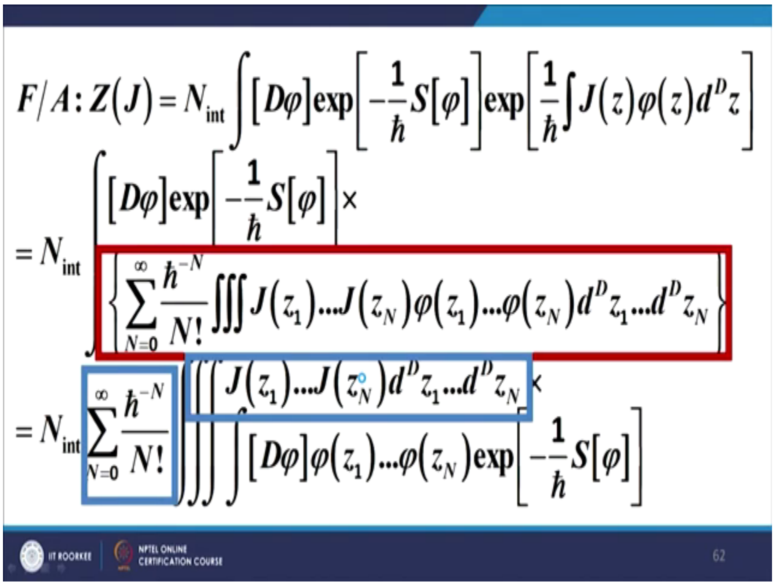

And what we get is the expression that is given in the red box here. On expanding this exponential which is the second exponential 1 upon h integral J z phi z d z we get this expression which is the which is in the red box. Again what we do is we flip the integral and the summation, the summation goes outside the integral and we get this expression inside the integral.

And if you look at it carefully, the expression that we have right at the bottom integral path integral of phi z 1 to phi z N exponential minus this is together with the normalization and interaction this whole expression is nothing but the expected value or the interaction point functions N point functions with respect to the interaction vacuum.

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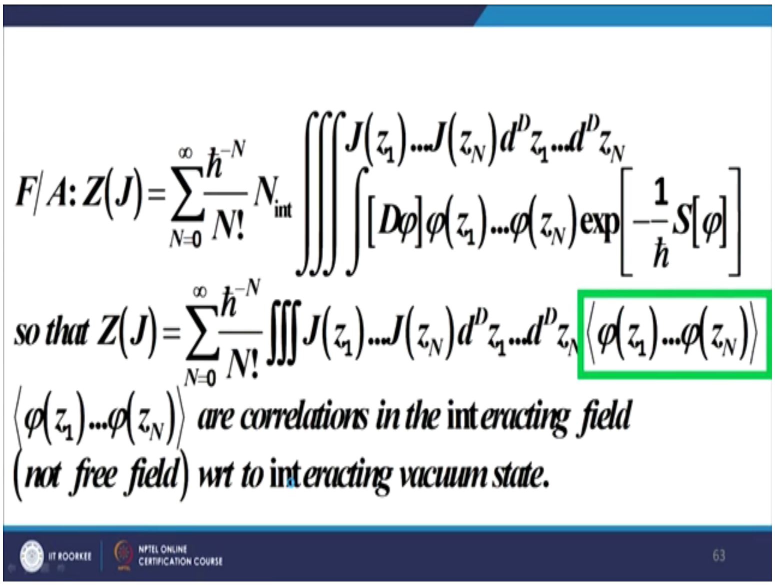

So, we write them as the interaction point functions with respect to the interaction vacuum. And we can recover the x y values of this interaction N point functions in the usual manner by functionally differentiating Z J with respect to J, and then substituting J equal to 0. So, that is precisely what is there on the slide.

So, in the earlier case, we had a relationship between the interaction point functions with respect to the free field vacuum being expressed as a power series in terms of the free field correlators. Here you are having an generating function which is generating the interaction field vacuum in terms interaction field N point functions with respect to the interaction vacuum.

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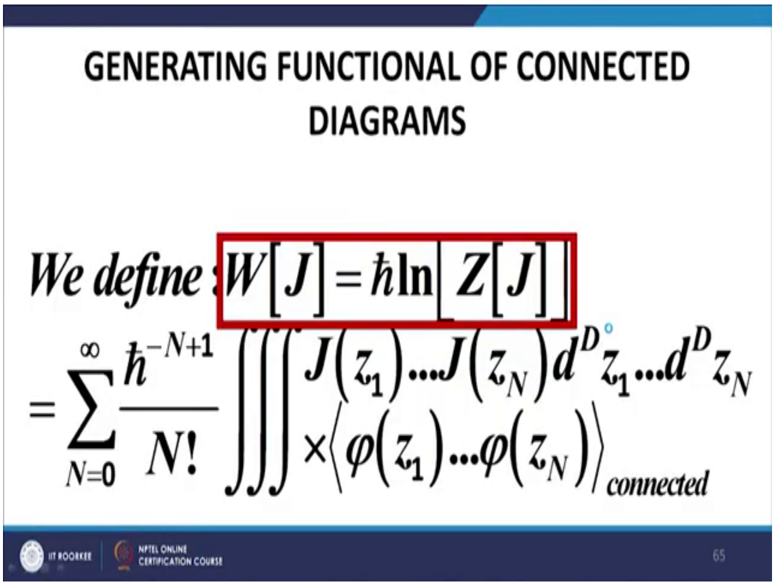

The connected, connected diagrams are given in the usual manner the with the generating function for the connected diagrams being given by the expression in the red box.

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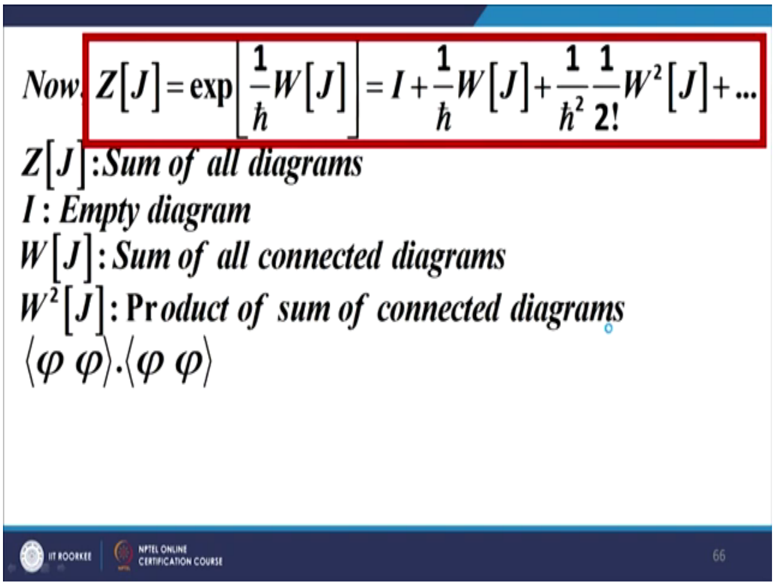

And Z J is equal to exponential of 1 upon W J which can be expanded as an exponential series. And each term having the usual meaning Z J is the sum of all diagrams 1 is the identity operator of the empty diagram, W J is the sum of all connected diagrams, W square J is the product of the sum of connected diagram pairs of connected diagrams and so on.

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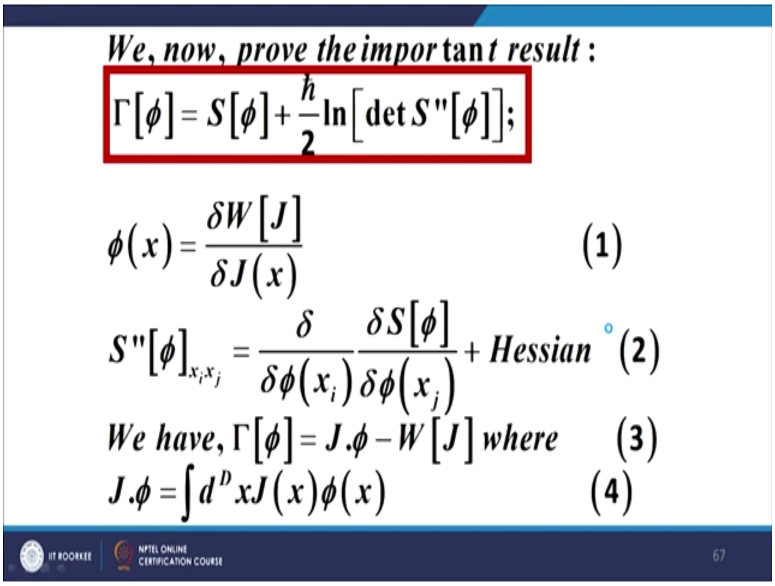

So, from here we will continue after the break.

Thank you.