Path Integral Methods in Physics & Finance
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Lecture - 42
Euclidian Field Theory [2]

So welcome back, we have just discussed the four point function. The lowest order contribution of the first four point function for the fourth diagram in the slide and we have arrived at this expression.

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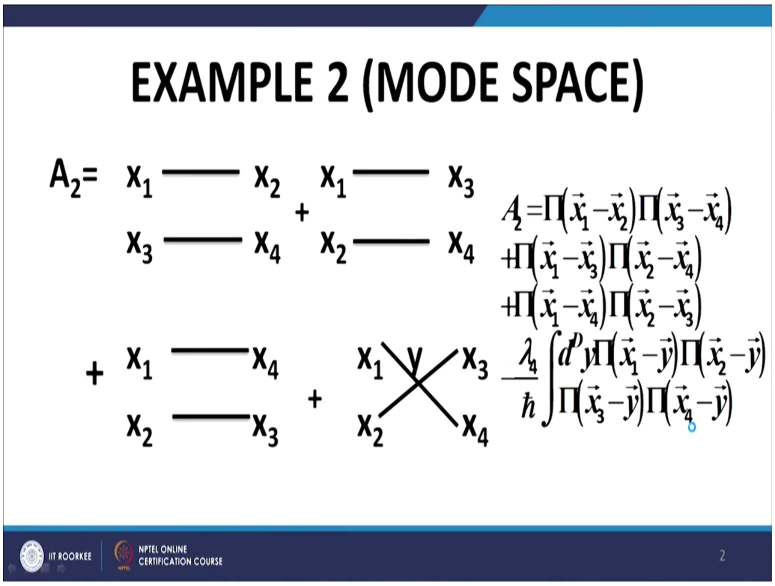

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$$= -\frac{\lambda_4 \hbar^3 (2\pi)^D \delta^D\left(\vec{k}_1 + \ldots + \vec{k}_4\right)}{\left(\left|\vec{k}_1\right|^2 + m^2\right)\ldots\left(\left|\vec{k}_4\right|^2 + m^2\right)}$$

The evaluation of the other diagrams is quite straightforward they represent product of propagators. So, we end up with the final expression for the lowest order Green function four point Green function as the expression that is given on this slide.

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The Green's function is:

$$A_2 = -\frac{\lambda_4 \hbar^3 (2\pi)^D \delta^D(\vec{k}_1 + \ldots + \vec{k}_4)}{\left(|\vec{k}_1|^2 + m^2\right)\ldots\left(|\vec{k}_4|^2 + m^2\right)}$$
$$+\left(\frac{(2\pi)^D \delta^D(\vec{k}_1 + \vec{k}_2)}{|\vec{k}_1|^2 + m^2}\right)\left(\frac{(2\pi)^D \delta^D(\vec{k}_3 + \vec{k}_4)}{|\vec{k}_3|^2 + m^2}\right)$$
$$+\left(\frac{(2\pi)^D \delta^D(\vec{k}_1 + \vec{k}_3)}{|\vec{k}_1|^2 + m^2}\right)\left(\frac{(2\pi)^D \delta^D(\vec{k}_2 + \vec{k}_4)}{|\vec{k}_2|^2 + m^2}\right)$$
$$+\left(\frac{(2\pi)^D \delta^D(\vec{k}_1 + \vec{k}_4)}{|\vec{k}_1|^2 + m^2}\right)\left(\frac{(2\pi)^D \delta^D(\vec{k}_2 + \vec{k}_3)}{|\vec{k}_2|^2 + m^2}\right)$$

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This is the total expression that we get for the Green functions for the four point four point function of the lowest order at the lowest order level.

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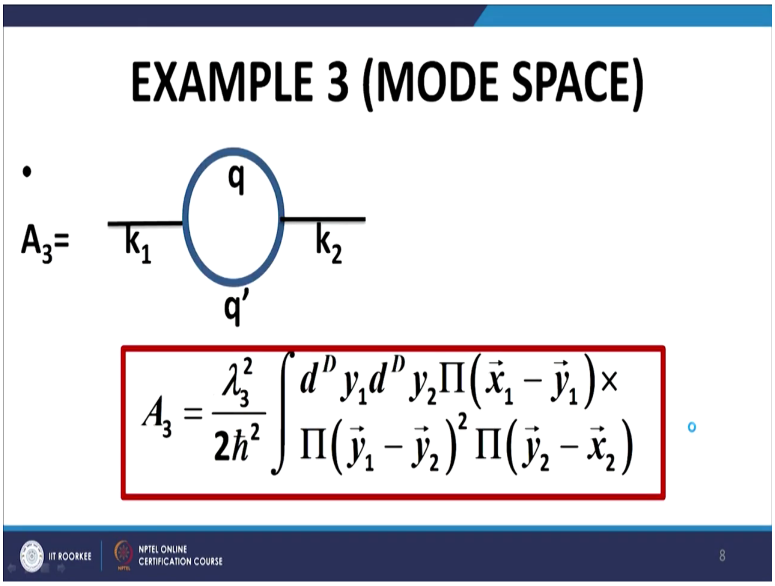

Now let us look at another example, this is in the case of a phi 3 theory. We have discussed this earlier we now talk about it in Fourier space in wave vector space. In the position space the expression worked out as we saw earlier to the expression that is given in the red box at the bottom of your slide.

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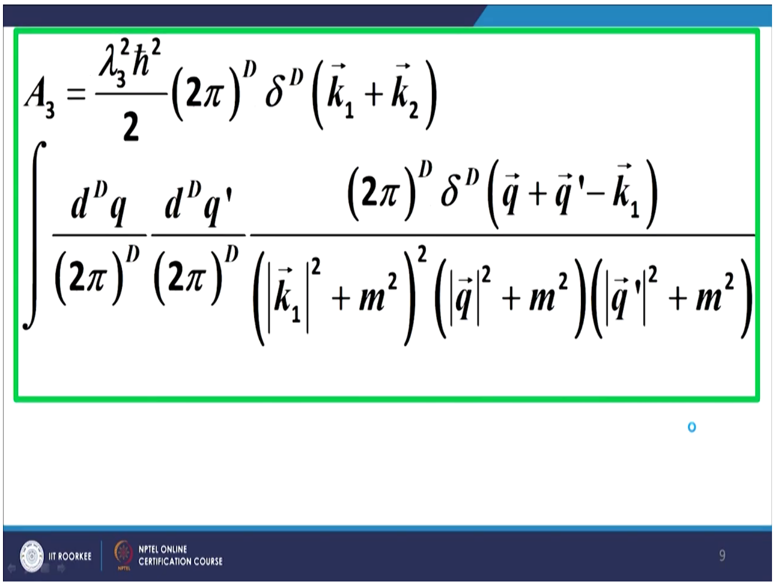

And we again adopt the same metrology, we transform the variables to their Fourier transforms to the wave vectors and then we use explicit expressions for the propagators and we arrive at the expression that is given in the green box on your slide.

Now the interesting issue is to we have to look at it carefully, we have got two integrals integrations here over q and q dash, but and we have just the one delta function. So, when we do let us say we do the q dash integration first, when we do the q dash integration first it enables us to fix q dash with respect to q and k 1.

However the variable q which is an internal momentum a remains unfixed or remains free in a sense. And therefore we need we are left with the task of integrating over q of the entire expression with no delta functions to define or to fix the value of q and q is an internal variable

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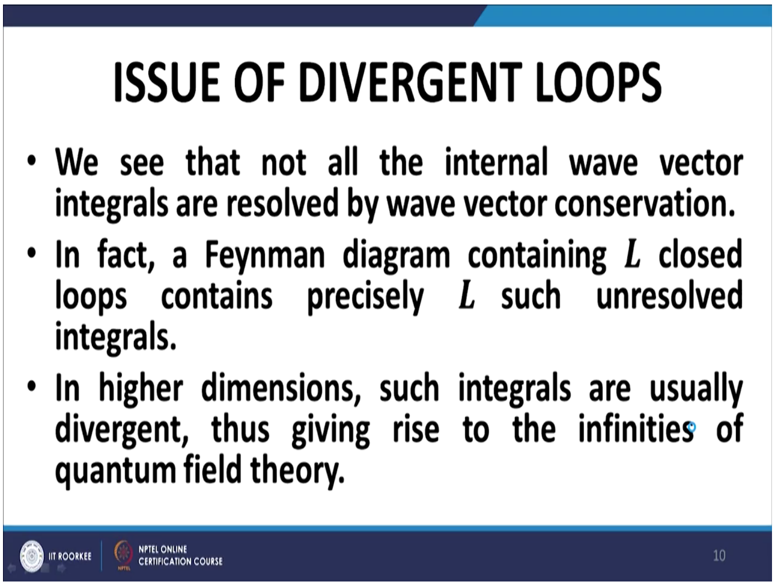

So, this is where the issue of Divergences come into play note and in fact you will observe here that if a Feynman diagram contains L closed loops. In this particular diagram you recall that we have just the one closed loop and if and we have just one unfixed or one free internal momentum.

So, if you have L our diagram with L closed loops, then we have L such un unresolved integrals or unresolved momenta. And in higher dimensions this particularly when we talk about Minkowski space and these integrals tend to be tend to diverge and we encounter the infinities of quantum field theory.

So, the quantum field theory the infinities or the issue of renormalization and infinities and all that that paraphernalia is related to the existence of closed loops in the Feynman diagrams.

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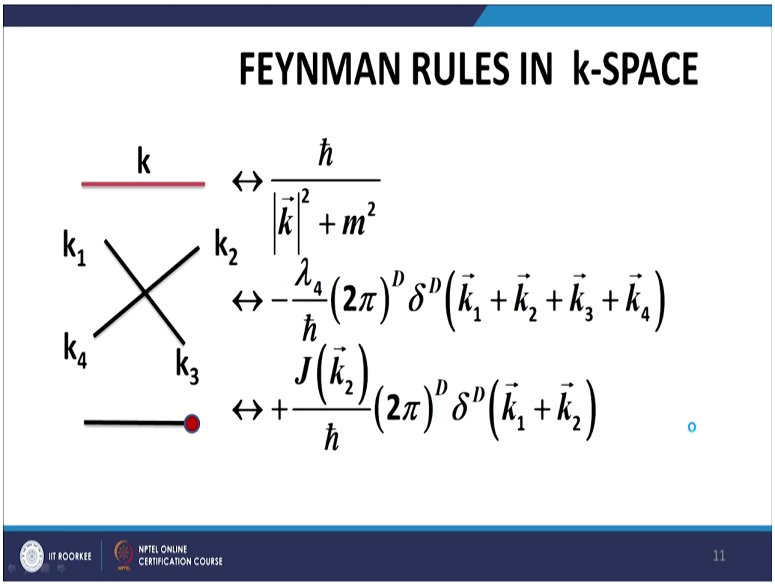

These are the Feynman rules in k space in wave vector space they are quite straightforward and really do not need much explanation.

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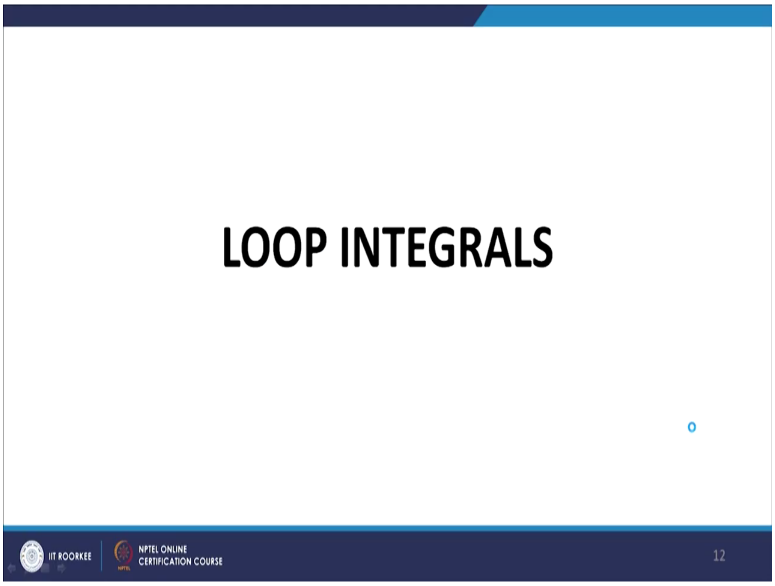

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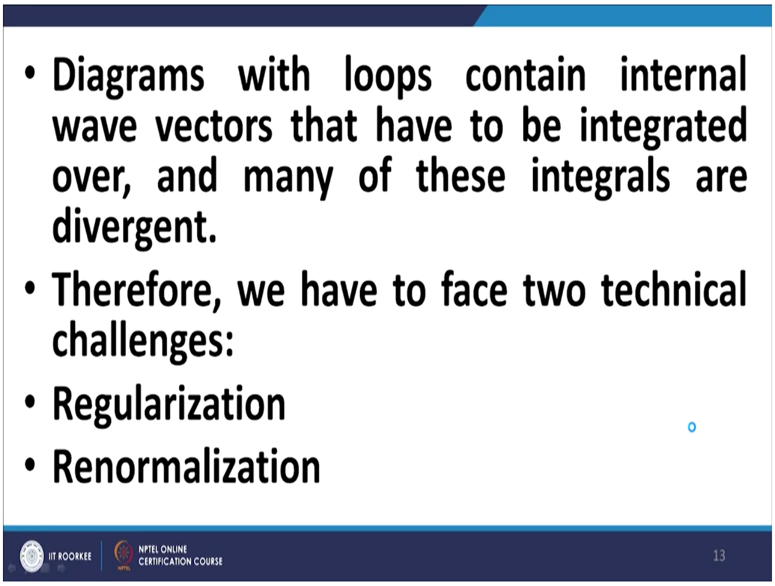

Now we come to that case of loop integrals, we carry forward from where we were. As you saw in the earlier example where we had one loop we had one integral which was unresolved or unencumbered in a sense and they we now take up this issue of how to manage such integrals.

And these loops as mentioned earlier, the loops in contained internal wave vectors internal momenta that have to be integrated over and in many cases the integration is divergent. So, now there are two issues in relation to this loop integrals one is Regularization and the second is Renormalization.

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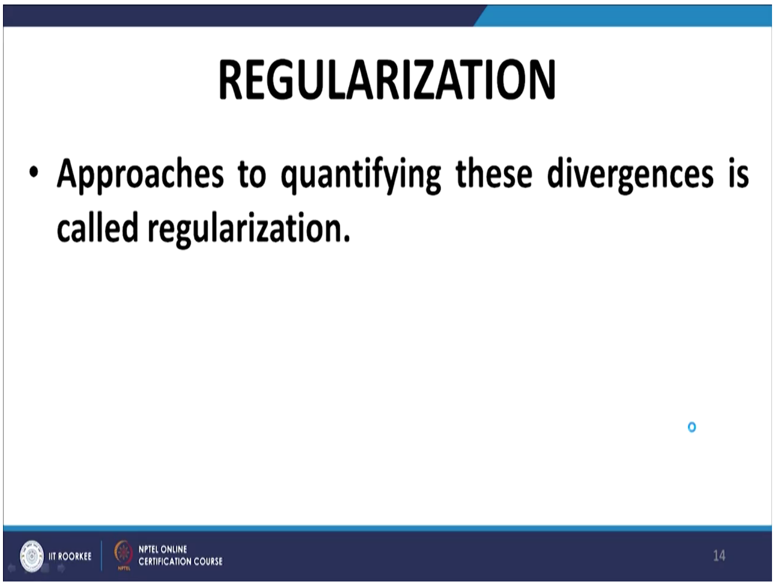

The Regularization attempts to quantifying these divergences to locate to identify these irregularities or these divergences. So, it introduces the parameter, such that the if the parameter takes a particular value. Then we identify the existence of divergences in the theory.

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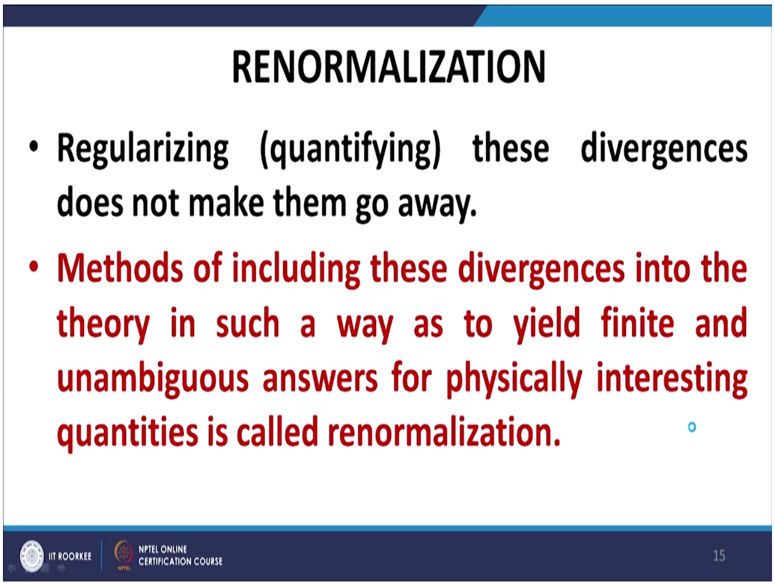

And then this however you see regularization does a enable us to identify the existence of irregularities or unresolved expressions integrals. But it does not eliminate that it simply identifies them. The second step is to develop approaches to eliminate or to manage these unresolved integrals in such a way that we have a finite and a constructive quantum field theory at the end of the day and that second process is called renormalization.

So, regularization relates to quantifying identifying the divergences and quantifying them and renormalization, then tries to attempt to resolve them in such a way to arrive at a as a reasonable and a finite quantum field theory.

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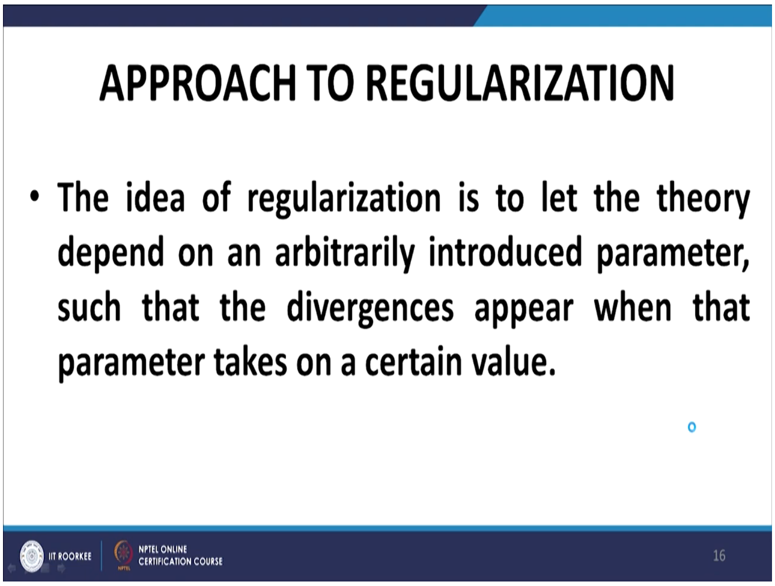

The approach to regularization as I mentioned earlier usually involves the introduction of an arbitrary parameter, such that the divergences may be deemed to exist or may appear when the arbitrary parameter takes a certain value. In other words that new that additional parameter is introduced in order to identify and quantify the divergences, it is taking of a its link to the divergences and if you take certain values the divergences would be assumed to exist.

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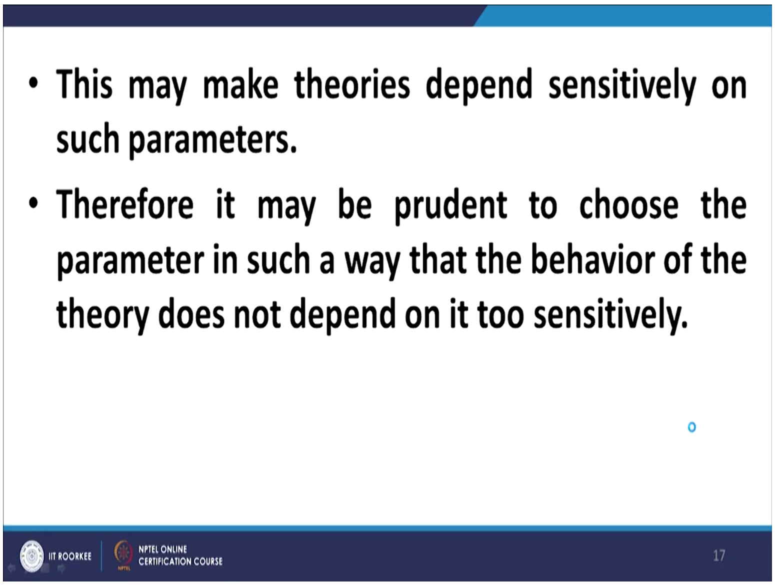

However we need to be careful in the context that the theories become, obviously the theory would become sensitive to or would be dependent on the values of that parameter and the this para and the sensitivity should not be so much as to affect the intrinsic behavior of the theory. That is most important the introduction of the parameter should not distort the intrinsic behavior of the theory.

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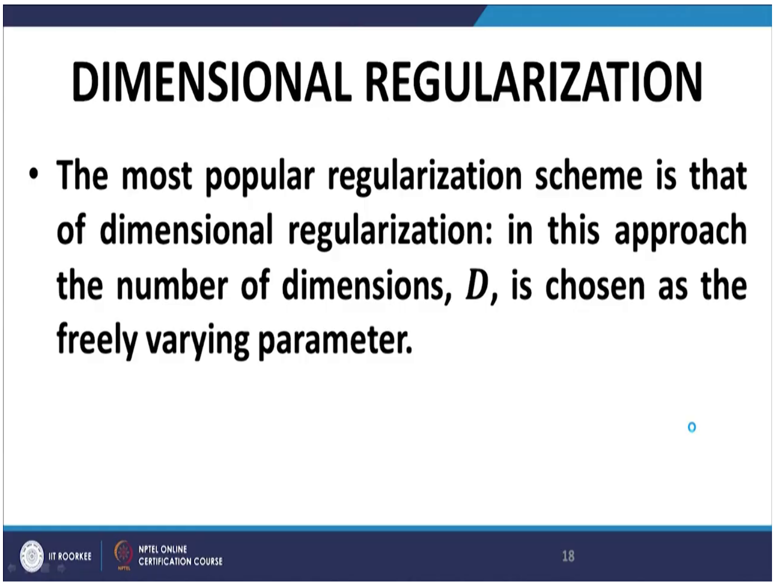

So, one of the most common regularization schemes is the Dimensional Regularization. In this case the number of dimensions is chosen as the varying parameter of freely varying parameter.

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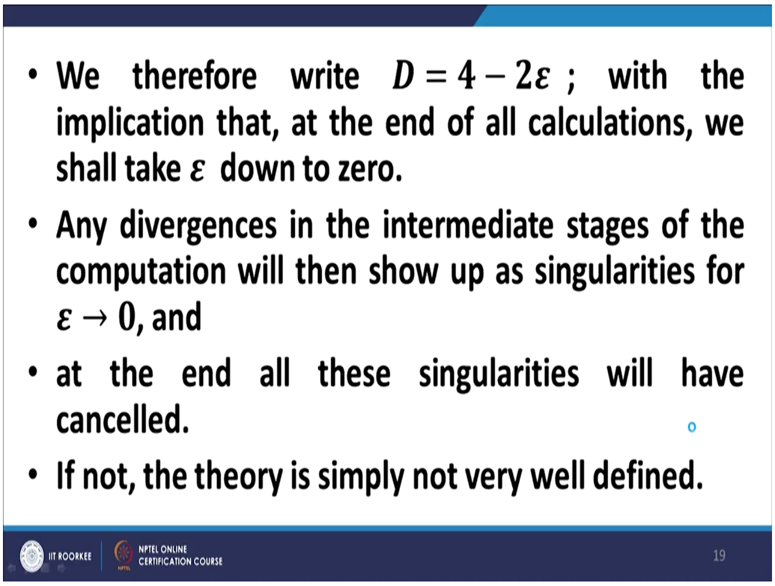

We write D is equal to 4 and that is the conventional space time dimension 4 minus 2 epsilon and at the end of the end of all calculations we shall take the limit that epsilon tends to 0. So, if there are any divergences in the intermediate stages and they will show up as singularities, when epsilon if we set the limit epsilon tend to 0. Then these divergences at intermediate stages will show up as divergences on taking the limit epsilon tending to 0.

But we do desire our theory should be such that at the end of the day all these divergences cancel out and we are left with a finite theory a well defined quantum field theory.

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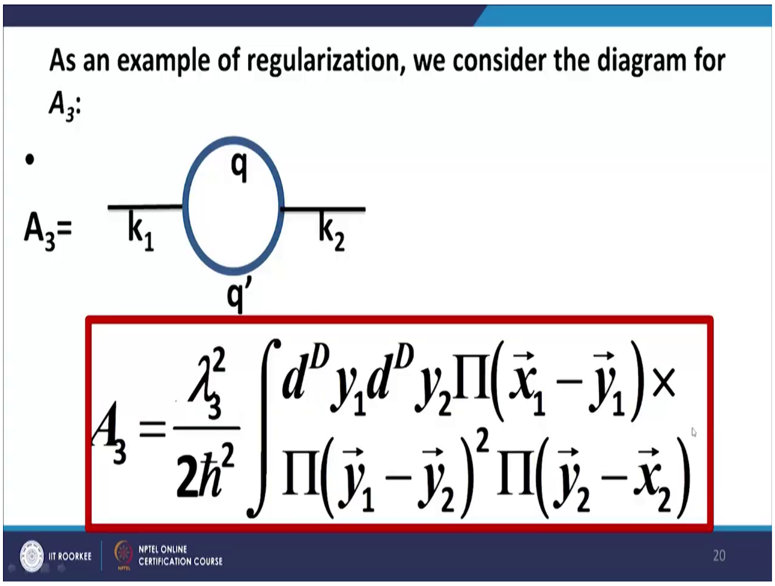

As an example of regularization we go back to the diagram that we discussed just now, the value of this diagram in a position space is given by this expression in the red box.

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$$A_3 = \frac{\lambda_3^2 \hbar^2}{2} (2\pi)^D \delta^D \left(\vec{k}_1 + \vec{k}_2\right) \int \frac{d^D q}{(2\pi)^D} \frac{d^D q'}{(2\pi)^D} \frac{(2\pi)^D \delta^D \left(\vec{q} + \vec{q}' - \vec{k}_1\right)}{\left(\left|\vec{k}_1\right|^2 + m^2\right)^2 \left(\left|\vec{q}\right|^2 + m^2\right)\left(\left|\vec{q}'\right|^2 + m^2\right)}$$

And when we transform this to the wave vector space k space, it takes the form expression given in the green box.

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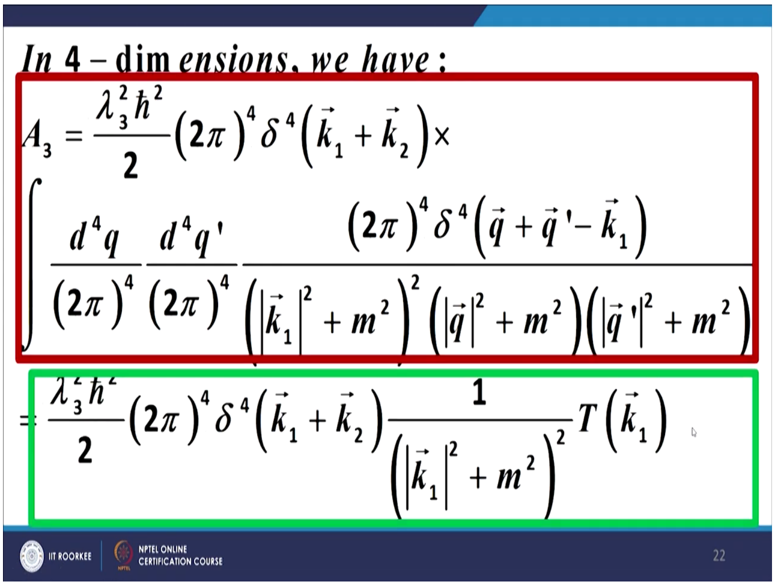

Now, what we do is we split this integral this extensive expression into two parts. We write it as a the term that includes that exclusive two integrals, that is independent of q and q 1 that is the free factor and the expression 1 upon mod k 1 square plus m square whole square this expression is taken on one side and the rest of it is labeled as T k 1, which depends on k 1. But which is taken as a separate term and the expression in the rest of the term is taken as a free factor to T k 1.

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$$F/A: A_3 = \frac{\lambda_3^2 \hbar^2}{2} (2\pi)^4 \delta^4\left(\vec{k}_1 + \vec{k}_2\right) \frac{1}{\left(\left|\vec{k}_1\right|^2 + m^2\right)^2} T\left(\vec{k}_1\right)$$

$$\text{where } T\left(\vec{k}\right) = \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 q'}{(2\pi)^4} \frac{(2\pi)^4 \delta^4\left(\vec{q} + \vec{q}\,' - \vec{k}\right)}{\left(\left|\vec{q}\right|^2 + m^2\right)\left(\left|\vec{q}\,'\right|^2 + m^2\right)}$$

$$= \int \frac{d^4 q}{(2\pi)^4} \frac{1}{\left(\left|\vec{q}\right|^2 + m^2\right)\left(\left|\vec{k} - \vec{q}\right|^2 + m^2\right)}$$

We now try to try to manipulate T k 1 to get a reasonable result. So, T k 1 is given by this expression, it has obviously the divergence embedded in it, because it involves one delta function, but it has two integrals and d q and d q dash.

If I integrate over q q dash I get eliminate one delta function, fix the value of q dash, but the value of q remains still free does not have another delta function to fix up the value of q and that is where the divergence arises. And this when the q dash integral is done we get the expression in the green box, this is after the q dash integral is completed and we get T k equal to this expression.

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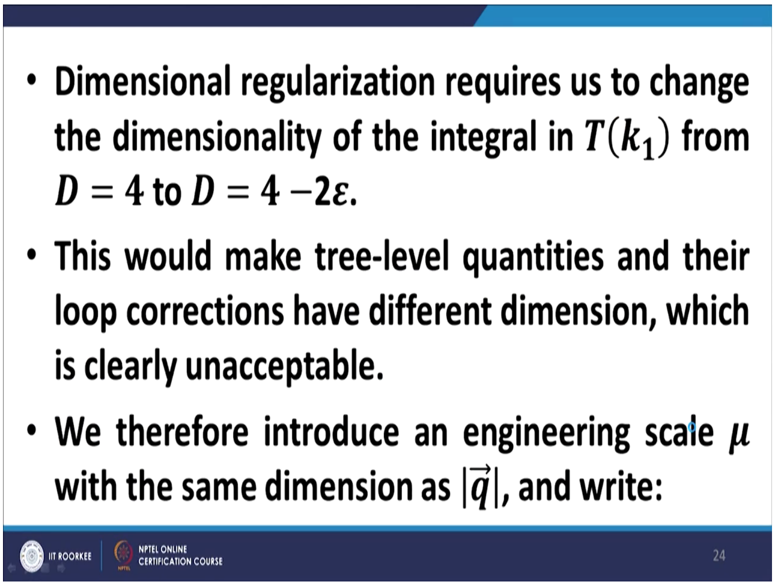

Now, what we do is we change the dimensionality of the integral in T k 1 from D equal to 4 to D equal to 4 minus 2 epsilon and we because this would introduce inconsistency. In the sense that the tree level quantities and loop corrections would have different dimensions, we also introduce a an engineering dimension with mu which has the same dimension as mod of q.

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$$T\left(\vec{k}\right)=\int\frac{d^4q}{(2\pi)^4}\frac{1}{\left(\left|\vec{q}\right|^2+m^2\right)\left(\left|\vec{k}-\vec{q}\right|^2+m^2\right)}$$

$$\rightarrow T\left(\vec{k}\right)=\mu^{2\varepsilon}\int\frac{d^{4-2\varepsilon}q}{(2\pi)^{4-2\varepsilon}}\frac{1}{\left(\left|\vec{q}\right|^2+m^2\right)\left(\left|\vec{k}-\vec{q}\right|^2+m^2\right)} \quad -(3)$$

And we write the integral T k by adding this engineering dimension factor mu 2 epsilon and the rest of it we transform the dimension from 4 to 4 minus 2 epsilon.

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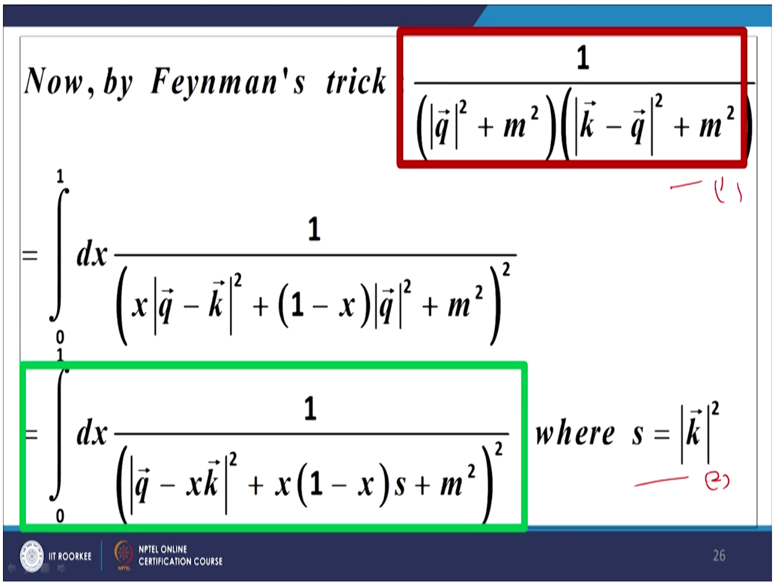

So, we get the expression that is in the bottom expression of your slide here. By applying Feynman's trick in this integral, the integral that appears here the at the bottom expression on your slide can be simplified. It can be simplified to the expression that is given in the green box at the bottom of your slide.

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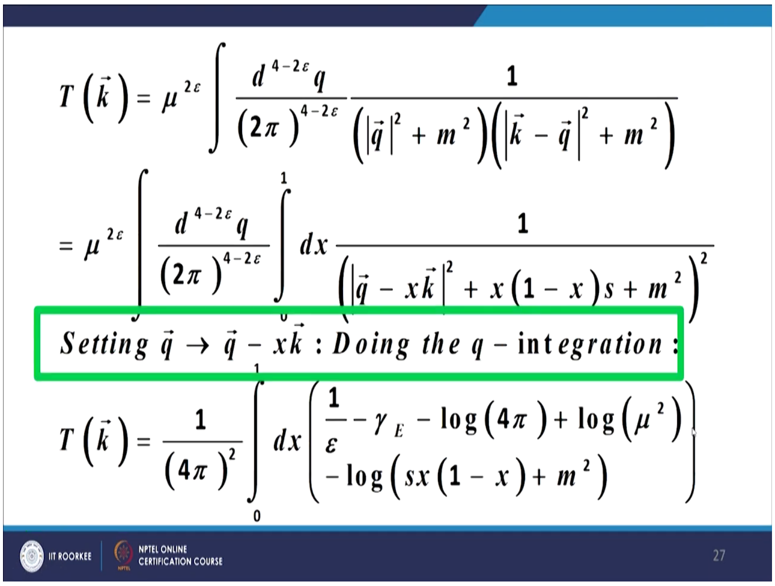

Now, once we have this integral we can perform, the now we substitute this value of this expression the expression in the red box which is now; which is now equal to the expression in the green box, let us call it 1 let us call this 2.

We substitute expression 2 in this expression and this is expression number 3 I am sorry this is expression number 3, we substitute from equation 2 in equation 3 and when we substitute from equation 2 in equation 3 we get the expression we get a q integral and an x integral. We form we first perform the q integral, when we perform the q integral we are left with the x integral and we get the expression that is in the round brackets here at the bottom equation of your slide.

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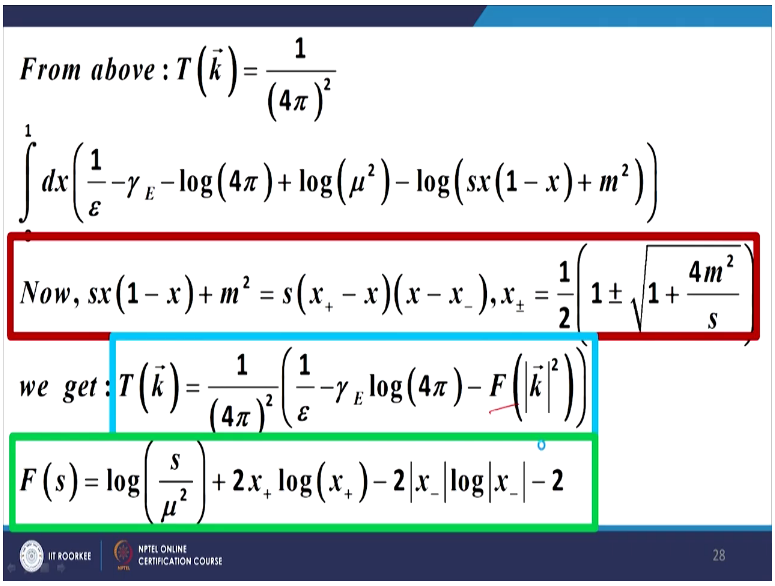

So, this is T k we still have not arrived at the original expression this is T k. So, this T k is this is brought forward from the previous slide T k is equal to this whole expression. Now it can be simplified further by writing in the form given in the red box s x 1 minus x plus m square is equal to this expression, where x plus s into x plus minus x into x minus minus x minus x minus, where x plus minus is given by the expression given here 1 plus minus under root 1 plus 4 m square upon s.

So, we get the expression for T as given in the blue box with F s this function F and given by the expression that is in the green box here.

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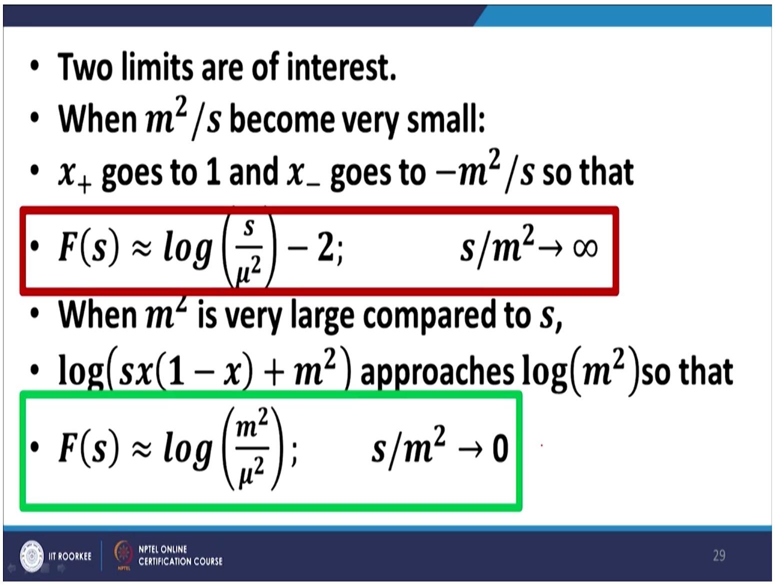

So slightly tedious calculations, but two limits are of interest when m square upon s becomes very small, that is x plus tends to 1 x minus goes to minus m square upon s. We have F s going to log of s upon mu square minus 2, this is when s upon m square tends to infinity in other words m square upon s becomes very small.

So, this is one extreme the second extreme is when m square becomes very large compared to s, then the log of this expression behaves as log m square. And we have this F s is equal to log m square upon mu square, this is in the case where s upon m square tends to 0. So, this is the other case. So, these are two extremes and this is the result that we arrive at for T k which we can substitute in the expression earlier.

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$$F/A: A_3 = \frac{\lambda_3^2 \hbar^2}{2}(2\pi)^4 \delta^4\left(\vec{k}_1 + \vec{k}_2\right)\frac{1}{\left(\left|\vec{k}_1\right|^2 + m^2\right)^2} T\left(\vec{k}_1\right) \quad -(3)$$

$$\text{where } T\left(\vec{k}\right) = \int \frac{d^4 q}{(2\pi)^4}\frac{d^4 q'}{(2\pi)^4}\frac{(2\pi)^4 \delta^4\left(\vec{q} + \vec{q}\,' - \vec{k}\right)}{\left(\left|\vec{q}\right|^2 + m^2\right)\left(\left|\vec{q}\,'\right|^2 + m^2\right)}$$

$$= \int \frac{d^4 q}{(2\pi)^4}\frac{1}{\left(\left|\vec{q}\right|^2 + m^2\right)\left(\left|\vec{k} - \vec{q}\right|^2 + m^2\right)}$$

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Expression for A 3 we can substitute here where expression of T k and arrive at the four point Green function expression with the loop.

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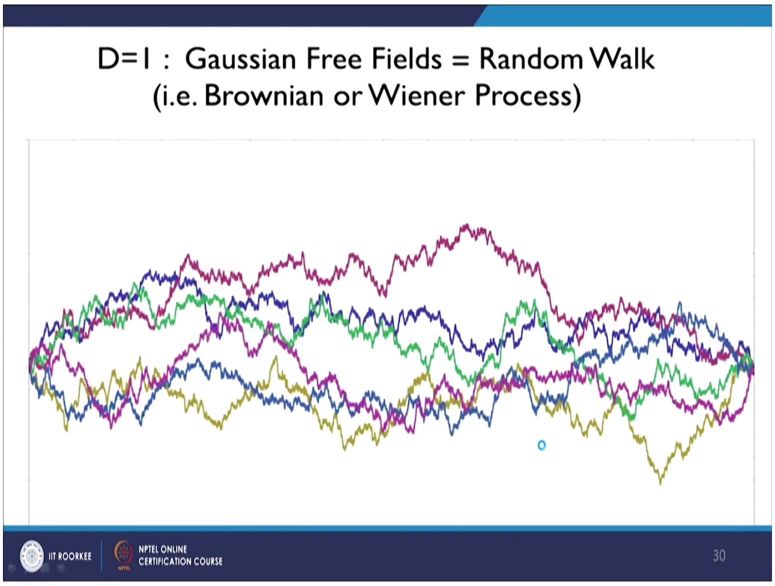

Now, these are examples some examples of how the path integrals and how the free fields Gaussian Free Fields look like. In the first case even D equal to 1, obviously in 1 dimension these Gaussian free fields are nothing but Brownian motion paths and this is what they look like.

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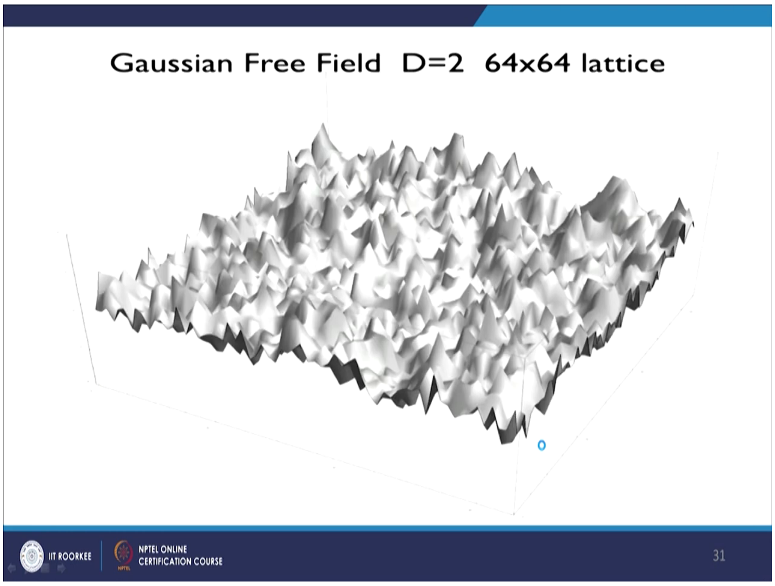

In 2 dimensions they appear in the form which is given here on your slide, 2 dimensions 64 cross 64 lattice.

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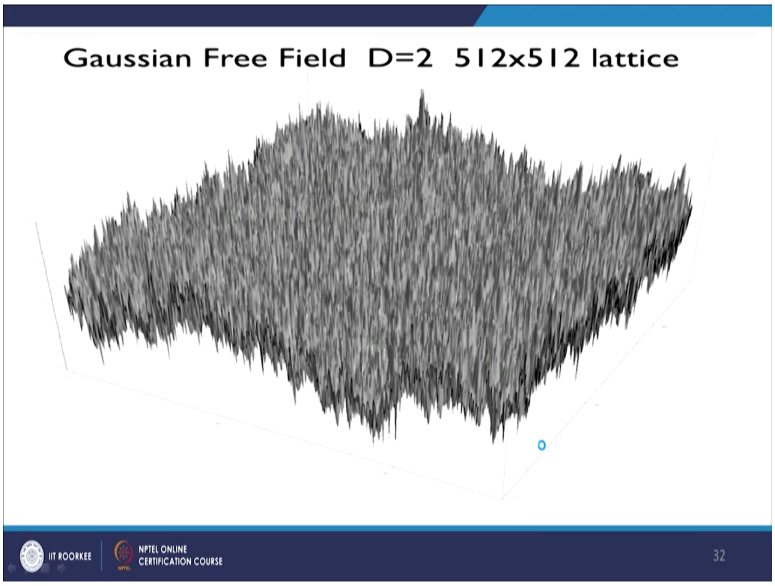

This is a slightly more involved lattice and you see the plot also shows greater irregularities greater zigzagging, because the lattice is now of 512 cross 512 lattice. This is in 2 dimensions note this.

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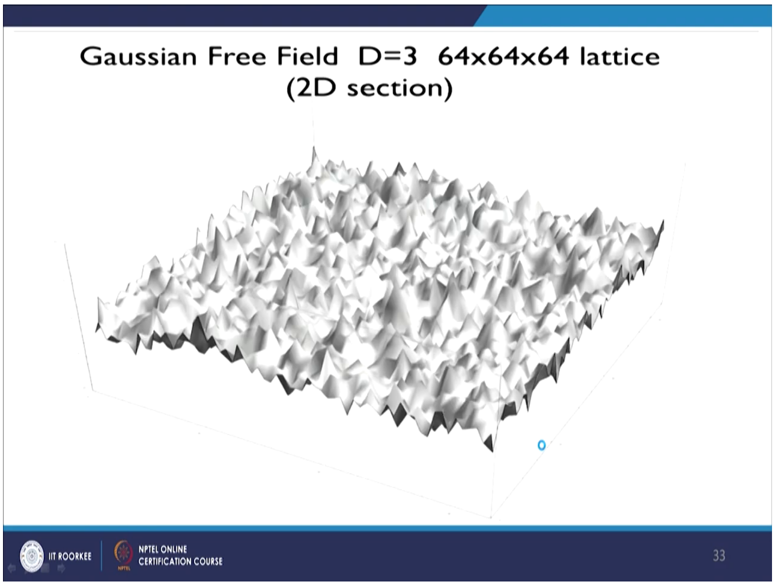

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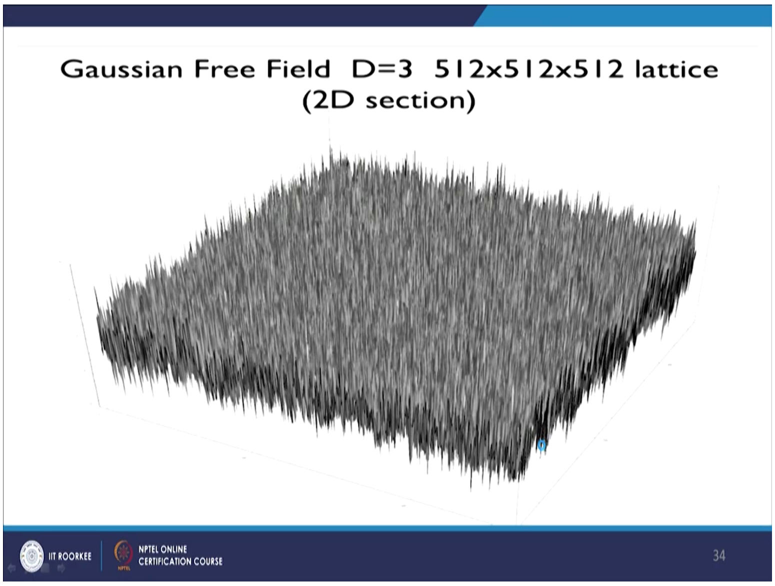

Now this is in 3 dimensions and 64 cross 64 lattice and this is in 3 dimensions, 512 cross 512 lattices, this is the 2 dimensional section of this 3 dimensional path integral 3 dimensional field Gaussian field free field.

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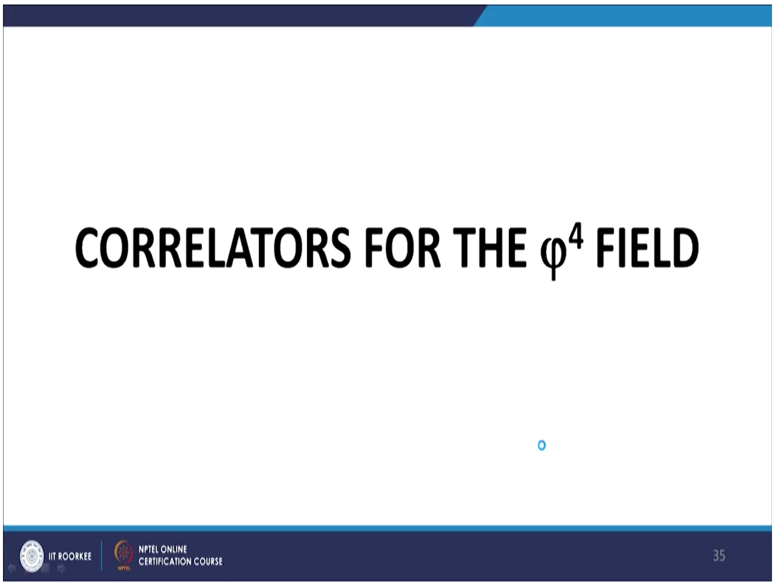

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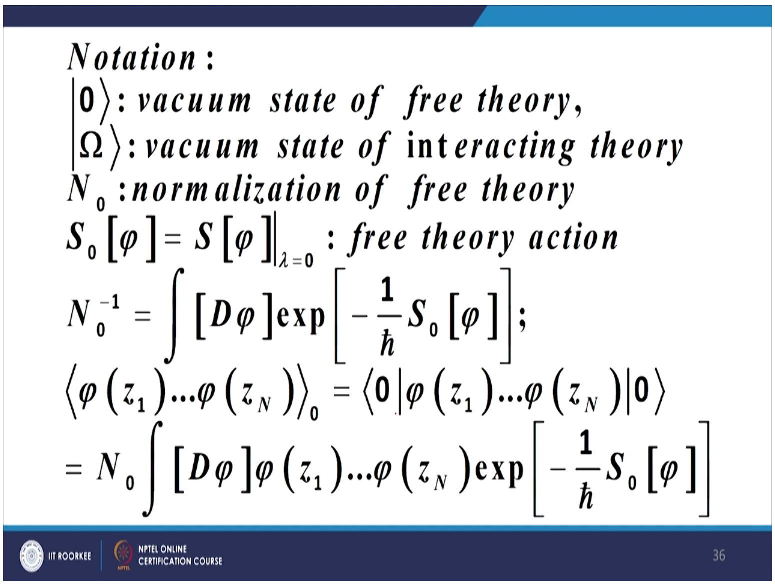

So now we talk about the correlators and the generating function generating function of the phi four field. These are some notation this in ket 0 represents the vacuum state and ket omega represents the vacuum state of the interacting field. 0 represents the vacuum state of the free theory free field and omega represents the vacuum state of the interacting theory.

N 0 is the normalization of the free theory N interaction N int is the normalization of the interaction theory, S 0 is the free action phi Z 1 expected value phi Z 1 to Z N is the with the. Subscript 0 is the expression for the Green function in the with respect to the free vacuum and that is obviously given by the expression by the equation right at the bottom of your slide. This is the expression for the N point function of the in the free vacuum with respect to the free vacuum.

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$$N_{\text{int}}^{-1} = \int [D\varphi] \exp\left[-\frac{1}{\hbar} S[\varphi]\right]$$

$$N^{-1} = \frac{N_0}{N_{\text{int}}} = \frac{\int [D\varphi] \exp\left[-\frac{1}{\hbar} S[\varphi]\right]}{\int [D\varphi] \exp\left[-\frac{1}{\hbar} S_0[\varphi]\right]}$$

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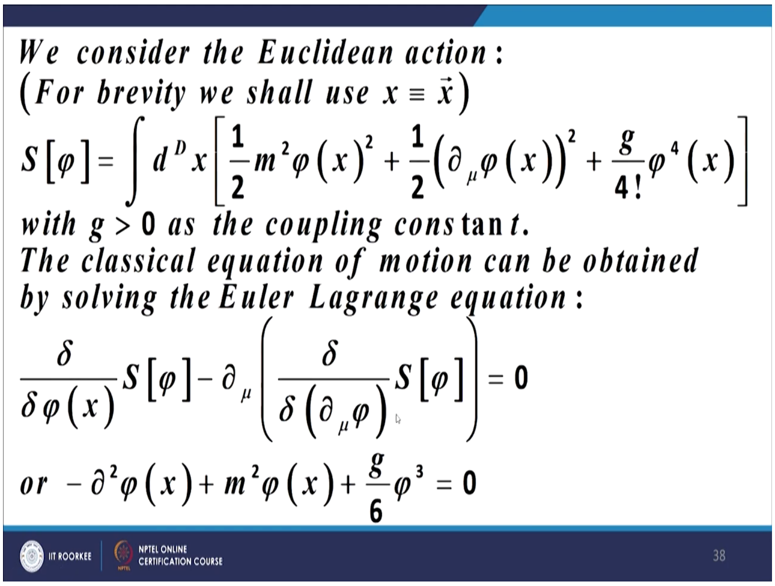

Similarly the expression for the N interaction is here. Now with the phi 4 theory involved here the action takes this form where g is the coupling constant here. The Euler Lagrangian equations takes the form that is given at the bottom equation of this slide and which simplifies to the expression that is the equation below the Euler Lagrange equation. And in this equation if you see this is the Klein Gordon equation with the coupling parameter incorporated therein.

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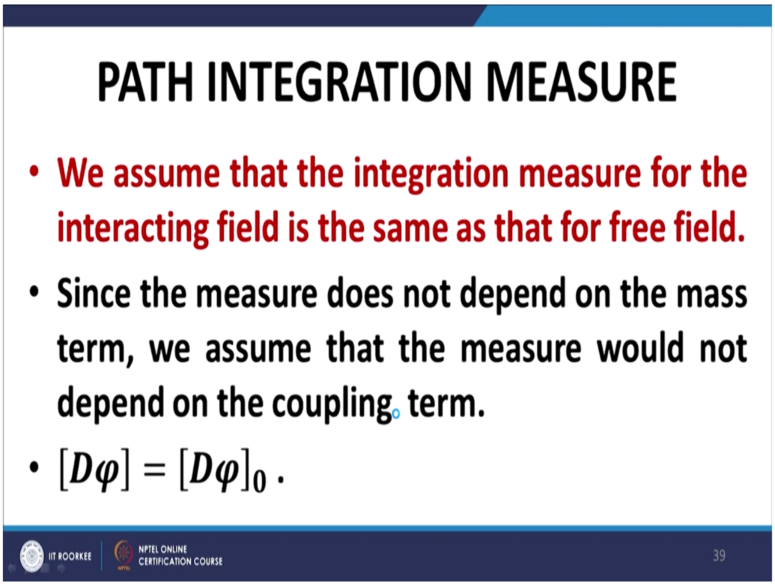

As far as the path of course in Euclidean space, the Path Integral Measure we have taken as the same as the path integral measure for the free theory. There is logic behind this, the logic is that since the path integral measure does not depend on mu it should also not depend on the coupling constant.

And that means that the path integral measure should be unchanged or independent of the interaction term and therefore we retain the path integral measure in the integration in the interacting theory to be the same as the path integral measure for the free theory.

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$$Let: Z(z_1,...,z_N) = N_0 \int [D\varphi]\varphi(z_1)...\varphi(z_N)\exp\left[-\frac{1}{\hbar}S[\varphi]\right]$$

$$= \frac{\int [D\varphi]\varphi(z_1)...\varphi(z_N)\exp\left[-\frac{1}{\hbar}S[\varphi]\right]}{\int [D\varphi]\exp\left[-\frac{1}{\hbar}S_0[\varphi]\right]}$$

$$= \langle \varphi(z_1)...\varphi(z_1) \rangle_{\text{int}}$$

And that gives us the expression for the N point function and this whole expression here on the first expression which when simplified gives us the expression phi z 1 and the expected value phi z 1 to phi z N in the interaction theory.

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$$\text{Now, } Z(z_1,\ldots,z_N) = N_0\int [D\varphi]\varphi(z_1)\ldots\varphi(z_N)\exp\left[-\frac{1}{\hbar}S[\varphi]\right]$$

$$= N_0\int [D\varphi]\varphi(z_1)\ldots\varphi(z_N)\exp\left[-\frac{1}{\hbar}\left(S_0[\varphi]+\frac{g}{4!}\int d^D x\varphi^4\right)\right]$$

$$= N_0\int [D\varphi]\varphi(z_1)\ldots\varphi(z_N)\exp\left[-\frac{1}{\hbar}S_0[\varphi]\right]\exp\left(-\frac{1}{\hbar}\frac{g}{4!}\int d^D x\varphi^4\right)$$

$$= N_0\int [D\phi]\varphi(z_1)\ldots\varphi(z_N)\exp\left[-\frac{1}{\hbar}S_0[\varphi]\right]\left\{\sum_{K=0}^{\infty}\frac{1}{K!}\left(-\frac{g}{4!\hbar}\right)^K\int d^D x_1\ldots d^D x_K\varphi^4(x_1)\ldots\varphi^4(x_K)\right\}$$

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41

Now, this is Z of z 1 z N this is N point N point function N point correlator, if you look at this gives us this can be written in terms of the this can be. If you simplify this if you expand the coupling constant term assuming that the coupling constant is small, if you expand this exponential involving the coupling constant in powers of the coupling constant we have this expression.

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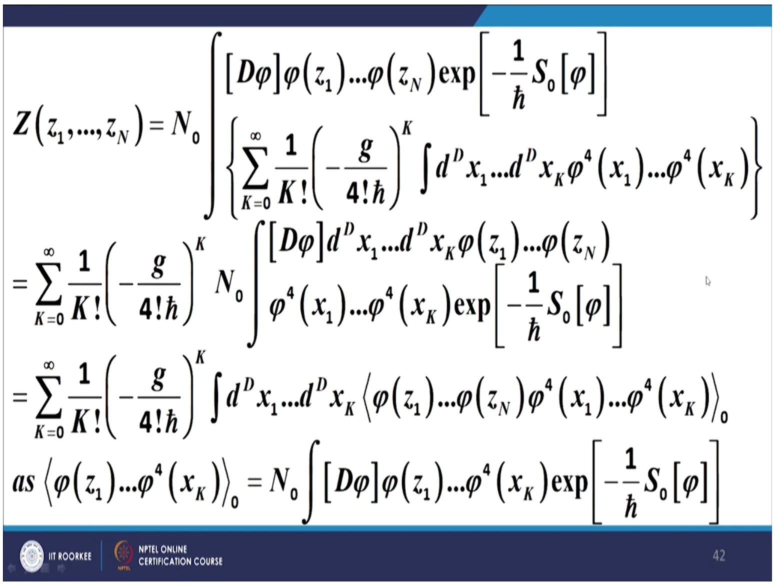

And then we shift flip the series involving the coupling constant and look at this N point this is an N point function z 1 to z N and then phi 4 1 phi 4 2 phi 4 phi K phi 4 K. So, phi phi z 1 phi z 2 phi z N and then phi 4 x 1 phi 4 x 2 phi for x K. This phi 4 x Ks please note this these are coming from this k which is out in the free factor and the z 1 z N are the N point correlators that we want to find.

But the important thing is how many terms are there are N terms of z and there are K terms of x. But each term of x has 4 vertices.

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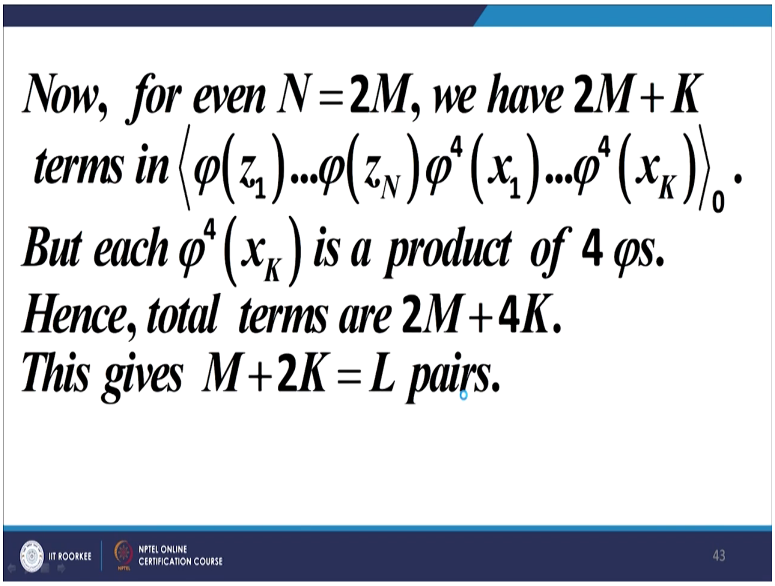

So, we have 4 K plus N N plus 4 K and if I write N as 2 M, then we have 2 M plus; 2 M plus 4 K total number of terms that is M plus 2 K equal to L pairs M plus 2 K equal to L pairs.

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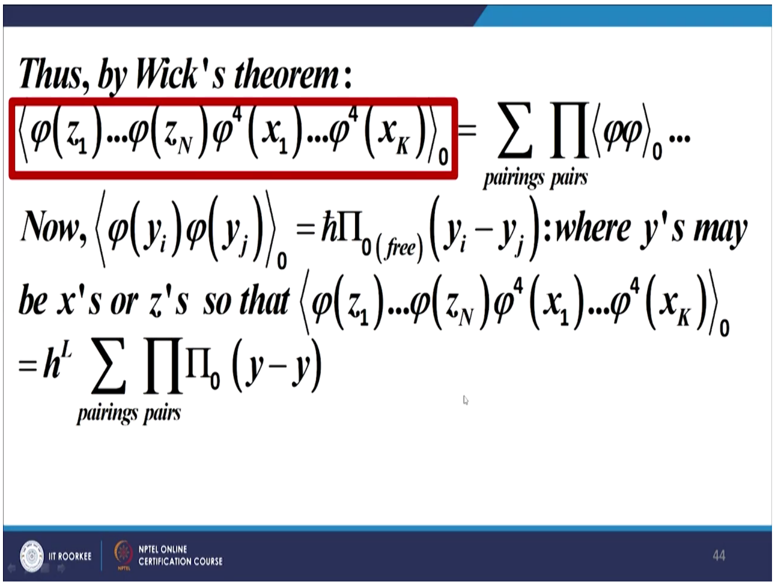

So, by Wick's theorem then the this correlator which is given on the left hand side with respect to the vacuum free vacuum is given by summation over pairings, product of two point functions phi phi 0 and phi phi 0 two point functions are nothing but the propagators of the free theory. And that is can be written as capital pi of y minus y j, y i minus y j where, y i y j can take values x z 1 z up to N and x 1 x 2 up to k. The total number of values would be obviously L pairs will be there.

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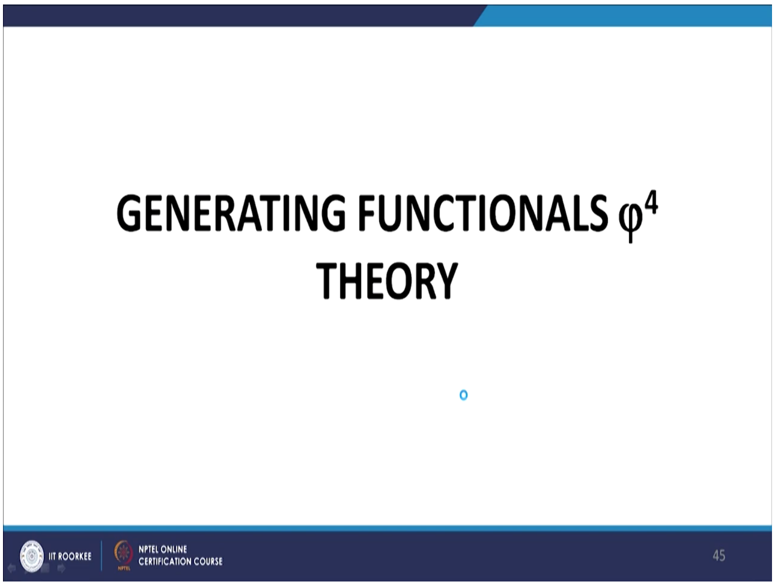

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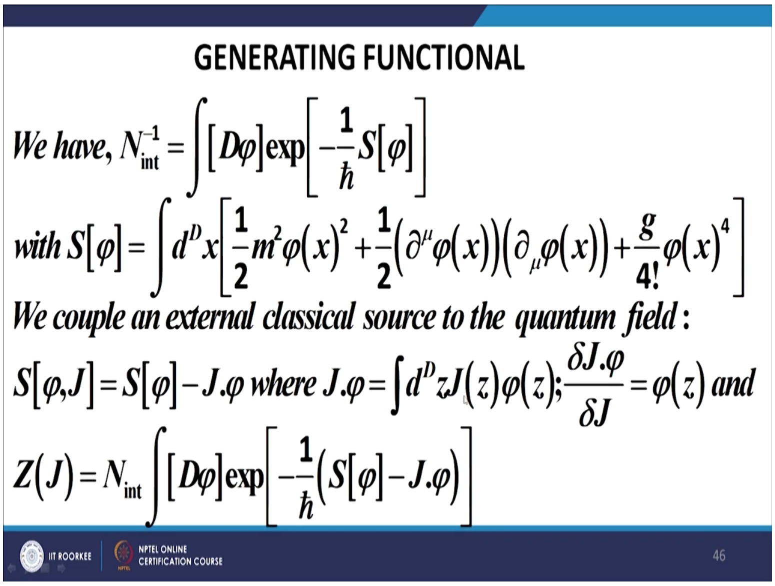

And Generating Functional of the phi 4 theory the generating functional of the phi 4 theory can you have the action here and we coupling an external source we get the generating functional, we get the revised action as the expression given here for S phi comma J, where J dot phi is equal to is the integral. Now because we are now working in D dimensional space not in 0 dimensional space, so Z J is equal to the expression that we already encountered in 0 dimensional space which is given at the last equation on this slide.

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$$\text{We have, } Z(J) = N_{\text{int}} \int [D\varphi] \exp\left[-\frac{1}{\hbar}\left(S[\varphi] - J.\varphi\right)\right]$$

$$= N_{\text{int}} \int [D\varphi] \exp\left[-\frac{1}{\hbar}\left(S[\varphi] - \int J(z)\varphi(z) d^D z\right)\right]$$

$$= N_{\text{int}} \int [D\varphi] \exp\left[-\frac{1}{\hbar} S[\varphi]\right] \exp\left[\frac{1}{\hbar}\int J(z)\varphi(z) d^D z\right]$$

So, Z J is equal to N integral this whole expression the path integral or the functional integral generating functional. And now what we do is we write this exponential of J as a separate exponential and we develop it or we expand it as a power series in J.

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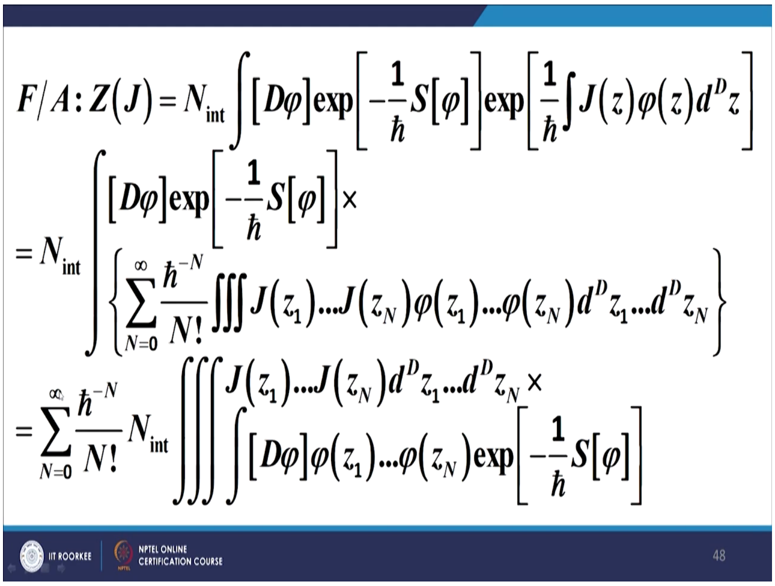

And please note it is not a power series in the coupling constant. The coupling constant is still within the action, it has not been expanded it has not been expanded as a perturbation series. The perturbation series has been done with respect to J the external sources and we were writing exponential 1 upon h integral J z phi z D D z; as a perturbation series we arrive at the second equation and we flip the integral and the summation to arrive at the third equation.

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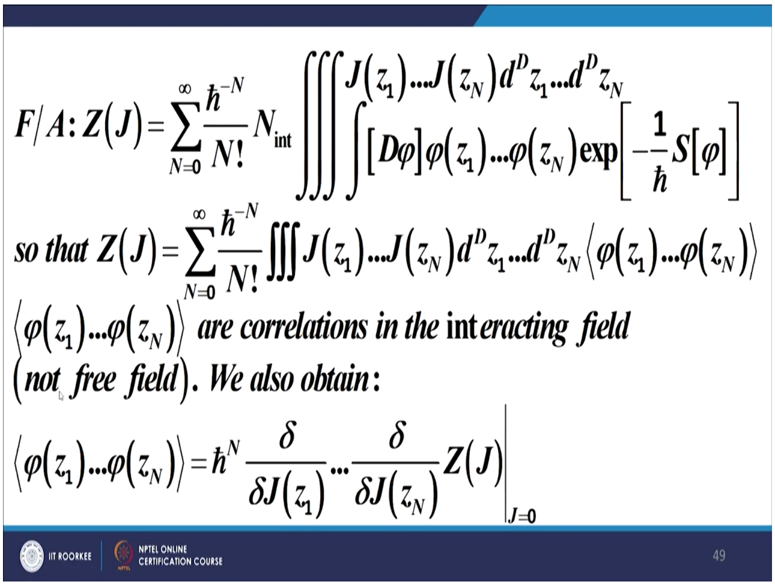

So, Z J is equal to this expression which is given in the top equation on this slide and if you see within this expression, within this integral within this integral is nothing but the N point function N point function. But with respect to the interacting vacuum not the free vacuum.

So, Z J can be written as a power series in h and with an integral involving the four point functions. In the interaction with the interaction vacuum and we can recover the we can recover the N point functions from the expression for Z J by functional differentiation as usual. But this is with respect to the interacting vacuum please note this.

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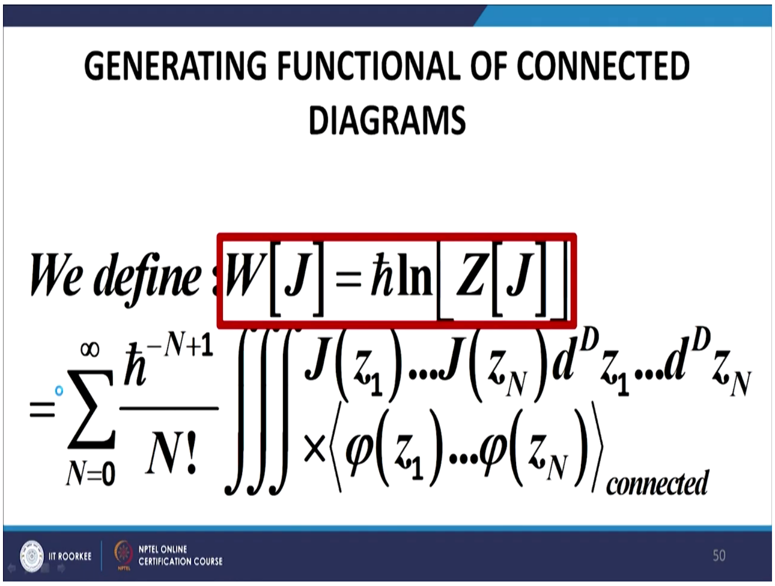

The generating function for the connected diagrams is given as usual by h bar log Z J and that takes this form, where we now have connected functions instead of the Green functions.

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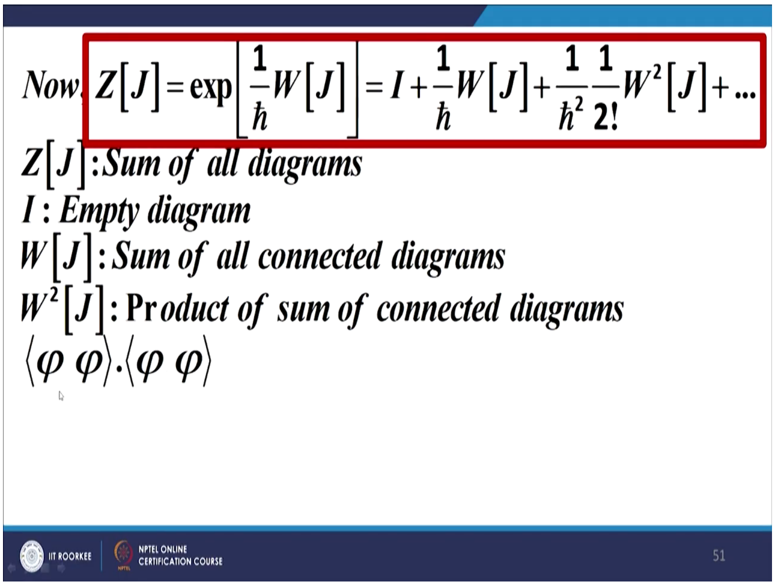

We can write an expansion for the Z J as exponential 1 upon h W J, which can be expanded as 1 plus 1 upon h W J plus 1 upon h square 2 factor exponential series; Z J is the sum of all diagrams here all Feynman diagrams, 1 is the empty diagram and W J is the sum of all connected diagrams, W square J is the product of sums of connected diagrams and so on. So, this is another way of representing the generating functional in terms of connected diagrams.

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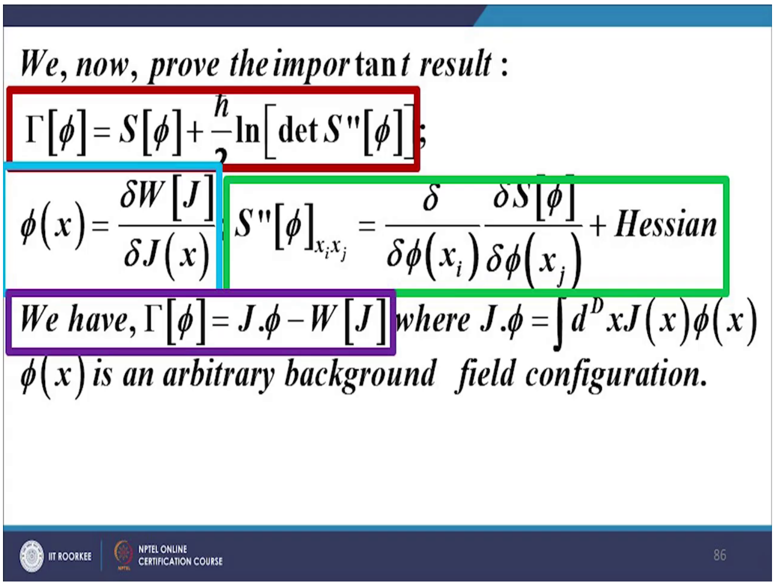

We now prove a very important identity this is the definition of phi x and the second derivative of S double dash of phi is given by delta of phi i delta of phi j of the functional derivative double functional derivative of S phi with respect to phi x i and phi x j plus Hessian terms. Gamma of phi is equal to J phi minus W J, J phi is equal to integral d integral over x J x phi x. So, this is these are definitions and nothing much.

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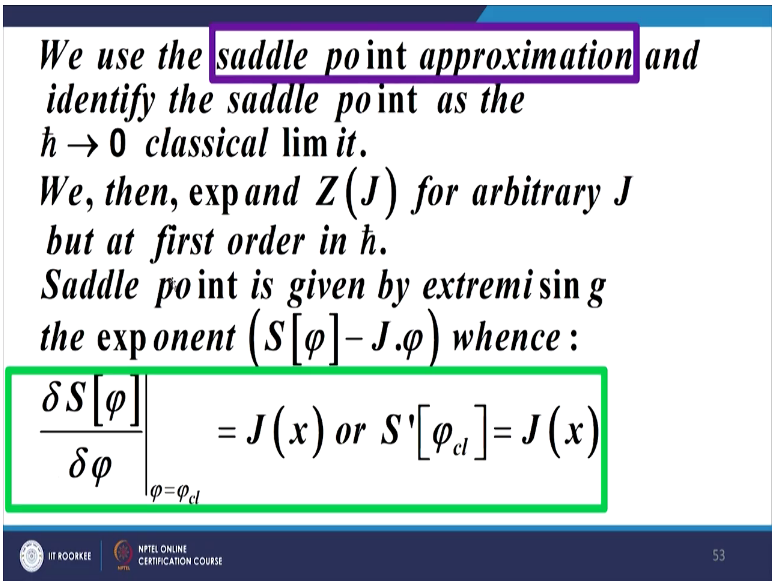

Then we have then we use the saddle point approximation to identify the saddle point which represents the classical limit, which represents recall that saddle point was approximated as the classical limit and it represent is the h tends to 0 classical limit that is the saddle point approximation.

So, we expand Z J for arbitrary J, but to first order in h bar. The saddle point is for the saddle point we know saddle point is obtained by S dash of phi minus J equal to 0. And functional derivative of S phi with respect to phi minus J is equal to 0 or functional derivative of S with respect to phi is equal to J and that occurs at phi is equal to phi classical in other words we have S dash phi classical is equal to J x.

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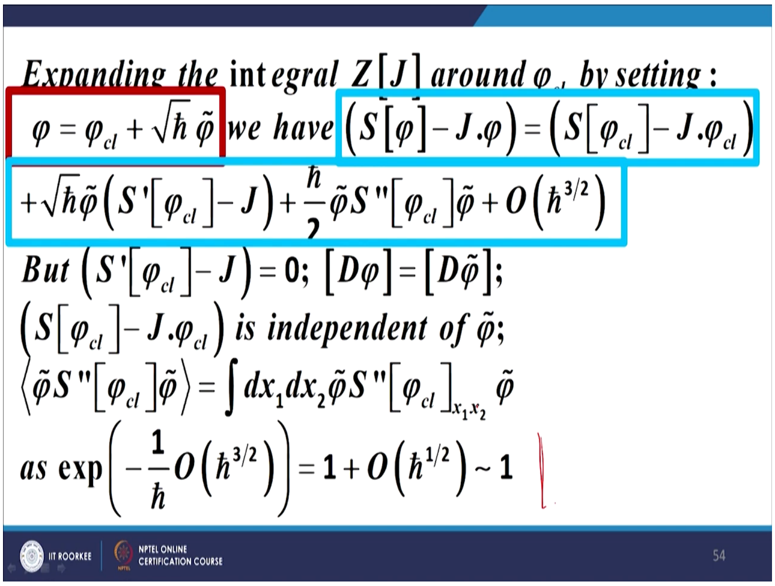

Now, we expand the integral Z J around phi classical by setting phi equal to phi classical plus under root h phi tilde. So, we write this as S phi minus J phi is equal to S phi classical minus J phi classical plus under root h bar phi tilde into S dash phi classical minus j plus h upon 2 phi tilde S double dash phi classical into phi tilde plus higher order terms. But S dash minus J is equal at phi classical is 0 by the saddle point approximation and we also have D phi is equal to D phi tilde because phi classical is constant.

So, that being the case S phi classical minus J phi classical is independent of phi tilde and we also have this expression, the expected value of phi tilde S double dash phi classical phi tilde. Now this is D x 1 D x 2 phi tilde S double dash phi tilde classical at x 1 x 2 into phi tilde, because exponential of minus 1 by h order of h to the power 3 by 2 is equal to 1 plus higher order terms which is approximately 1.

So we can approximate this expected value, because of this condition; because of this condition we can approximate this expected value by the expression given on the right hand side. And this functional differentiation is with respect to phi classical at the points x 1 and x 2.

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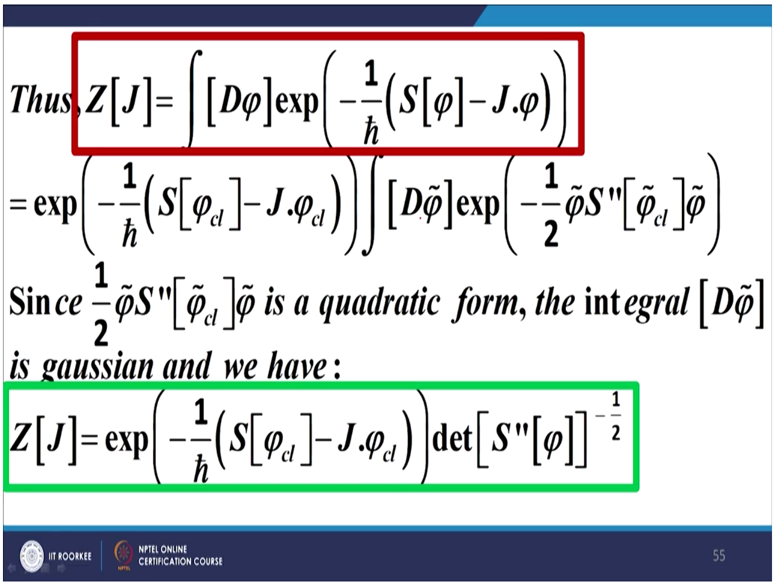

So, we have Z J is equal to integral D phi exponential this is the definition of Z J and now we put the respective things here. The expansion of this expression this expression here, this expression phi we have S phi minus J phi equal to this whole expression we use the expression in the blue box.

We use and we substitute in the expression in the red box here and this being independent of the integration element is taken outside the integration and within the integration we have this expression. Please note the first order derivative is 0 because of the saddle point condition.

Now, this is a quadratic form phi tilde S double dash, phi tilde is a quadratic form and the resultant integral is a Gaussian integral and we can do the Gaussian integral and get the expression which is here in the green box at the bottom of your slide.

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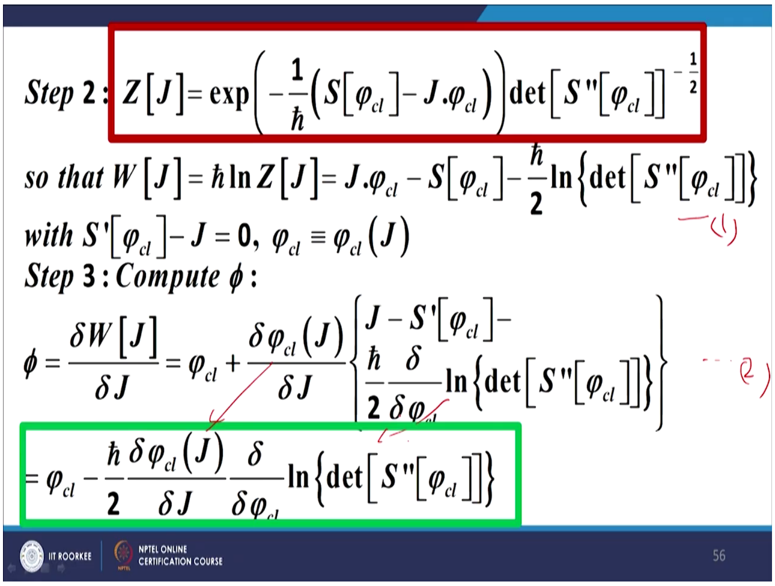

Now, using this is the expression which we have from the previous slide, which gives us for the generating functional of the connected green functions as h bar log Z J which gives us the expression on the right hand side of this equation.

Let us call it equation 1 with S dash phi classical minus J equal to 0 phi classical is equal to phi classical function of J. Now this is from here I can compute phi, phi is equal to the functional derivatives of W with respect to J and when we do the functional differentiation of this expression equation number 1. When we do the functional differentiation of equation number 1, the expression that we get is equation number 2.

But J minus S dash phi is 0 J minus S dash phi classically is 0. So, what we are left with is phi classical minus h by 2 this expression and then this delta by delta classical log of this expression. Because this expression is 0 and this expression comes here and this expression is here and h by 2 is of course here. So, this expression J minus S dash classical vanishes because of the saddle point condition.

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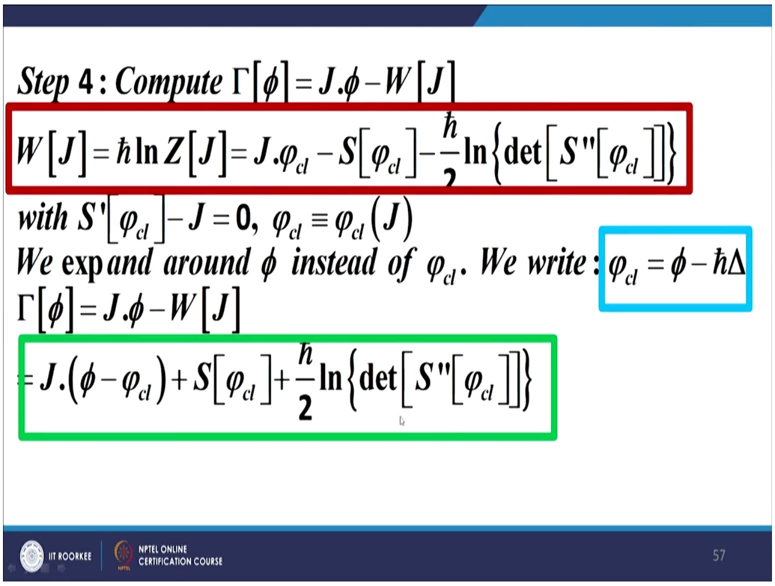

Now, we compute gamma phi that is the effective action which is given by J phi minus W J, W J we have already obtained W J is this expression with S dash phi vanish here that I have already mentioned phi classical as a function of J as well.

We expand around phi in the form we write phi classical is equal to phi minus h delta. If we expand it in this form we write phi classical we have J phi classical from here minus J phi minus J phi classical. That is J into phi minus phi classical that is the first term. The second term is obtained from here W J term and the third term is also obtained from here, so this is the expression in the green slide.

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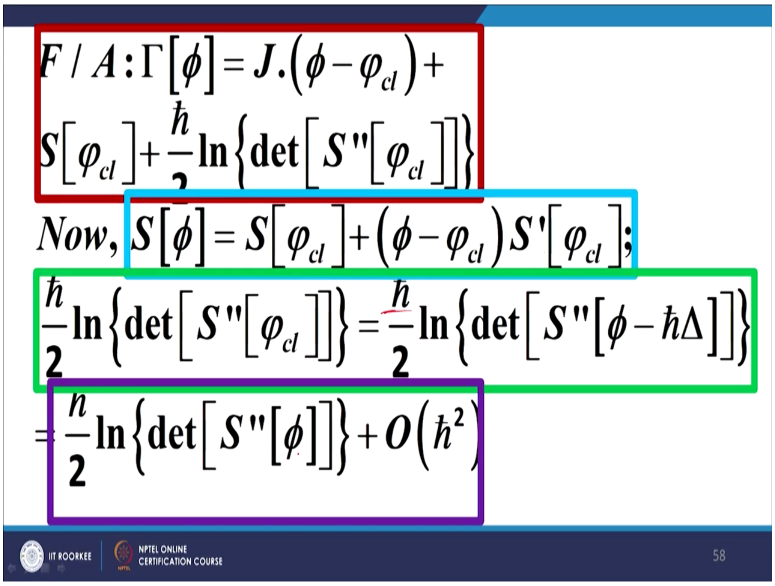

Now, this is the first expression is what we have brought forward and we note that J S phi is equal to S phi classical plus phi minus phi classical S dash phi classical and so on.

So, we substitute this here expanding S phi around S phi classical to first order, we have S phi classical plus phi minus phi classical S dash phi classical. So, that being the case we have now got for S phi classical, what do I have phi minus phi classical S dash phi classical and then I have this S phi here, phi minus phi classical S dash phi classical and J into phi minus phi classical. So, phi minus phi classical into S dash phi classical minus J this whole term goes to 0.

So, what we are left with is S phi classical S phi and this is for the first part. The second part is that this whole expression, now can be written phi classical can be written as phi minus h delta.

Now this determinant can be expanded, if you expand this the first order terms in h bar because of the existence of this h bar here the first order terms in h bar will contain phi and the rest of the terms will be of order h bar square which we ignore. So, we retain terms only up to first order which contain only phi no delta.

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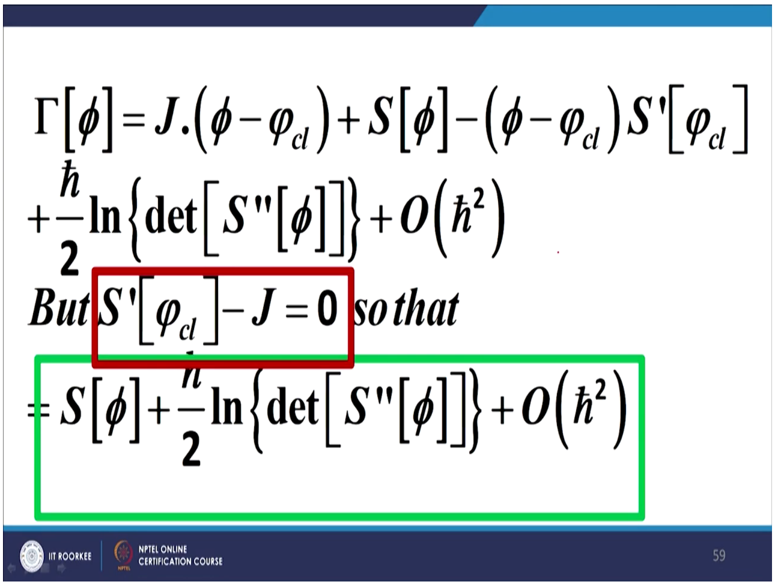

So, keeping the all these things we get gamma phi is equal to J phi minus phi classical plus S phi minus phi minus phi classical S dash phi that is what I mentioned here. This is the expansion that we have got from the previous slide and this we have already done. This is here S phi is equal to S phi S phi classical is equal to S phi minus phi minus phi classical S dash phi that is what we have here and J minus S dash phi becomes 0.

So, both these terms vanish we are left with S phi plus this expression and that is what we intended to prove.

Thank you.