Path Integral Methods in Physics & Finance
Course No 110107146
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Lecture 01: Setting the Scene

INTRODUCTION

PROPOSED COVERAGE

- General Theory of path integrals.
- Path integrals in quantum mechanics.
- Path integrals in Euclidean space and role of path integrals in statistical mechanics.
- Path integrals in quantum field theory (scalar fields & gauge fields).
- Renormalization approaches.
- Path integral applications in finance.

RECOMMENDED TEXTS

- Richard Feynman, Albert Hibbs & Daniel Styer, Quantum Mechanics & Path Integrals, Dover, 2010.
- Ulrich Mosel, Path Integrals in Field Theory, Springer, 2004.
- Lewis Ryder, Quantum Field Theory, Cambridge, 1996.
- Hagen Kleinert, Path Integrals in Quantum Mechanics, Statistics, Polymer Physics & Financial Markets, World Scientific, 2000.

COURSE UTILITY

- With the gradual acceptance of “path integral” based “string theory” as a strong candidate for unification, knowledge of the nuances of the “path integral” formalism of QFT is indispensable for academic progression in this area.
- The use of path integrals based innovative methods for the pricing of derivative instruments is also rapidly gaining acceptance in the financial services industry.

DEVELOPMENT OF PATH INTEGRAL TECHNIQUE

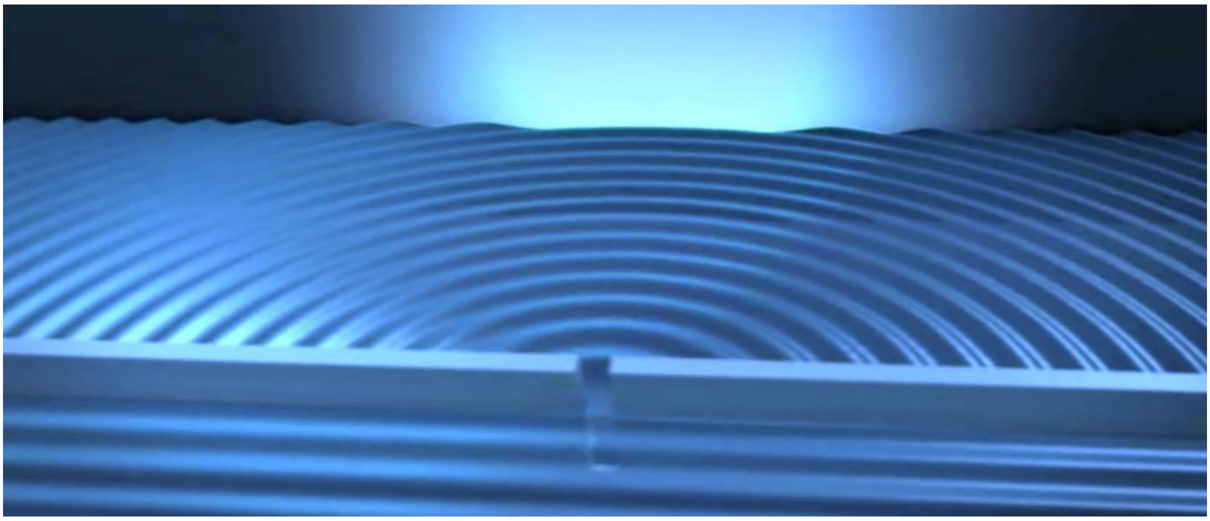
- This formal technique was developed by Wiener and Kac in context of stochastic processes.
- Richard Feynman developed it as a tool for studying quantum mechanics.

APPLICATIONS OF PATH INTEGRALS

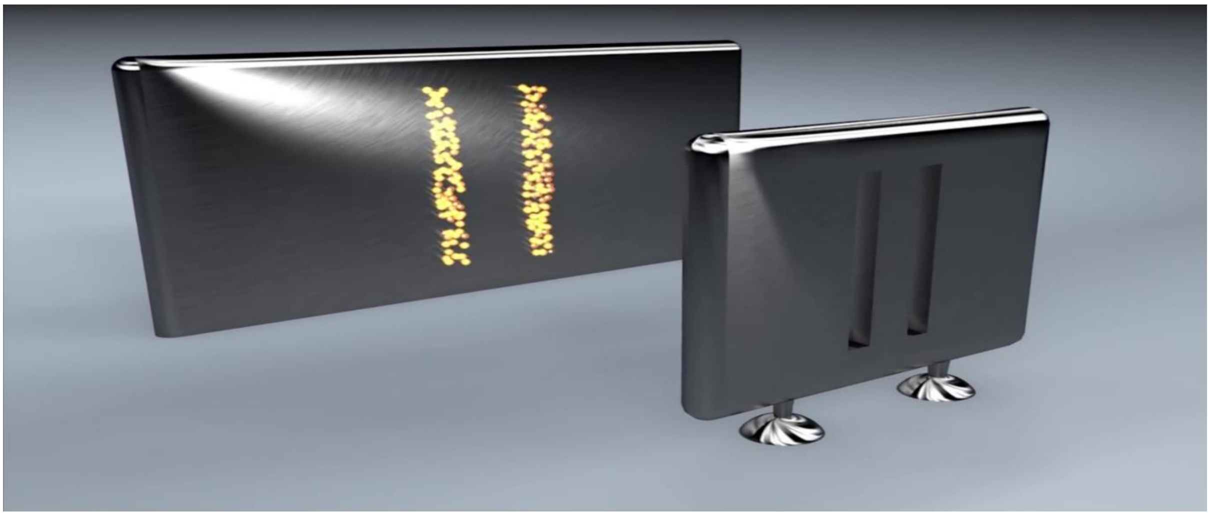
- Quantum field theory
- Statistical mechanics
- String theory
- Condensed matter physics
- Statistics & stochastic processes
- Polymer physics
- Financial markets

THE MOTIVATION:
"DIFFRACTION OF ELECTRONS BY A CRYSTAL OF NICKEL",
DAVISSON & GERMER PHYSICAL REVIEW, VOL 30, NO 6 (1927), pp 705.

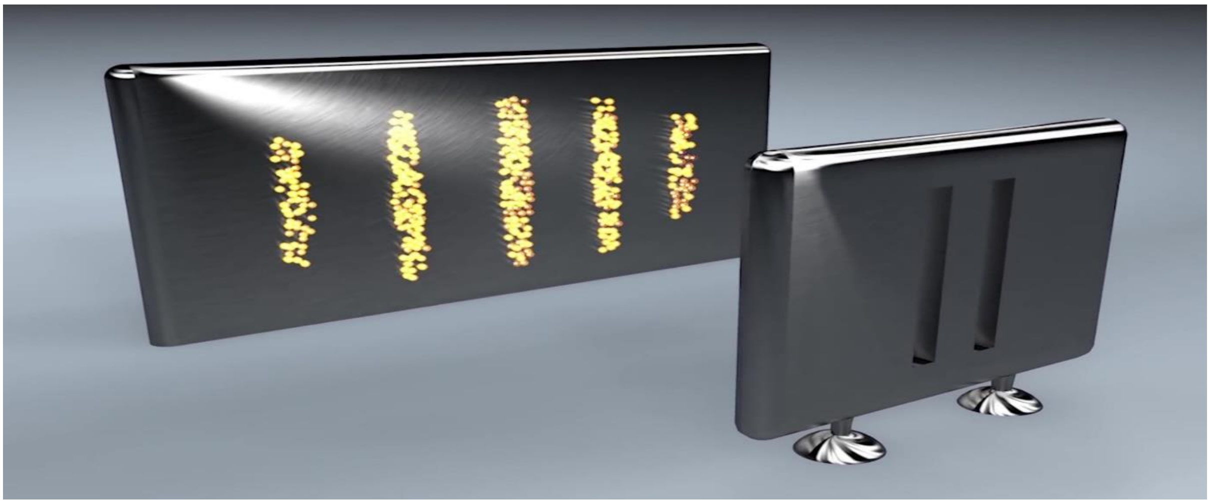
WATER WAVE THROUGH SINGLE SLIT



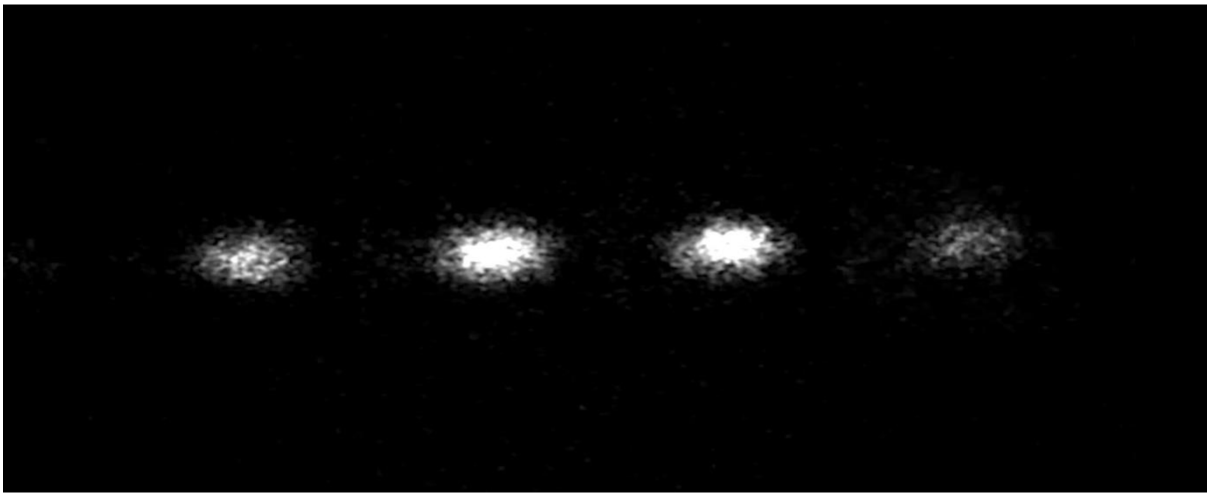
CLASSICAL MECHANICS OF ULTRA MICROSCOPIC BALLS



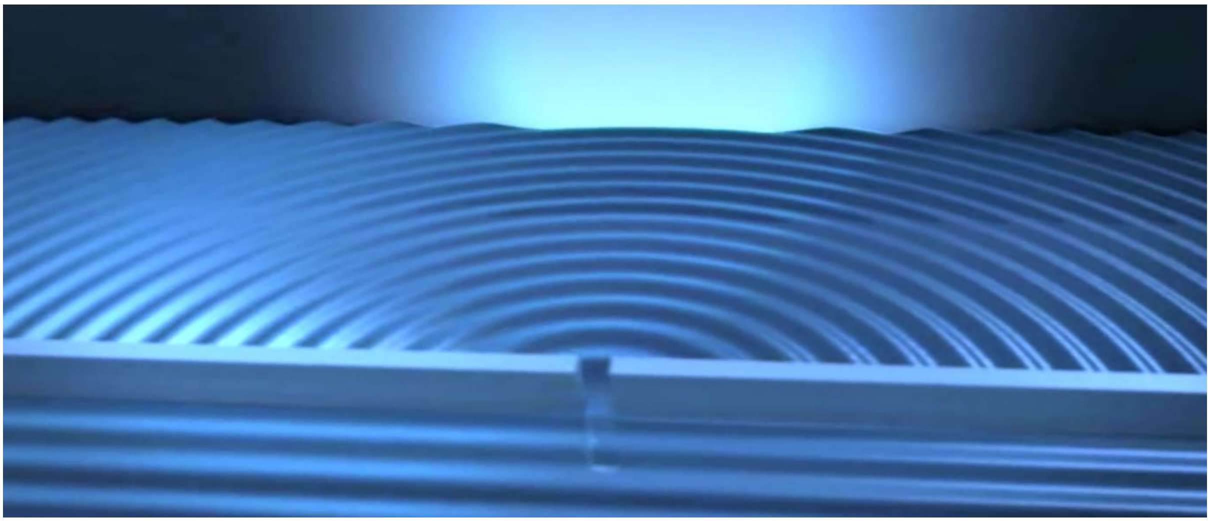
ACTUAL OUTCOME: ELECTRON BEAM



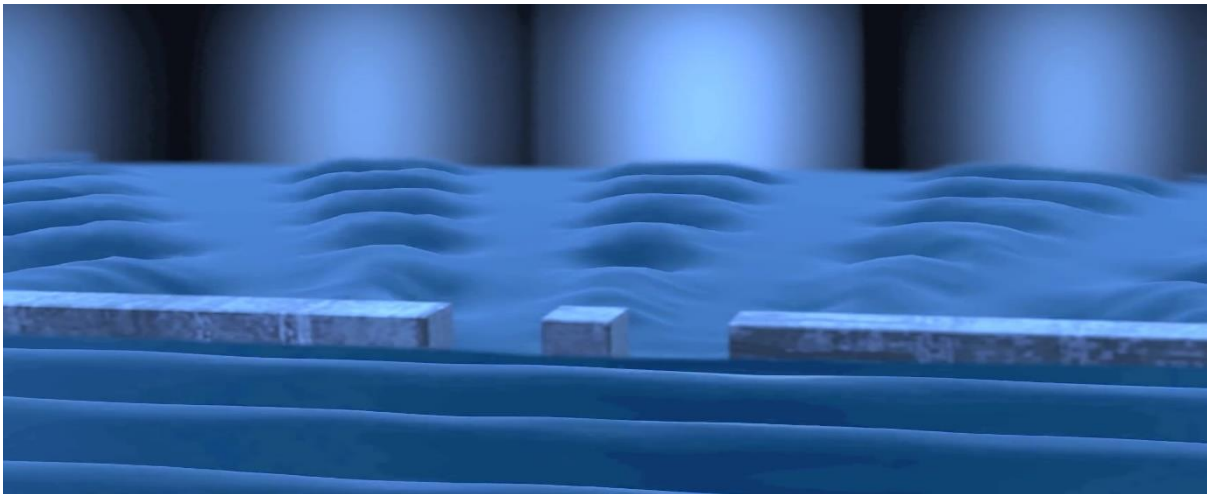
PHOTON LASERS



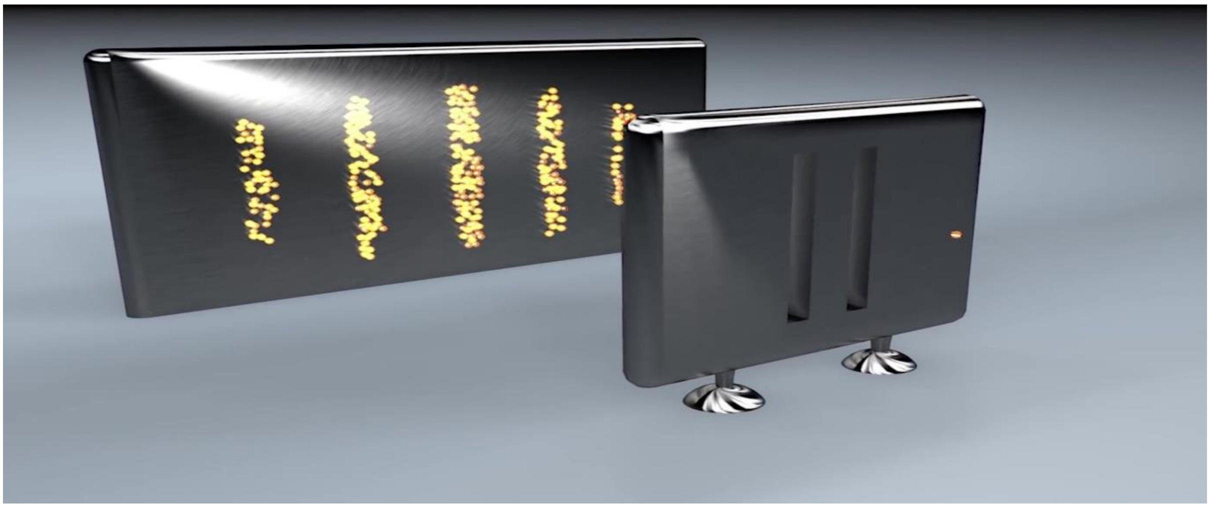
WATER WAVE THROUGH SINGLE SLIT



INTERFERING WATER WAVE THROUGH TWO SLITS



STREAM OF ISOLATED ELECTRONS

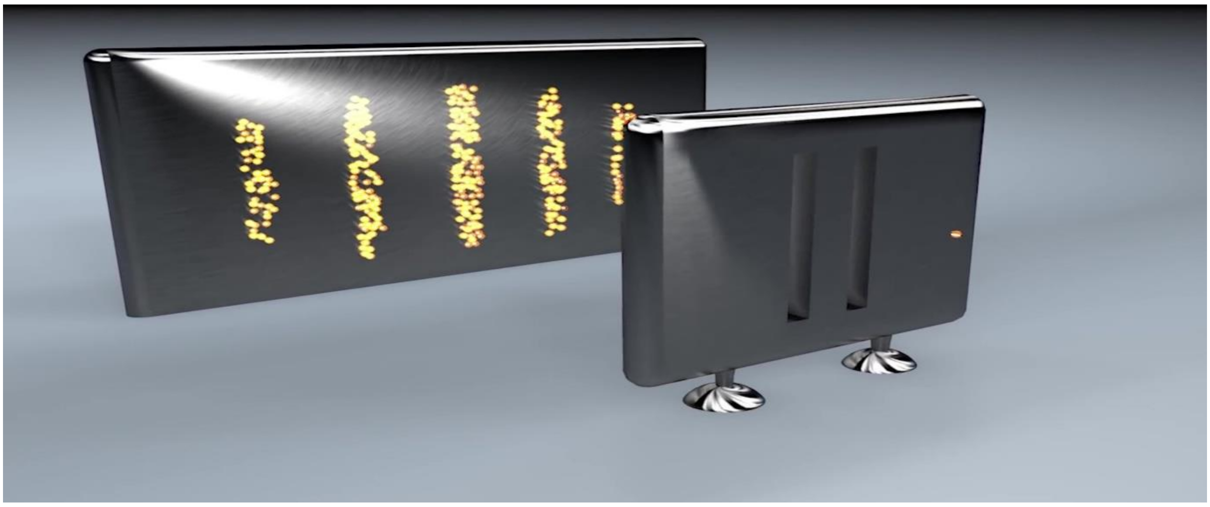


THE TWO SLIT EXPERIMENT

Open up only slit A_1	Interference bands pattern disappears
Open up only slit A_2	Same
Open up both slits	Interference bands appear
One particle at a time from the source, both slits open	Same

- **Only one photon goes through every hour and a half, we still see the effect.**
- **Maybe each individual particle breaks in half, and half of the particle goes through one slit, and half of the particle goes through another slit.**
- **Then we should be seeing half-strength detections. But that's not what we actually see.**
- **Each time a particle is sent through, it is detected in one place and one place only on the screen.**

PATH INTEGRAL: THE RATIONALE



- **Let S be the point source emitting micro-particles, O any arbitrary point on the detector screen under observation and let the two slits be $A_i; i = 1, 2$.**
- **Clearly, the amplitude of the wave at O is the sum of the amplitudes of the waves emerging from the two slits. We can write it as:**
- $A_o = \sum_{i=1,2} A(S \rightarrow A_i \rightarrow O)$.

- **Now, if the number of slits is increased to three the amplitude at O would be the sum of the amplitudes of the waves emerging from all the three slits. We can write it as: $A_o = \sum_{i=1,2,3} A(S \rightarrow A_i \rightarrow O)$.**

- **Now, it is not at all necessary that these paths be restricted to two-legged paths from S to A_i and A_i to O. In other words, instead of just one screen, we can have one more screen Bconsisting of other slits. The amplitude now is:**

$$A_o = \sum_{i,j} A\left(S \rightarrow A_i \rightarrow B_j \rightarrow O\right)$$

- **What happens when the no of slits in a screen tends to infinity. It simply vanishes.**
- **What happens when the no of slit screens, each with infinite slits, tends to infinity. The entire space between the source and detector screen simply becomes a vacuum.**
- **Hence, it follows that the total amplitude at O due to the propagation through vacuum of wave-particles emitted by source S is**
- $A_o = \sum_{i,j,k\ldots=1,\ldots,\infty} A(S \rightarrow A_i \rightarrow B_j \rightarrow C_k \ldots \rightarrow O)$
- **i.e. the sum of the contributions at O of amplitudes of all possible paths from source S to detector O.**

SUMMARY

- **In other words, to obtain the amplitude of wavefunction of a waveparticle at a detector point emitted from a point source, we have to sum the amplitudes of particle waves propagating from the source to the detector following all possible paths between the source and the detector.**

WHY QUANTUM EFFECTS PREDOMINANTLY MANIFEST IN MICROSCOPIC DYNAMICS

- **Dual nature of matter: Quantum physics describes matter and energy as quantum wavefunctions, which sometimes act like waves and sometimes act like particles.**
- **Quantum laws are universal: In reality, every object in the universe operates according to quantum physics. The laws of quantum physics are still operating in a baseball thrown across the field, but their operation is not obvious, so we say the system is non-quantum.**

- **What is quantum effect: A "quantum effect" is an effect that is not properly predicted by classical physics, but is properly predicted by quantum theory.**
- **A situation is described as quantum when its quantum behaviour becomes obvious, even though it is really always quantum.**

- **Classical physics describes matter as composed of little, solid particles.**
- **Therefore, anytime we get the pieces of matter to act like waves, we are demonstrating a quantum effect.**

- **Quantum effect in macroscopic bodies: To be a quantum effect, we have to get matter to act like waves.**
- **To be a macroscopic quantum effect, we have to get many bits of matter to act like waves in "an organized fashion".**
- **If all the bits of matter are acting like waves in a random, disjointed manner, then their waves interfere and average away to zero on the macroscopic scale.**

- **In physics, we refer to an organized wave-like behaviour as "coherence".**
- **The more the wave-like natures of the bits of matter are aligned, the more coherent is the object overall.**
- **And the more coherent an object, the more it acts like a wave overall.**

- **Bits of matter must not just have their motions aligned, the bits of matter must also have their quantum wave natures aligned.**

- **The key here is that a large-scale coherent state is improbable as long as the individual parts are behaving randomly. There are only a handful of possible ways to have a system of pieces act in a coordinated fashion, while there are far more ways to have the system act in an uncoordinated fashion. Therefore, coordinated behaviour is less likely than uncoordinated behaviour, although not impossible.**

- **For example, if you roll 5 traditional dice, there are six ways to get all the numbers to be the same in one roll. In contrast, there are thousands of ways to get all the numbers to not be the same (5^6-6). Getting the dice to show the same number is improbable but not impossible.**

- **In a similar way, quantum coherence on the macroscopic scale is improbable, but not impossible. If the quantum wave natures of the individual bits of matter can be aligned into a coherent state, then quantum effects will become evident on the macroscopic scale.**