Financial Management for Managers
Professor. Anil K. Sharma
Department of Management Studies
Indian Institute of Technology Roorkee.
Lecture 14
Time Value of Money Part 5

Welcome all. So, we are in the process of learning about the different aspects of the time value of money and in the previous class we were talking about some of the applications of the future value of annuity. So, we saw we discussed the some two, three, say applications of the future value of the annuity. Where we tried to find out that say for example, we want to find out that what is there in the store for you that, for example be deposit a certain sum every year and then for a given rate of interest how much that amount becomes at the end of his certain period of time.

Similarly, that for fulfilling one particular requirement of a house, how much should you save annually. So, that we can buy the house 5 years from now, which will be for the 2 million rupees 20 lakh rupees. Third say application we discuss was the sinking fund application, annual deposit into a sinking fund. So, that say, if any company has to redeem the bonds, worth rupees 500 million. So, how much they should say deposit every year to a sinking fund or into a sinking fund.

So, that at the end of the 6 years they have the sufficient amount means equivalent to 500 million rupees for returning back the say investment they have sought in the bonds from the different investors 1 or 2 more applications, we will discuss which are quite interesting in the series and then we will move to the say next part and the next part we will be talking about is the present value of the annuity.

So, we have talked about the future value of annuity and after discussing the future value of annuity, we will move to the next part that is a present value of annuity. But before that, let us discuss one or two more applications of the future value of annuity and when we talk about the one or two more applications of the future value of annuity.

For example, here in this case is finding the interest rate, sometimes what happens that some company advertises that if you say, keep on giving us a certain sum means a annuity sum, means the same sum every year, for a certain period of time, then we will return you this much amount. So, they know we are in this discussion, they are talking about the rate of interest, what rate of

interest they will be means returning the money back at what rate of interest, they will be returning the money back to the firm they are not talking about anything.

They are simply talking in the lump sum value, that every year you give us this much of the amount in the annuity form in the same amount every year and at the end of say this much number of years we will return with this much amount. Now, for example, if this situation arises, they are not telling you the interest rate.

Because normally what happens, the comparison of any investment is, means normally takes place with the help of the interest rates, that if I make investment in one important avenue, how much interest I am going to get back, or what rate of interest I am getting, if I invest my funds in the fixed deposits of the banks, how much interest I am getting back how much tax I am paying on that, if I am say investing into the bonds of certain companies, how much interest I am getting back how much interest tax, I will be paying back to the government.

So, it means every time we need to find out the, say, the percentage of interest that which we will be earning, our investment will be earning, but sometimes what happens that the company just to say you can call it as not to communicate the, say, the real picture, they simply say, give us this much every year for a period of this much a number of years, we will return you back this much amount.

So in this case, that important question is, million dollar question is to find out the interest rate which will be means earned by that investment, when we will give that investment in the form of annuity, annuity to accompany for a certain number of years and they will return again back us asset and lump sum amount, then what is the rate of return is at which our investment is growing while investing into that fund of that particular company.

(Refer Slide Time: 04:29)

For example, in this case, what is given a finance company advertises that it will pay a lump sum of rupees 8000 at the end of 6 years, a lump sum of 8000 rupees at the end of 6 years to investors who deposit annually rupees 1000 for 6 years, 1000 for 6 years are the 2 important things and one is 8000 rupees, and that number of years are 6 here. So, here they are only talking about the amount to be paid back at the end of how many 6 years and how much they are asking as annuity deposit, they are asking for 1000 rupees to be paid annually.

So, it means in the total number of 6 years, we will be ending up paying how much 6000 rupees to the company and at the end of the 6 years for a deposit of 6000 rupees given by the person or the investor, the company will return back to the investor 8000 rupees. Now, in this case, nowhere the company is talking about the rate of interest. At what rate that investment will be growing and that 6000 rupees will be becoming 8000 rupees.

So, what rate of interest the company's paying back to the investor nowhere the company is talking here about the interest rates. So, what we have to do is our job is to find out the interest rates and if you have to find out the interest rate here. So, there are the one step process, one is there is twostep process. So, the interest rate may be calculated in this two steps.

One is find the future value interest factor for annuity, find the future value interest factor for the annuity for this contract and you have to find it out how you can find it out for finding it out the processes explained here. How much they are returning back to us 8000 rupees, and how much is

we are paying to them, they are paying 1000 rupees every year, annuity, and future value of the annuity means we have to find out for example, if r and say this number of years n is given to us, then there is no problem we can easily find out the interest rate, but in this case, r is missing, r is not given to us and the number of years the n is given to us.

So, what is our job we have to find out the r. So, then you means solve this you find this situation that future value interest factor are unknown and number of years n is 6. So, that is equal to 8000 divided by 1000 and that value comes out as 8. Now, what is this meaning of this 8 it means it is the future value interest factor for that annuity, 8 is the future value interest factor for that annuity and now, what you have to do is how to find out the interest rate, you have to search for this particular value in the future value interest factor for the annuity table future value for the interest factor for an annuity table.

That table I told you which is given in the end of every book or with every good book of the financial management. For example, Prasanna Chandra, I am referring to you time and again if you buy that book and if you see at the end of the book, all the tables are given their present value interest factor, future value interest factor, present value interest factor for the annuity and the future value interest factor for the annuity all these tables are given.

So, you have found out this factor, feature value interest factor for annuity have found out and which has come up as 8. So, now you have to refer to that table and there in that table future value interest factor for annuity table you have to look at that in the same table, you have to go to the row number of years that is on the vertical side and on the horizontal side the rate of interest. So, against that 6 number of years because they are giving 1, 2, 3, 4, 5, 6, 7 like this.

(Refer Slide Time: 08:19)

So, against the say the table is given like this. So, here you are given the say your number of years 1, 2, 3, 4, 5, 6, and here you are given the rate of interest 2 percent, 4 percent, 6 percent, 8 percent then it is say you call it as maybe a 10 percent you take it as say it is 10 percent and then you can say for the extended 12 percent. So, what you have to do is against a 6 you have to find out some value here wherever it is in the table and that value against 6 number of years, the value have, you have to find out which is nearer to this value of 8, which is the nearest to this interest factor of the, this future value, interest factor of the 8.

You have to find out that value and if you move to the table in the table against that future value interest factor at the rate of 12 percent for a period of 6 years, if you find a value, this value comes up it is a table value, this value comes up as 8.115 this value comes up as 8.115 which is the nearest value to this factor 8.

So, you can conclude here that by giving 1000 rupees annually to a company for a period of 6 years, and if the company agrees to return 8000 rupees at the end of the period of 6 years, then this interest rate is slightly below the 12 percent because, this value is more than this factor this value is 8.115.

So, this and this is are the rate of 12 percent. So, it means this is the rate of interest is near about the 12 percent not 12 percent but slightly less than 12 percent. Because exactly 12 percent will be this value since this factor is this value is less than this value. So, this interest rate has to be not 12 percent but little lesser than that. So, you can call it as 11.95 or 98 percent something 11.9598 percent something like that will be there.

So, near about 12 percent rate of interest we are going to get by making say, investment in this particular annuity and you can calculate that this annuity fund, where are we are depositing 1000 rupees for a period of 6 years at the end of say 6 years you are going to get back 8000 rupees. So, you are going to get back this return of 8000 rupees at the rate of near about 12 percent near about 12 percent.

So, this is one important, another important application of the future value of annuity. Where with the help of this you can find out that if the interest rate on any investment decision is missing only the lump sum values are given only absolute values are given, then how to find out the interest rate interesting. Then you go to the another, this is the last application of the future value of annuity, then I will take you to the next important component that is the present value of the annuity.

So, in this case now, for example, you said the last one is how long should you wait? How long should you wait? Now for example, what is the objective here that you want to incur some expense. Any investment or any expense you want to make and the total amount required is say for that is any given say 1 million rupees you want to have acquire something any investment you want to acquire or maybe any say, where you want to go the cost of going there is a 1 million rupees.

Now in this case, you have to collect 1 million rupees. So, it means you can find out that what is my annual savings that is very easy to find out that depending upon my total annual income,

depending upon my say consumption. So, total income minus my total consumption or me and my family my consumption this much annually I can save or I normally save. I want to achieve some certain objective buying that investment or going somewhere.

So, that cost is approximately say this much say 1 million rupees my savings say for example is 50,000 rupees per month. So, how long should I wait to take my savings of 50,000 rupees per month up to that amount of 1 million rupees. So, the question here is the application of the future value of annuity is how long should you wait?

So, for example, we have taken the say another situation here we have created another example we have found out it is not I have found out but it is given in the book of Prasanna Chandra if you see the all these applications I have taken from the book that is a percentage and that is financial management by Prasanna Chandra which is a McGraw Hill publication.

So, this problem says that this example says that you want to take a trip to USA which costs 1 million rupees, which costs 10 lakh rupees and the cost is expected to remain unchanged in the nominal terms, you want to go to US over a period of time and expected a cost is going to be 1 million rupees or 10 lakh rupees, and this cost is going to remain constant, this is not going to change.

It is not that because of the time value of money, by the time we collect 1 million rupees. So, the cost will become, 1.2 million rupees that is not going to happen. By the time you collect 1, 1 million rupees, the cost will be 1.2 million or something like that that is not going to happen. So, you can save annually 50,000 to fulfill that desire, requirement is 1 million rupees 10 lakh rupees, and our annual saving rate is 50,000 rupees to fulfill that desire.

How long will you wait? You have to wait if your savings earn and interest of the 12 percent. How long do you have to wait? If you are saving earn an interest of the 12 percent and in this case, you have to find out the say, means the future value of an annuity, which is of the 50,000 rupees that earns a 12 percent interest and we have to make it equal to the 1 million rupees. So, how long will it take that that 50,000 rupees saved annually invested at the rate of 12 percent per annum, making it 1 million rupees or 10 lakh rupees, How much time will it take?

So, how long should you wait this is another application of the future value of an annuity. So, it means different diverse questions we are going to answer or different applications we are going

to means make us have to clarify the process of the future value of an annuity. So, in this case, this entire process has been explained how that value will be worked out and that how long the question of this how long will be answered and here the answer is that almost 11 years.

How it has been worked out? For example, what do you want to collect is 1 million rupees or the 10 lakh rupees and what is your annual saving 50,000 rupees and future value annuity say factor if you calculate in this case what is missing now, n is missing in the previous case if you go back, in the previous case r was missing and was given r was missing. In this case r is given, but n is missing.

So, if anything is missing out of this in this model. That to out of the total requirement if any item is missing provided all other items are given to us. So, the missing item can be found out. So, 50,000 into future value of interest factor for annuity n is missing we have to find out the n. So, and that has to be made equal to the 1 million rupees. So, if you means convert that into that say future value or interest factor of our annuity this kind of the factors and you want to equate it to 1 million rupees.

So, finally, this amount becomes like this if you solve this, this equation becomes this and this very comes up as 2.4 and finally, this amount comes down to 3.4 and the help of log converting these simple estimates to the log estimates finally we can solve it with the help of the logarithm and if you means convert into the log values, So, finally, we find out that this value is coming up as 10.8 years.

That within a period of 10.8 years, we will be able to collect a sum of rupees or x would be able to collect a sum of rupees 10 lakhs or 1 millions, if he saving 50,000 rupees annually, that investment is made into a fund and that fund gives that 12 percent rate of interest per annum. So, at the end of 11 years or within a period of 11 years or by a period of 11 years, that sum savings of 50,000 rupees annually saved for a period of 11 years at the rate of 12 percent will become 1 million rupees.

So, this is another application, so you can means see all these applications can help us that if has anything is missing, we can find out and all kinds of questions have been tried to be answered here that for any particular type of investment or good investment, how to make estimates, how to go ahead about it and how to find out the relevant values the future value of an annuity,

Because normally when we think of making investment in any say fund or in any investment avenue, we think of that say if we buy any investment fund or maybe insurance policy, which is basically the investment for the investment purposes not for the lifesaving purpose or maybe we buy any mutual fund or when we buy maybe any provident fund account, when we get opened in the banks, we try to find out that if we give this much of amount in this particular fund, annually means that fund that amount of saving remains the same.

So, how much will it become if the rate of interest is also known to us and if the number of years are also known to us, if one thing is not known to us, then it can be founded provided the other things are given to us and if all the things are given to us easily the all the questions can be answered whatever it is, we want to get answered or we want to means find the solution for.

So, future value of annuity is very, very important and everywhere in the say further discussions relating to the any aspect of the financial management of maybe in case of individuals or in case of the firms or the organization's this same concept that the future value of annuity is very, very important and will be making use of it.

So, that is why I am spending quite a good amount of time on the time value of money. So, that whenever we discuss it at the later stage for the evaluation of the different investment options or for evaluating the different financial decisions, then they are we have to make use of it and we do not find any kind of difficulty or I have not to explain it again to you that how the particular value has been worked out, how this investment has become this at the end of this much number of years.

So, for that, you should have the complete background knowledge and there, the time value of money and the different concepts of time value of money are in important. Now I will take you to the next part and that next part is quite interesting like, means we simply call it as the reverse of. We were talking about earlier the future value of an annuity and now we are going to talk reverse present value of an annuity, present value of an annuity.

For example, we make any investment today in the zero period in the current period, we make any investment in the lump sum amount in any investment fund and that investment fund says that for the next 3 years or next 5 years or next 10 years, we will give you annually this much amount back. So, it means we will have to find out that at the end of the first year I am getting

something at the end of second year I am getting something at the end of say, third year I am getting something we are going to talk about here is that, that return is annuity it is not uneven sum.

When we are going to get the same amount. Then we have to calculate now because we are making any investment in the current period in the zero period and that cash flow is going to occur that return is going to occur to us over the subsequent number of years maybe 3 years, 5 years, 7 years, 10 years, right.

So, we want to find it out that if I give this say 50,000 rupees or 1 lakh rupees or 10 lakh rupees for any investment fund, and they pay me annually a certain sum of annuity, then what is the present value of that total annuity which I am getting over the number of subsequent year or the future years.

So, this is totally the reverse completely the reverse of the future value of annuity, the present value of annuity from the previous concept you were trying to find out that any investment I start making today and for the next number of 5, 7 or 10 years, I keep on making the same investment I say amount I keep on investing somewhere at the end of that period of time, 5 years, 6 years, 7, 8, 9, 10 years, how much that amount will become. So, that is called as a future value of annuity.

Here we are going to talk about if I invest a certain sum in lump sum and over the next 3 years, next 5 years, I am going to get some amount back, but that 2 in the equal amount annuity fim, then what is going to be the present value of that total cash flow. I am going to get over the given number of years.

(Refer Slide Time: 21:18)

PRESENT VALUE OF AN ANNUITY

Suppose Ram expects to receive Rs 1000 annually for 3 years, each occurring at the end of the year. What is the present value of this stream of benefits if, the discount rate is 10 percent?

So, for example, here you are given small, say example here, that suppose that Ram expects to receive 1000 rupees annually for 3 years. Ram expects to receive 1000 annually for 3 years, each occurring at the end of the year, what is the present value of this stream of benefits, if the discount rate is 10 percent, if the discount rate is 10 percent. So, how much is going to be the present value?

Now, for example, in this case, maybe you can add one more thing here, that Ram wants to make investment of 2000 rupees today in lump sum the 2000 rupees investment today and that investment is going to fetch him and an annuity of 1000 rupees, annual sum of 1000 rupees for the period of next 3 years and that rate of interest is 10 percent.

So, that 3000 rupees, which will be received by Ram in the next 3 subsequent years, comparing that total inflow, some of that 3000s, 1 plus, 1 plus, 1000s becoming 3000 rupees, comparing that 3000 rupees coming over the 3 subsequent years with the present investment of 2000 rupees, whether it is a worthwhile investment or not. So, you have to you should not feel happy that I am giving 2000 rupees today, and next 3 years, I am going to get back 3000 rupees, 1000 rupees per year.

So, it means certainly, I am going to get 3000 rupees against the investment of 2000 rupees, it is not like that, you have to calculate the present value of that annuity, which you are going to get over the subsequent 3 years. So, in this case for evaluating this proposal, what you have to do is,

you have to calculate the present value of annuity and how can you calculate the present value of annuity, that inflow.

(Refer Slide Time: 22:59)

PRESENT VALUE OF AN ANNUITY

Suppose Ram expects to receive Rs 1000 annually for 3 years, each occurring at the end of the year. What is the present value of this stream of benefits if, the discount rate is 10 percent?

So, for calculating the present value of annuity, we have to calculate for this particular part, the present value of annuity and in this case, we have to use the discount factor process, we have to discount those future cash flows and try to find out whether there are 3000 rupees we are getting in the 3 next years is equal to 3000 rupees or something more or less and certainly it will be less because of the time value of money 1 lakh rupees 1000 rupees coming back to us 1 year from

now, is not equal to 1000 another 1000 coming 2 years from now is not equal to 1000 and another 1000 coming back to us, but 3 years from now is not equal to 1000.

So, let us discount it and try to find out here that how to find out the present value of that annuity So, how you have to do is what is the inflow coming up? It is 1000 rupee. If it is 1000 rupees so, you have to discount it; what is the discount rate? It is given to us and the discount rate here it is 10 percent and annually we are getting back is 1000 rupees period is 3 years, and here the investment part is missing.

So, for example, we assume that 2000 rupees we are investing and we are going to get back 3000 rupees. So, whether it is a worthwhile investment or not. So, in this case, you have to discount it. So, this is first year's discount that at the end of the 1 year when you are going to get back something. So, it is equal to 1000 into say, 1 divided by 1 plus, 1 divided at 1.10, because this is 1 plus r, this formula is basically 1 plus r. So, it is going to be 1.10, then second is rupees second 1000 is also going to come to us.

So, in this case, this is going to be again 1 divided by 1.10 this again we have to find out. So, this is we have to take this 1.10. So, here it is, but we have to we are getting it after, we are discounting it for 2 years. So, it means we are discounting it for 1 year, we are discounting it for 2 years and here we are again getting some amount back and that amount is how much again 1000 and again 1 divided by 1.10 that is a discount factor and how many years we are getting after 3 years.

So, we are discounting it for 3 years. So, we have found it out that if you want to find out this value. So, how much it is going to be this amount will become in the first case 1000 multiplied by this factor will become how much 0.9091. This is a factor value you can find out these values directly from the table also. Here this amount will become that is 1000 but it will be no less than something that is 0.8. This value is 8264.
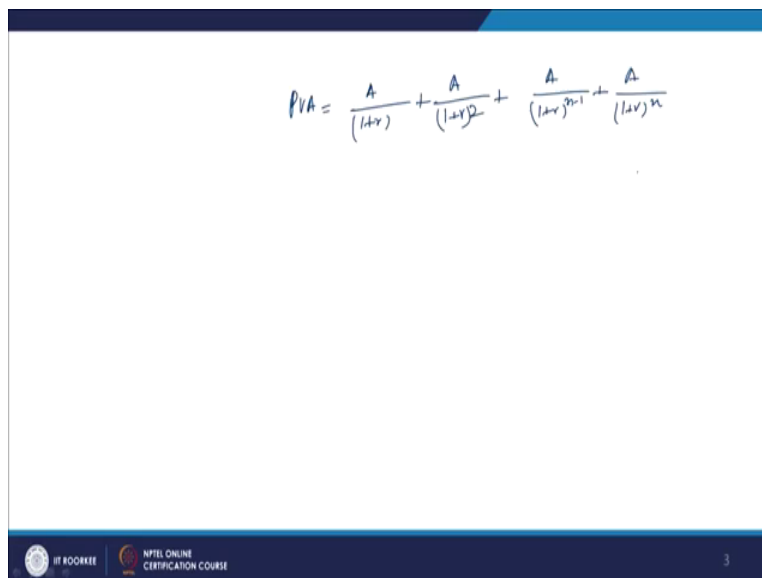
This value becomes 1000 and multiplied by that this factor value. If you find out from the table, this becomes 0.7513. Now, if a total of these values and means if you multiply and total up these values, you will find out the sum of these values will be how much? The sum of these values will be somewhere 2486.8, 2486.8. So, this is going to be the present value of the annuity.

This is the annuity you are going to get. This is the number of cash flows you are going to get and if you look at this cash flow, you are going to find out that 1000 rupees, which you are going to get at the end of every year for the period of next 3 years, then total value of that means the total inflow coming to us in the absolute value is 3000 rupees, but if you are discounting it against a discount rate or the cost of capital of how much 10 percent

So, finally you are going to get back something and that amount is 2486. So, as I told you that we can assume that we are making investment of how much rupees 2000. So, investment of 2000 rupees is made today and we are getting 3000 rupees over a period of next 3 years 1000 rupees each. So, the total sum value of that becomes 2486.8. So, it means you can say if you want to find out the NPV, this is the present value, we will call it as the present value and if you want to find out the net present value.

So, this will become how much total present value of the inflows minus the present value of outflows and this is 2000 rupees. So, this amount becomes how much 486.8. So, it means, this is a surplus you are going to get against 2000 rupees of investment you are going to get back 2486.8. So, this is a net present value of this investment we are going to get and this investment you can say is worth making and we should go ahead with this investment. So, we all means note that how the present value can be calculated.

(Refer Slide Time: 27:58)

So, in this case, I would like to explain it to you, what is a formula for the present value interest factor for calculating the present value of annuity, you have to run this model like this, this model is say 1 plus r and plus A. So, it means 1 plus r and A number of years are 2. So, this is a 2 plus, you can take it to further that is to the A divided by 1 plus r, n minus 1 and finally, taking it further A divided by 1 plus r 1 plus r power n.

So, this is the model which you can make use of if you make use of this model. So, with the help of this model, you will be able to find out the total present value of the annuity factor and for this factor, if you want to apply, this will find out the value of the say present, any present value of any annuity, you can easily find out. So, it means this very easy model and very you can call it as useful model for calculating this. So, in this case for the further estimates or evaluation of anything, if you want to go ahead about it finally, this model, what it becomes this model becomes something like this.

(Refer Slide Time: 29:14)

PRESENT VALUE OF AN ANNUITY

$$\text{Present value of an annuity} = A \left(\frac{1 - \frac{1}{(1+r)^n}}{r} \right)$$

Value of $PVIFA_{r,n}$ for Various Combinations of r and n

n/r	6 %	8 %	10 %	12 %	14 %
2	1.833	1.783	1.737	1.690	1.647
4	3.465	3.312	3.170	3.037	2.914
6	4.917	4.623	4.355	4.111	3.889
8	6.210	5.747	5.335	4.968	4.639
10	7.360	6.710	6.145	5.650	5.216
12	8.384	7.536	6.814	6.194	5.660

$$PVA = \frac{A}{(1+r)} + \frac{A}{(1+r)^2} + \frac{A}{(1+r)^{n-1}} + \frac{A}{(1+r)^n}$$

This is the model, because if you solve this particular case, if you carry it on taking A as the common outside. So, this will become something been divided by 1 plus r, 1 divided by 1 plus r square plus 1 divided by. So, taking the A common, so if you take this out A, so what it becomes this model becomes something like this A we have taken as a common outside so, this is into 1 plus 1 minus 1 by 1 plus r power n divided by r that is again the discount rate.

So, this is the way the process how the discount factors can be calculated how the present value, how the present value interest factors can be calculated, not discount factors but the how the present value and trust factors can be calculated, and for making the things easy for making the

life easy now, these are all interest factors are available to us in the readymade tables. Again with every book of financial management I am, I repeat that this table is available.

So, you can find our present value interest factor table is available at the end of every book and you can say readily you can pick the values here on this side you are given the number of years this side you are given the interest rate for any interest rate or any discount rate, discount rate is basically the cost of capital. So, for any discount rate, you want to discount the future value of the future cash flows all that which are normally coming in the annuity.

So, if you want to say discount it this table is only for the annuity not for that and the uneven sums for the any uneven sums of the different table is there. So, this is basically for this annuity part. So, you can make use of this to simply what is the value given here as the cash inflow for example, it is 1000 we have seen in our example, simply you take how many number of years 4 is a percentage, we are going to get 10 percentage.

So, you find out this is the 4 and 10, 3.170. So, 1000 if you multiplied by the 3.170 this amount will become 3170. So, it means very easily it can be done this table is available with us and right, state way, by picking up the value from the table the interest factor from the table, multiplying that sum we are going to receive as a annuity cash flow, we can find out the discounted value. So, this model we have means to make use of and this model is used for calculating the present value interest factors, all these factors are calculated with the help of this model. Thank you very much!