

Financial Management for Managers
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Lecture 13

Time Value of Money Part IV

Welcome all, so in the previous class we were talking about the future value of an annuity and we learned about that how to calculate the future value of an annuity and there also we talked about that we have, we have to use the concept of compounding. So, we saw it that when we go for a compounding and especially when the cash flow is say is occurring at the end of the year.

So, how to calculate it then you have to be very careful that only compounding has to be done for one year lesser. Total time period is five years in this case, but since we are giving our money, we are making the investment at the end of the year. So, it means what is the total time available for the first investment four years. So, second, third, fourth and fifth year.

So, we have we will not be compounding it for five years, we will be compounding it for the four years. But yes, if you are giving the investment in the beginning of the year, then for calculating the future value of an annuity, you have to compound it for the means compounding will done for the five years.

So that differences that in the beginning we discussed that the timeline of the, this time value of money. So, when we were talking about that timeline there, it's very important that at what point the point and the period two important things at what point we are making the investment and for what period we are making the investment if we are making in the beginning of the year, then total point is the beginning of the year.

So it means every year we have got the full five years. So it means the compounding of that will be done for a period of first amount which we are giving that will be compound for five years. Second for the four years, third for the three years second for the two years and the last one means that the fourth one for the two years and the last one for the one year.

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Future Value of an Annuity

Suppose X deposits Rs 1000 annually in a bank for 5 years and his deposits earn a compound interest @ the rate of 10 percent. What will be the value of this series of deposits (an annuity) at the end of 5 years? Assuming that each deposit occurs at the end of the year.

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FUTURE VALUE OF AN ANNUITY

• An annuity is a series of periodic cash flows (payments and receipts) of equal amounts (Ordinary or Deferred annuity & annuity due)

1	2	3	4	5
1,000	1,000	1,000	1,000	1,000
				+
				1,100
				+
				1,210
				+
				1,331
				+
				1,464
				Rs.6,105

• Future value of an annuity = $A \frac{[(1+r)^n - 1]}{r}$

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But if it is being given as in this case, we have shown that we are giving this investment at the end of the year it is clearly written that the investment is assuming that each deposit occurs at the end of the year, then what we are doing here is this value we have calculated here is that is 1464 is that is by compounding at the rate of given rate of interest and that rate of interest is 10 percent.

So, we have compound this for a period of four years, this for three years, two years, one year and zero years. So, this we have to be very, very careful about the timeline of the say, interest factors or for the compounding.

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WHAT LIES IN STORE FOR YOU

Suppose you have decided to deposit Rs.30,000 per year in your Public Provident Fund Account for 30 years. What will be the accumulated amount in your Public Provident Fund Account at the end of 30 years if the interest rate is 08 percent ?

The accumulated sum will be :

$$\begin{aligned} & \text{Rs.30,000 (FVIFA}_{8\%, 30\text{yrs}}) \\ &= \text{Rs.30,000} \left[\frac{(1.08)^{30} - 1}{.08} \right] \\ &= \text{Rs.30,000 [113.283]} \\ &= \text{Rs.33,98,490} \end{aligned}$$

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Now, I was talking to you in the previous class that we will discuss some applications of the say future value of an annuity. So here are means these are the very day to day decisions which we take in our life also, and we want to answer these questions or we want the answer of these questions to ourselves also, not necessarily that we are a finance manager, we in the normal terms also we want the answers to these questions and when we want the answers to these questions. So, means it is very helpful in our day to day life also, in the corporate life also or in any investment decision, we get the answers to very relevant questions.

For example, what lies in the store for you this is the first application how you can answer this question. Suppose, you have decided to deposit 30000 per year in your public provident fund or anybody xyz, any person has decided to deposit 30000 rupees per year in his public provident fund account, PPF fund account which is very common, we all make investment in the PP fund accounts or PPF accounts.

For a period of 30 years of means we can decide this period depending upon our life and then the service and the income level, what will be the accumulated amount in your public provident fund account at the end of 30 years, if the interest rate is 8 percent if the interest rate is 8 percent right. Now, how it has to be answered? How it has to be means (04:19) in and what how answer we have to get for this, because we have already got that future value interest factor annuity module.

This model we have discussed here this model is here, this model is already given by applying this model here simply we can get the answer to this question and here it is, the accumulated sum will be how much rupees 30000 and future value interest factor for annuity means at one important point of caution here I would like to share with you, when you look for the future value interest factor tables, there will be two tables in every book if you open Prasanna Chandra financial management who Prasanna Chandra the book which I am following here for all this discussion.

If you open that book at the end, the book two tables are given one table is with done which is giving you the FVIF future value interest factor, future value interest factor if you take if you see that interest factor that is only for a single amount, but if you want to find out the future value interest factor for an annuity, then there is annuity table which is called as future value FVIFA. So, you have to refer to that table because models are different. So, values are going to be different.

So, the say interest factors are also going to be different. So, be careful there is one table which is called as the FVIF table and annuity is called as the FVIFA table. So, for the different purposes if it is a single amount, then you see refer to FVIF table but it is annuity amount you refer to FVIFA table in every book of financial management two tables are given. So, for the no annuity is different, and for the single amount it is different. So, look for the right interest factor or if you want to apply the model here, we have seen the module.

So, here what is this $1 + R$ power 30, number of years is 30 minus 1 and divided by the interest rate, what we are expecting the interest rate these days roughly the interest rate on the PPF account is 7.8 or we are assuming it as a 8 percent. So, if you with the help of this model, if you want to calculate this rate of interest, this works out as something that 30000 multiplied by this interest factor PBIFA factor and this amount becomes as 3398490 rupees.

This is the total amount which will be becoming, so it means whatever the total return I am going to get back is this total return is going to be there with the help of this when we are investing a certain sum of money rate of interest has come down to 8 percent period is 30 years. So, what we are giving is this amount we are amount we are not giving, say for example, a period of 30 years and this amount is 30000, this amount works out has to be how much nine lakh rupees.

So, this nine lakh rupees, we are not giving in one go. This amount we are giving in how many this amount we are giving in 30 installments, this amount we are giving to the fund in 30 installments for a period of 30 years 30000 rupees invested and the rate of interest we are expecting is if it is 8 percent then how much this amount will become.

So, this is basically you have to calculate with the help of the present, future value interest factor for annuity process. And there you can apply the model also you can put the values in the model, but for the simplified purpose you put this amount and apply it by the interest factor and the interest factor if you look at it in the table, that interest factor becomes as 113.283 and total amount becomes as 3398490 rupees.

So, this is one very important question which we can answer with the help of the future value interest factor for annuity process. Then second important question second important application of the say future value of annuity. And here is again a very interesting question, how much should you save annually for a particular purpose, you want to achieve one particular purpose and to achieve that particular purpose, how much should you save annually? This is our question.

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HOW MUCH SHOULD YOU SAVE ANNUALLY

You want to buy a house after 5 years when it is expected to cost Rs.2 million. How much should you save annually if your savings earn a compound return of 12 percent ?

The future value interest factor for a 5 year annuity, given an interest rate of 12 percent, is :

$$FVIFA_{n=5, r=12\%} = \frac{(1+0.12)^5 - 1}{0.12} = 6.353$$

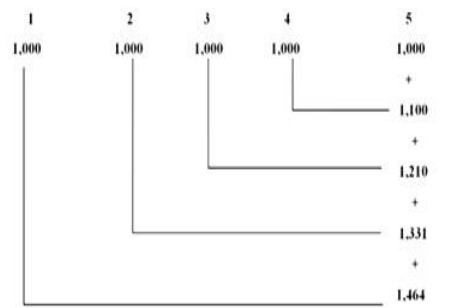
The annual savings should be :

$$\frac{\text{Rs.}2,000,000}{6.353} = \text{Rs.}3,14,812$$



FUTURE VALUE OF AN ANNUITY

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• Future value of an annuity = $A \frac{(1+r)^n - 1}{r}$

Rs.6,105



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The accumulated sum will be :

$$\begin{aligned} & \text{Rs.30,000 (FVIFA}_{8\%, 30\text{yrs}}) \\ &= \text{Rs.30,000} \left(\frac{(1.08)^{30} - 1}{.08} \right) \\ &= \text{Rs.30,000 [113.283]} \\ &= \text{Rs.33,98,490} \end{aligned}$$



Now, what is the question? You want to buy a house after five years or I want to buy a house after five years, when it is expected to cost me twenty lakh rupees or two million rupees. After five years I want to buy a house I know that what are the prices of the houses in the market today, I want to I am planning to buy a house after five years because today it do not have the money to invest in that say investment that purpose.

So, I will have the sufficient amount of the money or some time it may be possible, I do not require a house today, I will be requiring the house five years from from now. So, after five years, when I want to buy a house, that house will cost me expected to cost me for 2 million rupees or 20 lakh rupees. How much should you save? How much should I save annually, if my savings or your savings earn a compound return of 12 percent?

Interesting question. How much should you save? Or how much should I save annually? If my savings or your savings earn a compound interest of compound return of 12 percent? How much should I save? Now the future value interest factor for a period of five years annuity we have to apply here because we want to find out that amount, one single amount, which I want to save annually every year I want constantly I want to save the same amount. So, amount will remain consistent amount will remain the same.

So, if I want to save that amount, the future value interest factor for a five years and annuity and at the given rate of interest of 12 percent, we have to calculate at the given rate of interest of 12 percent we have to calculate. So, we are applying the concept of future value interest factor for

annuity, number of years n is 5 and then R is 12 percent number of years is 5 R 12 percent and then if you apply this again the same thing is coming again the same model we are applying, this model we are applying here. So, when you apply this model, so you are getting this and this is the $1 + 0.12$ power 5 minus 1 divided by 12 percent rate of interest.

And this factor comes up with this. This factor comes up with this that is 6.353, if you go to the table, future value interest factor for annuity table in any book, then you will find this factor is there, this factor is already calculated there provided your number of years means for different number of years you will find it out 1 2, may be next 20, 30 years the factors are given and the for the different rates of interest 2, 4, 5, 2, 4, 6, 8, 10, 12. So, 12 is covered and normally, which are the prevalent rates of interest in the market for that the interest factors are there given in the in these tables.

So, if you find this factor, we have calculated this factor our self here, we have calculated this factor our self here, but you can find it directly from the interest factor table also and the you can find out the total amount but that is twenty lakhs, two millions we want to save, I want to have means in my hands after 5 years. So, dividing it by this factor, it means this amount comes up as how much? This works out as 314812 rupees, 314821 rupees, I have to save annually.

I have to save annually to have a house after 5 years, which will cost me a sum for a sum of rupees two millions or twenty lakhs, interesting. So, it's a very important question, which we can answer in the previous case, but we wanted to find out that what lies in store for me, if I save a certain same amount 30000 rupees every year for a period of 30 years and the rate of interest is 8 percent.

This is a question in this case we answered how much should you save annually to achieve a one particular objective. Provided, you know that means, the cost of that objective and the rate of interest available on our investment, you can easily answer this question and this question is going to help us to find out that if you want to find a house of twenty lakh rupees, you can save annually 314812 rupees and after 5 years if you earn the interest of 12 percent on your, that a precondition, be careful.

That whatever the amount you are saving here, 314821 rupees that should earn a minimum rate of interest or 12 percent if it earns at 12 percent, then certainly this amount will become twenty lakhs or two millions at the end of the 5 years.

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ANNUAL DEPOSIT IN A SINKING FUND


Futura Limited has an obligation to redeem Rs.500 million bonds 6 years hence. How much should the company deposit annually in a sinking fund account wherein it earns 14 percent interest to accumulate Rs.500 million in 6 years time ?

The future value interest factor for a 5 year annuity, given an interest rate of 14 percent is :

$$FVIFA_{n=6, r=14\%} = \frac{(1+0.14)^6 - 1}{0.14} = 8.536$$

The annual sinking fund deposit should be :

$$\frac{\text{Rs.500 million}}{8.536} = \text{Rs.58.575 million}$$



Next question is annual deposit in a sinking fund, annual deposit in a sinking fund. Now, what happens many times, this problem comes to the companies which in the corporate life, this question is more relevant. Because corporate companies confront the problems like that when they borrow money from the market. And say for example, in the form of the bonds or debentures, when the bonds are issued in the mark market or the debentures are issued in the market, then these bonds or debentures are issued by the companies for a certain stipulated period of time.

For example, 5 years, 7 years 10 years. After ten years, those bonds have to be redeemed by the companies, those bonds have to be redeemed by the companies. Means, bonds have to be bought back and the investment made by the say buyers of the bonds has to be returned back with the interest. Normally, if it is a, different options are given by the companies, but if people choose to the cumulative interest option, then people do not means proffer to get to the interest annually back on that investment, but they choose to the option that at the end of the say at the time when these bonds will be redeemed,

I would like to get the in that principle amount along with the interest. So for that, what has to be there because company knows that we have to return back after five years or after 10 years a sum of rupees five million, fifty lakh rupees which we are borrowed from the market. So, it means, if you immediately at the end of the fifth year or the end of the tenth year, you have to arrange a sum of rupees fifty lakhs, it may not be possible for the companies that that amount is not available.

So, they plan it that every year they keep on depositing something in some fund which is called as a sinking fund, they keep on depositing either they buy some say investment product. So, they keep on depositing that money equal amount of sum in that, say investment account or in that investment product or within the firm if they use the funds then they keep a proper account of that and they provide every year the interest on that.

So, money keeps on accumulating in that account. They use that money internally but they create a proper account and they know it that after 5 years or after 10 years when we have to return fifty lakh rupees back with interest. So, it means, we need the money, so they have already created a sinking fund where they are annually saving certain amount. So, that amount becomes that amount in that fund at the end of that period becomes fifty lakhs plus interest on that, equivalent to the same amount so that you liquidated the fund at that time and then easily that liability can be paid off.

Second situation can be that in case of the say replacement of the fixed assets. For example, there is a plant which is purchased by the company, they are using it and the life of the plant is for example, ten years and say, the cost of the plant is say one crore say ten billion rupees. So, the total life of the this plant is one crore rupees, you know, that the life of the plant is 10 years only and after 10 years, we have to replace this existing plant with the new plant or the existing machinery with the new machinery.

So, we have to be very careful that we have to means we know the process of depreciation, every year we charge the depreciation on the existing plant of that investment of one crore rupees at certain say given method of depreciation, we debit with that amount of depreciation every year the profit and loss account and then from the profit and loss account. Because since it's a non

cash expense, so that money does not go anywhere that is taken out from the profit and loss account and is kept safe.

So, that kept safe is that can be invested in some fund and that fund is also called as a sinking fund at the end of the ten year, when this asset will come the value useful life of that plant will become zero or that plant will become unusable. So, we will have to replace it at that time. So, we have collected the sufficient amount through the process of depreciation.

So, and we are depositing that depreciation amount in some sinking fund account. So we will sell off that sinking fund at that time, we will liquidate the sinking fund at that time. So entire one crore plus interest on that will come back to us and probably that will be the price of the new plant along with the, see the basic amount of the one crore plus increase in the price because of the inflation. So, means the company is at a very comfortable position.

So, maybe in case of the redemption of the bonds or debentures or for the replacement of the existing plant and machinery, the firms require funds in the lump sum after a certain at the end of certain period of time and at that time the funds may not be available. So, they planned the things in such a way in advance that when that replacement or requirement of the fund arises, then they have already collected the fund by setting aside a certain amount every year plus interest on that, that at the end of that period that fund is liquidated and we get the funds readily available in our hands.

So, here is the question and will deposit in a sinking fund for example, you want to say a say replace a plant of one crore rupees for creating a sinking fund how much amount you have annually to deposit in that. So, that at the end of ten years, it becomes say the same amount One crore plus interest and that may be the same amount, which is the price of the plant at that time in the market.

Here it is, Futura Limited has an obligation to redeem 500 million bonds and a 500 million, 6 year, 500 million bonds 6 years hence. Futura Limited has an obligation to redeem 500 million bonds 6 years hence. How much should the company deposit annually in a sinking fund account, where in it earns 14 percent interest to accumulate rupees 5 million in 6 years of time.

They have to return the five million rupees these are the bonds issued that amount may become the total amount that is the say principal plus interest or it may be possible. The company is paying their interest annually from the revenue incomes and the principal amount of 500 millions will be paid back after this 6 years. How much should the company deposit annually in a sinking fund, now they know it that this liability will arise at the end of the sixth year.

So, we should be planned enough in advance we should be knowing that we have to create a fund. So, that every year from the profits we are earning, when we are apportioning that profits distributing their profits, we will have to set aside a certain amount, we will have to set aside a certain amount. So, that we are safe that are the time when the requirement of the funds that arises, we are very safe at that time because we have made the proper arrangement for the funds in the beginning you can call it as the financial planning also it is a part of the financial planning.

So we are financially very planned. We have done this financial planning beforehand and now we are very safe and secured. So after the six years at the end of the six years, when 500 million rupees will be required to be paid back to the bond subscribers. So, in 6 years period of time, that how much the company has to save? It means the question is how much should the company deposit annually or save annually in a sinking fund account wherein it was 14 percent interest to accumulate rupees 500 million in 6 years period of time.

So, it means, one condition here is interest is 14 percent interest is 14 percent. So, we have to be very careful, what is the rate of interest because the moment the rate of interest changes your this your installment to be deposited in the sinking fund will change. So, we apply the same concept of the future value of annuity, because we want to find out the same amount to be deposited every year in the sinking fund account.

So, if you apply here it the future value interest factor four or five years annuity given and interest given at the end interest rate of the 14 percent is you can calculate here is that is. So, it means when we are making the investment here, what is the total period? 6 years. But we are assuming here, because we are writing here five years annuity, five years annuity means every year when we will transfer the investment to sinking fund that will be invested at the end of the year.

So, at the end of the first year, you will transfer for some amount at the end of the second year you will transfer some amount. So, at the end of the sixth year, you will transfer some amount. So, we are going to create the interest factor for a period of 5 years annuity for a period of 6 years.

So, it means when you are going to create this annuity for 5 years, we are going to find out here that is the future value interest factor and n is 6 years R is the 14 percent and when you are means applying this model here you are getting effective here. When you are getting the factor here 8.536, this factor is coming up simply as we did it in the previous problem that you got this factor for a period of six years, we have charged the interest because the amount of the funds will be available, we are liquidating this say annuity at the end of the six year will be liquidating it, So, we will be calculating this for the period of 6 years.

So, it means if you try to find out this factor has come with us, and if you divide the 500 millions with this factor. So, it means this amount comes up as how much 58.575 million rupees you have to save to collect a total amount of the 500 million rupees. Annually, this amount has to go to the sinking fund 58.575 million rupees has to go to a sinking fund. So, that at the end of the period of six years, this amount becomes and the rate of interest they have already factored, we have already factored this.

So, they have calculated this factor, future value interest factor we have already calculated. So, rate of interest is already taken care of. So, now you have to save rupees fifty eight lakhs or 58.575 million every year and transfer it to the sinking fund. So, that at the end of the 6 years when there will be the need for the 500 million rupees for the redemption of these bonds or whatever the obligations of the different bond subscribers is this money is readily available with us.

So, very interesting question that whether it is the redemption of the bonds, whether it is the replacement of the plant or machinery, or whether it is any kind of other kind of the fund you require, we should be planned enough in the beginning. So, that we know that some financial requirement is going to arise, say 5 years from now, or maybe a 10 years down the line. So, we should be very clear about that we should be well planned in the beginning.

And if we know that this much is going to be a requirement, if we keep on setting aside certain amount every year, and plus, you know the interest we are going to earn on that fund every year than I think we are at a very safer position. So it does not put any pressure in the mind of the CFO or maybe in case of the individual investors in the mind of the individual investors that he has already planned for his future. And what is now is going to his financial requirement every he is setting aside a certain amount.

So, he will have the sufficient amount at the end of the given amount of the period or the number of years. So, now, there are some other kind of the say applications also. So, for example, finding the interest rate or maybe some other say how long should I wait? Or maybe there are some other important questions are there also which we can answer with the help of your this future value interest factor for annuity.

So, means we have discussed to some to three applications till. Now, say three applications we have discussed today in this class and remaining applications plus some other related concepts of the time value of money I would discuss with you in the next class. For this class, I will stop it here and thank you very much.