Financial Management for Mangers
Professor Anil K. Sharma
Department of Management Studies
Indian Institute of Technology, Roorkee
Lecture 12
Time Value of Money Part III

Welcome all. So, in the previous class we were learning about the power of compounding and we learned about that how we can calculate the future value of a single amount. And what is a basic difference between say, compound rate of interest and the simple rate of interest?

At what rate the money grows, the future value of the money grows when you are say investing at the compound rate of interest and at what say, pace the investment grows when we are investing at the simple rate of interest. So, this is the some basic difference we could discuss and we could learn the compounding and the simple rate of interest.

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So, here I would like to say explain it to you now that how you can graphically show or what is the difference between these two rates of interest. You can show it something like this. Here it is we are taking as a period and here, we are taking as the future value of any amount, of any investment this is a future value.

So, if you talking about the simple rate of interest, it goes like this. It increases but it goes like this. It increases like this but when you talk about the say, compound rate of interest, it goes like this. So, this is the difference in this case.

This is a difference, you can understand that initially it grows at a lesser pace but then later on because in this interest on the interest, interest on the interest, you can understand. So, that this growth pace is increasing and this is going at a simple pace at this period of time.

I am showing this that you money is growing at the simple rate of interest is that, this rate of interest is going up like this. But in this case, it is growing at a very spectacular rate, at a very faster rate because after some period of time, base becomes flatter, base becomes bigger. Base becomes flatter and when the base becomes bigger or flatter then automatically the growth rate becomes very fast.

We invested 1000 rupees for a period of 12 years, but after some period of time because we are not withdrawing the interest so at the say, you call it as, at the say your what do we say here is, that at the end of the 10 years or at the end of the 5 years, at the end of the 8 years, your total amount will become very high because the base of that investment is also becoming bigger and we are adding further interest on that.

So, this is how you can show the difference between the simple rate of interest. This is basically the simple rate of interest, this is a simple interest and this is the compound interest. So, the difference can be seen with the help of this structure and we have practically also seen this difference.

Now, I would like to discuss with you one shortcut here that if you do not want to put into the this all intricacies of the future value interest factor table or applying the future value say, interest factor model and that all calculations of future the present value into 1 plus R power n.

All these things if you do not want to do then we have got the very simple rule here also and these called, these are called as the say, doubling period rules. We call it as the doubling period rules or the rules of the thumb of the doubling period, right.

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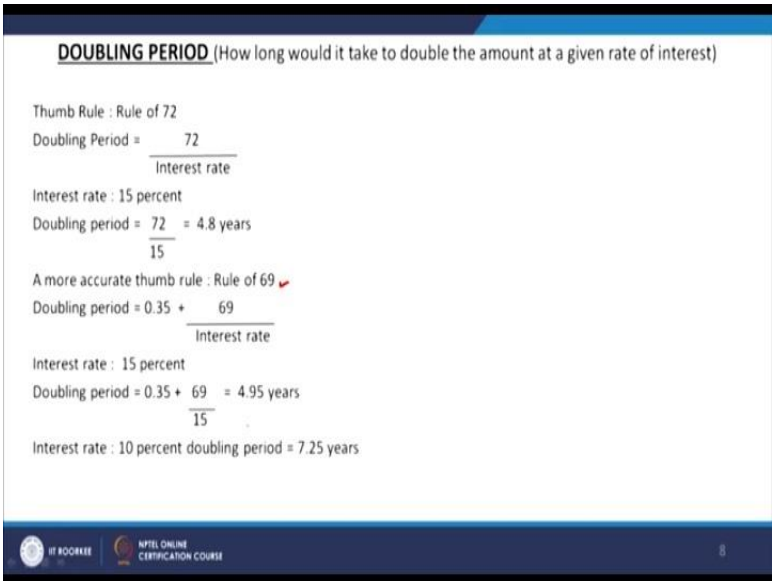

If you want to calculate the doubling period of something then the rules of thumb can be applied here and how you can apply the rule of thumb here, that for example the question here is, how long would it take to double the amount at a given rate of interest?

Any amount 1000, 2000, 10,000, 20,000, 100,000 or 1 million rupees. How long would it take to double the amount at a given rate of interest? If you want to find out that period, number of years, that in how many years your amount will be doubling or amount will become double?

If you know the say the present value of the investment or the investment which we are going to make and if you know the rate of interest then simply by applying this rule of thumb of the doubling period we can have, we cannot get the exact value, means the exact number of years but we can we have the approximate number of years that we can reach nearer to the exact value if you calculate the value by applying the future value interest factor table or you follow the proper say you can call it as mathematical process.

So, simple rule of thumb given here is, that is the doubling period rule of thumb is the rule of 72, rule of 72. So, whatever the investment we are making and if the rate of interest is given to us. Any investment which is invested or any amount with the present value, if it is invested in any avenue of investment and that avenue of investment is giving us 15 percent rate of interest.

So, what you have to do here is, you simply divide this is a standard rule of thumb and this rule is called as the rule of 72. This rule is called as the rule of 72 so it is a doubling period and you simply divide this 72 by that given rate of interest and you can find out that period by which your amount your investment will become double.

For example, doubling period here when we are calculating we are taking the rate of interest, assuming the rate of interest as 15 percent. So, you are dividing 72 by 15 percent, number of years are coming up as 4.8 years, about 5 years. About 5 years, within a period of 5 years, your investment will become double.

Whatever the amount of investment you take, if you know the rate of interest which is going to be paid by that avenue of investment then we can find out the doubling period. 4.8 will be the number of years, and we have seen means long back, 20 years back or say, 25 years back the rate of interest was something like this, 15 percent, 12 percent or 13 percent.

Even the banks in India were also paying this kind of the rate of interest and at that time, any investment we were giving to the banks, that was doubling in 5 years period of time. These days it has become more than 10 years, sometimes say 12 years also, because rate of interest has fallen.

But if the rate of interest is very high, 12 percent, 13 percent or sometime 15 percent then any amount given to any investment avenue can become double in this period of time. So, rule 72 says you divide the 72 the, this say 72 figure of 72 by the given rate of interest.

That figure which comes out is the period by which the given amount of investment will become double. And if you want to become more precise, if you become more nearer to the truth then you can call use the another rule and that rule is called as the rule of 69. That rule is called as the rule of 69.

This is a more accurate rule of thumb. More accurate as compared to the rule of 72 but here, you have to add something more in this, say, formula that is 0.35 plus 69 divided by the rate of interest. Earlier it was 72 now it is 69 divided by the rate of interest.

So, the interest rate is again 15 percent and doubling period we are calculating is, so we are, we are more precise. Earlier were saying approximately 4.8 years. But now,

by applying the rule of 69, we have been able to find out that the rule of doubling, sorry, the amount of period of doubling your investment, whatever the amount of investment is. If the rate of interest is 15 percent then by applying the rule of 69, our doubling period is again about 5 years and exactly it is 4.95 years, 4.95 years.

So, and if you lower down the rate of interest for example, it is not 15 percent, for example it comes down to 10 percent then what is happening here? Then the doubling period is increasing. That now, in the 7.25 years, the total amount of the present investment will become double. If it 10,000 rupees after 7.25 years, it will become 20,000 rupees from 10,000 to 20,000 rupees.

We are we are see, having or getting the interest at the rate of 10 percent. So, two rules are, very simple rules are available here in the literature, that rule of 72 and rule 69 so if we do not want to go into any kind of the intricacies of say, applying the future value interest factor and then say applying the future value interest factor model 1 plus R, present value 1 plus R power n.

If we do not want to say indulge into all this kind of the calculations then simply by these two direct rules, or the direct rules of thumb, you can calculate the or we can calculate the say amount by which our investment will become double. This is a another important corollary of the time value of money.

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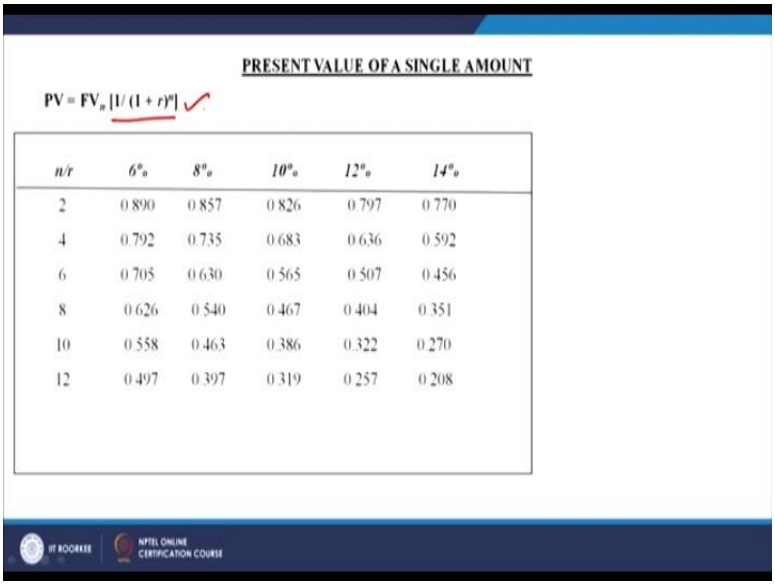

Now, we will go the next part and that next part is present value of a single amount. What we discussed earlier? We discussed earlier was the future value of a single amount. Now, we will discuss the present value of a single amount reverse. Earlier we

were calculating present investment is going to become how much after 5 years, after 10 years or after 8 years?

Now, we are going to learn reverse that the present value of the future cash flow, a single amount which we are going to get 5 years from now then what will be the value of that amount currently, if you want to compare it in today's terms then what is the value currently of that amount which you are going to receive after 5 years or 5 years down the line.

So, present value of a single amount and now if you look at this model, just carefully look at this model. If you look at this model, this has become reciprocal of what the model of the future value interest factor. There, in the future value interest factor it was, that is the 1 plus R power n.

1 plus R power n, and in this case it has become now 1 divided by 1 plus R power n,. So, now this has become the discount factor. That was the interest factor, future value interest factor. This is the present value discount factor you call it as. This is a present value interest factor of, we call it as the present value discount factor.

So, for any rate of interest at which we want to discount and for the number of years for which the, after which the cash is cash flow is going to come to us. If you want to convert, say any amount coming to us after 5 years and we know the our cost of capital.

Now, how you have to find out this discount rate? This should be minimum our cost of capital. This should be our minimum our cost of capital or opportunity cost of capital so if I know the discount rate, if I know the amount which I am going to get after that given period of time and if I know their period also exactly then I can calculate the present value of that amount and if somebody says that after 5 years I will pay you 10,000 rupees for any means, the work I am doing or any investment I am making so I should be means thinking in today's terms.

That after 5 years if I am getting some amount back to me, what is the value today to be. Because my efforts are going today, my services I am rendering today, my investment I am making today. So, how much I am getting back after 5 years? Let me discount it and try to find out the present value.

So, present value of a single amount, here also if you want to calculate with the help of the model, you can calculate with the help of a model. But simply means as we discussed the table in case of the future value interest factors of a calculating the future value of a single amount.

Similarly, we have calculated a say table of the values and here, these are the discount factors which are available here with this, again the number of years and again the interest rates. Interest rates are normally costs of capital. So, they are available with this and we have already calculated. Therefore example any say, cost of capital is 6 percent and number of years are 2. What is the discount factor? That is 0.89.

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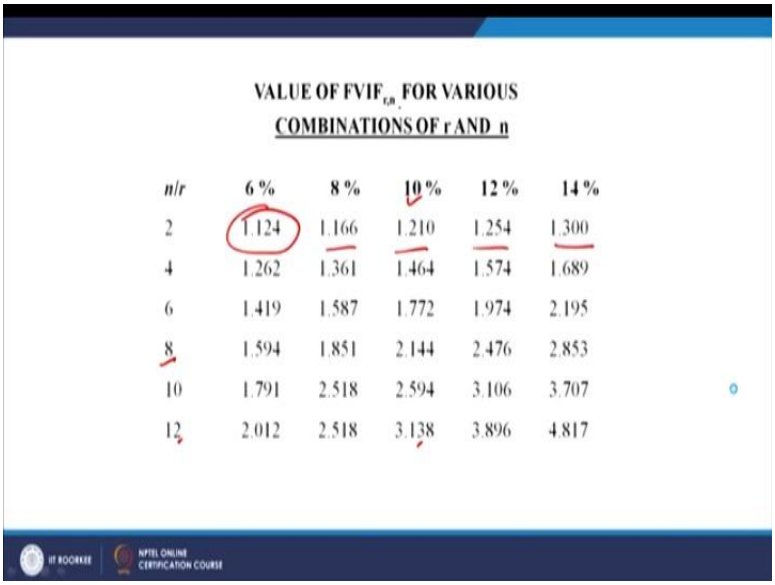

So, I was discussing with you that, for example if you go back then we have seen, I was talking to you that when you talk about the future value interest factor, this these values become more than 1 because you are say, converting the present value into the future value. And the future value will become present value plus interest. So, it should be more than 1.

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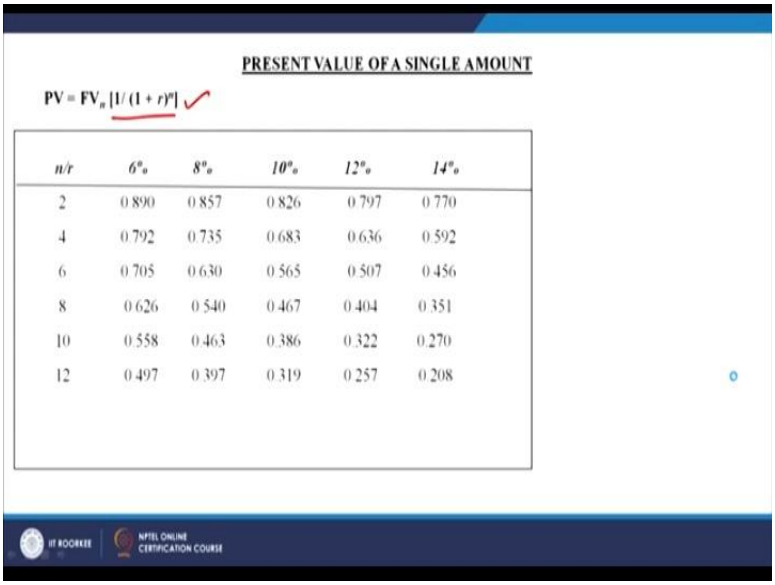

Whereas, in this case if you talk about that you are converting the future value into present value. So, future value though it will be very high in future but in present value terms, the value of that will be less than 1. What you are going to receive after 5 years 10,000 rupees.

Today that is not, say is going to be the value of that received 5 years 10,000 rupees from now, is not equal to 5 years. You have to convert that and it is something less than that. Because inflation will reduce the value of money or some other factors will play a role.

So, this in that value of that 10,000 rupees which you are going to receive after 5 years, it is going to be lesser than that. So, this discount factor is becoming always less than 1. So, it means this gives an indication that whatever the investment you are talking about, that will be certainly less than what we are expecting after 5 years.

So, this is a very clear and simpler way for the method of calculating the say, the present value of a single amount or any amount which we are going to receive in future, that present value of that can easily be calculated.

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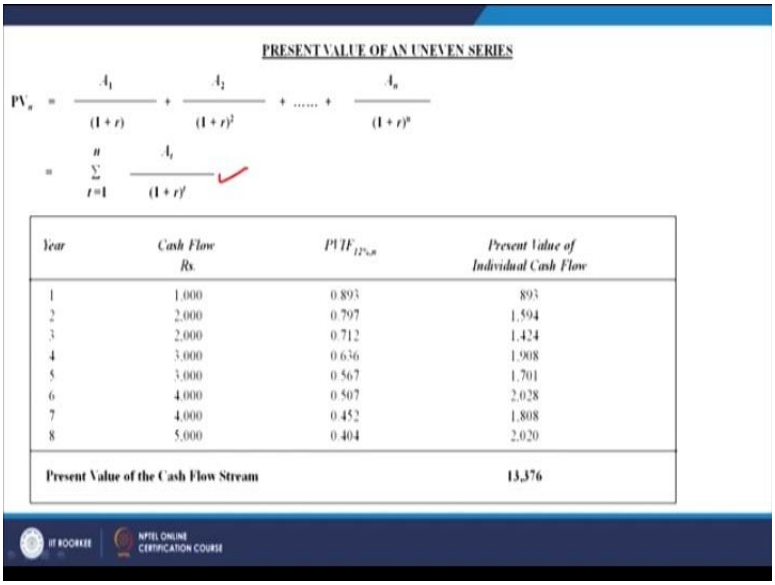

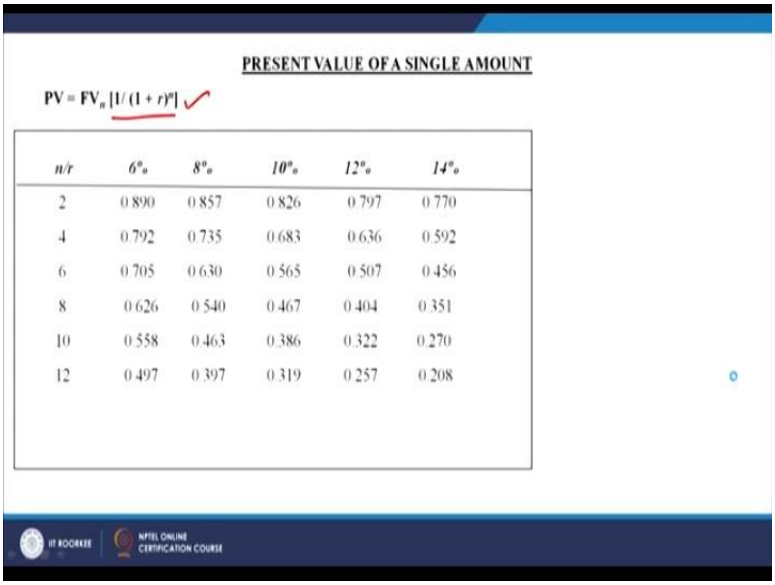

Similarly, now we talk about the next part of the present value and if you talk about the present value of an uneven series, present value of an uneven series. So, if you talk about the present value of an uneven series. Because in that basic any investment projects, the cash inflow which occurs over a future period of time, that normally remains as the uneven. That cash flow, cash inflow remains, largely it remains uneven because if any investment we are making.

For example, we invested 50 lakh rupees, 5 million rupees in any project today, any say business project today. After say, building of the project and starting the

production, at the end of the first year we got some cash flow back. I am not talking about the profits. I am talking about the cash flows back.

So, what is that cash flow? Second year, the cash flow's value may be different as compared to the cash flow we are getting at the end of the first year. Third year, the cash flow's value will be different, what we are getting at the end of the first and the second year.

So, all the 5 years or all the ten years, the cash flow's values may be different. They are not even in series, they are uneven. They are in coming in series. Every year we are getting something but they are coming as uneven cash flows. So, if we get the uneven cash flows and if we want to compare it against some, say, investment.

Then we can find out how much is the investment we are making in the current period which is called as 0 period, and how much cash flows are going to occur to us after the period of 1 years, 2 years, 3 years, 4 years any foreseeable period. So, you can easily calculate the sum total of that and then you can find out that whatever the total cash flows we are getting over the next number of the 5, 8 or 10 years, the total discounted value is going to be this much.

For example, now we have this model here. Now, the how this model applies? This model is basically if you talk about present value interest factor model is this, if you want to use it. And this is a sigma of the number of years and time periods t1. If you take this A t, A t is basically the cash flows occurring over the different periods of time and again discounted by 1 plus r power t the number of years.

If you are discounting it for 1 year then it will be 1 plus r. If it is for the 2 year then 1 plus r power 1or 3 years is so 1 plus r power 2. Like that, so power will keep on increasing as the number of years means in future the cash flow is going to, we calculated.

So, finally this way you can extend this formula and if you have seen this, finally the summarized form model of this is the present value of the uneven series can be calculated this. So, again by using this model, we already have converted that into some factors and these factors are already available with us.

Those factors from this table, we will not go into long calculations. We will use these factors from the table and if you use these factors from the table, so you can easily understand that how we can calculate the present value of the uneven series.

For example, in this case we have taken how, we have taken the 8 years. 1, 2, 3, 4, 5, 6, 7, 8 years period of time. Cash flows we have taken here is, that is 1000, 2000, 2000, 3000, 3000, 4 4 and 5000. Now, if you total it up, if you total it up how much it becomes? It becomes 3 plus 2, 5, plus 6, 11, then 15, 4 is 15.

Then it is 19 that is 24. It is becoming 24 thousand rupees. If you calculated this total amount, this becomes is the 24. Again we can check it up. This is 5 plus 4, 9 plus 4, 13, plus it is 16, 19, 19 and 5, 24. This total amount if you sum up it becomes 24000 rupees.

So, somebody may tell that if you make any investment of say 20,000 rupees or not 20,000 rupees for example, any investment of say 10,000 rupees. If you make a investment of 10,000 rupees today. After 8 years, after 8 years, I will give you return you the total amount of 24,000 rupees. So, we will be very happy that we are investing 10,000 rupees today and after 8 years we are going to get back say 24,000 rupees but we have not to feel happy.

As a student of finance you must be knowing about, we have to convert that future value of the different cash flows amounting to 24,000 rupees into the present value and that annually we have to discount the annual cash flows.

So, if you discount it, we know the, this interest factors present values interest factors are known to us and here we are discounting it at the rate of 12 percent. My cost of capital is 12 percent and n is the number of years. So, here when we calculate these factors given to us 1000 multiplied by this, this value becomes this. 2000 it is and it is this this is a factor, the value becomes this. So, finally, I am not going to get back 24,000 rupees. I am going to get back equal and to 13,376 of today.

I am giving 10,000 today so I want that, for example, after 8 years I am to get, going to get 24,000 rupees. As per today's investment, my outflow cash outflow how much I am going to get back? For example, if I give today and I have to return back today only, then how much I am going to get back.

So, I am going to get back for an investment of 10,000 rupees, I am going to get back 13,376. So, the present value of those cash flows which is going to occur to me over a period of 8 years, is going to be this much. So, this is called as the present value of the future cash flows.

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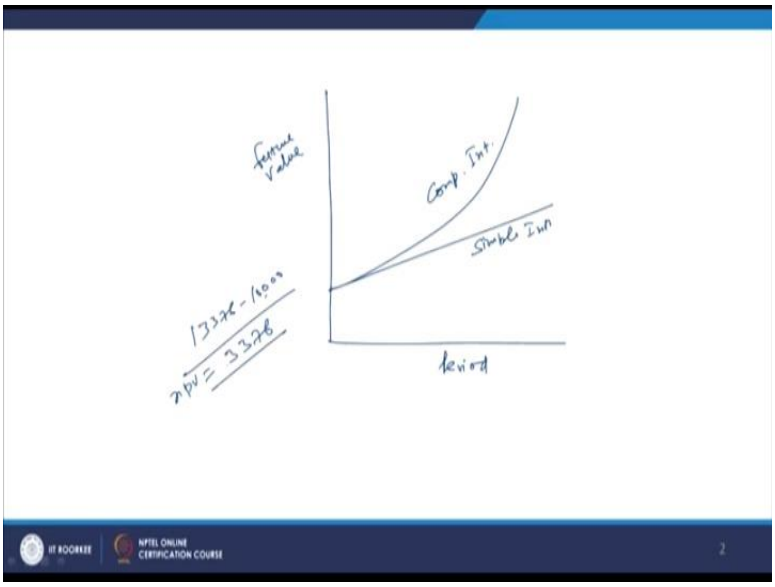

And if I calculated the net present value, so I have to calculate is the net present value will be 13,376 minus initial investment is 10,000. So, this amount will become how much? 3,376 which is in the literary terms called as the net present value. This is called as a NPV or the net present value.

So, we will calculate this net present value and this analysis we will do while we will talk about the capital budgeting. So, at that time we will discuss all those things but currently say this is the process, how we can calculate the present value of the cash flows which is coming to us in a series but the inflows are uneven. That is a series is a uneven series.

Now, here we talk about something is the future value of an annuity. Now, we talk about the future value of an annuity and if you talk about the future value of an annuity say, future value of an annuity is means we confront this kind of problem in our daily lives, in day-to-day lives also as a common investor, as a very small level investor.

We also confront with this kind of the problems and future value of an annuity that for example, I buy any investment say product and every year I keep on investing 1000

rupee in that product, in that investment fund. So, at the end of 5 years, how much amount for a given rate of interest, how much amount I am going to get back.

So, it may be possible that I know the interest rate so I want to know the total amount. I know the investment also. I am making investment, annuity means same investment you are making every year, you are not changing. That investment is not uneven but that investment is even when you are making the even amount of investment, same amount of investment every year. Then for a given rate of interest, after a certain number of years, how much that amount will become? So, future value of an annuity.

Other way round the question can be means asked is that for example, I am depositing or anybody is depositing 1000 rupee in any investment fund for a period of 10,000 rupees. 1000 is annuity means same amount every year. 10,000, 1000 rupees for a period of 10 years and somebody is promising that after a period of 10 years.

After a period of 10 years, I will be returning you back 15000 rupees. It means, in total period of 10 years how much I deposited? 1000 into 10 is the 10,000 rupees and after 10000 somebody is promising to return me 15000 rupees. So, in the question, the question will become as that what is the rate of return.

What is the interest rate I am going to get that against 10000 rupees of annual investment, if I am going to get back 15000 rupees, how much rate of interest I am going to earn on my annuity, my investment which is the annuity.

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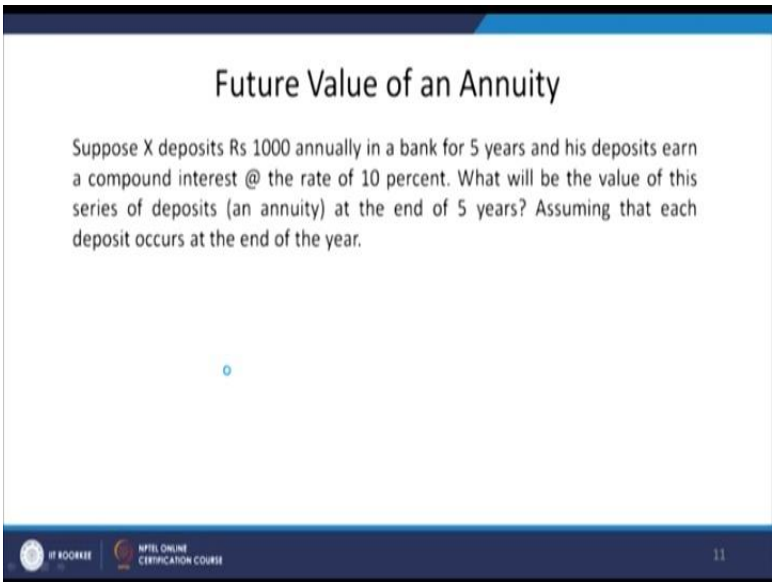

So, future value of an annuity can be calculated. So, how we can calculate the future value of annuity. For example, we have kept here is the one problem that is a, say suppose X deposits rupees 1000 annually in a bank for a period of 5 years and his deposit earns a compound interest at the rate of 10 percent.

What will be the value of this series of deposits, an annuity at the end of the 5 years? Assuming that each deposit occurs at the end of the year, assuming that each deposit occurs at the end of the year. So, it means suppose X deposits rupees 1000 annually in a bank for a period of 5 years and his deposits earn a compound interest, the interest is compound at the rate of 10 percent. What will be the value of this series of deposits, which is an annuity, at the end of the 5 years assuming that each deposit occurs at the end of the year.

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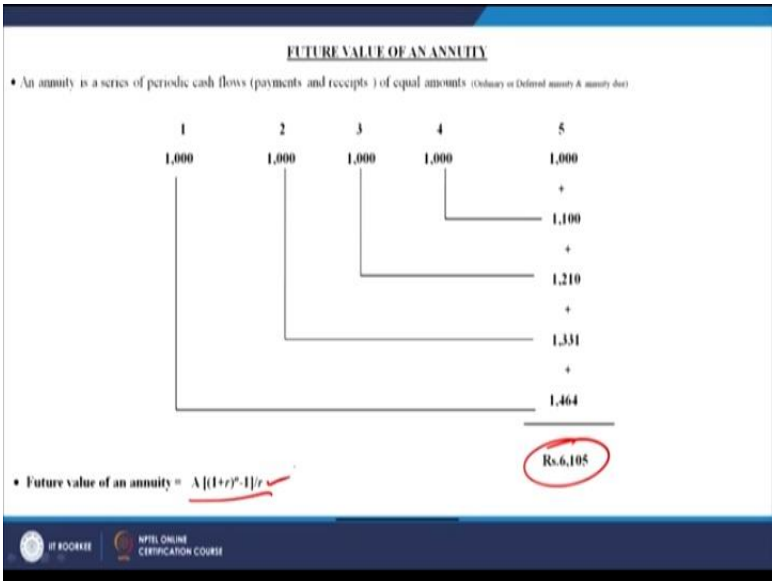

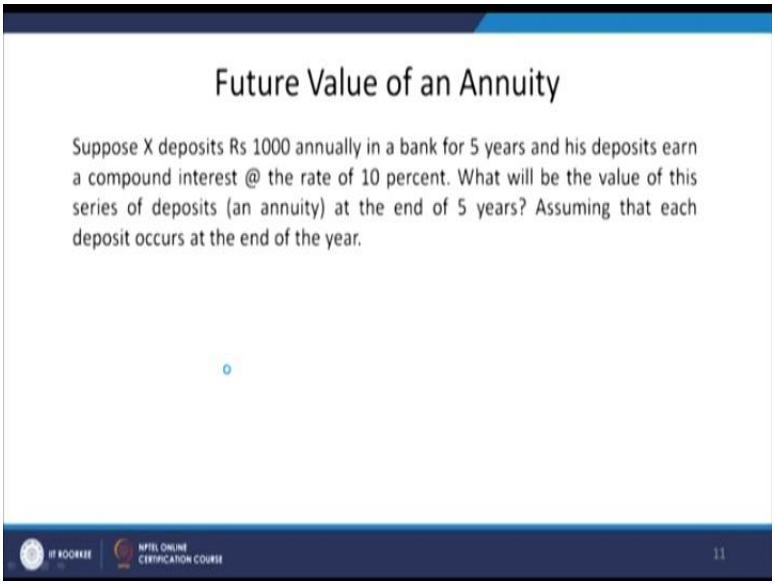

Now, how we have to explain it? The amount will become how much? 6,100 and, 6,105, this amount will become at the end of the 5 years. When compounded annually, compounding is annually, it is already given to us that when you are talking about the compounding amount, this amount is given to us.

This is the compounded annually when we are ())(24:42) What will be the value of this series of deposits at the end of 5 years and the compounding we are doing here in this case is the interest rate at the rate of 10 percent and it is a compound rate of interest.

So, when you talk about this compound rate of interest, how it has been calculated? Future value of an annuity. An annuity is a series of productive, periodic cash flows. An annuity is a series of the periodic cash flows, payments or receipts. It can be payment also, it can be receipt also, of equal amounts which we will discuss later on, ordinary or deferred annuity and the annuity due.

So, these three kind of the annuities we will discuss later on, but simply you understand at this moment that an annuity is a series of the periodic cash flows, payments and receipts, of the equal amounts.

So, how we will that amount will become? How much we are investing? We are investing 1000 rupees for a period of how many years? 5 years, and the rate of interest is 10 percent. So, if you look at this model, this model is given to you, that is the, the amount that is the cash flow here, 1 plus r power n minus 1 divided by r.

This is the model, little different model is here for calculating the future value of an annuity. Earlier what was the model for calculating the future value of a single amount? It was the present value into 1 plus r power n.

So, we are doing the same thing. Present value is A, into 1 plus r power n but now we are doing it minus 1 and whole is divided by r, that is a interest rate for calculating the annuity, that is the future value of an annuity. So, what is the number of years? We are getting 1, 2, 3, 4, 5 number of years. How much investment we are making here? 1000 each every 5 year. We are making the investment of the 5000 rupees.

Now, when we have invested first 1000 rupees, after 5 years, it is becoming, it is the say invested at the end of the year. It is clearly given in the question, assume that each deposit occurs at the end of the year. It means, first 1000 rupees has to be compounded for how many years? 4 years.

Second amount has to be compounded for how many years? 3 years. Third amount has to be compounded for how many years? 2 years. This has to be compounded for 1 year because we invested at the end of the fourth year, and fourth to fifth year, only one year is there. So, it means only one year's compounding. So when you are doing one year's compounding, what is happening? This 1000 is becoming 1100 rupees. And final amount, which you gave at the end of the fifth year, so this amount is 1000 is equal to1000 because no period has elapsed so no interest is due on this.

So, finally if you sum it up, you can calculate the future value of an annuity that that 5000 rupees which we gave, we did not gave the total amount in the beginning as 5000 rupees. We gave 1000 rupees every year in the say, same amount that is called as annuity, and at the rate of 10 percent compound rate of interest, this amount has become 6105 at the end of the 5 years.

So, this is the magic of compounding here and with the help of the say this future value of annuity process with the help of this model, which is a very important and interesting model, you can find out that if you, means keep on depositing in any fund, a certain amount every year, at the end of every year then at the certain number of years, at the given rate of interest and normally, the rate of interest are compound these days, compounded these days.

Only difference comes up is whether the compounding is annually or the compounding is at the say, bi-annually or quarterly or a monthly or daily. That difference comes up but normally, whatever the return we get these days, they are the compound interest, provided you have not opted for that you will take the interest back every year.

So, it means if you take the interest back every year then it will maybe a say, it will be a it will be a simple rate of interest but if you are not taking the interest back at the end of every year then certainly the total interest will be added back into the principle amount. So, next year the base will be going up.

Now, for example, if you look at this base of 1000 rupees at the end of the fifth year, it is becoming 1464. This base of 1000 rupees is becoming 1331, 1210 then it is 1100 and this is same 1000 because it is a the end of the fifth year. So, total amount will become 6105. So, this is how we can calculate the future value of annuity.

Please remember all these concepts because they will be very useful when we will discuss the other advance concepts in this course, means the some other parts of the syllabus when we will be, or the other parts of the course plan. When we will be discussing the other topics everywhere we will be using these terms, future value or present value, future value of a single amount, present value of a single amount, future value of an annuity, present value of uneven series.

All these things are useful, so I am say very means carefully discussing all these concepts with you so that you remain very clear when we start applying these concepts for the, say different kind of the business decision making.

Now, here are some of the say, application of the say, future value of an annuity. Some of the applications of the future value of an annuity which I would like to discuss with you, and we will discuss 4-5 applications which are very interesting applications and would answer the questions pertaining to your day-to-day life, means we all make investment and normally we select some investment products where the investment has to be made, say in the even series and at the end of certain number of years.

For example, you talk about that when we take the say, insurance policy and that is not a term policy but the endowment policy which is basically for the both, your say,

the lifesaving also plus the money back also, part of the money also comes back. So, every year the amount of premium remains the same.

When the amount of premium remains same, you pay at the end of every year or maybe twice a year, whatever it is. And for a certain number of years we take that policy for 20 years 30 years, and the end of 30 years, they return us some amount. They the life of a person also remains insured but if the person remains safe and nothing wrong untoward happens, then at the end of that period of the insurance that amount comes back

So, there we apply the concept of the future value of an annuity and there, means the part of the amount which is invested in the market for the saving purpose by the insurance company, that amount has to be returned back after means say subtracting their own expenses, administrative expenses and their commission, it has to be returned back. Or anywhere when you say buy, maybe for example, you open up an say recurring deposit.

So, if you take the open up an RD account in the bank, it may be possible that every year we decide that I will deposit 10,000 rupees in that say, recurring deposit account, and for a period of next 10 years I will keep on depositing 10,000 rupees. So, after the period of 10 years, it will become 1 lakh rupees 100,000 rupees plus the interest on that.

So all this, these are the kind of the questions we are going to discuss here and we are going to means learn about the applications of the future value an annuity. So, these are very interesting applications, very interesting say day to day means, answer to the day to day problems.

So, when I will discuss all these applications, you will find them quite interesting and you can use them also for answering so many questions which pertain to your daily life. So, all these applications, 4-5 applications plus some other related concepts of the time value of money, I will discuss with you in the next class. Thank you very much.