Production and Operation Management
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Lecture 09
Time Series Forecasting - Exponential Smoothing-I (Brief)

Welcome friends, now, we are entering into the ninth session of this course. In last few sessions, we started discussions on forecasting, which is one of the most important input for various decision making activities. We discussed about various characteristics of our past data. And based on those characteristics, we discussed the development of different models for forecasting purpose.

The models which are based on past data are known as time series models or extrapolative models, we discussed two very simple type of models in our previous session, these are simple moving average method and weighted moving average method. We also discussed some of the challenges related to these two types of models.

In this particular session, we will move to slightly advanced models of time series analysis and these are known as exponential smoothing models. Now, the name exponential smoothing is derived from weighted moving average method systems. As you remember, in the weighted moving average method, we used to assign weights to the demand data of previous periods.

Now, if the weights are decreasing exponentially, it becomes exponential smoothing model. So, it is a type of weighted moving average method. So, in this exponential smoothing method, we assign weights, but these weights are decreasing in the exponential order from the present period to the past periods.

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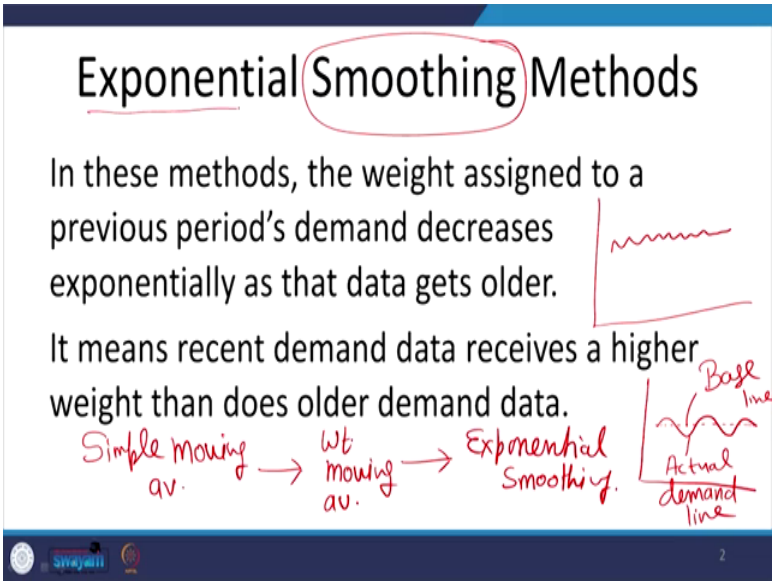

So, you can say that the simplest model is simple moving average method then from simple moving average, we go to weighted moving average and then from weighted moving average, we go to exponential smoothing. Now, this is one reason that this name is given exponential smoothing that the weight is decreasing in the exponential order. There is one more reason about exponential smoothing that what is the reason of using this word smoothing in these models.

Now, the issue is that if you remember, when we discuss the characteristics of our past data, we use some data and in that, we saw that there is no smooth curve available for historical data. Even in the case of horizontal demand data, the curve is like this. So, there are fluctuations, there is a zigzag movement in my past data, it is argued that these zigzag movements are around a baseline, this is the baseline and around this baseline, this zigzag movement is taking place.

Now, when we are using this word smoothing. So, what we are trying to do? We are trying to smooth this zigzag line so that it can touch this baseline this dotted line, this is my baseline and this thick line is the actual demand line. So, whenever actual demand data will be there, there will be some kind of fluctuations and these fluctuations are beyond control of anybody, you cannot control these fluctuations, but what we can do for the purpose of forecasting, we can smooth these fluctuations and therefore, this zigzag curve this actual demand data curve can match this baseline curve.

So, with this idea, this word smoothing comes into this model. So, the exponential comes because the weight are reducing in the exponential order. And the second is we want to smooth the fluctuations of this actual demand line. So, that actual demand line can coincide with this baseline.

So, for this purpose, these types of models are very, very efficient. And we will see the power of these models that how our computation becomes much easier with the help of these exponential smoothing models. We have some more reasons, some more conditions that helps us to understand where to use these exponential smoothing model.

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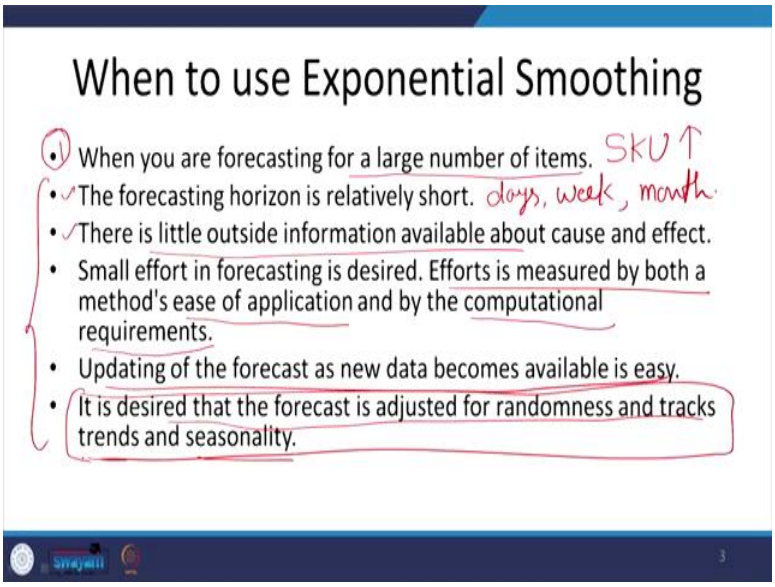

Now, the first important thing we need to realize, we already discussed that SKU are increasing SKU are continuously increasing, stock keeping units are on the rise, and therefore, we need to forecast for a large number of items. You visit a small retailer close to your house and even that small retailer houses thousands of different types of items and you can understand when a retailer houses such large number of items.

So, think of a wholesaler, think of a factory, they also need to forecast for large number of items and continuously with more and more customization, these SKUs are increasing and therefore, we have to forecast for large number of items. And therefore, this exponential smoothing becomes very, very suitable, why the answer is available in the following points.

The forecasting horizon is relatively short when we are forecasting, maybe for days, for weeks or maybe up to month if this type of forecasting horizon is there, the exponential smoothing model is best suited.

So, in our last session, we discussed various forecasting horizons. So, the short term forecasting horizon where we have immediate requirement for the demand at that time this exponential model works very well. The second important thing is there is little outside information available, what are the other qualitative factors? What are the other variables which can affect the demand of your product?

So, you do not have the cause effect relationships. So, when the regression analysis those outside information are not available, at that point also the exponential smoothing methods are best suited. Then third point is when we want to put a small efforts in the forecasting, small efforts are measured by two things.

One is ease of application that how conveniently you can apply that algorithm that model that is one dimension of ease of efforts and the second dimension is computational requirement, whether you require a special server for keeping the data whether you require high speed processor for processing the algorithm.

So, these are the additional requirements, but if your normal machine can perform these computations, it means it has ease of computation also. So, ease of computation and ease of application these are the two dimensions which are measuring the effort aspect. So, when we are using exponential smoothing, our efforts are less.

So, whenever we want to put less effort we can go with this exponential smoothing method. Then another very important thing is when we want to upgrade the forecast whenever some new data is available, new information is available, you need to continuously update the inform, your forecast.

So, updating our forecast is also very convenient with the help of exponential smoothing. In my moving average methods and other methods, you need to do a lot of calculation for updating the forecast, but that updation is very convenient in case of exponential smoothing method.

And then when you want to adjust the forecast for the randomness and you also want to track what is the trend and seasonality that is also possible with explanation smoothing methods, which is not possible with earlier moving average methods.

So, the adjust, this is particularly the most important thing that to track the forecast with respect to trend, with respect to seasonality, that is possible with the help of exponential

smoothing. So, when I see that my data is exhibiting some kind of trend, it is exhibiting some kind of seasonality, you should use exponential smoothing methods.

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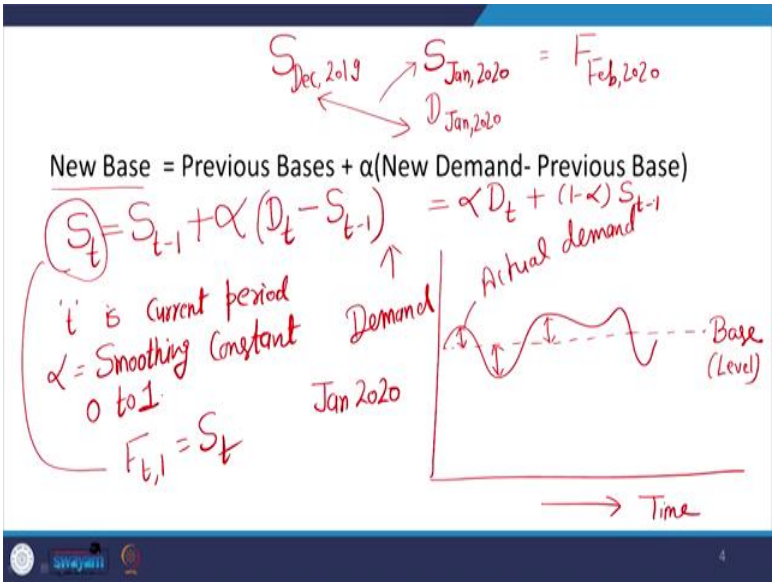

Now, when we are discussing exponential smoothing method, it is important to understand that how to apply it as I am saying that it is a simple method, where you have this type of arrangement that this horizontal line is representing my base, in some books you will find a word level also for this, so some author uses base word some author uses level word.

Now, the demand is moving like this, it is a fluctuating value. So, this zigzag line is representing the actual demand and this is the base value on x axis you have time, on y axis you have demand. Now, what is happening, these demand values we want to smoothen their fluctuation. So, that whatever these randomnesses are there, these are the randomness.

If we smoothen that means, we are removing this randomness from the demand data and when we are able to remove this randomness from the demand data, we will be able to achieve this base value. So, we in this particular method of exponential smoothing, we continuously try to update the base value, we continuously try to upgrade the base value and that base value becomes the forecast for the next period.

So, what is the formula we apply for updating the base value this is the new base value. So, this is in the symbolic form, we write it like St that is the base value for the current period previous base that is the base value for the previous period t minus 1, t is current period. So, t minus 1 becomes immediately previous period.

Now, you see we are introducing one constant alpha here and then new demand, new demand means the demand of the current period that is Dt minus previous base that is St minus 1. Here alpha is my smoothing constant, alpha is my smoothing constant. And the values of alpha can range from 0 to 1, values of alpha can range from 0 to 1.

Now, if you see this equation St equals to St minus 1 plus alpha Dt minus St minus 1, here, I am updating the base value, updating the base value means I am trying to find out the current base value and this current base value is nothing but the forecast for the next period. So, if the meaning if I explain you, if I am presently in January 2020 so if I get the value of S January 2020 this is actually the forecast for February 2020.

So, I continuously want to get and in this case, St minus 1 will be S of December 2019. So, when I am in the period of January 2020, I know what is the demand of January 2020 and with the help of these two things S of December 2019 and D of January 2020, I will determine the value of S of January 2020 and the value of S of January 2020 is forecast for February 2020.

So, this is how the calculation goes. And if you just simplify this if you just simplify this, you can also get a very simplified mathematical equation slight mathematical rearrangement you do this becomes alpha Dt plus 1 minus alpha St minus 1, that is the formula, you can say for calculation of St.

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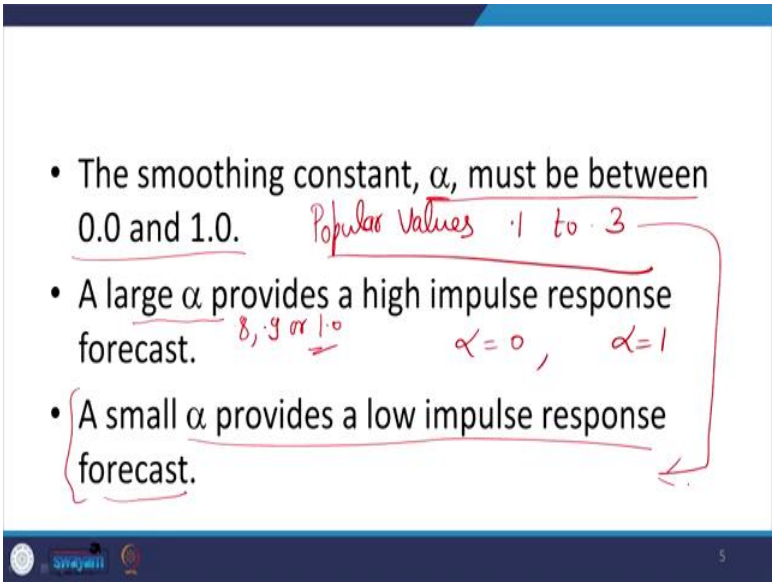

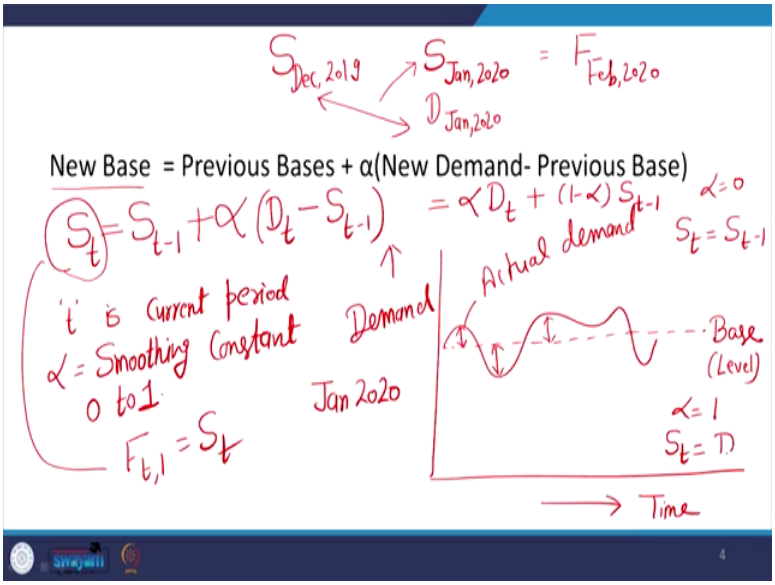

Now, we will take two minutes of time to discuss the values of alpha, as I say that alpha can range from 0 to 1, the value of alpha can range from 0 to 1 popular values of alpha, normally 0.1 to 0.3. This is the popular range of alpha. In most of the practical situations, we use alpha either from 0.1 to 0.3.

Again, it is a matter of your experience and there are some software help also available which can help us what is the right value of alpha for my case. But I will like to take you to two very special cases of alpha, one when alpha becomes 0 and when alpha becomes 1. Now, if you see our formula is alpha Dt plus 1 minus alpha St minus 1.

Now, if you put alpha equals to 0 here, what becomes your St becomes St minus 1 and when you put alpha equals to 1, it becomes St equals to Dt. Now, these are two extreme situations, when alpha equals to 0, my previous base is the current base. It means, it means there is something, there is something in the current period's demand which I do not want to include for my forecast.

There is some fluctuation, there is some fluctuation in the current period's demand, which is not of permanent nature, which is exceptional and it is not going to happen again and again. Therefore, I want to totally discard that fluctuation and in that situation I am taking alpha equals to 0, where because of some very unnatural reason, some fluctuation has taken place and this is not of permanent nature.

Therefore, I want to keep my previous base as the base for current period and therefore, I am totally neglecting those fluctuations that is the case of alpha equals to 0. Now, when alpha equals to 1 St becomes Dt. I want to take 100 percent fluctuation for my future demand. My

forecast is now going to be based 100 percent on the demand of current period, I totally want to ignore the base value of previous period.

So, now, my base has permanently shifted to a new value, this is a case where the base is permanently shifted to a new value. And in this particular case, the change which has happened is of permanent nature. For an example, when there is a new Pay Commission, from sixth Pay Commission to seventh Pay Commission, the salary of people have increased permanently.

So, therefore, there is a change, which is going to be there permanently in the consumption pattern of the people. So, I do not want to keep the old base or old calculations for my new forecast. And in that case, I may like to have alpha equals to 1. So, these are the two extreme situations when I want to keep my old base as the new base that is alpha equals to 0, because the fluctuations are very, very temporary, we do not want to include these fluctuations for my next forecast.

Alpha equals to 1 when the fluctuation is permanent, and I totally want to disconnect from my previous base. So, these are the two cases where alpha equals to 0 and alpha equals to 1 are applicable. Now, therefore, if you see, here we have written that a large alpha, a large alpha means, alpha equals to 0.8, 0.9 or maybe up to 1.0 provides a high impulse response forecast.

That means, you are totally taking new demand into consideration. A small alpha provides a low impulse response forecast that means, it is giving you a more smoothing effect. A low value of alpha gives you more smoothing effect, you are not going to change your forecast too much, whatever is the forecast of previous period, there will be some slight changes, but there will not be a quantum jump in the new forecast.

So, low value of alpha gives you more a smoothing effect, high value of alpha gives you a high impulsive forecast, that is, so, you get more a smoothing effect, you get more smoothing effect with lower values of alpha. Therefore, 0.1 to 0.3 are more popular values of alpha because of their ability to give you higher smoothing effect.

Now, once we have discussed this particular theory of our basic exponential smoothing model, whatever we are discussing, this is the part of basic exponential smoothing model. So, now with the help of one example, we will see that how to implement, this basic exponential smoothing model.

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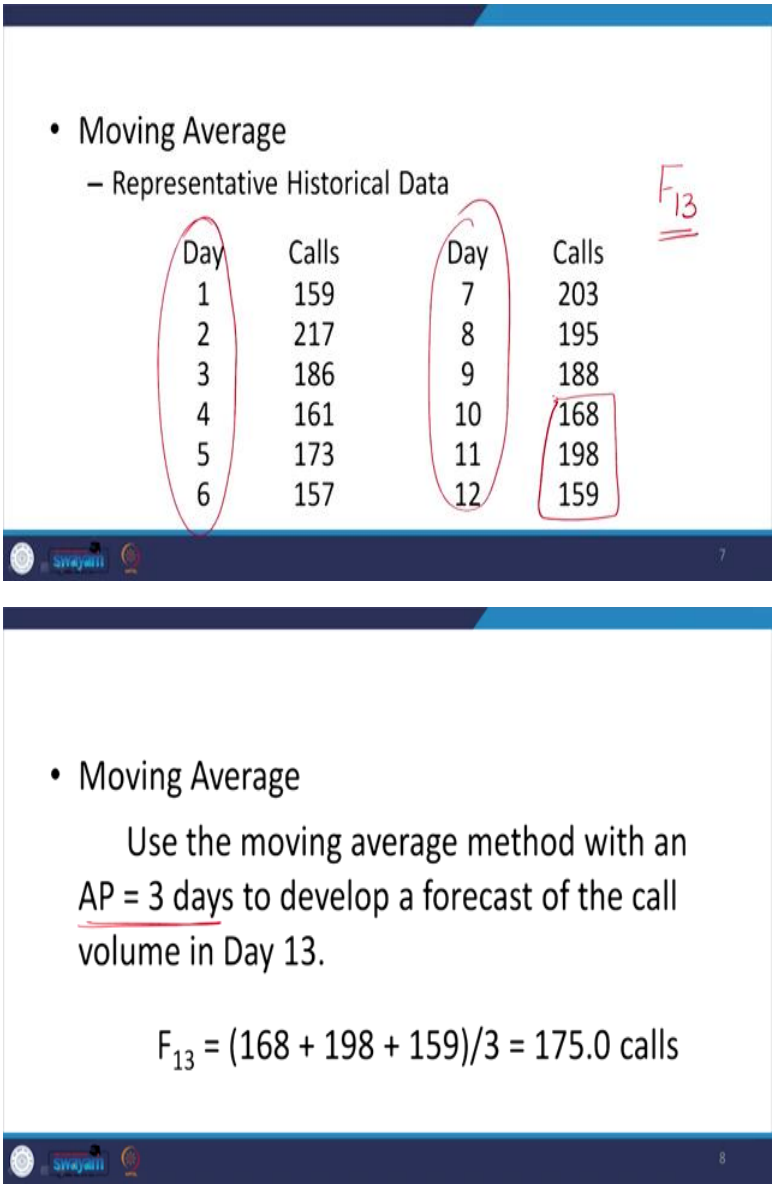

So, we have one example and in this example, we have some historical data available with us. And this historical data is available for past 12 days, from day 1 to day 12 these are the periods and these are the number of calls, which a company is making on a particular day. Now, we can solve this question, we want to have forecast for 13th day that how many calls will be there on 13th day? You can apply different types of forecasting models for the same data.

We can apply simple moving average, we can apply weighted moving average and we can also apply what we recently discuss basic exponential smoothing model. Now, when we are applying this simple moving average method. So, first important requirement is we need to have a period of moving average, we need to have a period of moving average.

So, now, let see that the period of moving average we are taking 3, days moving average period is 3 days.

So, what we are going to do, we will take the average of most recent 3 days. So, these are the three days most recent 10th 11th, 12th. So, we will take the average of 168, 198 and 159. So, 168, 198 and 159 we are taking the average of these 3 days calls. So, on 13th day, we can expect 175 calls, we can expect 175 calls on 13th today, so, this is the simple moving average method.

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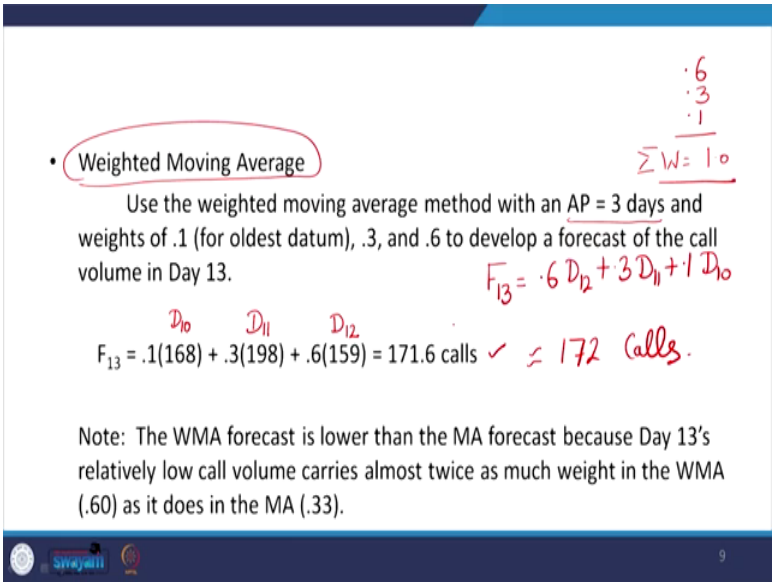

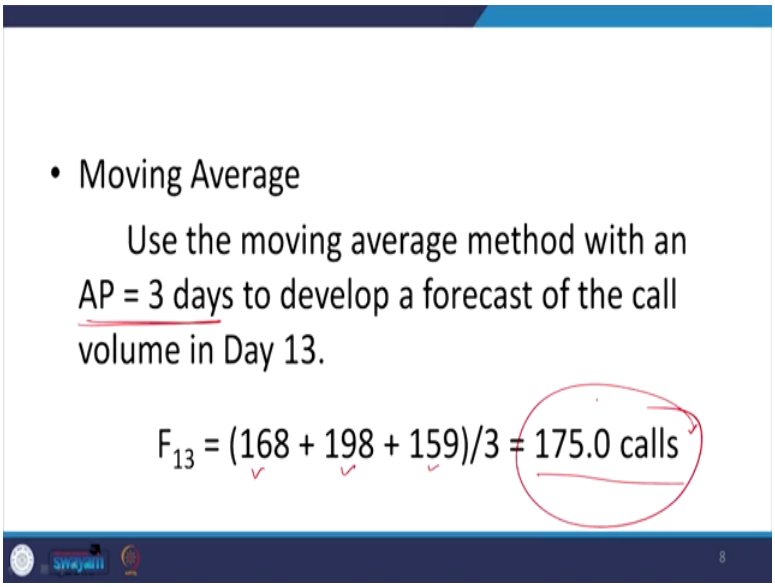

Now, we can move further be weighted moving average as we have already discussed in the weighted moving average method, we assigned weights and we assigned weight the highest weight for the most recent period and as we move away, weight can decrease. So, here we have again the period of moving average as 3 days and we have three weight, one weight is 0.6 or another weight is 0.3, another weight is 0.1, the sum of the weights should be equal to 1 so, that is one.

So, that condition is satisfied 0.6, 0.3, 0.1 equals to 1, now, you will give highest weight to the most recent period. So, this is the demand of 12th period 159. This 198 you remember, is the demand of 11th day, 168 the demand of 10^{th} day. So, the formula becomes F13 equals to 0.6 into D 12 plus 0.3 into D 11 plus 0.1 into D 10 and that formula is executed here and this is 171.6 calls or you can say calls cannot be in points so, it can be 172 calls.

So, my simple moving average method gave me 175 calls as my forecast. Now, 172 calls is the result of my weighted moving average method. So, now your forecast has decreased by 3. Now, let us see the application of exponential smoothing for doing the forecast. Now, in the exponential smoothing, we do not require we do not require the period of moving average, we have only one requirement, what is the value of alpha?

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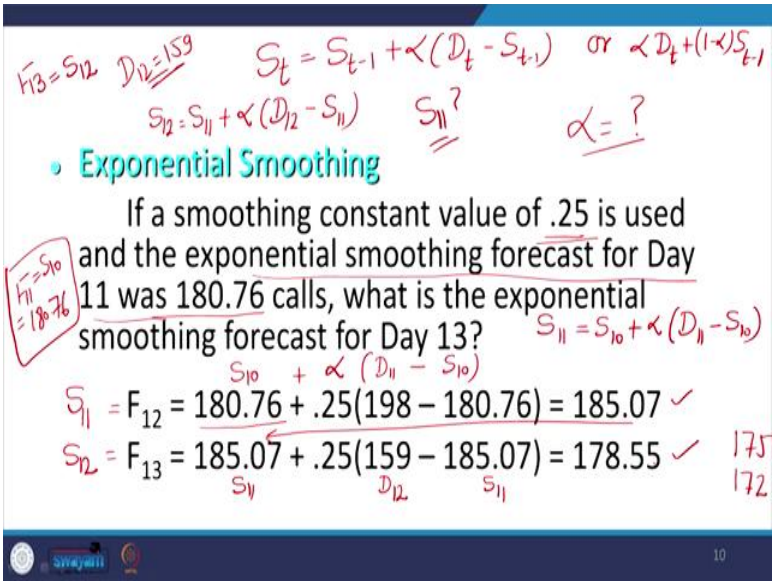

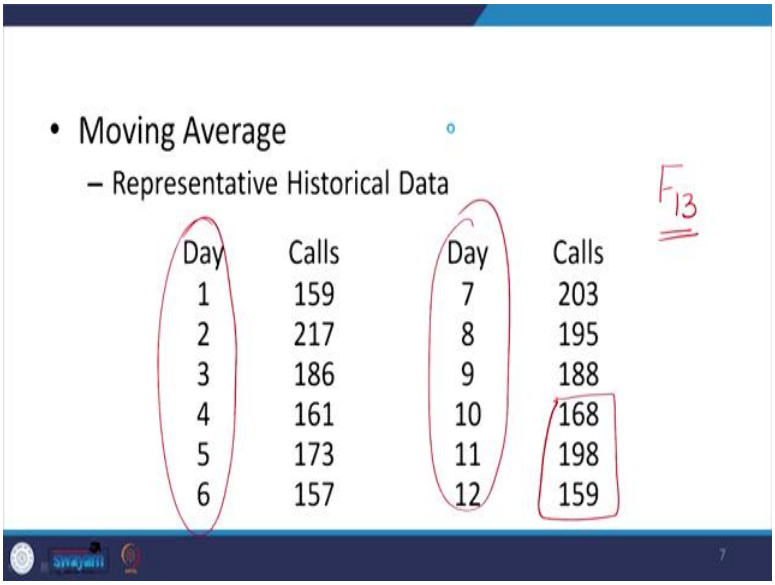

The value of smoothing constant and the value of smoothing constant given to us in this situation is 0.25 and it is between 0.1 to 0.3 the popular values of alpha. So, now, we are going to start the discussion of exponential smoothing with the help of this 0.25 as our smoothing value.

The formula if you remember is St equals to St minus 1 plus alpha into Dt minus St minus 1 or you can also write it as alpha Dt plus 1 minus alpha St minus 1. So, these are the two ways in which this formula can be written for determining the updated value of base. Now, for calculation of F13 for calculation of F13 we require value of S12, S12 is F13, now for calculation of S12, if I write this first in the form of a formula, it will be S11 plus alpha D12 minus S11.

These are the conversion of formula for our purpose. So, for this formula, we require the value of S11. To apply this formula, we require the value of S11 if you go back D12 is available to us.

D 12 is 159. So, D 12 is given to us as 159 but S 11 is not available to us, so, we have here the value of S 11 that is 180.76. Now, if you read this line, the exponential smoothing forecast for day 11 this is not S 11 this is F11. F11 given to us is 180.76. F 11 is as 180.76, now F 11 is actually, F 11 is actually the result of S 10, S 10 gives you F 11.

Now, with the help of F 11 we will calculate, we will calculate the value of S 11 what will be the formula? For calculation of S 11 it will be S 10 plus alpha into D 11 minus S 10 and F 11 is nothing but S 10. F 11 is nothing but S 10. So, using this formula, we calculated the value of F 12, F 12 is nothing but S 11. So, this is S 11, S 11 is equals to this is given to you as S 10 plus alpha into D 11 minus S 10, D 11 was 198.

So, we calculated the value of S 11 which comes to be 185.07. Now, we will use this value of S 11 for calculation of F 13 that means, S 12. We have already used this equation that is S 11. So, this value of 185.07 is used here, 0.25 is the alpha, D 12 is 159 minus S 11 and this gives you 178.55 as the forecast for F 13.

Now, we have three different forecast from three different methods, the simple moving average method gave us 175, weighted moving average gave us 172, and now, exponential smoothing gave us 178.55. So, there will be a very important discussion, we need to do that out of these three forecast which forecast is more accurate?

Because there is a sufficient difference between three forecasts 175, 172, and 179 around. So, there is a concept, we will be discussing in our coming classes, that is the forecasting errors and the measure of forecasting error will help us in identifying the right method for my forecast, even if this simple exponential smoothing method, we are going to use, the use of forecasting error will help us to identify which value of alpha is more suitable for my data.

So, I have taken 0.25 as the forecasting as the value of alpha here, but maybe somebody takes 0.20 or somebody takes 0.30, their forecast may be more accurate than the value of alpha taken as 0.25.

So, all these things are the matter of iteration and with the help of our calculations on forecasting error, we will see that how to understand the right value of alpha and how to understand the right use of a particular type of forecasting model. So, with this, we come to end of this session. Thank you very much.