

Production and Operations Management
Professor Rajat Agrawal
Department of Management Studies
Indian Institute of Technology, Roorkee
Lecture 08

Time Series Forecasting

Welcome friends. So, in our last session, we discussed the importance of forecasting. We discussed various categories of forecasting. We discussed particularly one classification on the basis of time span where we discussed current requirements. We discussed intermediate requirements and we also discussed long range requirements. Then on the basis of various methods of forecasting, we discussed quantitative forecasting and qualitative forecasting.

In quantitative forecasting again we have two categories, one is Time Series forecasting and the second is causal forecasting. We started discussions in our last session for Time Series forecasting which are also known as extrapolative methods. You have historical data and you extrapolate that historical data to obtain future demand, and now we will go into some kind of specific methods for my Time Series analysis.

Now, in this particular case if you remember, we discussed various components of our Time Series data. Now, you will see that how those components, components means the pattern of my Time Series data affect my choice of a particular type of analysis or a particular type of, you can say algorithm which I am going to use for taking the forecast. Now, for the Time Series forecasting there are some fundamental requirements. Irrespective of type of method you are going to use, you need to know few very important things.

(Refer Slide Time: 02:16)

Time Series Forecasting

You can pick models based on:

- ① 1. Time horizon to forecast 6 periods → 7th (Good)
- ② 2. Data availability 4 periods, 12 periods, 3 periods
- ③ 3. Accuracy required Actual demand, Forecast (Estimation)
- ④ 4. Size of forecasting budget 1%, 5%, 10%, 12%
5. Availability of qualified personnel

NPTEL ONLINE
CERTIFICATION COURSE

2

And these are, first is the time horizon to forecast. Now, if data is available for, let us say 6 periods, data is available for 6 periods, so with the help of this data of 6 periods I can forecast for the seventh period, I can forecast for the eighth period also. Or even I can forecast for the twelfth period also.

So, for which particular period I want to forecast using my historical data that is the first important thing. And here I will like to submit that if I am going to forecast for the seventh period my quality of forecast would be good. And as I am going away from my most recent period, if I am going to eighth period, I am going to twelfth period, the quality of forecast will deteriorate. That means there will be more chances of errors. Quality of forecast is nothing but how much errors are there in the forecast.

So, forecast you understand, is an estimation for future demand. It is not actually the demand, it is just an estimation. So, there will always be some difference between actual demand and your estimation. So, if your estimation is close to actual demand you can say that forecasting is good.

But if there is a wide difference between your forecasted values and the actual demand then I will say that forecasting is not good. It is a poor forecasting and as a result of poor forecasting we will discuss we may incur different types of losses, and therefore I am saying that when I am forecasting for my immediate periods with my historical data the quality of forecast will be good and if time horizon is more, then the forecast quality will be poor. So, I need to first see that for how much period I am going to forecast.

Second thing is availability of data. I gave the example of 6 periods. So, for one product I have the data for 6 periods. For other product I have the data for 4 periods. For some other product I have the data for 3 periods. So, how much data is available, how much past data is available to me?

More past data means better forecast. If you have data of 10 periods, 12 periods, more than that, your quality of forecast will improve; if you do not have enough past data, if you have data of just 1, 2, 3 periods and if you are doing your forecast on limited availability of data, again the quality issue will come. It will not be able to have a very good forecast.

So, data availability is another very important thing and it is a very costly thing also because in your organization you are not having only one item. You may be dealing with hundreds of items. And now it is the time of customization. We discussed in our one of the sessions that it

is the time of customization where continuously SKUs are increasing, the stock keeping units are increasing. So, you need to forecast for large number of items and therefore you need to keep historical data for large amount of period, for large amount of items.

So, stocking that amount of data is again a challenge and nowadays, thanks again to IT we have systems like cloud computing, etc. where we can store huge amount of data. But that requires some amount of investment. That requires cost. So, you have to see whether forecasting is becoming a very costly activity or not.

So, there is a trade-off. How much accuracy is required? So, you require to store only that amount of optimal data which can give you the required accuracy. If you store excess amount of data which is not going to serve any purpose so that is just going to increase the cost without improving the accuracy of your forecast. So, how much data is available and how much data is required, that is the second important characteristic on the basis of which your model will be developed.

Third important thing is accuracy. Accuracy means forecasting error. So, we discussed that accuracy is a important phenomenon because you have one actual demand and the forecast which is estimation. So, there is going to be some difference between actual demand and estimation.

Now, depending upon organization to organization you can handle 5 percent forecasting error, 10 percent deviation, 12 percent deviation or 1 percent deviation. If unit value is very high for a product you will like to have very small deviation. Or in other words you will like to have very high accuracy.

But if the unit value is not so important, it is not so high, for an example in case of FMCGs where unit value is not very high; you may have a little accurate forecast. Or you can expect higher forecasting errors. But in case of jewelry, where the unit value is very high so you cannot afford to have high forecasting errors. You must have good accuracy because of the high unit value products.

So, you need to see from organization to organization. I take an example of Wal-Mart here. The success of Wal-Mart, because tagline of Wal-Mart is every day, low pricing. Now, everyday low pricing is only possible because of the accurate forecast they are able to do and as a result of that they have minimum wastage for inventory in their organizations.

And across Wal-Mart stores you will find that they are very effectively using information technology for doing appropriate forecast and that appropriate forecast is helping them reducing the cost of their products. Otherwise, we all have this kind of fear, whether we are able to fulfill the customer's requirement and in that we unnecessarily stock more items in our store and that results into the dead stock and that dead stock increases our cost and it reduces our competitiveness.

So, this whole issue of forecasting is not just forecasting but it is related to various other aspects of your business. So, accuracy of the forecasting is a very critical issue and obviously all organizations aspire for higher level of accuracy. But higher level of accuracy may require some additional investment so you need to see whether the accuracy which is required in your particular business, in your particular supply chain, whether enough capital investment is justified for that level of accuracy or not.

Then size of forecasting budget, because forecasting is giving you an estimation for future and as we just discussed it requires some amount of investment with respect to IT infrastructure and we require some specialist also who can handle that modeling part for the forecasting, so the budget allocation for forecasting is also important activity.

So, but this is more generic in nature. This is not very specific to Time Series but it is more generic in nature that for any kind of forecasting activity whether it is quantitative or qualitative, how much budget you have? Depending on that you can do the forecasting.

We in India just had our general elections and there were good amount of organizations doing exit poll. Now, the exit poll data was collected from, some organizations collected from 5 lakh people, some organization from 7 lakh, some collected from 10 lakh.

Now, the collection of data from 5, 7 or 10 lakh people depends upon your budget. How much budget you have, how much manpower you can hire for collecting the data? And this is particularly more important in qualitative forecasting because there, the data collection requires manpower and manpower is directly affecting your budgetary issues.

Then availability of qualified personnel, now it is again a very important issue; that we primarily discussing this type of courses in industrial engineering, in MBA courses but many a time for doing modeling for Time Series analysis we require people from mathematics background; those who know extrapolation and what type of better numerical analysis

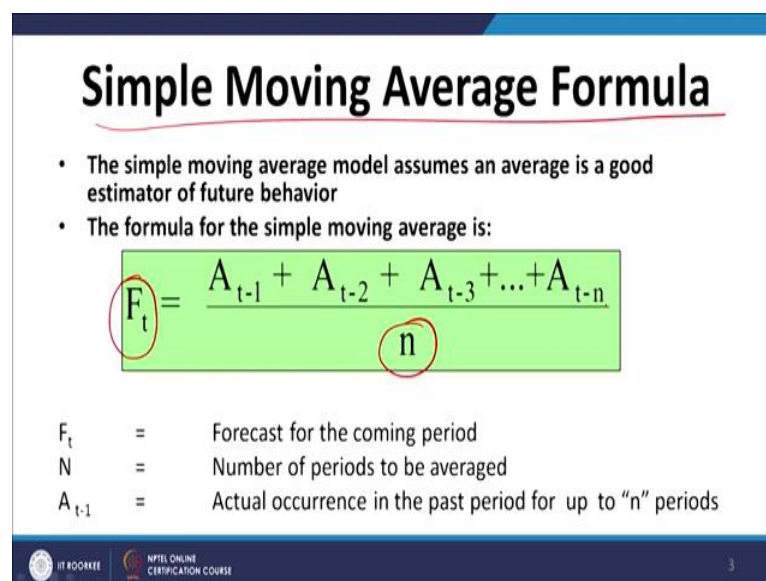
techniques can be used for efficient extrapolation. So, qualified personnel are required and availability of qualified personnel ensures higher accuracy of your forecasting.

So, your ability to do modeling on the basis of raw data is a very, very important art of Time Series forecasting. Particularly those who are coming from industrial engineering or management background they can work with available algorithms but development of new algorithms, development of new models, this is mostly done by students of mathematics and physics.

So, the qualified personnel are also playing very important role in Time Series forecasting because each data may have its own characteristics. In our course we will discuss some limited amount of examples but in reality, in practical you may find variety of situations which can be modeled uniquely so how you need to have a modeling system, for that unique kind of arrangement that requires good amount of knowledge of algorithm and model building. And therefore, availability of qualified personnel is also an important aspect of Time Series forecasting.

Now, we will start our discussion with some very simple type of modeling activity. We will start with very simple type of modeling activity. The purpose of this simple starting is just to give you an exposure that how models are developed, and I will also like to tell you at this point that simple things are not used practically.

(Refer Slide Time: 14:27)



Simple Moving Average Formula

- The simple moving average model assumes an average is a good estimator of future behavior
- The formula for the simple moving average is:

$$F_t = \frac{A_{t-1} + A_{t-2} + A_{t-3} + \dots + A_{t-n}}{n}$$

F_t = Forecast for the coming period
 N = Number of periods to be averaged
 A_{t-1} = Actual occurrence in the past period for up to "n" periods

NPTEL ONLINE CERTIFICATION COURSE

So, this particular method which we are now going to start that is known as Simple Moving Average Formula, this is the simplest way of extrapolation but this is the beginning of our

modeling activity for extrapolation methods. Now, what do we do in this particular method, that we take a simple moving average of our historical data and that simple moving average becomes the forecast for my coming period.

So, what happens, this is a formula where the F_t that is the average of the demand of past n periods so the period of moving average of n . This is the period of moving average and these values A_{t-1} , A_{t-2} and A_{t-n} , these are the actual demands of previous periods, previous n periods.

So, I am taking average of these period n periods and that average, that is coming as F_t , this is the forecast for my coming period that is $t+1$. And we will see with the help of one example that how you will be calculating demand, forecast with the help of this simple moving average method.

(Refer Slide Time: 15:54)

The slide displays a table of weekly demand data for 12 weeks. To the right of the table, a question asks for 3-week and 6-week moving average forecasts. Handwritten in red ink are the formulas for these forecasts. The 3-week moving average formula is $F_{13} = \frac{D_{12} + D_{11} + D_{10}}{3}$. The 6-week moving average formula is $F_{13} = \frac{D_{12} + D_{11} + D_{10} + D_9 + D_8 + D_7}{6}$. A red arrow points from the 'Actual' column of the table to the 'Demand' column, and another red arrow points from the 'Demand' column to the 'Forecast' column.

Week	Demand
1	650
2	678
3	720
4	785
5	859
6	920
7	850
8	758
9	892
10	920
11	789
12	844

Question: What are the 3-week and 6-week moving average forecasts for demand?

Handwritten formulas:

$$F_{13} = \frac{D_{12} + D_{11} + D_{10}}{3}$$

$$F_{13} = \frac{D_{12} + D_{11} + D_{10} + D_9 + D_8 + D_7}{6}$$

Now, we have the demand for, this is the actual values; this is the actual demand for past 12 periods. This is the actual demand for past 12 periods and these periods are in the form of weeks.

So, you can take any period, you are free to take period of your choice, may be it can be a week, may be it can be a day, may be it can be a quarter, may be it can be a month, so depending upon your frame of discussion you are free to take a period of your choice. So, here data is available in the form of weekly data. So, this data is available for 12 weeks. The demand is available in the units.

Now, the question is what are the 3-week and 6-week moving average forecast for the demand? Now, when I am saying a 3-week moving average forecast, so now I want to determine forecast for the thirteenth period because for the 12 period demand is already available to me, and I am using a 3 period moving average method. So, what I am going to do, I will take demand of most recent 3 periods, D12, D11 and D10 and average of these 3 periods demand is the forecast for the thirteenth.

When I am saying 6-week moving average, in that case again F13 will be calculated but now F13 will be the result of average of 6 periods demand. So, you see that F13 can be calculated as a moving average of 3 weeks or 6 weeks or 4 weeks, 5 weeks that is also possible. So, if you have data of 6 periods then only you can go for 6-weeks moving average. But if data of 6 week is not available you cannot have the second formula.

So, depending upon, if you remember we just discussed data availability so based on data availability only you can decide your algorithm. So, if data is available for 5 periods, maximum moving average you can calculate for 5 periods. If data is available for 4 periods, maximum moving average you can calculate for 4 periods. So, that is the demand.

Another important thing, when we are using 6-week moving average, this forecast is better than this forecast because you are using more past data. You are using more past data and this will help you in giving more accurate forecast for the coming period.

When you have less past data, here you have in the first case, past data of just 3 periods. So, chances of errors are high. Or you may say accuracy may be low. But when you have 6-week moving average accuracy improves but the cost also increases because you need to store data for 6 weeks.

The other problem in the case of this simple moving average method is there, whether it is a 3-week example or a 6-week example you will see that you are giving equal weightage to all past periods.

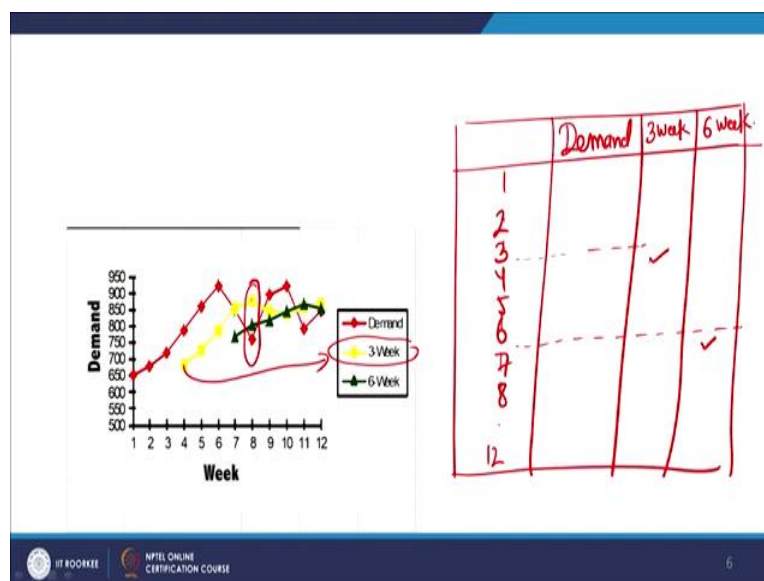
In case of 3-week moving average, we are giving equal weightage to D12, D11, and D10. In case of 6-week moving average we are giving equal weightage to D12 to D7. So, it looks very obvious to us that we should give more weightage to most recent period and that weightage can decrease as we move away from the most recent period.

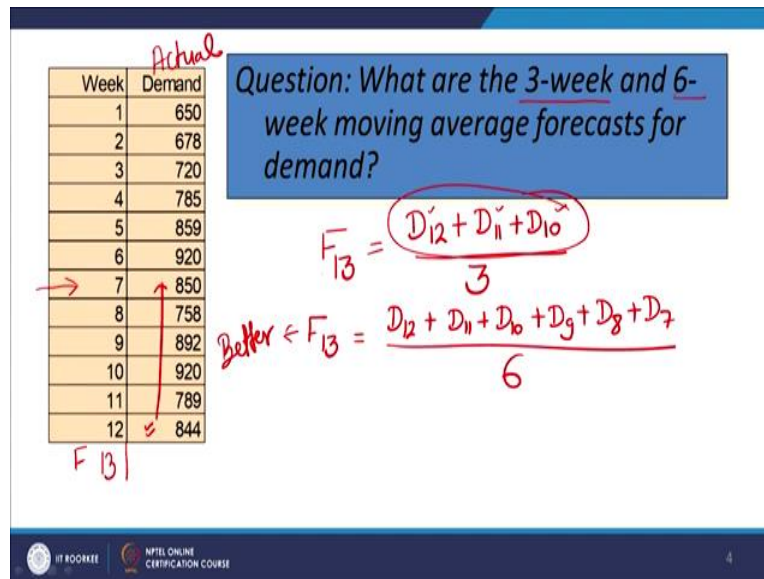
So, here something which has happened in the seventh period that is also equally affecting the demand in the thirteenth period. It looks obvious that something which is in the twelfth week that should affect demand most and as we are moving away from twelfth period to seventh period the weightage of this period should decrease.

But in the case of simple moving average method we are giving equal weightage to all past periods. So, that is also a challenge in the case of this simple moving average method. So, two challenges we identified, one is you require more historical data for a better forecast. That is challenge number 1, and storing data is a challenge in itself.

The second point we discussed that we are giving equal weightage to all the past periods. Whatever is the moving average period, it is 3 week, 4 week, 5 week or 6 week but we are giving, that once we have decided that this is the period of moving average, we are giving equal weight to all those past periods. So, these are the two challenges in case of simple moving average method.

(Refer Slide Time: 21:51)





Now, you can also do one thing that you can plot this graph with the help of this data which we had from period number 1 to period number 12 we have made a graph where this red line is indicating the movement of actual demand data from period number 1 to period number 12, that how actually the demand is moving.

And then we have two more curves. One curve is in the form of yellow, this, which is giving you the forecasting values. Like you have the demand data for period number 1 to 12. Now, what I am trying to say and you can do this exercise that you create a table for 3-week and 6-week forecast.

Now, when you have up to 3-week data this fourth week onwards you may start forecasting using past 3 weeks data. And when you have 6-week data you can start using the 6-week data from the seventh week onwards. So, that is what we have, that this yellow line is indicating the forecast which started available from fourth week onwards and this black line is indicating the forecast which started available to us from seventh weeks onwards.

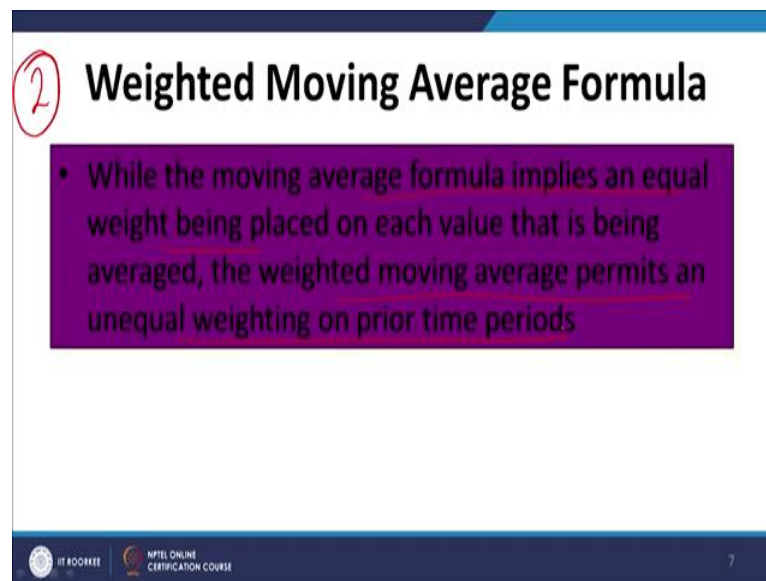
Now, you see in the case of 3-week forecast though there are some smoothening as compared to actual demand data but the smoothening is more in case of this black data which you can see is more closer to the actual demand as compared to yellow lines.

For example, if you see this particular period where the red line is giving you a demand of around 750 and this black line is giving you a demand of around 775 while the yellow line is giving you the demand of 850. So, the difference between actual demand and yellow line, yellow line is a result of 3-week forecast, black line is a result of 6-week forecast. So, when you increase the period of moving average your forecasting error has also improved.

So, with the help of, I request all the students that please do this calculation on your own and this will give you an idea that how the, once you come to the seventh periods onward you will see that demand is closer 6-week forecast than to 3-week forecast.

So, the point of keeping large amount of historical data for accurate forecast is proved here. So, now we move to one improved method of forecast. We discussed equal weightage problem in our simple moving average method.

(Refer Slide Time: 25:24)



2 **Weighted Moving Average Formula**

- While the moving average formula implies an equal weight being placed on each value that is being averaged, the weighted moving average permits an unequal weighting on prior time periods

IT KODAKKE | NPTEL ONLINE CERTIFICATION COURSE | 7

Now, we are moving to second type of moving average method which is known as weighted moving average method. Now, weighted moving average method is a method which is differentiating between different periods. It gives different weight to the different periods demand. So, while the simple moving average formula gives equal weight, now what we are doing, the weighted moving average method permits an unequal weighting on prior time periods. So, you can give different weights to different periods demand.

(Refer Slide Time: 26:04)

$$F_t = w_1 A_{t-1} + w_2 A_{t-2} + w_3 A_{t-3} + \dots + w_n A_{t-n}$$

w_i = weight given to time period "t" occurrence (weights must add to one)

$$\sum_{i=1}^n w_i = 1$$

$w_1 + w_2 + w_3 + \dots + w_n = 1$
 $w_1 \geq w_2 \geq w_3 \geq \dots \geq w_n$

And here you see this formula. So, now you are not calculating the average. Rather you have started giving these weights to different periods demand, w_1 , w_2 , w_3 and w_n these are the weights and A_{t-1} , A_{t-2} , A_{t-3} , A_{t-n} , these are actual demands, and the product of these weights and demand, and then sum of all these products is actually the forecast for next period.

Now, there are certain conditions which we need to discuss and these conditions are that, that the sum of all these weights, sum of all these weights, that w_1 , w_2 , w_3 , that sum of all these weights should be equal to 1. So, that is one important thing because it is replacing that average, so either in case of average the weights are there, w_1 , w_2 , w_3 are there but if you are having a phenomena like a 4-week simple moving formula you are using, in that case w_1 equals to w_2 equals to w_3 equals to w_4 equals to 0.25.

So, equal weights are there in case of simple moving formula but here we are not having equal weights. These weights are different. And normally what happens? We give maximum weightage to the most recent period. w_1 will be the highest, then w_2 , then w_3 and then in the last it is w_n . So, you give weight in the order of decreasing as you are moving away from the most recent period. Now, for that purpose to understand this working of this weighted moving average system we have a very simple example available with us.

(Refer Slide Time: 28:13)

• **Weighted Moving Average Problem (1) Data**

Question: Given the weekly demand and weights, what is the forecast for the 4th period or Week 4?

Week	Demand
1	650
2	678
3	720
4	?

Weights:

t-1	0.5
t-2	0.3
t-3	0.2

$\Sigma = 1.0$

$F_4 = 720 \times 0.5 + 678 \times 0.3 + 650 \times 0.2$

Now, we have the weekly demand and weights and we want to forecast for this fourth period. Now, for three periods we have the weekly demand and this is the demand and these are the weights.

Now you see t minus 1 is 0.5, 0.3 and 0.2. If you do the total of these weights, sigma of these weights, 0.3 plus 0.2 plus 0.5 that becomes 1.0. So, this is one condition we have satisfied. The second, we need to calculate F_4 as per this question. So, you will multiply the most recent period with the highest weight. Then 678 will be multiplied by 0.32, 0.3 and 650 will be multiplied by 0.2.

So, this is important thing you need to remember that the highest weight is given to the most recent period. So, your forecast will become 720 into 0.5 plus 678 into 0.3 plus 650 into 0.2. So, this multiplication and their product will give you the forecast.

So, here we have reduced the problem of equal weightage. Now, you are assigning weight as per your requirement. Now, how these weights have decided, whether we should give 0.5, 0.3, 0.2, this is based on our expertise.

If you are more experienced you will give more accurate weights. Otherwise you can take the help of experts; or now we want to go into more complex mathematics, so fuzzy systems are also available. You can involve fuzzy mathematics for assigning these weights because appropriate weight will change your entire calculation. If the weight is not appropriately defined then the forecasting error will be high. Then the purpose of the weighted moving

average will be lost. So, it will be better to go back to that simple moving average system if you are not able to assign appropriate weights.

So, somebody may say that this is 0.5, 0.3, 0.2; somebody may argue that I want to have a different weight system. In place of 0.5 I want to have 0.6 here, then 0.3 here and then 0.1 here. So, fine, people may argue but then we may need to take a consensus view of all the experts available with us that what should be the weight for these particular periods.

Somebody may say that, no, no, no, no, it is 0.7 and then these two periods are having similar weight of 0.15, 0.15. So, people may say different things so the experts need to decide that what should be the weight and this is one area which is debatable, which is not very fixed that how to decide the weight.

People use different type of methods. Somebody go with purely judgmental call, some people go with some fuzzy mathematics, some people may take the weight from 2, 3 experts and then take the average of that weight and go with those average values. So, we are not going to say that this particular method is the best method to give the weight. But giving weight is the most crucial thing for a good weighted moving average forecasting.

So, with this you can see that we have discussed two very important and fundamental methods of forecasting using our historical data, simple moving average method and weighted moving average method. So, with this we come to the end of this session. Thank you very much.