Production and Operation Management

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Lecture 53

PERT and Crashing

Welcome friends. In our last session, we started discussions about another very important aspects of operations management, and that is about project management. We discussed that different characteristics of the project, that how project is bounded by 3 important dimensions. One is, you have a specific time within which a project has to be completed. Then you have the budgetary constraints, you have limited amount of budget, within which you have to complete the project. And then, there are some specific goals which you have to achieve in a project.

And, we discussed that based on amount of efforts, the level of efforts; there are different phases of a project lifecycle. We have maximum effort during the execution stage of the project. So therefore, it has to be properly done, it has to be properly managed, the execution stage, so that you can take maximum advantage of your efforts. And for that purpose, project monitoring, project control is very essential.

And for that purpose, we discussed that there are two very popular methodologies through which you can monitor the performance of your project. These are PERT and CPM, which are also known as network techniques to handle the projects. Under that, we already discussed in our last session the CPM, where we discuss that how a project is composed of various activities.

And if you remember, in our work breakdown structure we discussed the making of that kind of various schedule of activities which are required to complete a project. Once you have that WBS with you, Work Breakdown Structure, then you know what is the precedence of different activities. And based on that precedence, we make a diagram which is known as network diagram or the precedence diagram. And that network diagram helps us in monitoring the performance of the project.

In our last session, we discussed the CPM methodology, Critical Path Methodology for monitoring the projects. And, one very important thing we discussed in that, in that CPM methodology, we had only single time estimate for each activity.

But it is quite possible that you may not be having the single time estimation for a particular activity, rather you have more than one type of time estimation. And that is what we are going to discuss in today's session that is based on PERT and the Crashing, that is the title of the session. And as we will progress in this session, you will understand the meaning of PERT and crashing.

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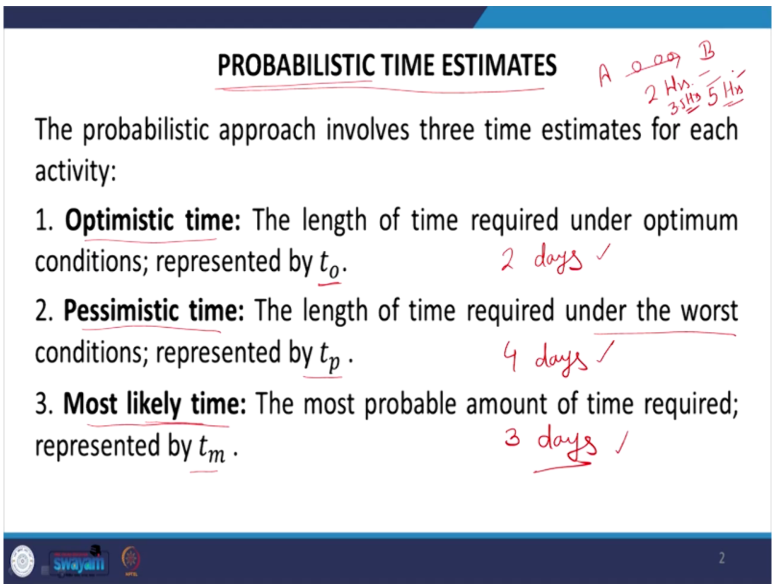

So, let us start the discussion of different time estimates that is probabilistic time estimations. Now as I just told you, in CPM we have only single time estimation. So, that was an example of deterministic time estimation.

Here, we will have 3 time estimations. So, there are, this is known as probabilistic time estimation case. And these time estimations are optimistic time, pessimistic time, and most likely time. That if all conditions are favorable, for a particular activity if all conditions are favorable, in how much time the activity will be completed, that is the optimistic time and it is written as t naught.

The pessimistic time, if there are multiple time of uncertainties and if all those uncertainties actually delay your project, delay your activity to be completed, then it is the tp. Under the worst scenario, then what is going to be the time of completion of the activity, that is pessimistic time. So, this is represented by tp.

And, most likely time, that because happening of any activity is a combination of optimistic and pessimistic type estimations. So, some conditions may be favorable, some conditions may not be favorable. So, based on that we also know that what is the most likely time estimation of completing a particular activity that is represented by tm.

For an example, for a particular activity the to maybe 2 days, tp maybe the 4 days, and tm will be 3 days. So, everything is favorable, everything is optimistic; you will complete this activity in 2 days. In the worst scenario, you will complete activity in 4 days. But most likely, you will complete activity in 3 days. So, these are the 3 time estimations which are possible for a particular activity.

And, we also know that if I am traveling from place A to place B, people will say that if you find roads are free, there is no traffic on the roads, you will travel this distance in 2 hours. But there are possibilities that you may find different patches where situation of jam may be there, so it may take 5 hours to reach you.

But in any case, most of the time, most of the time generally you will find some places some jam, some places are empty, so maybe in 3.5 hours you will reach. So, that is different types of time estimations you have for a particular activity.

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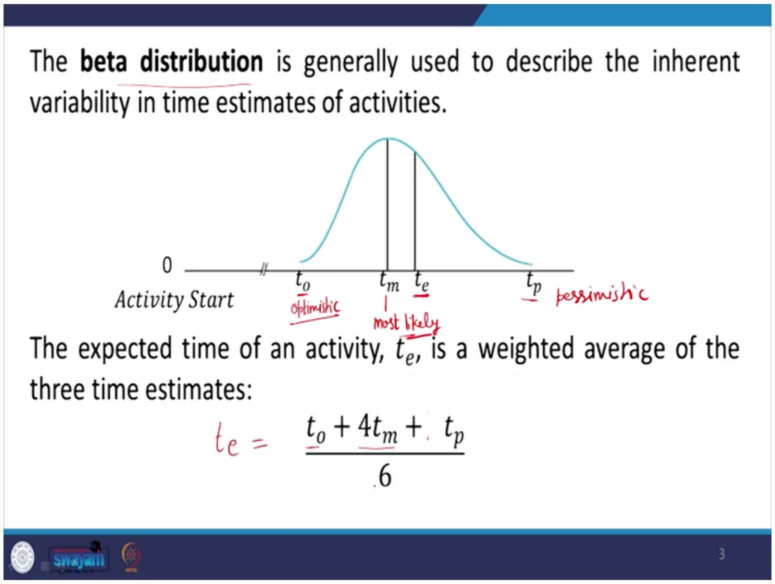

Based on that, you also calculate the time completion and that is known as expected time of completion of these activities. So, this is t naught plus 40 tm plus tp divided by 6. And the basis of this particular formula is that the completion time is during this activity is distributed like beta distribution. And here, these are the different time estimation, t naught is the optimistic, this is pessimistic, and this is most likely.

Now here you see that te, the expected time of completion of this activity is very close to tm, but it may be slightly higher than tm. So, this is the distribution we follow in activity completion time. And based on this, we have this formula for determining the value of te, that is the expected time of completion of this particular activity.

And now, once you have 3 time estimations for each activity, based on that how much variation is possible in each activity that is also you can calculate.

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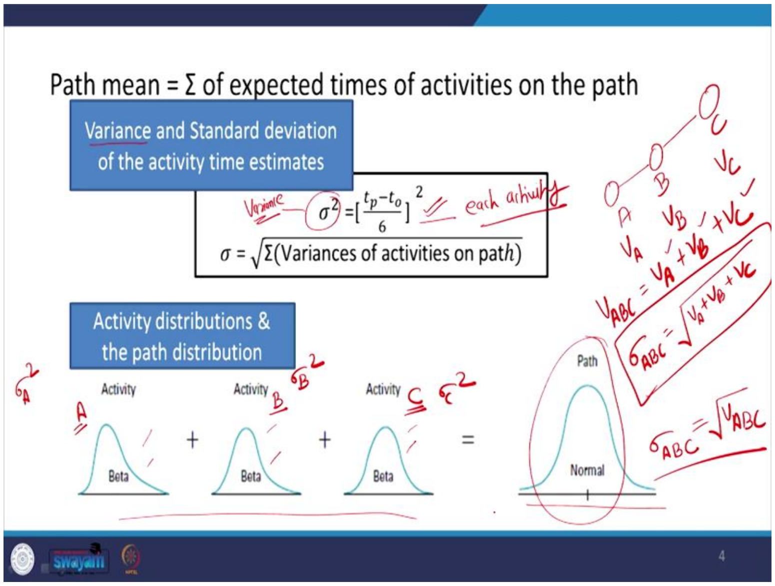

And for that purpose, for that purpose we first calculate the variance of each activities time. And variance is, this formula is used to calculate the variance that is the pessimistic time minus optimistic time divided by 6 the whole square. So, this sigma square it is variance, this is known as variance. So, the variance of any particular path, if this is a path, where 3 activities are taking place A, B, C.

Now, the variance of a variance of A, variance of B, variance of C. So, if I want to determine the variance of path A, B, C, it will be VA plus VB plus VC. And then, if I want to determine sigma of A, B, C, that what is the standard deviation of path ABC, it will be the under root of VABC or VA plus VB plus VC. So, this formula is very useful in our further PERT analysis of the network diagram.

So, you calculate the variance of individual activity, this is for each activity. And then, we do the sigma of though variances of activities on a particular path, we take the under root of that and then you get, calculate the standard deviation of that particular path.

Now, these are the different types of activity distributions and the path distributions which are possible. Now, you see this is skewed to one direction, this is skewed to another direction, and this is more or less like a normal distribution. So, when we combine, when we combine all these different types of distributions it becomes actually the normal distribution.

The point is that one activity A, this is activity B, and this is activity C. So, activity A plus activity B's and activities C's variances we have calculated, like this is sigma A square, sigma B square, sigma C square. These are the variances of different activities, as I have just mentioned here.

And then, the final variance, final variance of all these are added that is taken under root and that gives you the variance of ABC and the under root of that variance is actually the standard deviation. So, that is, the standard deviation is actually following the normal distribution curve. The variance of individual activity may follow the beta distribution but the standard deviation of a complete path is as per the normal distribution.

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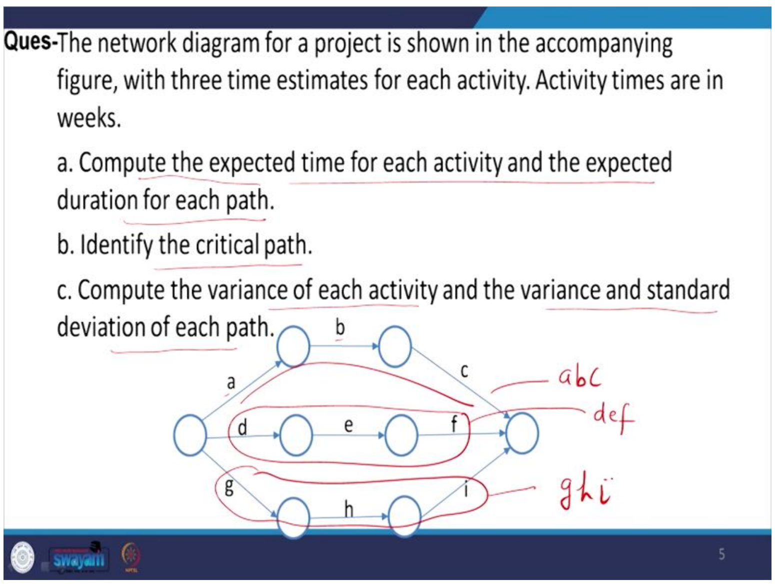

Now, here we have a very simple question and which will help us in understanding the entire process of this analysis of PERT. Now, in this particular case we have this diagram. The network diagram is shown to you that these are the activities a, b, c, d, e, f, g, h, i, etc. These are the different types of activities which are happening.

Now, you have to follow the, calculate the expected time for each activity and the expected duration for each path. Then, you have to identify the critical path, and then you have to find the variance of each activity and the variance and standard deviation of each path. So, now let us see what are the different time estimations for these different activities.

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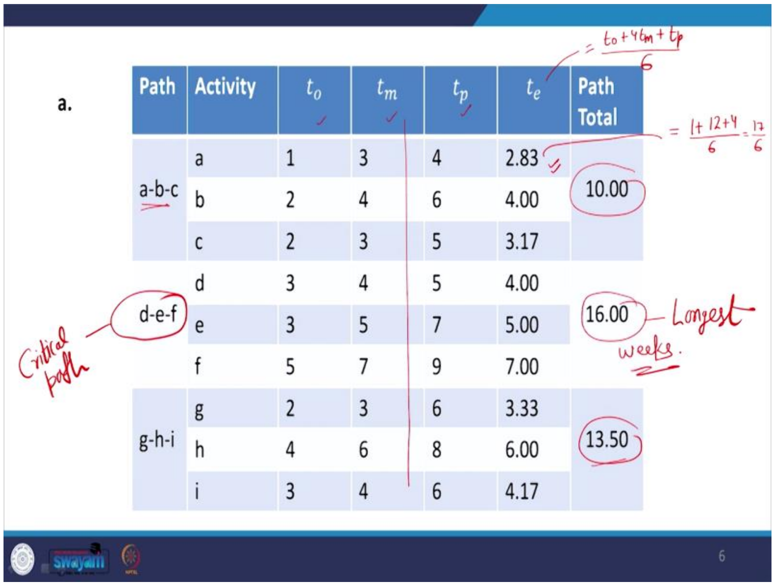
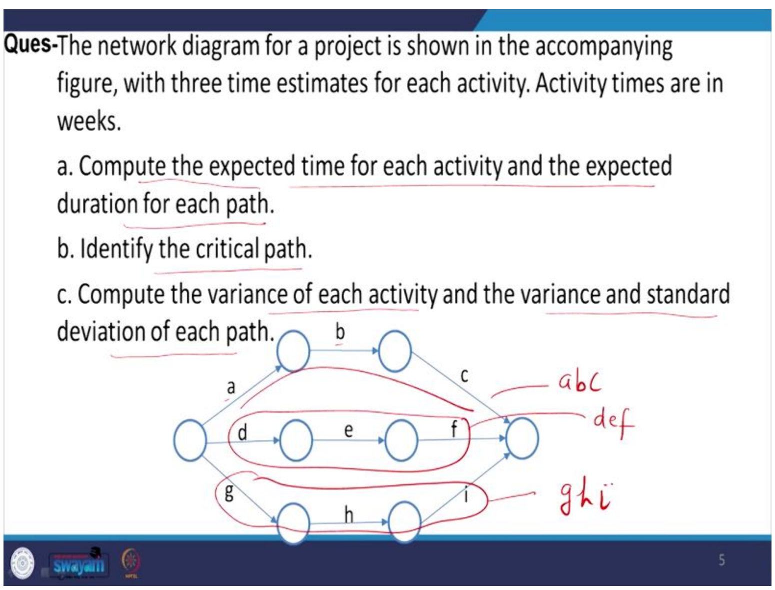


a.

Path	Activity	t_o	t_m	t_p	t_e	Path Total
a-b-c	a	1	3	4	2.83	10.00
	b	2	4	6	4.00	
	c	2	3	5	3.17	
d-e-f	d	3	4	5	4.00	16.00
	e	3	5	7	5.00	
	f	5	7	9	7.00	
g-h-i	g	2	3	6	3.33	13.50
	h	4	6	8	6.00	
	i	3	4	6	4.17	



b. The path that has the longest expected duration is the critical path. Because path d-e-f has the largest path total, it is the critical path.

c.

Path	Activity	t_o	t_m	t_p	$S^2_{act.} = (tp - to)2 / 36$	σ^2_{path}	σ_{path}
a-b-c	a	1	3	4	$(4-1)^2/36 = 9/36$	34/36 = 0.944	0.97
	b	2	4	6	$(6-2)^2/36 = 16/36$		
	c	2	3	5	$(5-2)^2/36 = 9/36$		
d-e-f	d	3	4	5	$(5-3)^2/36 = 4/36$	36/36 = 1.00	1.0
	e	3	5	7	$(7-3)^2/36 = 16/36$		
	f	5	7	9	$(9-5)^2/36 = 16/36$		
g-h-i	g	2	3	6	$(6-2)^2/36 = 16/36$	41/36 = 1.139	1.07
	h	4	6	8	$(8-4)^2/36 = 16/36$		
	i	3	4	6	$(6-3)^2/36 = 9/36$		

The activity are a, b, c, d, e, f, g, h, i, these are the different activities. And optimistic times are given in this column, these are the most likely times, and these are the pessimistic time. So, using this formula that is t naught plus 4 tm plus tp divided by 6, we calculated expected time for each activity. Like for this particular calculation, if I go, it is 1 plus 12 plus 4 divided by 6. So, it becomes 16 plus 1 17 by 6. So, the value is coming 2.83. And so on, we have calculated the values of te for each of the paths.

Now, there are different types of paths. Like one path is a-b-c. Another is, this is one path that is a-b-c. This is another path which is d-e-f. It is another path that is g-h-i. So, these are the 3 different paths, a-b-c, d-e-f, and g-h-i. And for a-b-c we have calculated the total time, for d-e-f also we have calculated the total time, and for g-h-i also we have calculated the total time.

Now out of these 3, the longest time expected duration is the critical path. So, now if you see, if you compare 10, 16 and 13.50, so this is longest. So therefore, d-e-f is your critical path. Since it is longest, therefore d-e-f is your the critical path.

And that is, we have already mentioned that d-e-f has the largest path total, it is the critical path. Now further is, we have to calculate the variance and standard deviation for all the activities. So, like this is tp minus to divided by 6, the formula if you remember it was tp minus t naught divided by 6 the whole square, that is the variance or sigma square.

So, we have calculated the variances for each activity, for activity a, b, c, d, and so on. We have calculated the variance for each activity. And then, for a particular path a-b-c if I add up the variance of these 3, it will give me the variance of path a-b-c. So, this is sigma square abc equals to 0.944. This is sigma square def that is 1.0. This is sigma is square ghi equals to 1.139.

And this, sigma of abc will be the under root of this value of variance, so it is 0.944 and that is coming this 0.97. This sigma of def equals to 1.00, so this is coming this. And sigma of ghi is equals to under root of 1.139, that is 1.07. So, you have calculated the sigma for each of the path, standard deviation.

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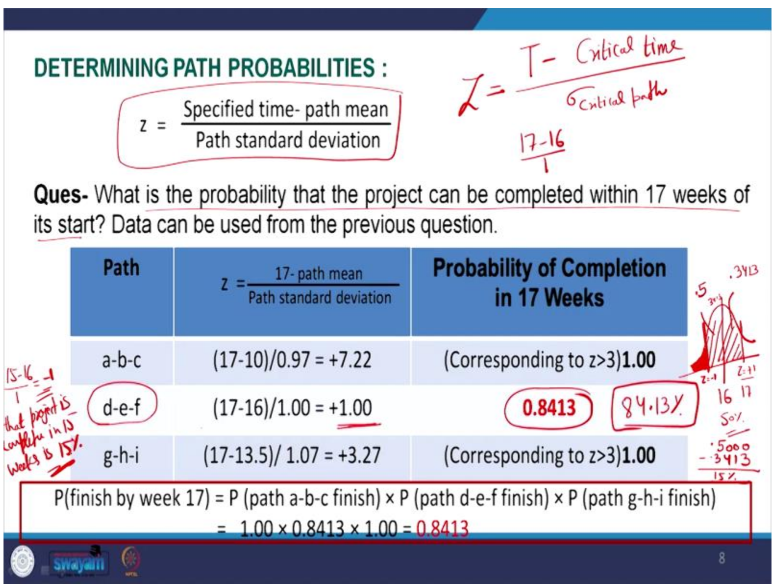

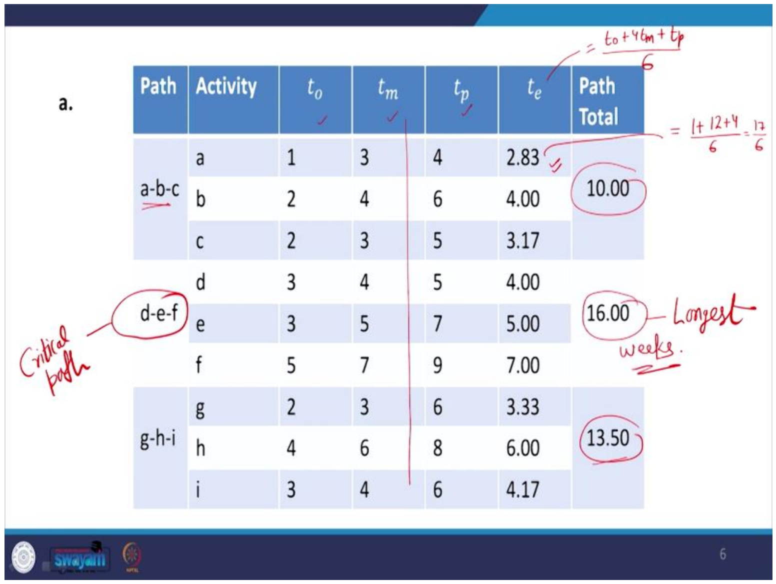

a.

Path	Activity	t_o	t_m	t_p	t_e	Path Total
a-b-c	a	1	3	4	2.83	10.00
	b	2	4	6	4.00	
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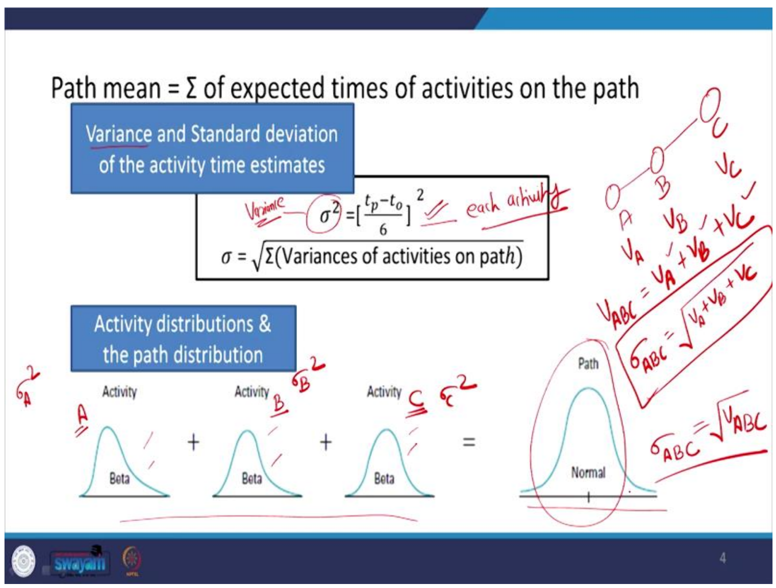
	e	3	5	7	5.00	
	f	5	7	9	7.00	
g-h-i	g	2	3	6	3.33	13.50
	h	4	6	8	6.00	
	i	3	4	6	4.17	



Now, what is the benefit of calculating the standard deviation for each of these path, now it is going to help us in determining the probability of completing the project within a particular time limit.

Now, what is the probability that the project can be completed within 17 weeks of its start? Now for that purpose, let us see the critical path was 16 week. According, to this data the project should be completed in 16 weeks.

Now, the questions is that what is the probability that the project will be completed in 17 weeks? So, without understanding of these calculations somebody may say that the longest time it is going to take 16 weeks, so if you are asking the project will be completed in 17 weeks, so obviously 100 percent probability is there that the project will be completed in 17 weeks because the longest time it can take that is the 16 weeks. But dear students this is not the correct answer.

Let us see, that how it is required to have a further calculation to determine whether the project can be completed in 17 weeks, and how much probability is associated with that. Now, for that purpose I am only doing the calculation with respect to my critical path, the critical path is 16 weeks.

So here, we will calculate the value of Z coefficients. Now, the Z value is, Z value is the time in which we want to determine minus the critical time divided by standard deviation of critical path. So here if you see, I have done this calculation, 17 minus 16, I want to determine whether the project is will complete in 17 weeks, 17 minus 16 divided by 1. So, the value of Z is coming 1.

Now, this value of Z coming 1 is a very good indicator that we discussed in our one of the slides earlier that the distribution of your path, the time completion of your path can be represented by normal distribution. So, this will be represented by the normal distribution, where it is like that. Where this 16 is the mean value, that the probability of completing the project in 16 weeks is just 50 percent, it is just 50 percent that is the probability of completing.

Now, 17 give you the Z value as plus 1. So, plus 1 is 17 and that is Z equals to plus 1. So, the probability will be this entire area plus this area around right-hand side of the mean value up to plus one of Z. And this area is, this is 0.5 already and this is additional that is 0.3413. So, the total probability of completing the project in 17 weeks is 0.8413 or 84 percent.

That is the answer; that the, and similarly, similarly if somebody says that what is the probability of completing the project in 15 weeks? Let us say, we just changed the question. What is the probability of completing the project in 15 weeks? So, then it becomes 15 minus 16 divided by 1 and the value of Z will be minus 1, value of Z will be minus 1. So, we will go to this side of Z and then it is up to here. Let me change the color, so that you can understand it better. So now, the new probability is, it is not changing. So, let me change this, let me darken it. So, this part is representing Z equals to minus 1.

Now, this area is again 0.3413. So, the probability of project getting completed in 15 week will be 0.5000 minus 0.3413, that becomes somewhere around 15 percent. So, the probability that project is completed in 15 weeks is 15 percent. So, now you understood that how with the help of this probabilistic arrangement you can calculate the probabilities of completing the project in different time durance.

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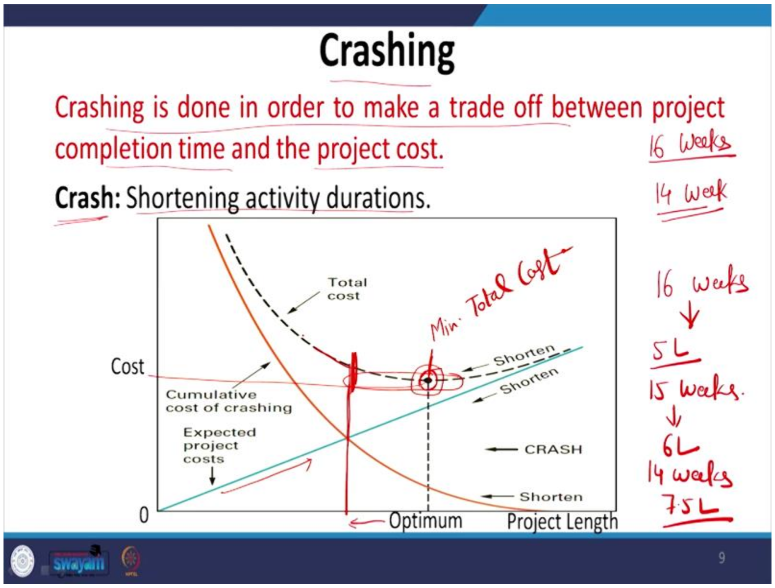

Now, another important thing which we need to discuss is the concept of crashing. Sometime it is possible that the critical path is 16 weeks. But, because of some requirement, because of some urgency, you want to complete project in 14 weeks. And you are ready to invest more money, you want to put more resources for completing the project in 14 weeks.

So, you will crash, you will reduce, you will excuse the duration of completion of some of these activities. So, that is the known as crashing, where you will use more resources to complete your project within less amount of time. And, which particular activity you should crash, which particular activity you should crash, that is important issue. And let us see, that how are we going to handle that particular point.

Now, crashing is done in order to make a tradeoff between project completion time and the project cost. The meaning of crashing is to reduce the activity duration time; that is the meaning of crashing. Now, how it is happening, you see this particular line, this is representing the project cost; expected project cost this is like this.

Now, this is the line which is representing the cumulative cost of crashing. Because when you start crashing, the cost of project starts increasing. There is a normal duration of completion of project and for 16 weeks it is taking, let us say 5 lakh rupees to complete the project.

Now, if you want to complete the project in 15 weeks, cost may increase to 6 lakh. If you want to complete the project in 14 weeks, cost may increase to 7.5 lakhs. So, as you reduce the duration of your completion of project, the cost of your entire project will increase.

So therefore, therefore, we look for some optimum, that this curve combines the crashing cost and the normal cost of completion of the project. And where this, is coming at the lowest level; where this is coming at the lowest level, this is the point which is the minimum total cost. This is giving you the minimum total cost and this is the optimum project length, this is the optimum project duration.

If you are going to reduce the project duration further, you will start increasing your total costs tremendously. If you go to this side; if you go to this side, then cost will increase drastically. So, you can think of whether you want to remain at this level or if you want to reduce your duration further, then cost will increase in a very high manner after a particular time.

You can see that around this minimum total cost, there is a sense of stability. That if you further reduce the cost, if you further reduce the duration there will not be sufficient escalation in the cost. But beyond a particular point, like in this case here around, somewhere around, the increase in cost is very high.

So, the rate at which cost will increase beyond a particular point is very, very high. And therefore, you need to see that what is this point, so this is the point like this, from where the cost of shortening and your actual cost they will not match. And there will be a rapid increase in the cost of your project.

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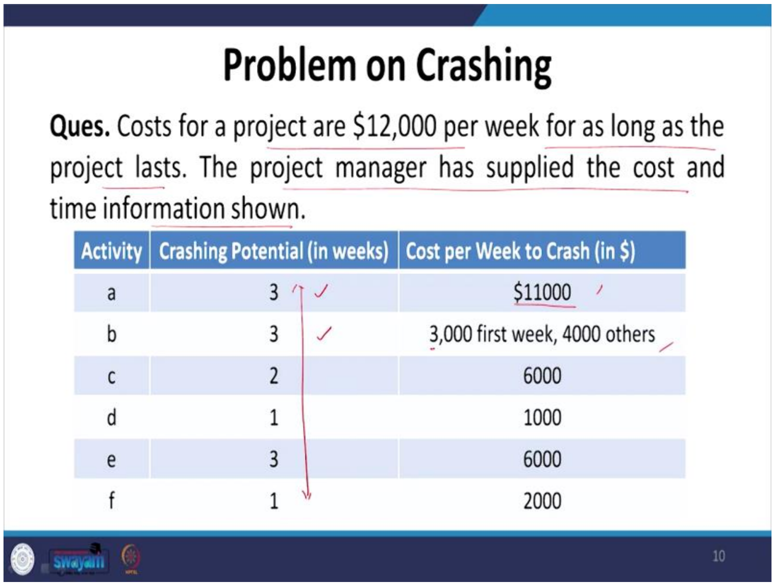

So, let us see with the help of some data which will help us in understanding this concept of crashing. Now, these are the different activities a, b, c, d, e, f, and these are the crushing potential of these activities. That you can reduce the activity a by 3 weeks, activity b by 3 weeks, c by 2 weeks, d by 1 week, e by 3 weeks, and f by 1 week.

And, cost of crashing, cost of crashing these activities per week is 11,000. Then, for first week in the case of b is 3,000 but beyond that it is going to charge you 4000. And so on for other activities 6000, 1000, 6000 and 2000. Now, cost of a projects are 12000 per week, for as long as the project last. The project manager has supplied the cost and time information as shown below.

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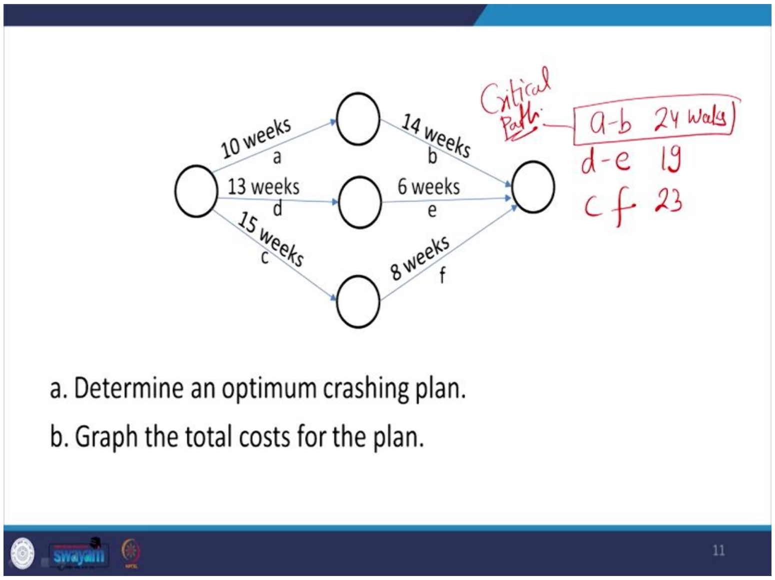

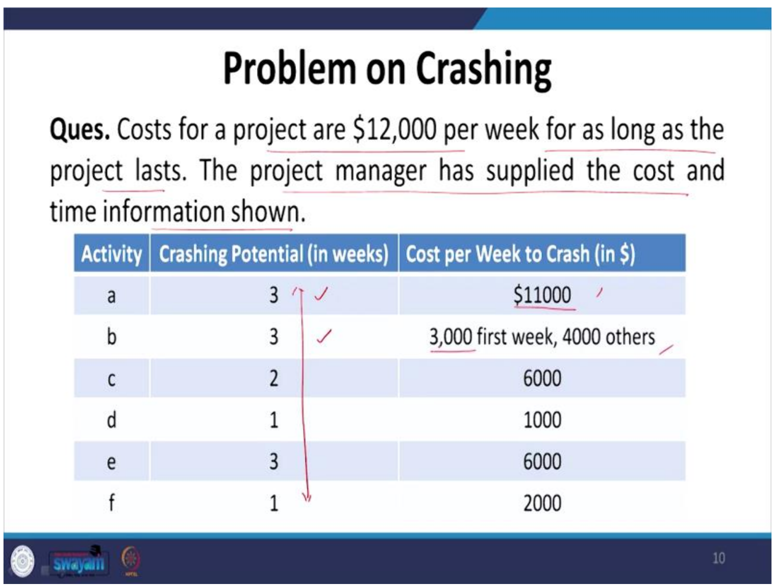

Now, this is the diagram for this particular project. There are a-b, d-e, c-f. So, now you can see that a-b is taking how much time, 24 weeks. d-e 13 plus 6 19, c-f is taking 15 plus 8 23. So, you can understand that a-b is the most optimal that is taking 24 weeks, that my critical path. Optimal means, critical path here.

So, if you want to reduce the completion time of this project, you should excuse the activity time around a-b; a and b. Now, in a and b you see that activity a can be reduced by 3 weeks, activity b can be reduced by 3 weeks, and their charges are these. So, first I will like to reduce activity b because reducing activity b will take only 3000 dollars in the first week. So now let us go for crashing.

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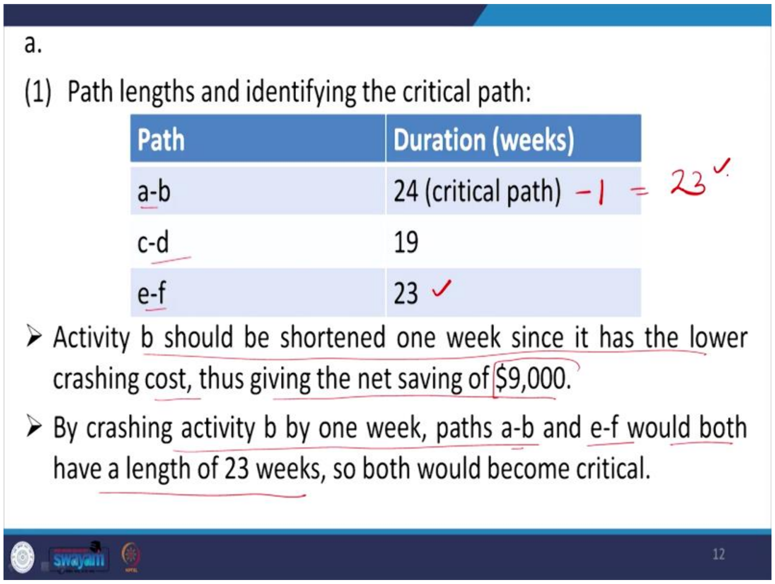

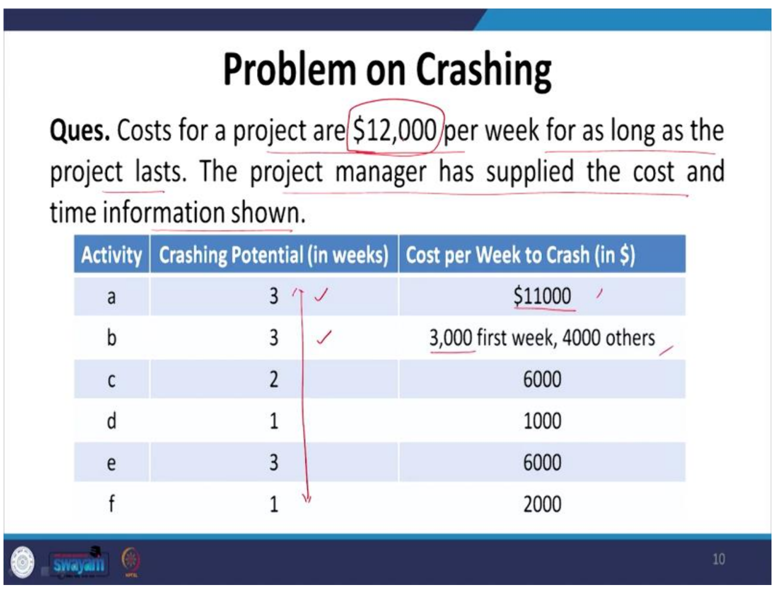

So, a-b, c-d, e-f we have already calculated. Now, activity b should be shortened 1 week since it has the lower crashing cost. Thus, it can give us the saving of 9000 dollars. So, you have the crashing cost of 3000s. And if you reduce it by 1 week, because your per week cost of project is 12000. So, if you are going to reduce your activity duration by 1 week, you will save, because you will put 3000 additional, 12000 at the cost of project per week for maintaining the infrastructure, labor all those things. So, you can save net 9000.

Now, by crashing activity b by 1 week path a-b and e-f, both have the same length of 23 weeks. Here, you are doing minus 1, it becomes 23, it is already 23. So now both these are becoming the critical path, a-b and e-f. Let us resolve them.

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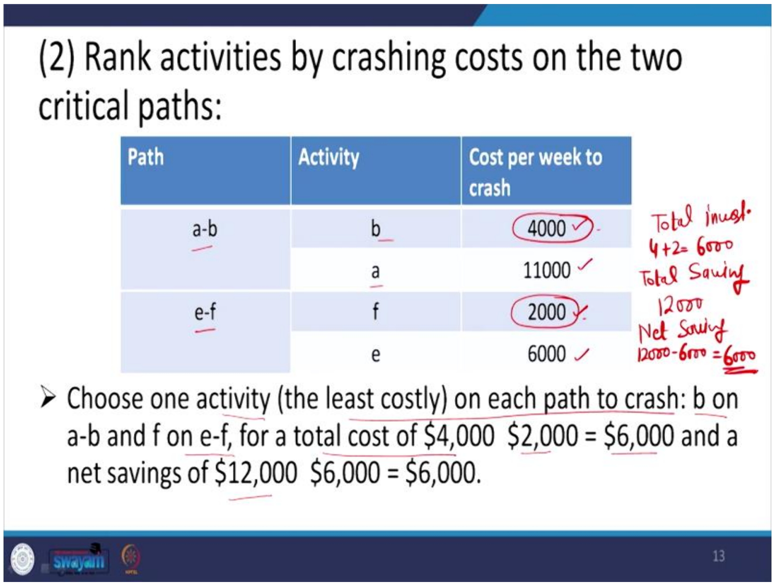

a-b has b and a, it has taking 4000. Now, and it will take 11000, e-f 2000 and 6000. Now, you can see that how to choose the, choose one activity which is the least costly on each path to crash. Here, the least costly is 4000, and in the second case the least costly is 2000. B on a-b and f on e-f, for a total cost of 4000 and 2000, that is the 6000 and a net savings of 12000.

So, we are going to have by reducing it by 1 week and it again by 1 week, you are going to put total investment, 4 plus 2 that is 6000. Total saving, because you are saving your project duration by one week, so that is 12000. So, net saving is 12000 minus 6000 equals to 6000. So, you are still saving 6000 rupees. So, you will like to crash both these activities b and f by 1 week.

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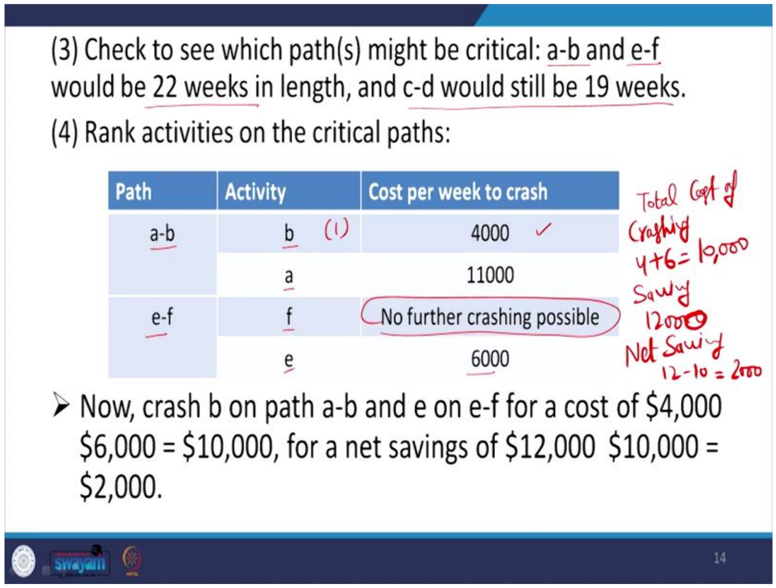

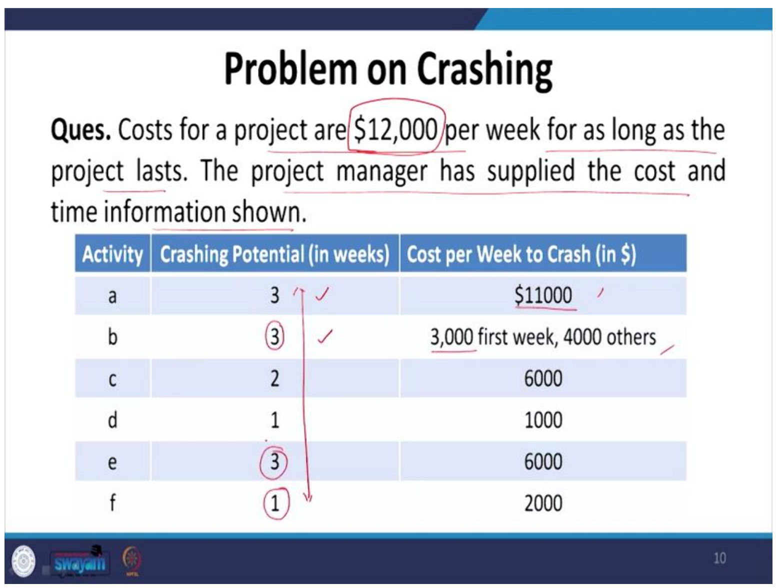

Now, when you are crashing both b and e, b and f your time has reduced to 22 weeks in total. And, c-d is still 19 weeks, so a-b and e-f, now again b and a, f and e. You also need to keep an eye, that how much you can reduce a particular activity. You can reduce b by 3 weeks. Already, we have exhausted 2 weeks of b. And, f you can do only for 1 week, that we have just used. Now, the e we can use for 3 weeks.

So, when we are going for next level, so no further crashing is possible in case of f because already we have exhausted 1 week which is available. In b, we can still have the scope of 1 week. So now, you will crash b on path a-b by 4000 and on e-f you will crash e by 6000. So, your total cost of crashing, total cost of crashing 4 plus 6 that is 10000, saving 12000. So, net saving 12 minus 10 2000. So, still you are able to save 2000. Now, the project duration has reduced to 21 weeks.

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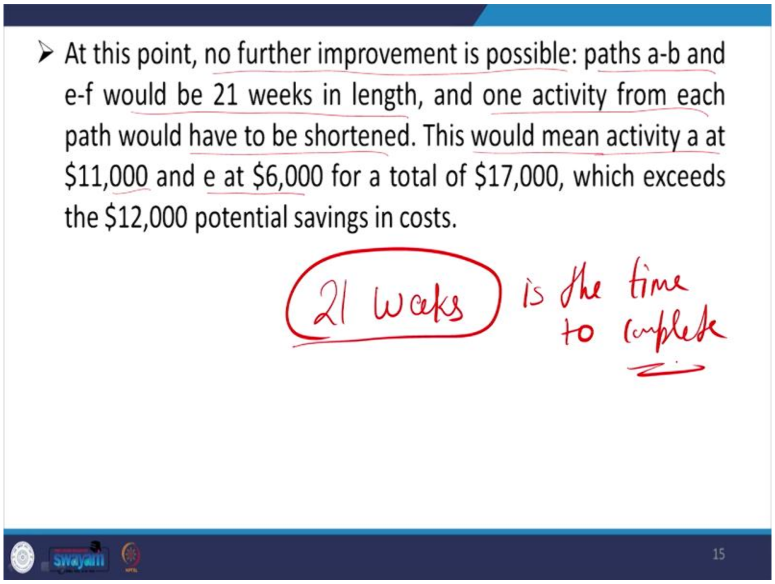

So, at this point no further improvement is possible. Path a-b and e-f would we have 21 weeks in length and one activity from each path would have to be shortened. This would mean activity a at 11000 and activity e at 6000. Now 11000 and 6000 makes 17000, you are going to save only 12000. So, it is not an economical decision to go beyond this point.

So finally, 21 weeks is the time to complete the project and that we achieved by applying the concept of crashing into our projects. That how much benefit you are having by reducing the duration per week? And, how much additional investment you have to do for shortening the activities?

If the investment is less than the benefit, then you will go for crashing, that you will go for additional investment. But, if the benefit is not as per the investment, if the benefit is small, investment is more; then obviously, it is not recommended to go for reducing the duration of the project. So, with this example, we also saw the concept of crashing that how project durations can be shortened to a particular optimal level.

So, with this we come to end of the discussions of project management network techniques. We discussed about various characteristics of project and then we discussed in last two sessions, what are the different types of network management techniques, particularly PERT and CPM.

And, we also discuss the concept of how to determine the probability of completing a project at different project durations, and also discussed about the concept of crashing or how to reduce the project duration by investing more into the activities. So, with this we come to the end of the project management discussions. Thank you very much.