Production and Operation Management
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Lecture 52
PERT and CPM

Welcome friends. In our last session, we discussed about the introduction to project management. We discussed that how projects are very important and very unique kind of activities. We discussed the some of the important terminologies related to project that how work breakdown structure, cost breakdown structure, and organization breakdown structures are initial part of development of a project. We discussed that project has different phases, which we represented in the form of project lifecycle.

Then we also discuss that maximum effort in that project lifecycle is required during the execution stage. So, to monitor execution stage very closely, we have some specific tools available which operation managers must know. And these tools are very effective in managing your project, in your activities in such a manner that we are able to achieve the project trilogy.

Project trilogy, if you remember, we discuss a combination of time and cost, so that you can achieve your specific objectives of the project within a specific time and within a given budget. So, that entire thing is very, very specific and very much time bound, very much resource bound.

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Now, for achieving those things within a given time, we discuss the concept of PERT and CPM. And in this particular session, we are going to make the foundation of PERT and CPM, which are two very popular tools for monitoring and controlling the project activities.

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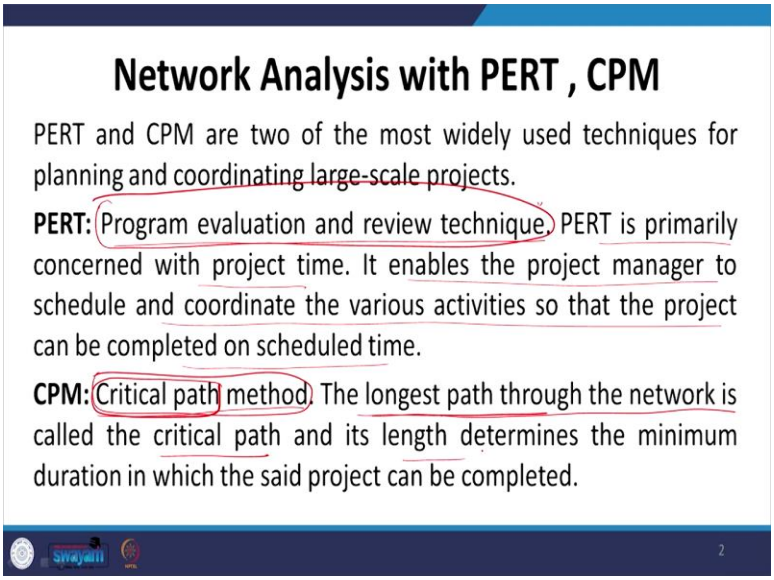

Now, PERT is known as, Program Evaluation and Review Technique. So, PERT is abbreviation for program evaluation and review technique. What we do that PERT is

primarily concerned with project time and it enables the project manager to schedule and coordinate the various activities, so that the project can be completed on the scheduled time. So, there are multiple activities.

We will see with the help of one example, that these are multiple activities and these are the times for these particular activities. So, how to complete your project within a given time, for that purpose PERT is a very useful tool.

Similar to PERT is another important tool that is known as Critical Path Method, which is popularly known as CPM. Now, in this critical path method, we try to identify the critical path, as the name itself suggests, that we are interested in this critical path. Now, the critical path is that path which is the longest path through the network. And, this longest path is having all those activities which are very, very significant for the completion of the project. If any of the activity on that critical path is delayed, it is going to delay the completion of your entire project.

So, the critical path determination and identification of those activities which are on critical path is very crucial for the monitoring of your project. You may have some kind of relaxation; you may enjoy some liberty in those activities which are not on critical path. But those activities which are on critical path require your special attention for project completion in due time.

So, the identification of critical path and the critical path, as I told you is the longest path in the network; the longest path from start to end of the network, that is known as critical path. And, during this all the activities are known as critical activities.

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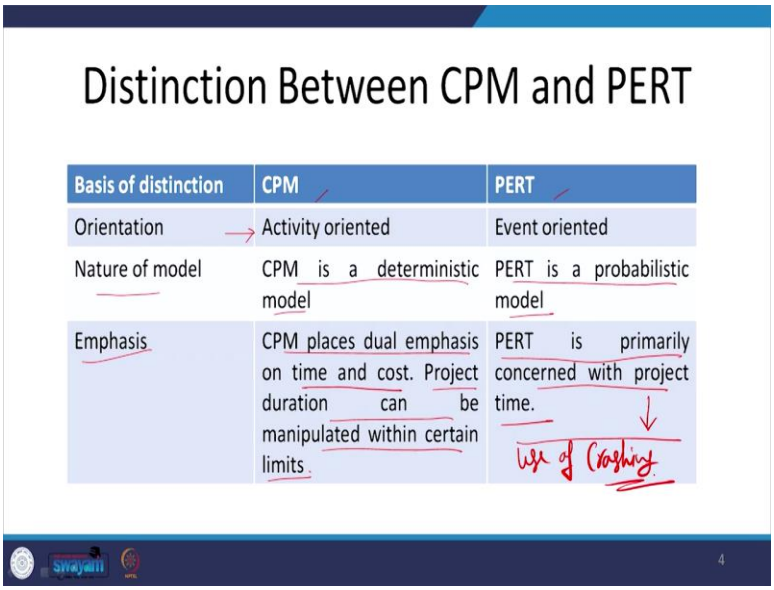

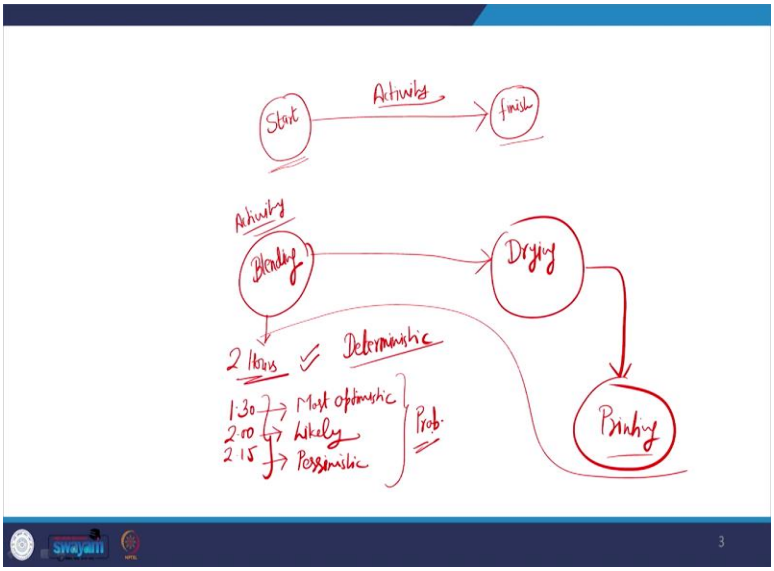

Now, because sometimes we get confused, whether to use CPM for project monitoring or to use PERT for the project monitoring. So, just to have some kind of theoretical understanding, there are issues like this is 2 diagrams. In one diagram, this is the start of activity, this is the finish of activity. So, here you have events, where events are representing the start and finish. The other thing is that blending, then drying, then printing.

Now, these are two types of examples. In first example, the activity is on the arrow and in the second example, activity is on the node. So, these are the two examples. So, one particular thing is based on the event that these are the events, which are guiding the development of your model. And these are the activities, which are guiding the development of your model.

So, based on whether you are focusing on activities or you are focusing on events, we have CPM and PERT. When we are focusing on activities, it is a CPM case. So, like the second example, this is where we are focusing on activities blending, after that drying, then printing, etc. And that is an example of CPM type.

The nature of model CPM is deterministic model, while PERT is a probabilistic model. The meaning is that, when I am talking of any activity, let us say blending. If you say that blending will be completed in 2 hours, I am giving you the time that blending will be completed in 2 hours. So, I am giving you one time. This is deterministic.

The other person says, blending may be completed in 1 hour 30 minutes, it may be completed in some time, in 2 hours also, but some time it may take up to 2 hours 15 minutes. So, here you have different time estimates. So, one time estimate is the most optimistic, other is the pessimistic, and other is likely.

So, there are different types of time estimates you can give for the blending operation. So, based on whether it is optimistic, likely, or pessimistic, you have these as probabilistic time estimations. So, there are different levels of probability, whether it is going to complete in most optimistic, or in pessimistic, or in most likelihood times. And on the basis of that, you will calculate the expected time to complete this activity.

So, when you have multiple time estimations given, that is the case of PERT and when you have a single time estimation, that is the deterministic time estimation, that is the case of CPM.

The emphasis; that is another basis of distinction and the emphasis in case of CPM places on time and cost. That, how to complete the project in minimum time and with minimum cost. The project duration can be manipulated within certain limits because you have only

one time estimate in this particular case. And, PERT is primarily concerned with project time, here our focus is continuously only on reducing the project time.

And therefore, we will see in the case of PERT use of crashing, where we will see that can we reduce the completion of project time further and because we have optimistic time estimations, likely time estimations, pessimistic time estimation, so can all the activities be completed in their optimistic time estimations.

So, the focus is more on the project time and cost is not considered that important. You are ready to pull; you are ready to invest more resources if you can reduce the project completion time. So, that is the important distinctions between PERT and CPM.

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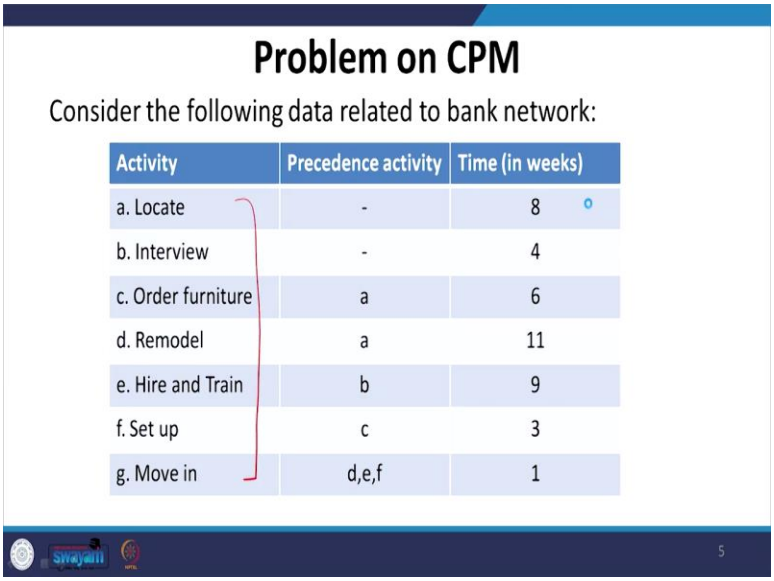

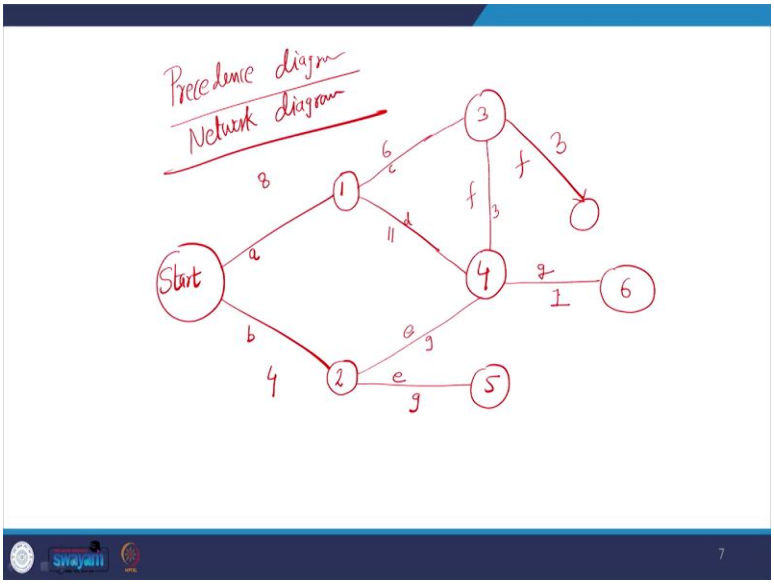

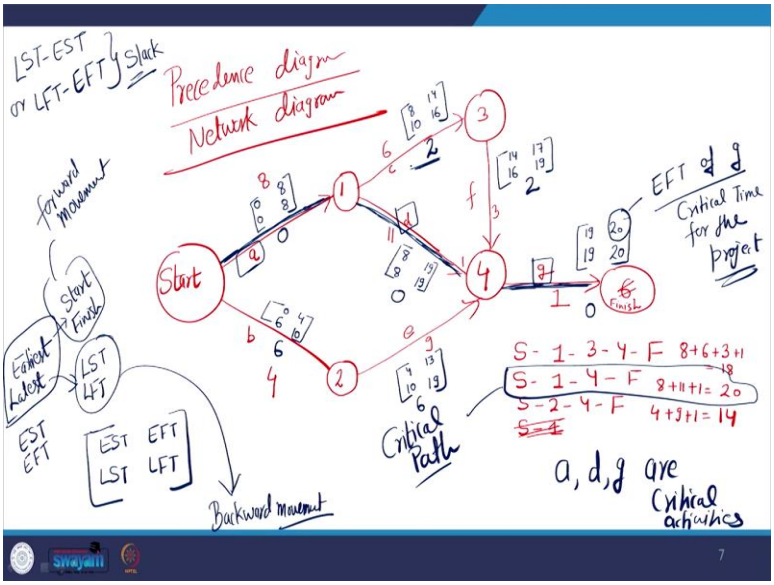

Now, to understand that how these things are actually done, for that purpose we have one numerical problem. And with the help of this small example, we will see that how we have the project monitoring problems.

Now, in this particular case, there are activities from a to g. The first activity is to locate, then to take interview, then the order furniture, remodel, hiring and training, setting up, and moving in, these are the different activities from a to g. And there is a precedence activities which are given to us that activity c can only be started when activity a is there, activity d only after a, e after b, f after c, and g after d, e, f.

So, the first duty is to make a precedence diagram or you can also call it as network diagram. Now, if you go to this question, you see that the first 2 activities are a and b, and activities a and b are not requiring any kind of precedence. So, activity 1 and 2 can start without any precedence. So, locate and interview, these are the 2 activities which are starting without any precedence.

So, let us see that it is the start and 2 activities are there, that is activity a and activity b. Activity a is taking 8 days, activity b is taking 4 days. Now, with this these 2 activities are completed. Then you see your activity c which is only possible after a, d is also possible after a. And therefore, once this activity is finished, then your new node, so start is this, so this you can name as 1, this you can name as 2.

So, from here you have two new activities which are starting. And one activity is taking 6 weeks time, another is taking 11 weeks time. And you know their names, the 6 week activity is c and 11 is d, so this is c activity and this is d activity. And you can put these names as 3 and 4, so that is further.

Now, up to d we have reached. Then e, e is only possible when b has completed. So, b has completed, so e will start and e is taking 9 weeks time. And, now we have come to this very point. Now, you see f is only possible when c is completed, f is only possible when c is completed. So, c is there and now f will start from here, and f will take time that is 3 weeks.

Now finally, you will see that the activity g is remaining and that is only possible when activities d, e, f are completed. So, you have final activity g which is going to, we completed. And g is going to happen, when all these 3 activities are going to be completed. So, you can think of merging these 3 into 1 step.

So, in place of e going from here to here, we can create e in this way, we can create f also in this way. And then, d, e, f, finally they will come into this particular fashion, where you have this particular block and that is your last activity, that is g. And this g is taking only 1 particular day.

And now, I am going to erase these extra lines which I have just drawn, because these are not required. So, this way our diagram is now made, that we have started or you can write in place of 6, finish. So, that also you can write.

So now you see, in this diagram, we have multiple paths S-1-3-4-F, finish. You have S-1-4-finish. You have S-2-4-finish. You have S-1-3-4-finish, that we have already taken. So, these are the 3 alternative paths we have in this particular case.

Now, if you see simply for this S-1-3-4-5. So, S-1 it is taking 8 plus 6 plus 3 plus 1, that means 8 plus 6 14 plus 3 17 plus 1 18. S-1-4-5, S-1 4, that means 8 1 to 4 11 plus 1. That means 18, 19 plus 1 20. S-2-4-5, 4-F, S-2-4, 2 2 4 9 plus 1, that means 14.

Now, these are the 3 paths which are emerging out of this diagram. Out of this you can easily see, that path S-1-4-F, that is this path- S-1-4-F, this is the path which is taking the longest time out of 3 paths. So therefore, this becomes my critical path, out of 3 paths which is taking the longest path, that becomes my critical path.

Now, all the activities on this critical path are known as critical activities. So, 3 activities are there a, d, g; a, d, g these are the critical activities. This is a, this is d, and this is g. These are the critical activities; you cannot think of any delay in the execution of these activities. So therefore, these are known as critical activities.

Now, there are few more important things. As I discussed, that the other activities, like you have to do all the activities c, f, b, e all those activities are required to be completing this particular project. But you may not be very, you can say, attentive or you may relax a bit with respect to these other activities. But how much can you relax, how much can you relax, that is the next important question.

And for that purpose, we need to do some more calculation. And for doing that additional calculation, we calculate 2 things. One is the earliest times and the second is the latest times. So, these two calculations we need to do. The earliest times will be calculated when we are moving in the forward direction. From start to finish, when we are moving in the forward direction, those calculations will give us earliest time. And when we will

move in the backward direction, from the last, that is the finish to start that will give us the latest calculations. So, let us start doing that calculation.

Now, the calculation will have 2 components. The earliest calculation will have 2 components; the start time and the finish time. For each activity, you will calculate Earliest Start Time, which is known as EST, that is the earliest start time and you will also calculate the finish time, that is earliest finish time.

Similarly, in our latest calculations also, we will calculate latest start time and latest finish time. So, you will calculate these 4 values for each activity. In this diagram, for each activity you will calculate these 4 values; earliest start time, earliest finish time. And let us start with this forward calculation first.

So, the activity a will start on day 0. So, I am writing in this way, so that we can have a very simple understanding, EST, EFT and below that we will write LST and LFT. So, these 4 values we will write in the bracket and, so by placing these values you can understand which is EST, which is EFT, which is LST, and which is LFT.

Now, activity a will start on day 0 and the earliest finish time is 8, activity b will also start on day 0 and the earliest finish time is 4. Now, activity c can start earliest on day eighth and it will finish by adding 6 into it on fourteenth. The activity d will start earliest on 8 and it takes 11 days, so it will finish on nineteenth. Now, activity f will start on fourteenth and it will finish on seventeenth. Activity e will start on 4 and finish on nineteenth; 9 plus 4 thirteenth.

Now, activity g can only start when all f, d and e are completed. When f, d, and e these 3 activities are completed. One activity is completed on seventeenth day, another is completed on thirteenth day, and another is completed on nineteenth day. So, you can easily understand g can earliest start on nineteenth day, g can earliest start on the nineteenth day. Before that, g cannot start because all 3 activities will available on the nineteenth day for the first time. And it takes one day, so it will finish on the twentieth day.

So, with this you have calculated EFT of g, this is the EFT of g and that becomes the critical time for the project. So, this becomes the critical time for the project, that your g can earliest finish on twentieth day.

Now, this the last value of EFT, that is for g in this particular case, that is 20. This is taken as LFT for g. So, this will automatically be placed here. And then, now we will go into the backward calculation. Earlier for calculation of LST, EST and EFT, you have forward movement, and for LST and LFT, we have backward movement. So, now we will start moving back from last stage to the start stage.

So, the EFT of g will be the LFT of g. And, now the LST will be minus LFT minus 1, LFT minus the activity duration that is 19. So, your this LFT, this LFT and LST calculation will give us LFT for those activities which are just before this particular activity. That means, this 19 will go as LFT for f, 19 will go as LFT for d and 19 will go as LFT for e. For all these 3 activities, the LFT will be this 19.

Now, f will be completed in 3 days. So, 16 can be the LST for f, d will be completed in 11 days, so 19 minus 11 8, will be the LST, for that is Latest Start Time for d. e is completed in 9 days, so 19 minus 9, 10 is the LST for e. So, you have got now LST's for d, e, f. Now, for activity 3, from activity f, you can go to activity c because that is just before activity f.

Now, the LST of activity f will be the LFT of activity c that is 16. And the activity is taking 6 days, so it is going to have the latest start time of 10. Then, from activity e, you can come to activity b, the latest start time of activity e will come as latest finish time for activity b, that is 10. Activity b is taking 4 days, so the latest start time of activity b will be 6.

Now, you see that activity a, activity a is having 2 activities as its follower activities, c and d, c has the latest start time of 10 and d has the latest start time of 8. So, you have to finish your activity a by eighth day or eighth week, so that activity d can start on the eighth day. So, the out of these 2, 10 and 8, the lower value will be my latest finish time for activity a. And activity a is taking 8 days, so the latest start time of activity a is 0.

Now, you can do calculation like LST minus EST or LFT minus EFT, you can do any of these calculations and that will give you slack of the activity. And slack is the answer of that question, that how much I can relax.

So, now you see the calculation of slack I am putting below this our matrix, here it is 0, here it is you see 2 days, here it is coming 2 days, here it is coming 0, here it is coming 6 days, here it is coming 10 minus 4 6 days, here it is coming 19 minus 19 0. So, all those activities which are on critical path activities a, activity d, and activity g; a, d, g are critical activities.

It is being proved again and again, because your path a-d-g or that is the second path S-1-4-F that is the critical path. And all those activities which are on critical path should be the critical activities. And our calculation has also demonstrated it, that the slack in all these activities are 0. So, for the critical activities, slack is always 0. So, that we have just seen by the calculation.

Now, we see in the other calculations that, here in this case, my activity on this particular path, where we have activities like c. So, in this activity c, we have a slack of 2 days; means the start of this activity c can be delayed by 2 days. If you do not have enough resources, you can delay the start of this activity c by 2 days. Activity f can also be delayed by 2 days. You do not have the resource; you can delay the activity f by 2 days.

Similarly, if you see the activity b and a do not have the precedence. So normally, anybody can think that okay, we can start both activity and b, a and b simultaneously. But now the calculations shows, that b has a slack of 6 days. So, it is not necessary to start a and b simultaneously, you can start a today but you have a time of up to 6 weeks to start activity b. So, that gives you enough idea of resource sharing that how you can use your resources more effectively.

Because if you start a and b together, your b will consume some resources. But you will not be able to complete the project before 20 weeks. So, you can delay the involvement of your resources by 6 weeks, so that your overall cost of project goes down. So, that is the idea of this critical path method that how we can combine the involvement of time

and cost. And, we have done this entire calculation. And now, we will show you finally the summary of our, this entire calculation.

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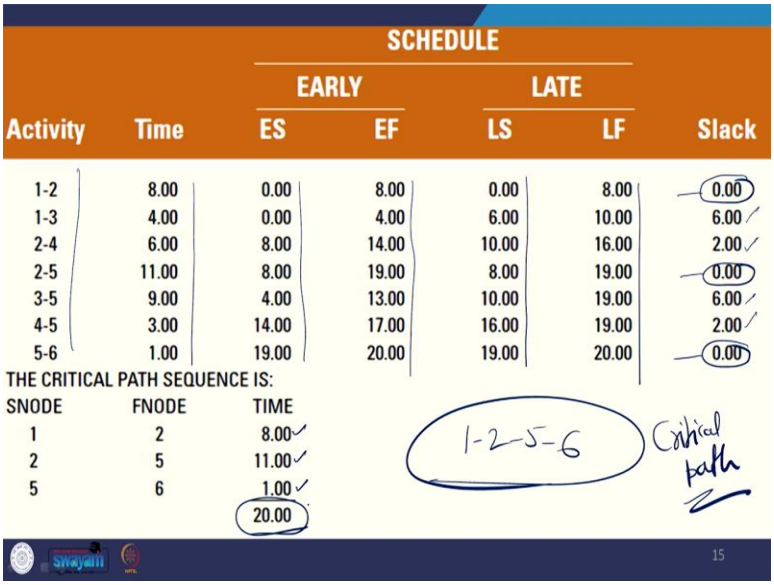

That these are the various activities we just discussed in the question. These are the times given to us, and for each of them we calculated our earliest start time and earliest finish time. So, we have shown them on that diagram. And, these are the latest start time and latest finish time.

Now, you see that 3 activities, this particular activity, this particular activity, and this particular activity; these 3 activities are having 0 slack. So, these 3 activities are making my critical path. All those activities which are having 0 slack, so that is the activity 1-2, this is 2-5, and then 5-6, so this becomes my critical path.

And the availability of the slack in various activities, 2 activities are having slack of 6 each, 2 activities are having slack of 2 each. That means, you can delay the start of these activities by this much amount of time. So, that is the critical path sequence that is 8, 11, and 1. So, that is the total time which is being consumed in my critical part.

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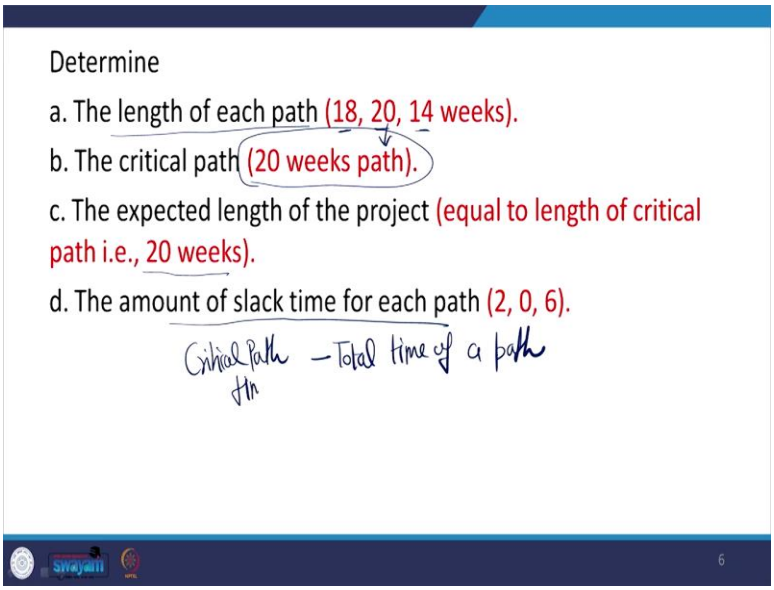

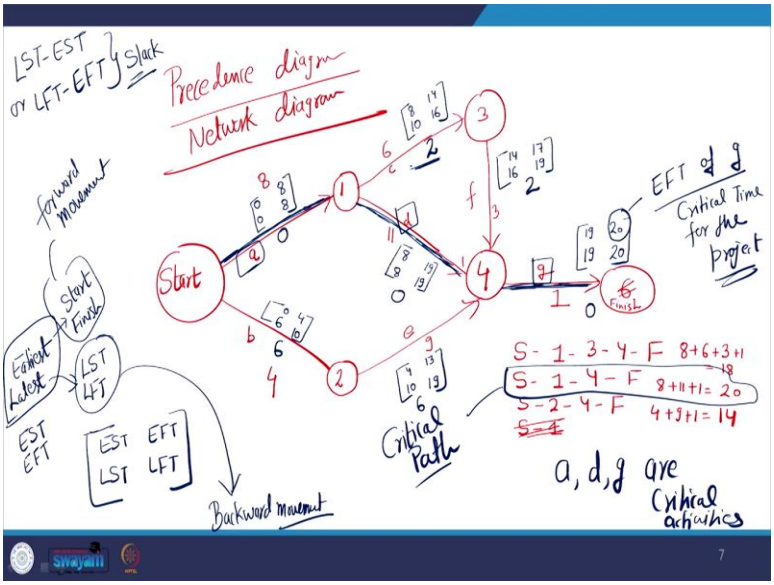

So, if I see this questions various issues, you will find that the length of 3 times, 3 paths which we have in this question, one path was having 18 weeks, another is 20, and another is 14. The critical path is of 20 weeks, where you are going to have maximum time. And therefore, it is expected that the project will be completed in 20 weeks.

And the total time of a path when it is subtracted from the critical path, time, so you get the slack time for a particular path. So, if you see, for one of the path, the total time of

completion is 18 weeks. So, the critical time, that slack time in this particular path is 2 weeks.

In another path, the total time to completion is 14 weeks, so 20 minus 14 6 weeks. So, we have the slack available for 3 different paths are 2 weeks, 0 weeks, that is the critical path itself, and 6 weeks is the another path. So, with this we calculated the important characteristics of a particular path with respect to critical path method.

In our next session, we will discuss about the probabilistic assignment of the time. And, with the help of those probabilistic assignments, we will try to answer more questions related to project management. Thank you very much.