Production and Operation Management
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Lecture No. 45
Process Capability

Welcome friends. So, in our last session, we discussed the about the acceptance sampling and we introduced two terms, one is producer's risk and another is consumer's risk. Because we are discussing all these things under the laws of probabilities. So, whatever you are doing there are chances of some errors. Now, if you are taking some sample there are possibilities that some defective pieces may go into the hands of the customer or it is also possible that some okay pieces may be declared as defective or rejected. So, that is producer and consumer's risk.

Now, moving ahead, we need to discuss that, how we can minimize the variations in our processes, very time, we have discussed that variations are natural phenomena and you cannot eliminate the variations. But can we minimize the variations? Yes, we can minimize the variations, if we work on improving the process capabilities, if our processes are better capable, we can minimize. The variation in our processes and when we can minimize the variation in the process, we will give more satisfaction to our customer, our cost of quality will also decrease, our rejection rates will also decrease.

So, in this particular session, we are going to discuss about process quality and this discussion of process quality will lead to the discussion of another very important topic that is Six Sigma. So, nowadays many of us whenever we talk of quality or statistical quality control, we immediately jump to Six Sigma discussions. And there are special training programs only based on Six Sigma. A lot of research is happening across the globe on implementation of Six Sigma in the organizations, not only manufacturing, but service organizations also.

But before discussing the six sigma, before moving to that concept, it is important to understand what is process capability? How process capability is measured? What are the different types of issues related to process capability? Then only we can discuss Six Sigma with right inputs.

So, let us start with the process capability discussion, in our discussions of quality control charts or in the acceptance sampling, we have discussed that we are taking some sample whether it is a quality control chart. So, on a regular interval, we take some sample of four or five items, we check the parameters of those samples. And based on that, we continuously track the movement of those parameters. In acceptance sampling also, we take a sample and we test the quality of the sample and if sample is fulfilling the required predefined level of acceptance, we are going to accept the sample otherwise, we are going to reject it.

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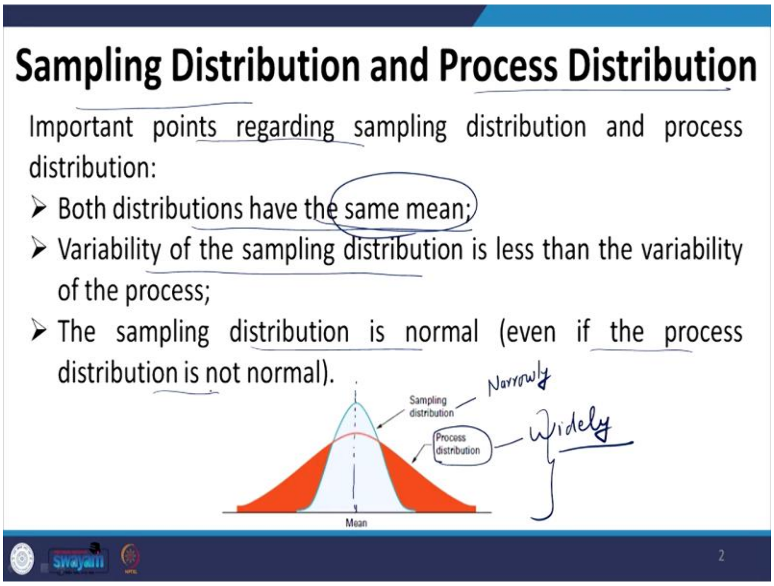

So, sampling distribution and process distribution are the first important thing, which we need to understand. Now, some important points which are worth considering, worth, you can say remembering with respect to sampling and process distributions are that both distributions have the same mean; our process and sampling both these distributions have same mean; here, if you see in this diagram, we have two shapes of the normal distribution curve.

Now, one shape, which is slightly, widely scattered and another is narrowly scattered. So, this shape which is in the orange color, this is determining my process distribution and shape which is in the light blue color, which is narrowly scattered around the mean value, this is the mean value, this is giving the sampling distribution, both these distributions are normally distributed around the mean value, but their spread is difference, the sampling distribution spread is low, the process distribution spread is wide, but, mean is same, they both have same mean value this is that mean value line.

Variability of the sampling distribution is less. So, it is narrowly disk, narrowly spread and this is widely spread. The sampling distribution is normal, even if the process distribution is not normal. So, maybe, maybe the process distribution is not normal sometime, but sampling distribution is always normal.

Here, we are considering that for the sake of understanding that both these distributions are normal as per the given conditions. Otherwise, sometime the process distribution may follow some other kind of statistical distribution, but most of the time it has a normal distribution. So, these are about process and sampling distribution.

Now, here a very important concept is to be highlighted and that helps us in generalization of various statistical test and that theorem is central limit theorem. We will not discuss this threat central limit theorem in great depth, because this is a subject matter of class of business statistics.

So, in that we discuss whenever we are going for testing of hypothesis and before that we need to discuss this central limit theorem that how the samples are determining something about the population. So, generalization of results obtained for a particular sample can be done for the entire population using this central limit theorem.

So, this is a very, very useful or rather you can say fundamental of entire hypothesis testing. And we also in this case are assuming that whatever sample we are taking, whatever calculations we are doing on the samples will be applicable to the entire population. So, therefore, the introduction of central limit theorem is very important here.

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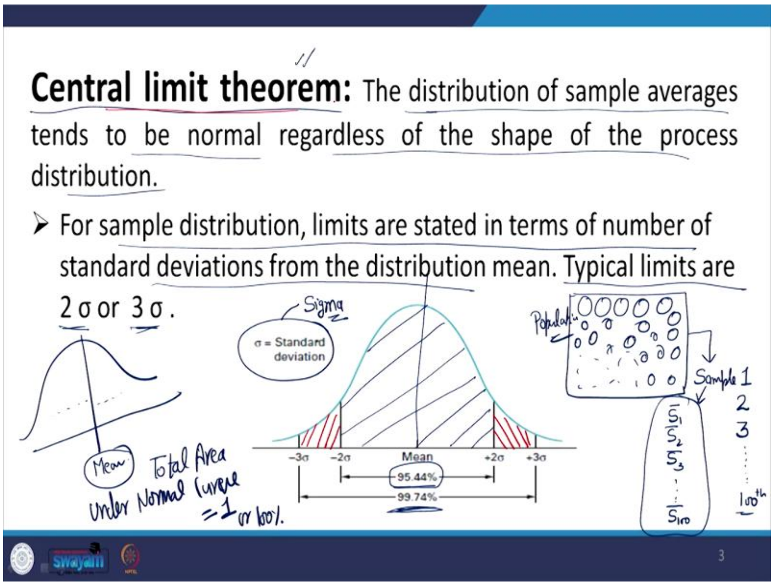

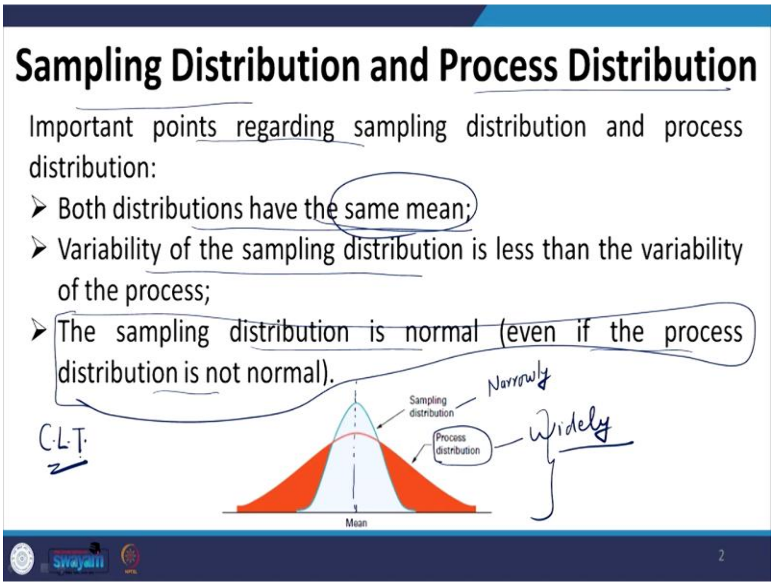

Now, what does it say that the distribution of sample averages tends to be the normal regardless of the shape of the process distribution. So, you are taking different samples, for an example, what does it mean to say that you have a process, this is the process output, where large number of items are there. Now, out of this process output, the process distribution may follow any kind of distribution, but you have sample 1, you have sample 2, then you take a sample 3 and then you take 100 sample out of this particular population.

Now, for each sample there is a sample mean, let us write it as S1, S2, S3 bar and so on S100 bar. This says, central limit theorem says, that the distribution of these sample averages tends to be normal. The shape of these simple averages can be represented as a normal distribution curve like this. So, whatever is the shape of process distribution, irrespective of that the distribution which is formed because of sample means, that will always have normal distribution.

So, some of these samples will have values on mean; some samples will come this side; some samples will come this side. Most of the samples will be around this mean value; most of the sample averages are around mean value and some will be on the left side, some will be on the right side. So, that is the central limit theorem. Same principle we have in this process and sample output that this the process output is there and you are taking samples from the process output.

So, if you are taking sample from the process, so, the averages of those samples will always follow the normal. So, this particular line which we just discussed is the result of central limit theorem that the sampling distribution is normal irrespective of shape of distribution for the process output, your this particular theorem, this particular theorem, you can say is the foundation of most of the statistical test, because we considered everything on the basis of sample. And the sample averages, they will form a shape which is of the normal distribution curve. So, that is a very important thing we need to understand.

For sample distribution limits are stated in terms of number of standard deviation from the distribution mean. Typically, these limits are two standard deviation or three standard deviation. Standard deviation is known as, this standard deviation is known as Sigma. So, use a 2 sigma or 3 sigma that is the, so, here if it is the mean value.

So, how much is your sample distribution varying around the mean value? So, that is either 2 sigma plus minus 2 sigma this much and this plus minus 2 sigma is covering the total area under normal distribution curve, total area under normal curve is equals to 1 or 100 percent that is the total area.

Now, this total area is equally divided on both the sides of the mean value. So, 50 percent of the area is on the left side of the mean value and 50 percent area is on the right side of the mean value. Now, when I am saying the mean values of sample are distributed within plus minus 2 sigma, it means from minus 2 sigma to plus 2 sigma, these values are distributed and this covers 95.44 percent area of the mean, of the normal distribution curve. So, around 5 point; 3.5 percent area, which is beyond plus minus 2 sigma will not have any mean value of the samples.

Similarly, in some other case I can say that the area which is covering all the means of samples that is plus minus 3 sigma. So, the area under plus minus 3 sigma is 99.74. Please remember this value 99.74, we will be using this particular value again and again in our discussions.

So, it is having a bigger area and this is also added in this particular case. So, that is 2 sigma and now, I have added 1 sigma, 1 sigma more on the either side of mean value. So, this becomes total Six Sigma and it is 99.74 percent area. So, only around 0.26 percent of area is beyond my Six Sigma limits. So, most of the area I have already covered. So, this is the usefulness of Central Limit Theorem, that irrespective of.

So, the point which is to remember that sample distribution and mean of those sample distribution will always follow a normal distribution curve, whatever is the shape of process distribution, irrespective of that, the shape of means of sample distribution will always be actually the normal distribution. So, you please remember this theorem and it will be used again and again, not only in our course of operations management, but in the courses of statistics and probabilities also.

Now, the after understanding what is sample mean? What is the process mean? What is the central limit theorem? The other important concept which we need to know before we jump to Six Sigma that is a process capability analysis. The process capability analysis that how capable, how much capable is your process? How much variation your process is producing?

So, sometime you must have seen that if you are going on a bicycle, so, for some bicycle, the wheel moves in a perfect straight-line direction, but sometime there is some side movement also in the wheel, it is not moving in a perfect straight-line direction. So, when some side movements are there it means that process is less capable, when the wheel is moving in a perfect straight-line direction, that means the process is more capable. So, how to identify that process is capable or less capable or more capable for that purpose process capability analysis is done.

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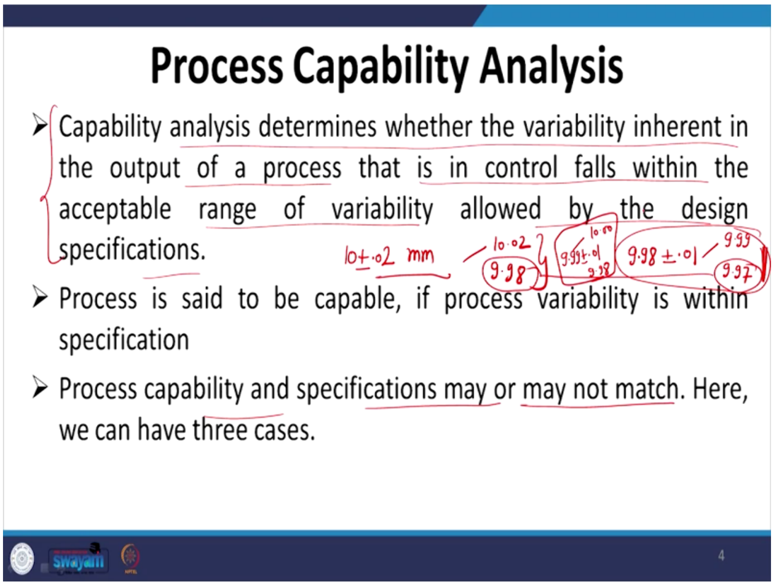

Now, let us see how do we do this process capability analysis and what are the different steps involved? Now, the capability analysis determines whether the variability inherent in the output of a process that is in control falls within the acceptable range of variability allowed by the design specifications. So, if the inherent variations in the process are within the design specification. For an example, what does it mean to say that I want to make a product, where the dimension is 10 millimetre and I have some tolerance available of 0.02 millimetre.

So, I can make up to 10.02 to 9.98 millimetres. So, these are by upper control and lower control limit, I have a process which is producing the mean value or which is producing the product of 9.98 that is the mean value and it has a capability of plus minus 0.01. So, it can produce from 9.99 to 9.97. Now, though the variation which by process is producing is lesser than the variation which are allowed by the specification, but still, because the mean value is 9.98 of the, my process output, there are some output, which are below the lower limit of specification.

So, here I will say that this process is not capable, because there will be few outputs, which are of 9.97 also. And these outputs are below the lower limit of specification, but, in place of this, but if in place of this, you have something like 9.99 plus minus 0.01. Then it will produce 10.00 and 9.98.

Now, you will say that the process is completely capable to produce as per the given specification because all outputs, all outputs of this process will fall within the specification given by the design people, design department. So, this process is now a capable process, the earlier process, which was also having the variability of only 0.01 millimetre, but that variability because of its mean value was not able to give me the desired process capability. So, process is said to be capable if process variability is within the specification.

So, now, in this particular case, it is not within the specification that I say that this is not a capable process, process capability and specification may or may not match. So, in these three cases we just saw that process capability and specifications are not matching, here we can have three different cases.

So, because many a time the specifications are decided on the basis of customer requirement, what customer expects on the basis of that product specifications are decided. But the process specifications, the machines which are producing those output are not purchased for each different products, you have a machine which is with you for long years.

So, every time you want to produce some new product, you have to see whether my machine is capable to produce that particular product or not. But it will be very rare combination that the machine capabilities or process capabilities. So, process capabilities are similar to your product capacities.

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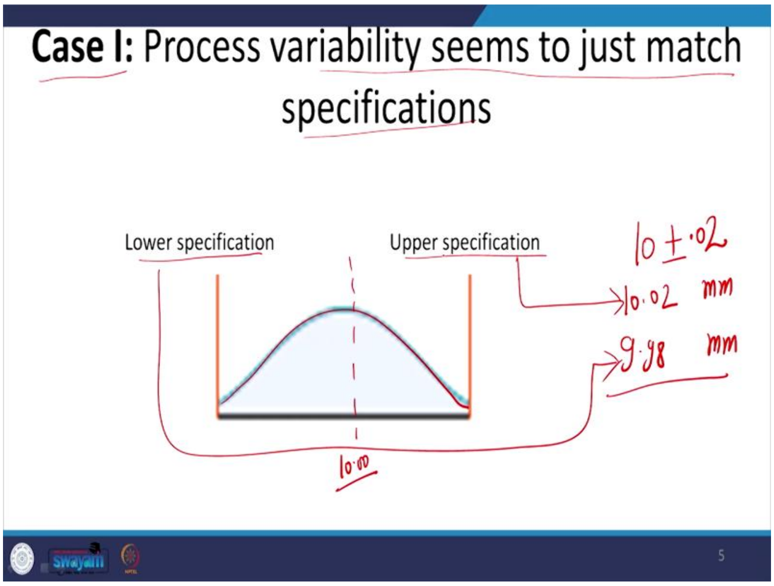

So, three cases are possible. So, case 1, what is this case 1? The process variability seems to just match the specific. So, here the specifications are the upper and lower limits of your specification. So, you have plus minus 0.02. So, this is 10.02 and 9.98 let us say millimetre. So, this is upper specification and this is lower specification.

Now, in this particular case, my process variability, this process variability is exactly matching the mean value of the process that is 10.00 millimetre that is matching exactly with the output or that is required in the product. So, this is the simplest case where you have perfect match. But the perfect match is a very rare phenomenon.

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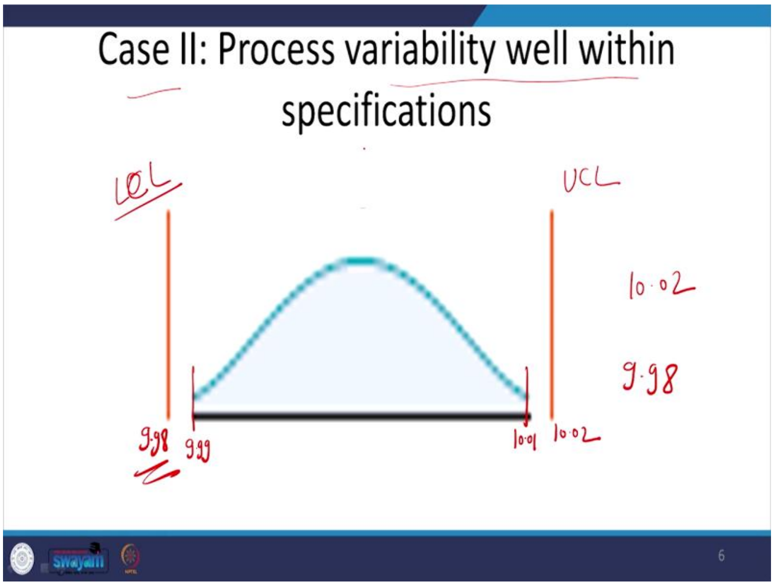

Then the second case, what is happening in the second case, process variability well within the specification. So, this is upper control limit, this is the lower control limit. Now, your upper control limit and lower control limits are too wide like it is 10.02 and 9.98 and your process capability is so good that it is producing only between 10.01 and 9.99 this is 10.02 and 9.98. So, here your process variability well within the specification. So, again whatever this process is producing that will be acceptable because these are within the UCL and LCL for this particular case.

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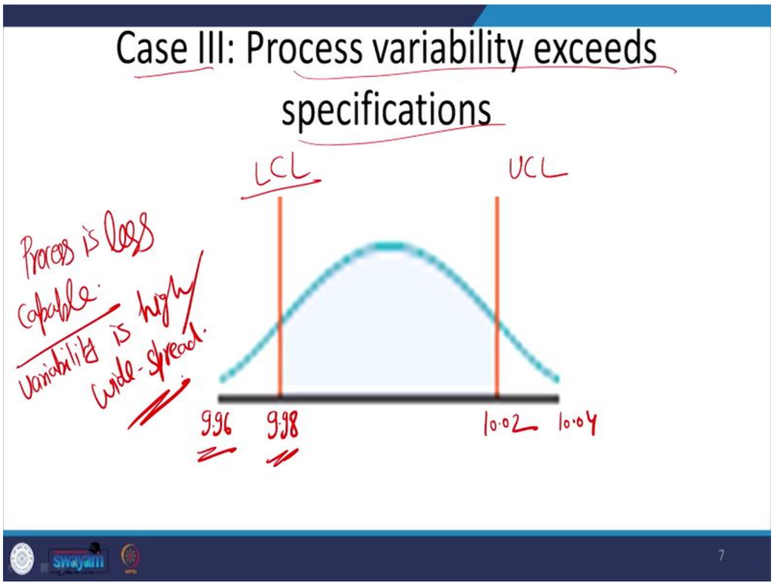

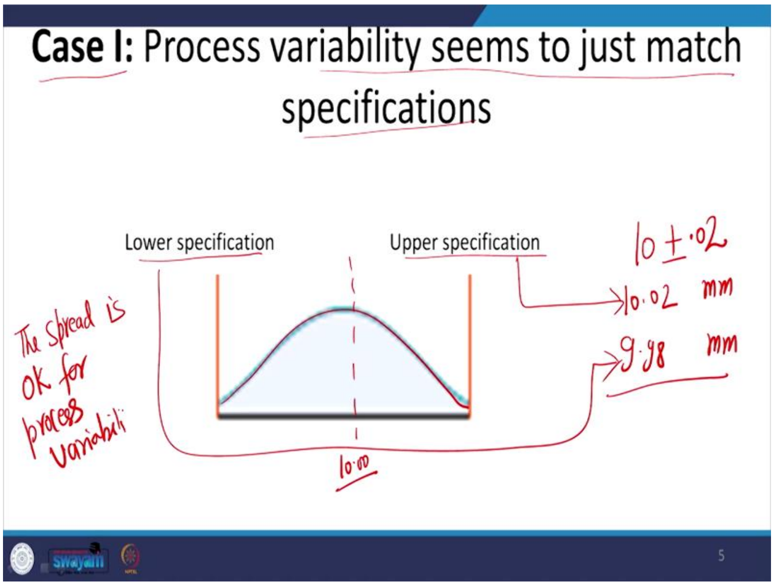

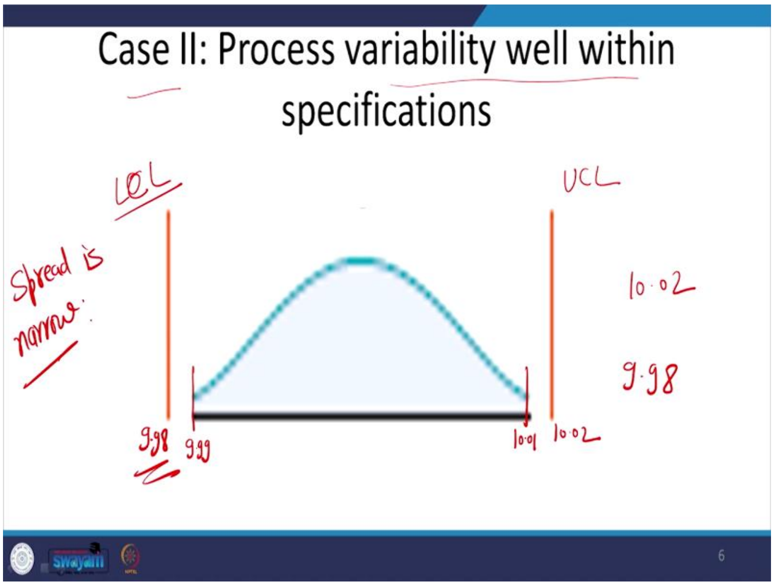

Now, the third case is interesting and that is a worrisome case for us here, UCL and LCL, it is 10.02 and 9.98. But now the process output, the process output is beyond these limits, here the process output is producing up to 10.04 and it is up to 9.96. Here, the process variability exceeds the specification, there are few output which will be beyond 10.02. But you will not be accepting those products which are having the dimension offer higher than 10.02. Then there will be few outputs which will have their dimension less than 9.98 maybe 9.97, 9.96. So, you will be again rejecting these products, where the dimension is less than lower control limit.

So, this particular case may result into various rejections because the process variability is more than the specification, here your process is widely spread. So, you can say the spread is okay for process variability. The second case spread is narrow and therefore, it has scope of increasing the variability in the process, so that you can have some kind of extra resources which you can take away from the process capability and put it to some other place.

But this third case is a case of deep thinking, it is a case of giving you some stress that your process is less capable, process is less capable because here the variability is high or widespread. So, that is the third particular case and based on these three, we actually cannot do every time this kind of graphical representation. So, with the help of some numerical data, we will try to see, we will try to understand the concept of process capability.

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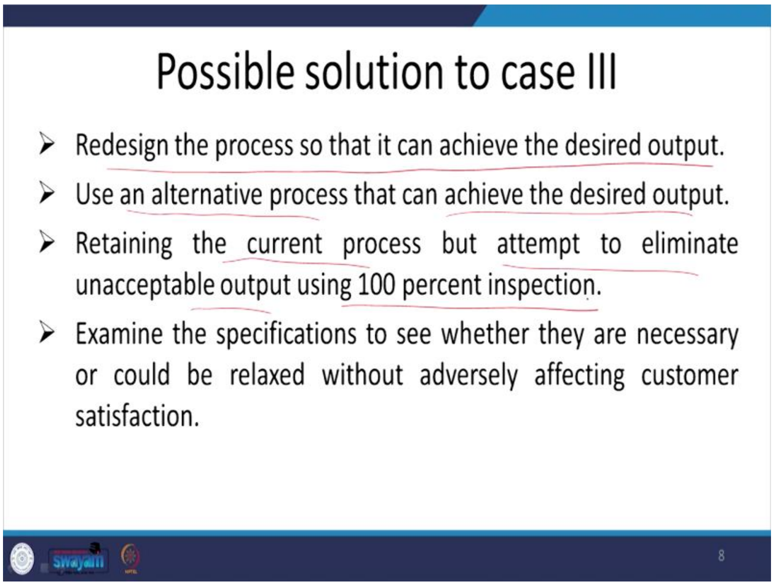

And first, let us see what are the possible solutions for case3. Now, for case 3, first you redesign the process, so that it can achieve the desired output, you can do that. Use an alternative process that can achieve the desired output, you can find some more capable process which can give the output within the specifications

Retaining the current process, but attempt to eliminate unacceptable output using 100 percent inspection. If the cost of inspection is low, then you can go for 100 percent inspection also, if other alternatives are not available. Examine the specifications to see whether they are necessary or could we relaxed without adversely affecting customer satisfaction.

Sometime, we very tightly placed our specifications and customer actually may not be interested to have that tightly held specification. So, if possible, you can relax the specification which can meet the process capabilities. So, in that case, if these specifications are not very critical and if these specifications are not affecting the customer satisfaction much you can relax to some extent these specifications.

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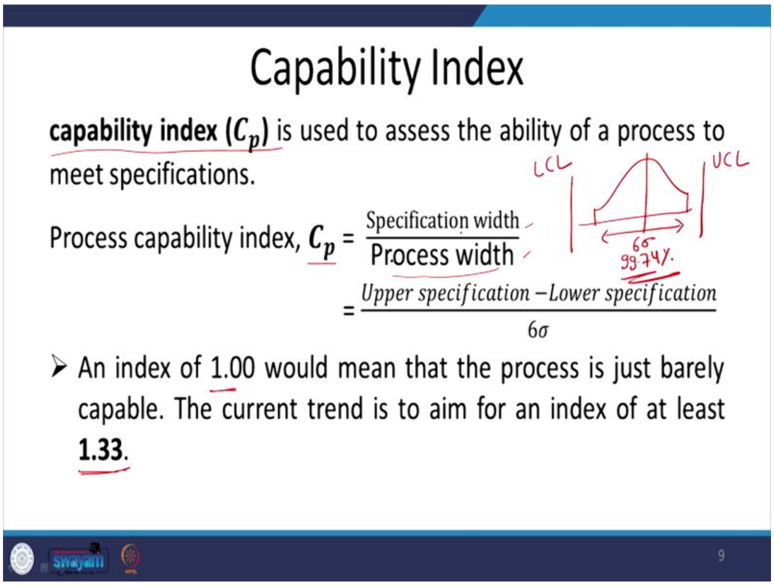

Now, the calculation part of this process capability for that purpose, we have a very good indicator which is known as a capability index and which is written as C subscript P. So, since CP is used to assess the ability of a process to meet specification. So, now you see the specification width divided by the process width that is the calculation of your process capability index.

So, the specification width is, as you remember any diagram, we have this diagram of process, case number 2. So, in that this is the upper control limit or upper specification, this is the lower control limit the lower specification and this is the width of the process. So, you have as numerator, the specification width upper specification minus lower specification and this is the process width which is considered to be Six Sigma plus minus 3 standard deviation on either side of the mean value which covers the 99.74 percent area of the normal distribution.

So, if you have an value of CP as 1, so that value of CP equals to 1 means your case number 1, where your process is just capable of providing that specification, it is just fulfilling the requirement or providing that specification. If it is less than 1, if it is less than 1, it means it is not capable of providing that specification.

And then nowadays the more acceptable value of process capability index is around 1.33. So, if 1.33 is the process capability index it is considered to be a good capable process that means the specification width are like this, process width is smaller than the specification width. Therefore, the denominator is less than the numerator. So, obviously, you understand this calculation will result the CP more than 1 and if CP is around 1.33, it is considered to be a good index for our purpose. So, this is the formula for calculation of that CP.

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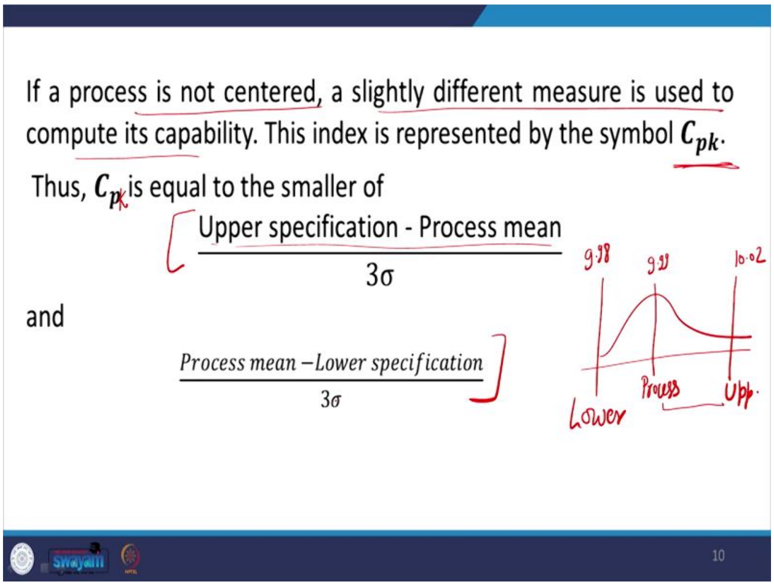

Now, there is one more formula, where process mean and the sampling mean are not same. In this previous case, the process mean and sampling mean are same, but it is possible like this particular situation that you have 10.02 9.98. So, there is a mean value of 10. But, what is happening, that the mean of my sample, mean of my sample is not at 10 it is at 9.99 that is the case. So, in this particular case, when process is not centered is slightly different measure of process capability index is used and that is represented as CPK.

So, CPK, this is CPK, it is equal to the smaller of, these are the two values, upper specification minus process mean. So, you have upper specification, this is process mean, this is upper, this is process and this is lower. So, from upper to process, you take one difference of this divided by 3 sigma and then the second is process to lower divided by 3 sigma. And the smaller of these two values is taken as your process capability index.

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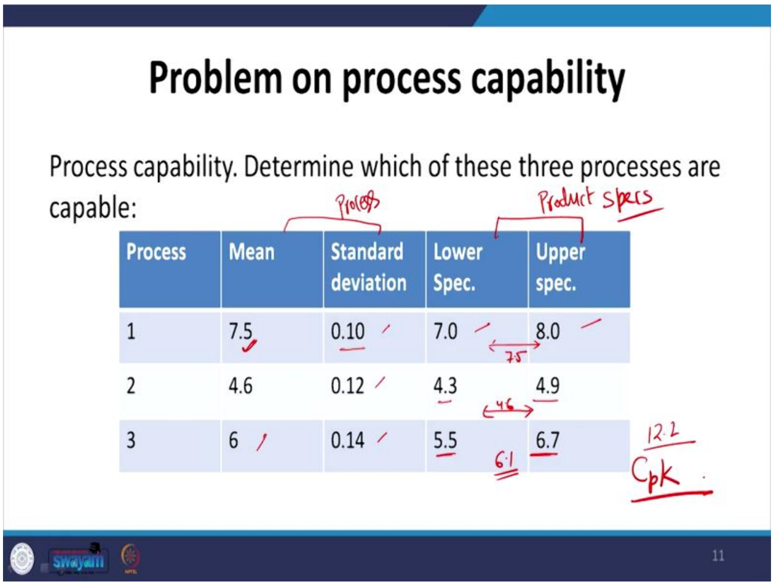

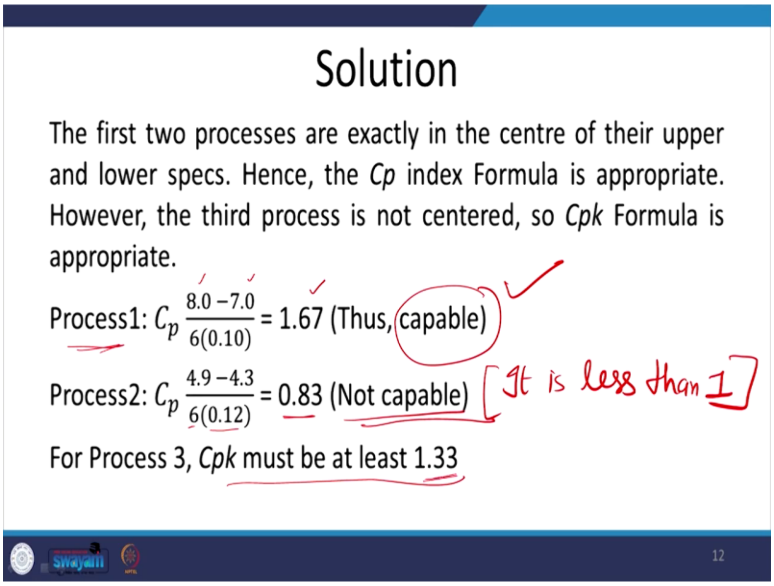

So, now, with the help of one numerical example, we can understand that how to use this process capability calculations. So, there are three processes 1, 2, 3, the mean values are given 7.5, 4.6 and 6, the standard deviations are 0.10, 0.12 and 0.14. The product which we are making, these are process parameters and these are product specification. So, the 7, 8, 4.3, 4.9, 5.5 and 6.7 in their respective units are the values given for lower and upper specifications.

Now, let us see how are we using this? For process 1, for process 1, the upper and lower, upper is 8, lower is 7, upper is 8, lower is 7. And here you see the mean value of upper and lower is 7.5 and that is the mean value of the process also.

So, here we are going to use our formula number 1 that is in the denominator it will be 6 into standard deviation, the standard deviation is 0.10. So, this calculation becomes 1.67 and it is more than 1. So, you will say that process is capable. Now, coming to the second case, here it is 4.3 plus 4.9. So, 4.3 and 4.9 the average is 4.6.

So, you see 4.9 minus 4.3 6 into the standard deviation 0.83, it is less than 1. Therefore, not capable, the specifications which you want to produce from this particular process, that process will not be able to deliver the required specifications, many rejections will take place.

And now, coming to third, in this particular case it is 5.5 and 6.7. Now, 5.5 and 6.7 the mean is around, you can calculate this is 6 plus 5 11, 12.2. So, the mean is 6.1 and the mean given for this process is 6. So, the process mean and the specification mean are not same. So, we have to use CPK formula in this particular case.

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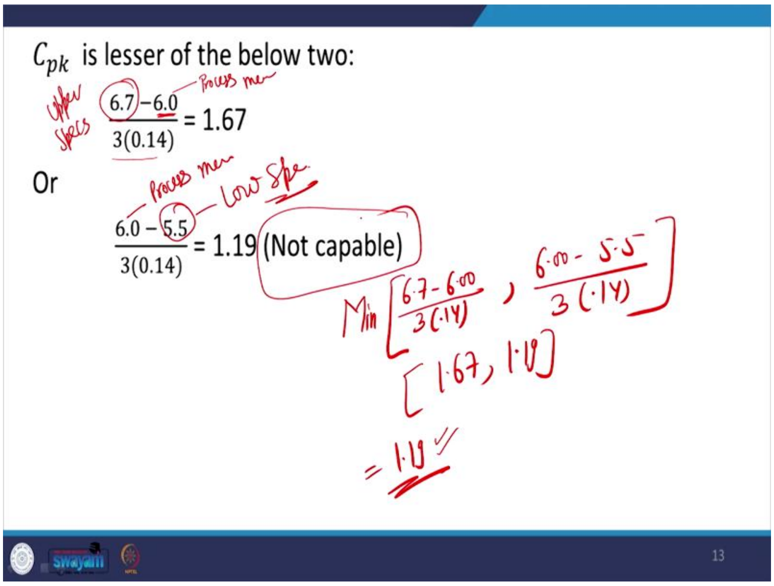

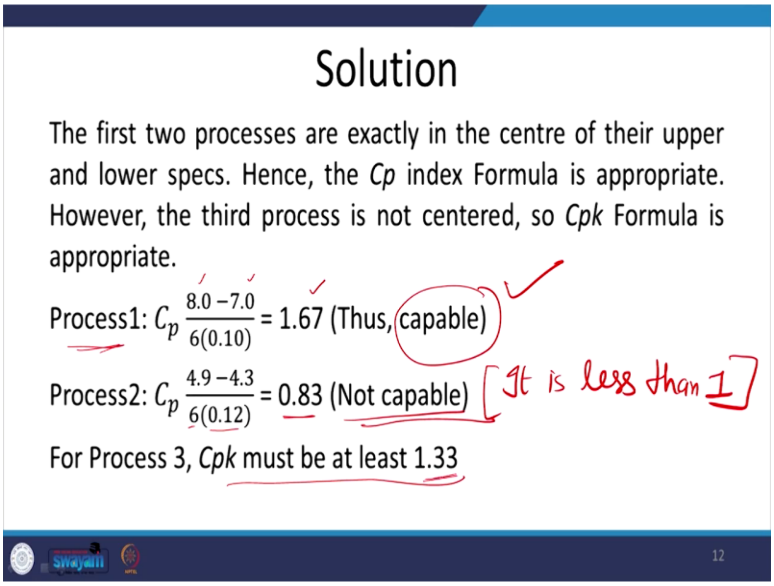

So, we have used the CPK formula and when we have used the CPK formula, so, out of these two the upper control that is 6.7 minus the mean value divided by 3 sigma, mean value, this mean is the process mean, this is process mean. And these are upper specification, this is lower specification and you have calculated. So, you can see it can be written simply in this way that is 6.7 minus 6 divided by 3 into 0.14, comma 6 minus 5.5 divided by 3 into 0.14.

So, out of these two, the minimum is you will see 1.19, the minimum acceptable value should be more than 1.33, but it is coming to 1.19. So, we will say that the process is not capable, process is not capable. So, with this, we are now able to understand that how to handle different types of cases for process capability, if the value or process capability is 1 in the case when the mean is matching the, mean both the means process and sample means are matching, it is acceptable capable, though the desired value is 1.33.

In the case of when process and sample means are not matching, we have different formula that is of CPK and normally the CPK value should be around 1.33. So, in this particular case, the CPK value came 1.19 and therefore, we can say that this process is also not capable of giving the desired specification. So, this becomes the entire concept of process capability analysis, we will use this information of process capability analysis for developing the Six Sigma concepts that we are going to discuss in our next session. Thank you very much.