Production and Operation Management
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Lecture No. 44
Acceptance Sampling

Welcome friends, we were discussing in our last few sessions about statistical quality control. We discussed that there are two main aspects of statistical quality control, one is appraisal and another is process quality control, under process quality control, we discussed in detail about different types of quality control charts and we also discussed about how to identify biasness in your parameters with the help have run test examples.

So, that is when you are doing a production activity, when some value addition is going on. So, how to control the quality during that value addition process? But sometime it is possible that you have some end products available with you, finished products are available with you and you have to check the quality of those finished products.

For an example, you are going to market to purchase some spare parts for your organization. Now, you do not have any scope of process quality control for determining the quality of those spare parts. We see many a times, vendors in Europe or in America, they purchase huge amount of readymade garments from Bangladesh, from Sri Lanka, from Vietnam, from India. And then the large consignments are sent to those retailers.

They just check few items out of that large consignment. And based on quality of those few items, they decide whether to accept or reject the consignment and this is another very interesting part of statistical quality control, which is known as acceptance sampling. So, in this particular session, we are going to discuss the process of acceptance sampling, we see how to decide the acceptance sampling criteria and what are the important terminologies related to acceptance sampling.

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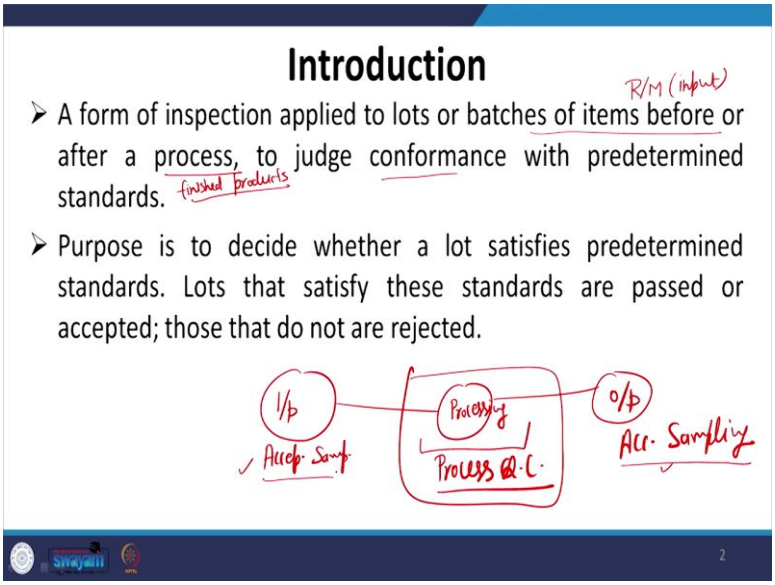

Now, when we are talking about acceptance sampling. So, it is a form of inspection, which is applied to the lots or batches and where you are going to use them in a particular process. So, when these are in the form of raw material or in the form of input or when they are becoming the finished products. So, at both these cases like you take this example, when we are at the input stage, then it is processing and then it is output.

So, at input you do acceptance sampling, at output you do acceptance sampling and during this processing, this value addition, we do process quality control, where we discuss the application of different types of quality control charts. So, this we have already done. Now, in today's session, we are going to discuss the process of these acceptance sampling.

So, that we, if we have the confidence that input which we are going to provide for the processing, if it is of good quality our processing will also happen of proper quality. Similarly, when we are doing the acceptance sampling at the output level, it gives us a confidence that the cost of external failure will be very less. Because only the okay components, only the okay products will go to the hands of the customer.

So, the costs related to guarantee, warranty, repair, replacements, etc will be minimum. So, acceptance sampling is also very much is used a very common example of acceptance sampling, we all very much is familiar, though we do not give that name to it that it is acceptance sampling.

But actually, it is that example is whenever food is cooked. So, particularly in case of rice, when you are cooking rice, so, out of that entire pot, where rice is cooked, you take a few grains and you just push those grains with your fingers. And by crushing those grains by your fingers, you determine whether the rice in that entire pot is properly cooked or not.

So, that is an example of a very simple crude example of acceptance sampling that we are not going to test the entire lot. We are just taking a very small sample of the lot and based on that small sample you are going to decide whether the food is cooked or not cooked.

So, in many situations, you will not be taking the 100 percent sample because it may not be economical, it may not be economical to take 100 percent sample and when it is not economical, why will you take 100 percent sample and you go with a very small sample of the population. So, let me explain you what is the meaning of this economics of sample.

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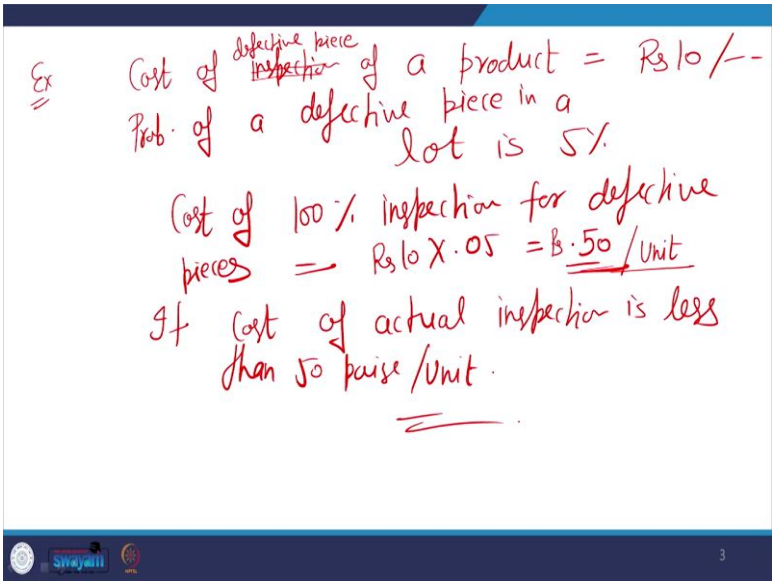

Let us say for example, let me tell you that the cost of inspection of a product is let us say rupees 10 that is the cost of inspection of a particular product. And the probability of a defective piece in a lot is 5 percent that is the probability of a defective piece in a lot is 5 percent. Now, the cost of 100 percent inspection for defective pieces, because cost of a defective piece, cost of not inspection cost of a defective piece is 10 rupees. So, rupees 10 into 0.05 that is 50 paisa.

Now, when you are going for 100 percent inspection of your lot. So, cost of a defective piece is going to come 50 paisa. Now, if the cost of inspection of per piece, cost of inspection, this

is the not cost of inspection, this is in fact the cost of defect, cost of a defective piece is 10 rupees and the probability of a defective piece in a lot is 5 percent. So, cost of 100 percent inspection for defective pieces is rupees 0.50 or 50 paisa per unit.

Now, if cost of actual inspection is less than 50 paisa per unit, then only you can take a economic decision to go for 100 percent inspection. But normally, it will be much higher than that. And therefore, it is not advisable to go for 100 percent inspection, we go with a limited sample, because you can very well generalize, you can very well generalize the results of that limited sample for your entire population. So, it is not economically viable for doing the 100 percent inspection.

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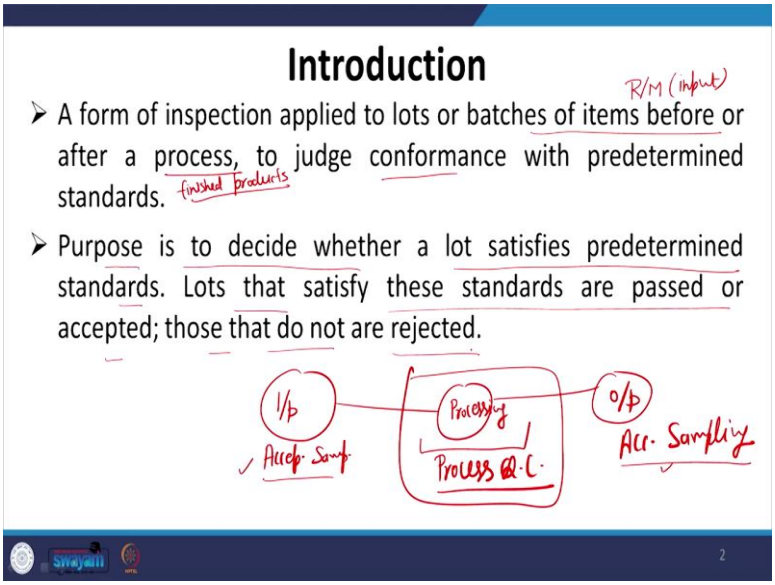

So, now what we do in this acceptance sampling. The purpose is to decide whether a lot satisfies predetermined standards, lots that satisfy these standards are passed or accepted, those that do not are rejected. So, you have a predetermined level of acceptance and if your sample is crossing that predetermined level of acceptance, then you are going to accept the lot.

If your lot is not coming to that particular predetermined level of acceptance, then you will reject the lot and sometimes we can take multiple samples. So, you may take another sample to decide whether to accept or reject the lot. So, that is in brief the process of acceptance sampling.

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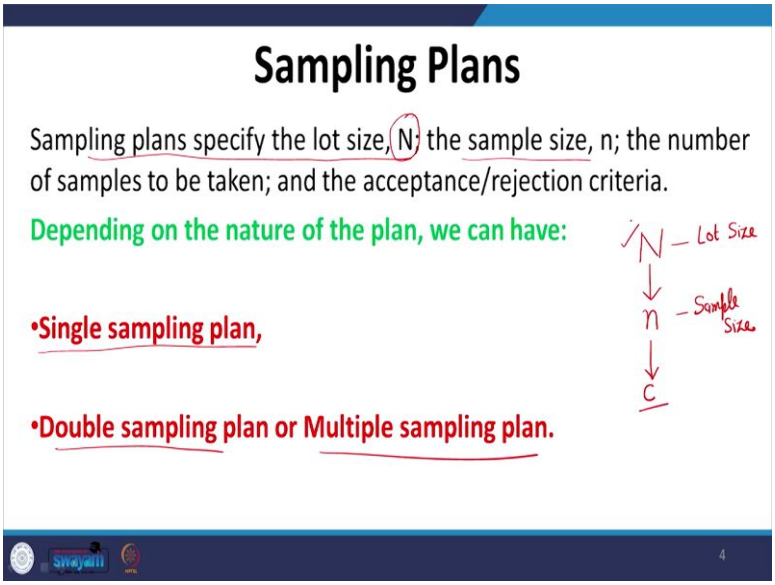

Now, based on our ability to spend money and then the criticality of the item, how critical is the item? How valuable is the item for my production purpose? We have different types of sampling plan, single sampling, double sampling, and then the extension of double sampling is multiple sampling plan. So, these are the different types of sampling plan.

Now what we do that sampling plan is specify the lot size, N; that what should be the lot size, so, if we are taking the input from the customer, from our vendor in the size of EOQ, so that EOQ may be my lot size. So, that is determined, that is denoted as capital N. Then the second important thing is, so there is a lot size of N; from this lot size, you take a sample of a small n. So, this is lot size, this is sample size. And then you have a predetermined level of acceptance that predetermined level of acceptance maybe C.

So, whether out of N number of units which are there in your sample size, if C number of units are able to fulfil the requirement, if C number of units are okay, then you are going to accept this lot of N units, capital N units, but if number of units which are failing that is more or if number of okay items are less than this is small C, then you may reject the lot of capital N. So, that is the sampling process, which we are going to do.

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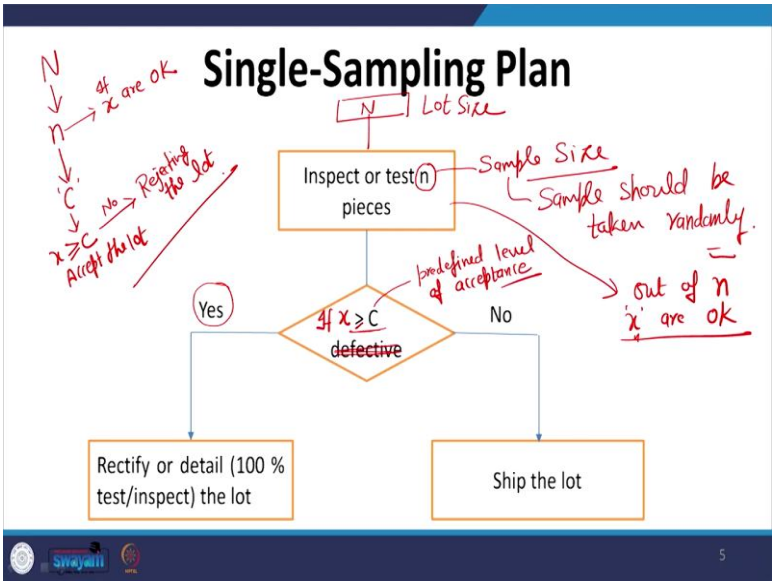

Now, let us see the diagrammatic form of this sampling plan. So, you have taken a sample of a small N items from the lot size of capital N; this is lot size. And this small n; that is the sample size, so, you test these small n items. Normally, we should take if it is a pile of items, if items are staged in a particular formation.

So, it is advisable that the samples should be taken randomly from different sides of that pile, otherwise, there is a chance of biasness. So, if you take one sample one unit from the top, another unit from the bottom, then one from the left side, another from the right side. So, there has to be some kind of equal representation from the population in your sample. If you take all units from the same place, there are chances of some kind of biasness.

So, the sample should be taken, sample should be taken randomly. So, that there is no biasness in your sample. Now, you are testing these small n items and then you have a decision rule, this type of shape, this diamond shape is known as decision activity. If you remember in our quality control tools, we discussed a flowchart, in that flowchart also we had this type of diamond shape process and that is known as decision making activity. Now, we have to make a decision out of these small n.

So, now, if your number of okay parts are more than C. So, C is our pre-defined level of acceptance. So, if our number of okay parts let us say out of n, let me be more specific, out of n, x are okay. And if x is more than or equal to C, if x is more than or equal to C, we are going to accept a lot. And if more number, if defective number of pieces are more, if x is more, if x are okay, so here we are doing that we are going to accept the lot and if the number

are more, if number are, this x are more and in that case, this is going to help us in accepting the lot, but if this number is less than C that is our predefined level of acceptance then we are going to reject the lot.

So, that means you have a predefined level. So, three important things are there, first you have to decide the lot size, out of that lot size you have to decide a sample size, from the sample size you have to decide that C that is the predefined level of acceptance. And then in this n, you are testing these items and how many of them are okay? So, if x are okay and if x is more than or equal to C accept the lot and if no if x is less than C, then we are rejecting the lot.

So, this is the summary of our single sampling plan that you have a predefined level of acceptance and if your number of okay components are more than predefined level of acceptance, then you are going to accept the lot, otherwise you are going to reject the lot. But many times, it is quite possible that item is of some critical nature.

If you are purchasing some item and you are doing the acceptance sampling, and that item is going to be used in your production process. Now, you have rejected the lot, because you are not satisfied with this acceptance sampling. In that case, it will adversely affect your production process also. So, you have to be careful that before rejecting you are 100 percent sure, that this is worth rejecting. Otherwise, it will affect your production process also.

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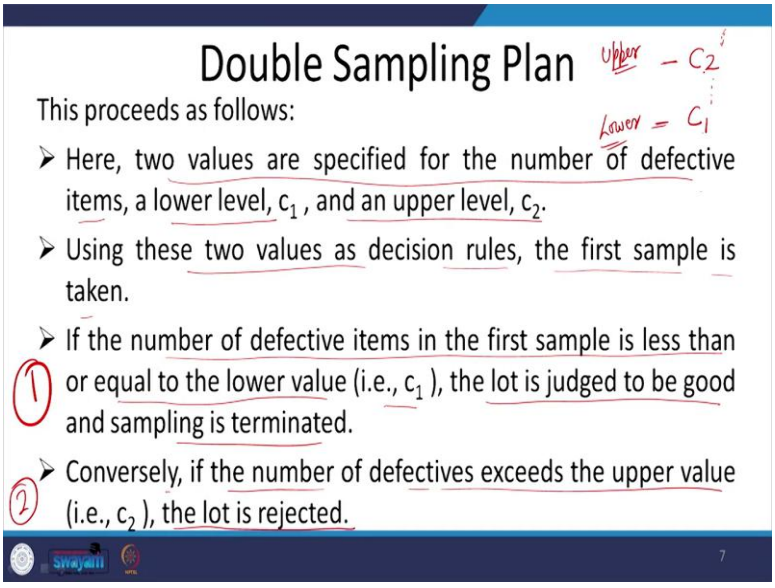

So, for that purpose, we go with double sampling plan or multiple sampling plans also. So, now, when we are not getting a proper result, what we do in double sampling plan. Here, two values are specified for the number of defective items; a lower level that is C1 and an upper level C2. In the single sampling plan, we have only one value of acceptance that is C, here we have a upper control and lower control limit like our quality control charts.

So, lower level we are writing a C1, upper level we are writing as C2. Using these two values, the decision rules are framed, the first sample is now taken, if the number of defective items in the first sample is less than the or equal to the lower value that is the C1 the lot is considered to be good and sampling is terminated. Then there is no need to go for resampling. So, you have the upper limit C2 lower limits C1, these are the level of defective pieces.

Now, if number of defective items are less than C1 in your first sample if number of this is lower, this is upper, if number of defective items are less than C1. In your first sample, it means it is a good quality of sample and then you do not go for any kind of next sampling and you are going to accept the entire lot. But it is quite possible that your value of defective number of pieces may fall somewhere between C1 and C2. So, let us see what we do in that case.

So, this is the first situation, the first situation is this, where we are going to accept the lot, we are not going for further sampling, if the value of defective pieces are less than C1. Now further, if the number of defectives exceed the upper value, the lot is rejected. So, the second case is you have a value somewhere here and it means you have more number of defective pieces than the upper acceptance limit.

So, that is also an extreme case. So, there is no need for second sample and you are going to reject the lot. So, there are two cases, one case where you have values less than C1 going to accept the lot, values more than C2 going to reject the lot. Now, the third case will come when value may fall between C1 and C2.

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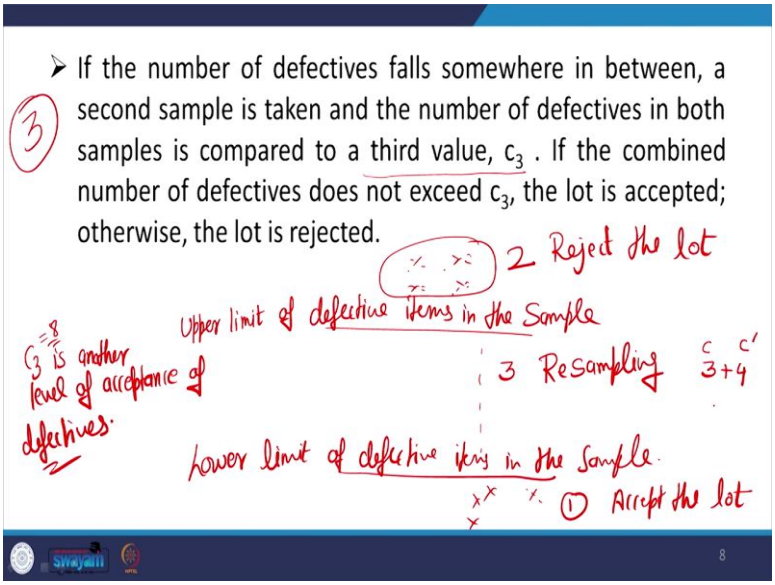

So, if that is the third case. So, what we are doing that upper limit of defective items in the sample, this is lower limit of defective items in the sample. So, this if I am somewhere here, this is my case **one**, I am going to accept the lot, if I am here, this is my case **two,** reject the lot.

Now, let us see the third case, if the number of defectives fall somewhere in between a second sample is taken. If here I am then it is the third case, and you need to do resampling, you do resampling. And the number of defective pieces in both these samples are compared the first sample because of which you have taken a second sample and then you calculate the number of defective pieces in the second sample also, in both these is compared to a third value C3, which is compared with a third value C3.

And if the combined number of defectives does not exceed C3. So, that C3 is also a predefined level. So, if the combined number of defectives does not exceed C3, the lot is accepted otherwise the lot is rejected. So, you also set one third level where you can combine C3 is another level of acceptance of defectives.

And C3 will be, let us say, if in one case, you got three defectives. In another case you got four defectives, my C3 is let us say 8. So, in that case, I am going to accept the lot because two samples in first sample, my value is of C is 3. Then in the second sample it is 4, and the C3 value is 8. So, I will say that the combined value of two C is less than the predefined level of combined defects, that is 8, so, I am going to accept, otherwise, I am going to reject the lot.

So, this double sampling gives you opportunity to take a second sample and that opportunity helps you in taking more confident decision about your sampling decisions. Then in some cases, where you can say the cost of products or cost of rejection actually is very high. And particularly cost of rejection is very high, when we are going to supply a finished product, and when we are willing to supply a finished product, I will not like to reject my own production lot. So, in that case, we like to go for multiple sampling plan.

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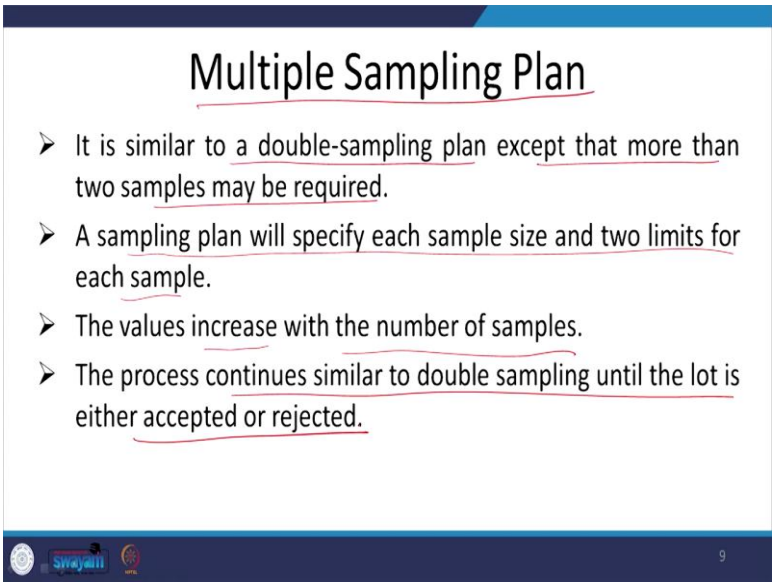

And in that multiple sampling plan, it is similar to a double sampling plan, except that more than two samples may be required. So, in that final decision is based in our double sampling, either our decision is based on a single sample or maximum two samples. But in case of multiple sampling, more than two samples, may be required to take a final decision. So, two sampling, double sampling is a typical case of multiple sampling, where only maximum two samples are required.

Now, a sampling plan will specify each sample size, a two limits of the samples that what are the upper limit and what are the lower limits of accepting the defects, the values increase with the number of samples. So, here the values of upper and lower limits are not static, but these values may change with the number of samples, the process continues similar to double sampling until the lot is either accepted or rejected.

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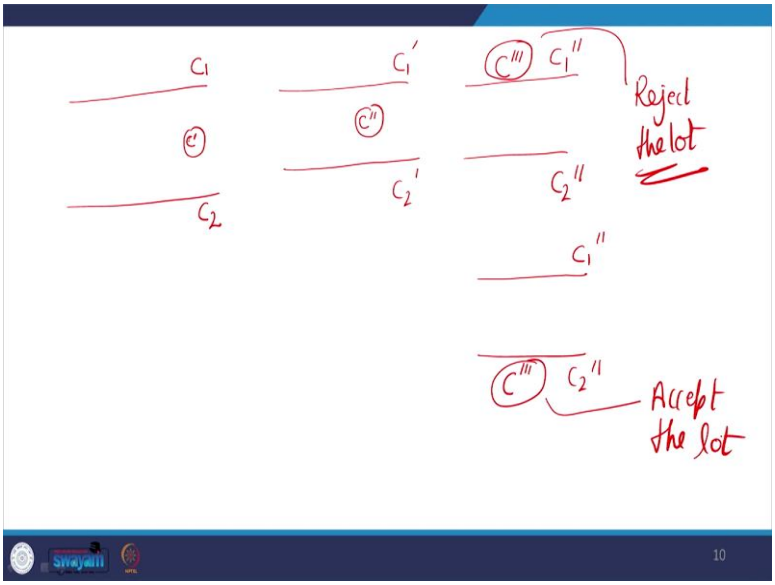

So, here the system is that way, first, you have upper limit C1 and C2; and if you are having a particular value of C for your first sample, which is coming somewhere here, let us say C dash. Now C dash is coming between C1 and C2. So, it says that you should take a second sample.

Now, in the second sample the values of C1 and C2 will change to C1 dash and C2 dash that is the dynamic process of doing this multiple sampling and here the value of C is moved to C double dash, again since C double dash is falling between C1 dash and C2 dash, so it will say that again you need to take a new sample. So, you will take a new sample and again the limits will change to C1 double dash and C2 double dash.

Now, you will take a new sample and in that sample, it is possible that the values of C double dash will change to C triple dash. And C triple dash is above to upper limit and that means, reject the lot. So, now, you have taken a decision after your third sample, that lot is to be rejected. And once you have taken this decision, then there is no need to take further samples.

So, that is the process of multiple sampling. It is also possible that in the third sample when your limits are C1 double dash and C2 double dash, C3, C triple dash is below the lower limits. And it means accept the lot. So, both these things are possible in your third sample, maybe a four sample is required.

So, unless you are able to finalize the decision whether you are accepting or rejecting the lot, you will keep taking the samples. So, that is a very advanced system and it is normally done

when you are supplying a product to the customer. Because the customer may not be interested to take multiple samples, but as a supplier because if we are rejecting our own lot, we will incur huge loss. So, we may take multiple samples to finally ensure the customer that yes this is a okay lot.

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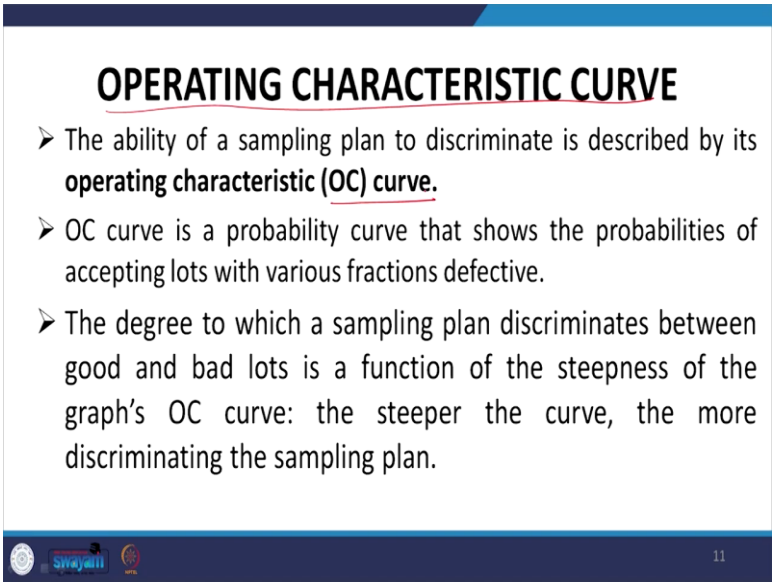

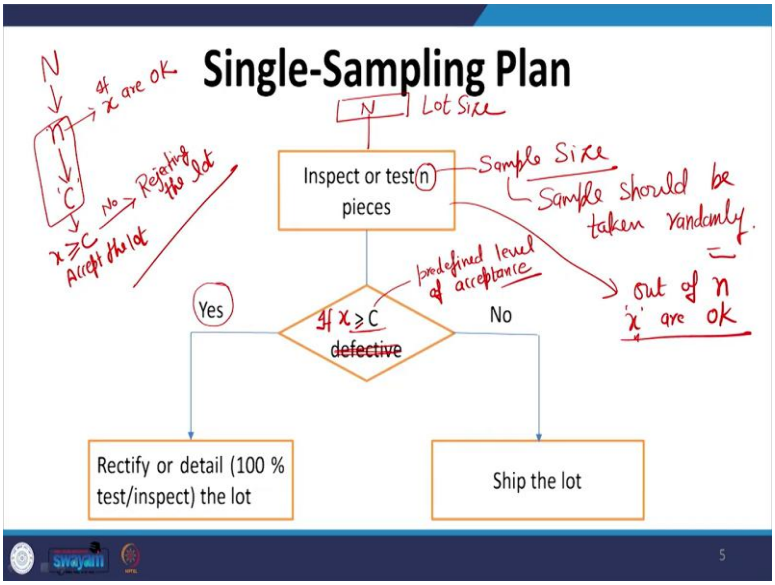

Now, for acceptance sampling to understand there is a very interesting graphical representation also. And particularly because, as you remember, we discuss that in a acceptance sampling, these are the two very important things that out of a particular lot size, what should be the value of sample size and what should be the value of your acceptance limit.

And based on these things, we have this curve, this graphical representation which is known as operating characteristic curve or in our day to day language, we call it as OC curve. Now, this OC curve is a very useful way to understand that how different types of N and C values will give you different level of quality acceptance.

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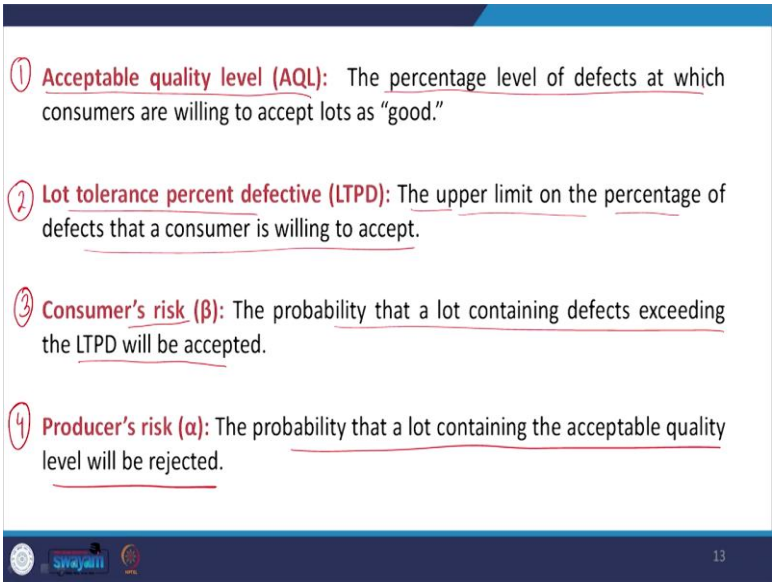

Now, first we need to understand that these are the four important terms AQL that is the first. The second is LTPD that is Lot Tolerance Percent Defective, third is consumers risk and fourth is producers risk, consumer means the one who is purchasing, producer means who is supplying. So, these are the four important terms, let us quickly understand the meaning of these terms, the producer's risk is you are a producer and it is very simple to understand that you have made good things.

Now, if because of some statistical issues, your good things are rejected, that is your producer's risk. So, the probability that a lot containing the acceptable quality level will be rejected. So, probability of getting rejected a good lot is the producer's risk, as a consumer risk what is your risk?

You are accepting or you are purchasing a lot which is of bad quality that is your risk, you do not want to purchase a product which is of bad quality. But because of some statistical issue, if you purchase a product, if you purchase a lot, where poor quality products are there, that is the consumer's risk.

So, the probability that a lot containing defective pieces, which is exceeding the LTPD limit are accepted. The third is LTPD, Lot Tolerance Percent Defective. Now, Lot Tolerance Percent Defective is the upper limit on the percentage of defects that a consumer is willing to accept that you will say that I am not going to accept more than this number of defective pieces in my supply. So, that is LTPD.

Now, the acceptable quality limit, acceptable quality level AQL is the percentage level for defects at which consumers are willing to accept lots as good that consumer may say that okay, I am accepting 2 percent AQL that means, out of 100 if **two** pieces are rejectable or **two** pieces are defective that is a good quality for me. So, these are the four important definitions, which are basically required to understand this operating characteristic curve.

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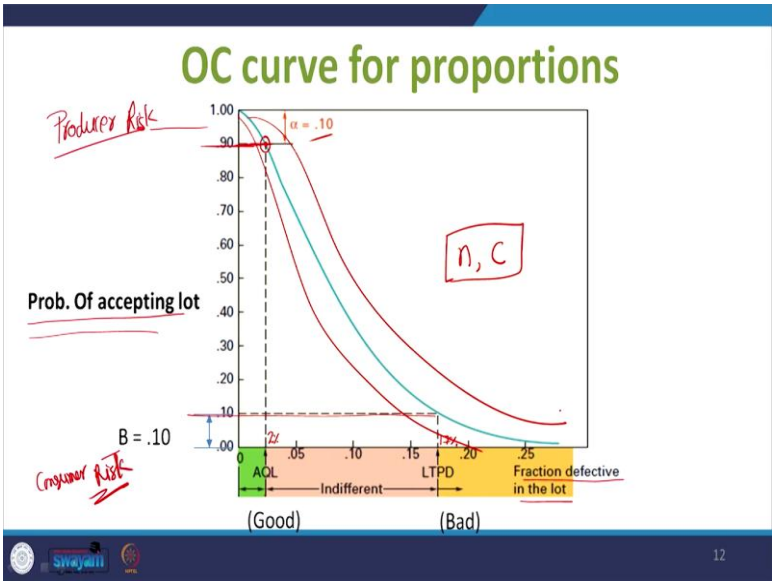

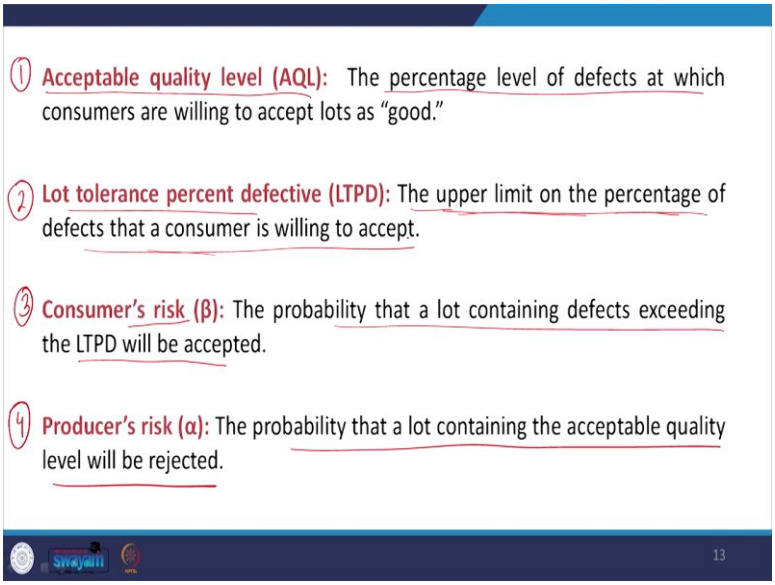

Now, if you see the shape of this operating characteristic curve here, we have these two axes, now, on the X axis, the fractured defectives in a particular lot this is mentioned that how many percentage? How many fraction of defective pieces are there in a particular lot? And on the Y axis, we have probability of accepting a particular lot, probability of accepting a particular lot. Now, the shape of this curve is dependent, shape of this curve is dependent on a small n and C, these are the two variables, which are defining the shape of this type of curve. So, this is a generic shape which we have represented here.

Now, if you see this diagram, there are different levels of issues, you whenever some purchase negotiations are taking place, whenever some purchase negotiations are taking place. Normally, those negotiations are about deciding the AQL and LTPD that what is the AQL and what is LTP?

And in our operations management, there is a very humorous story between Hewlett Packard and Japanese vendors. So, when Hewlett Packard was asking some electronic cables from Japanese suppliers. So, HP people were very much adamant that they want 2 percent AQL. And Japanese vendors were not willing to provide that 2 percent AQL.

But it resulted into a very hot it arguments, but Japanese had to accept the demand of HP because HP were the customers, so they finally accepted their demand. When the supply was made to HP, so in that box, there were two packets, in that box two packets were there. And in one packet, hundred good cables were packed. And in another packet, two defective cables were packed. And there was a note written on that small packet, that as per Purchase

Agreement you required 2 present AQL so we are keeping two defective pieces in this box, but we do not understand why do you want these two defective pieces.

So, that type of humorous story is there with respect to AQL and LTPD, because people many a time do not understand that what is the meaning of AQL and what is the meaning of LTPD. AQL is the number which you consider, so, they wanted, the meaning of HP was that we will maximum accept only **two** defective pieces in 100.

Japanese were very much obsessed about the quality. So, for them getting **two** defective pieces in a size of, in a lot size of 100 is a too much of thing. So, they thought that we can provide 100 good quality products in a size of 100. So, why they are asking **two** defective pieces? So, because of non-ability to understand the you can say capital abilities of others, we are sometime got into the wrong negotiation.

Now, if I see this operating characteristic curve, here we see that the probability of acceptance are from 0 to 100 and these are the number of defective pieces or the fraction defectives in this lot which is from 0 to 25 percent. Now, you have a particular level of AQL. Now, this particular level of AQL is somewhere between let us say 2 percent and here if I take this line, this is touching somewhere here.

And then the second issue is to set about LTPD. LTPD means if this much of lot, if this much of defective pieces are there, we are going to reject the lot, so LTPD is that the upper limit on the percentage of defects, that the consumer is willing to accept that beyond this, we are not going to accept it.

So, that LTPD is somewhere set as 17 percent, if defects are less than 2 percent, it is a good quality and if defects are more than 17 percent then it is certainly a poor quality. Now, if you see this shape of a diagram, here, there is a limit that 2 percent is a good quality, it means, it is a well-designed lot, but still the probability of acceptance is around 90 percent.

The probability of accent acceptance is 90 percent and this alpha that is 0.10 is giving you the producer's risk. The lot is good, it is following the AQL, but still you are not 100 percent sure that a lot which is following the AQL of 2 percent will be accepted because of statistical issues. So, there is a 10 percent probability that you are going to reject the accepted lot and this becomes the producer's risk. So, a lot which is coming under AQL can still be rejected and that we have seen here.

Similarly, this LTPD is 17 percent, so, any lot which is having more than 17 percent defective item ideally should be rejected. But if you see this diagram there is a 10 percent probability, there is a 10 percent probability that it can still be accepted. So, this becomes consumer risk, though it is having a poor-quality lot more defects are there, but again because of statistical issues, the sample which you are taking from there, it is possible that it will be accepted.

So, this operating characteristic curve says that, there will always be some producer risk and some consumer risk whatever level of AQL or whatever level of LTPD you set only 100 percent inspection can eliminate the producer risk, only 100 percent inspection can eliminate the producer risk, depending upon different levels of N and C you will have different shapes of these curves, but all those shapes will have similar kind of movement. So, whatever combination of N and C you take, there will be a producer risk, there will be a consumer risk.

So, acceptance sampling is advisable, whenever it is an economical decision and 100 percent inspection is not desired. If you take the values of AQL and LTPD correctly, then you can minimize the producer risk and consumer risk to a great extent and that is what the acceptance sampling tries to do. Thank you very much, we will see more discussions about other issues of quality in our next session.